Freie Universität Berlin

# State of the Art Overview on Automatic Railway Timetable Generation and Optimization

Julian Reisch

School of Business & Economics
Discussion Paper

Information Systems

2020/20

# State of the Art Overview on Automatic Railway Timetable Generation and Optimization

Julian Reisch*

November 24, 2020

**Abstract**

In railway transportation, each train needs to have a timetable that specifies which track at which time will be occupied by it. This task can be addressed by automatization techniques both in generating a timetable and in optimizing an existing one. In this paper, we give an overview on the state of the art of these techniques. We study the computation of a technically valid slot for a train that guarantees a (short) spatial and temporal way through the network. Furthermore, the construction of a cyclic timetable where trains operate e.g. every 60 minutes, and the simultaneous construction of timetables for multiple trains are considered in this paper. Finally, timetables also need to be robust against minor delays. We will review the state of the art in the literature for these aspects of railway timetabling with respect to models, solution algorithms, complexity results and applications in practice.

## 1 Introduction

Before a train can operate, it is mandatory that it has a timetable. Therefore, railway undertakers request timetables at the infrastructure manager for their train operations. The task of the infrastructure manager is to coordinate these requests and create a timetable for all operating trains. In this paper, we review the literature on the state of the art on how this coordination and generation process can be enhanced by automatization. That is, for the different planing horizons, periodicities and objectives, we present mathematical models, solution algorithms, complexity results and application tools.

**Single Slot Construction.** The timetable of a single train is called a slot. Most of the European infrastructure managers use the blocking time model to construct a slot (Hansen and Pachl, 2014). In the blocking time model, a slot is a sequence of block segments. Each block segment is defined from one signal to the next one and has a temporal expansion that reflects the trains driving dynamics. If two slots of different trains have overlapping block segments, we say that the trains have a conflict.

**Planning Horizons.** Depending on the planning horizon, the infrastructure manager has different degrees of flexibility to schedule the requested trains. In the annual timetable, all train operation requests, both for passenger and freight trains, are collected and then considered simultaneously. The request to operate a train comes with a desired planning period, for example, only on weekdays. Due to requests from other trains, or due to infrastructure restrictions,

*External doctoral researcher at the department of Information Systems at Freie Universität Berlin.

the infrastructure manager can split the planning period and construct for each part a separate slot. No two trains can have conflicting slots. For shorter planning horizons and especially in the intraday ad-hoc timetabling, there already exist the timetables from the annual timetable and the new slots need to be constructed individually and without any conflicts to existing slots.

**Periodicities.** Especially for passenger trains, the railway undertakers often wish to operate their train periodically, that is, for example, every 60 minutes. Periodical timetabling is mostly relevant for the annual timetable and ad-hoc train operations run aperiodically.

**Objectives.** As most infrastructure managers construct their timetables manually, one objective in automatic timetable generation simply is to generate a timetable without any manual work but by automatized algorithms. Moreover, in the models for timetable generation, typically three objectives are considered and balanced out against each other. First, the capacity utilization shall be maximized, that is, as many trains as possible should operate. Second, the timetable quality shall be maximized which means that the slots should have minimal running times. Third, the timetables robustness shall be maximized. That is, the expected follow-up delays shall be minimized given minor disturbances.

**Generation versus Adaption.** Timetable optimization can take place in the generation of the timetable, or in adapting an existing timetable to improve some objective function.

The outline of this paper is as follows. In Section 2, we review models to construct single slots in a time-space network that is restricted by capacity utilization. This task comes in ad-hoc planning, for example, when other trains already use some parts of the infrastructure. Then, in Section 3, we point out the difference between aperiodic and periodic timetabling. In particular, we present the Periodic Event Scheduling Problem (PESP) and analyze its complexity. Another complexity in timetabling is planning multiple trains simultaneously, as needs to be done in the annual timetable. Section 4 is about modelling and solving this problem, which is called the Train Path Assignment Problem (TPAP). In Section 5, we review timetabling optimization models, for both generation and adaption. Finally, in Section 6, we conclude this paper.

## 2 Slot Construction

A slot $y = ((x,t)_i)_{i=0}^{n}$ of a train is a trajectory through time and space. It specifies the time $t$ a position $x$ (mostly a signal) is passed. The time difference $t' - t$ between $x$ and $x'$ depends on the infrastructure and the train characteristics. Mostly, the starting time interval, the starting position and the target position are given. Then, in this simplest model, a slot can be computed as a constrained path through the time-discretized time-expanded infrastructure graph (cf. e.g. Caprara et al. (2002), Zhang et al. (2019)). For a mathematical formulation **MIP-**$slot$, let $\mathbf{F}$ denote its incidence matrix. That is, $\mathbf{F}$ is $-1$ for each arc entering a particular position $x$ at a particular time $t$ and $+1$ for each arc leaving $x$ at $t$. The set of arcs that can be reached from an arc depends on the train characteristics, such as maximum speed, and the infrastructure graph. Furthermore, the indicator vector $I$ is 1 for each arc in the starting position in the starting interval in time, $-1$ for the destination arcs, and zero elsewhere. Thus, (1b) ensures flow conservation. The conflict matrix $\mathbf{C}$ ensures that the slot $y$ can only occupy tracks at times that are not occupied, yet. Finally, (1d) indicates which arcs the slot uses. The objective (1a) minimizes the total travel time of the slot.

$$\begin{array}{rlr}\textbf{MIP-}\mathit{slot}\ \text{Minimize} & t_n - t_0 & \text{(1a)}\\ \text{s.t.} & \mathbf{F}y = I & \text{(1b)}\\ & \mathbf{C}y \leq \mathbf{1} & \text{(1c)}\\ & y_i \in \{0,1\} \qquad \text{for each } y_i = (x,t)_i & \text{(1d)}\end{array}$$

In particular, the slot construction **MIP-*slot*** is polynomially solvable when applying a shortest path or flow algorithm (Ford and Fulkerson, 1956). However, modelling the driving on a track only by the time span the train takes to pass the track is too rough for most infrastructure managers. A more sophisticated model describes a slot as a sequence of block segments as defined in Hansen and Pachl (2014). A block segment is the time span a track is utilized by a train in which no other train can enter the track including overlaps and headway times and depending on the concrete driving dynamics of a train. For example, if a train halts, depending on its load, it might block the succeeding track for some time because there might be an overlap. Figure 1 illustrates the construction of a slot as a sequence of block segments in the capacity that remains after other slots have been planned already. The German infrastructure manager DB Netz works with the block segment model. Therefore, Dahms et al. (2019) propose a heuristic shortest path algorithm to compute a slot with its block segments automatically. In the application *click and ride*, this algorithm comes into practice for the construction of ad-hoc slots for freight trains.

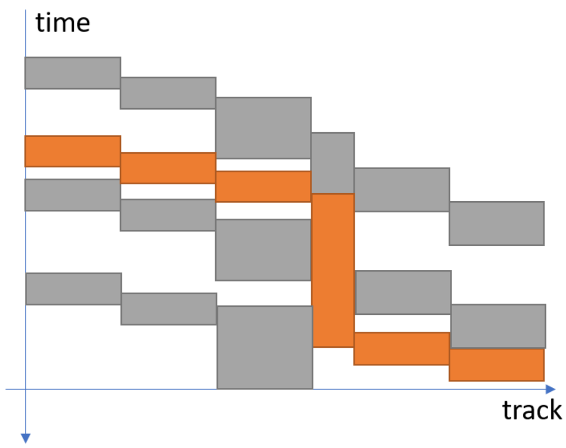



Figure 1: Block segments in the time-track diagram: The orange train can be scheduled in the capacity that remains after three other trains (grey) have been scheduled.

## 3 (A-)Periodic Timetabling

In this section, we will discuss the additional task of a timetable to satisfy periodicity constraints. That is, a train is supposed to be operated every $T$ minutes with the same timetable. We denote $T$ by the period. Following Serafini and Ukovich (1989), the periodic timetabling problem is modelled in the event network.

**Definition 3.1.** An *event network* is a tuple $\mathcal{N} = (D, l, u)$ where $D = (V, A)$ is a directed graph and $l, u \in \mathbb{Q}^{|A|}$ such that $l_a \leq u_a$ for all $a \in A$. The vertices $V$ and arcs $A$ are called events and activities, respectively.

Events are e.g. passings of a train at a given station or signal. Activities are the driving from one signal to the next one or the headway times from one train to a succeeding train. Hence, multiple trains can be scheduled simultaneously. Unlike in the time-discretized time-expanded infrastructure graph, the events are fixed. That is, deviations in the spatial way a train takes or different overtaking opportunities are not considered. The only flexibility is the time differences between the events.

**Definition 3.2.** Given an event network $\mathcal{N} = (D, l, u)$. Then, an *aperiodic timetable* for $\mathcal{N}$ is a vector $\pi \in \mathbb{Q}^{|V|}$ that satisfies

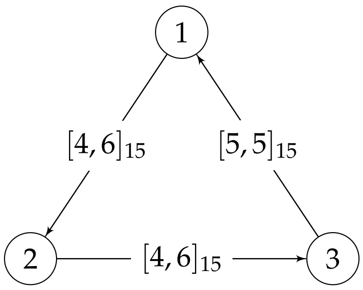
$$\pi_w - \pi_v \in [l_a, u_a] \ \forall \ a = (v, w) \in A$$

A vector $\pi \in \mathbb{Q}^{|V|}$ is called a *periodic timetable* for $\mathcal{N}$ with period $T$ if there exists $p \in \mathbb{Z}^{|A|}$ such that for all $a = (v, w) \in A$ we have

$$l_a \leq \pi_w - \pi_v + p_a T \leq u_a$$

Periodicity is not only practical for passengers but also has operational purposes. For example, a whole tour of a train is supposed to be a multiple of the period. The following example illustrates this situation.

**Example 3.3.** An event network with cyclic constraints and possible solution $(\pi(1), \pi(2), \pi(3)) = (2, 7, 12)$.




The aperiodic timetable $\pi$ or the periodic timetable $\pi$ *mod* $T$ together with fixed positions of the events forms a slot. The acyclic timetabling problem is polynomially solvable using a shortest paths algorithm (Liebchen, 2006), as is the slot construction. However, the *Periodic Event Scheduling Problem (PESP)*, i.e. the problem of finding a periodic timetable in the event network, is NP-complete for $T \geq 3$ (Odijk, 1994). The following MIP formulation additionally minimizes the weighted activity times.

$$\textbf{MIP-}\textit{PESP} \text{ Minimize} \quad \sum_{a \in A} \omega_a(\pi_w - \pi_v + p_a T - l_a) \tag{2a}$$

$$\text{s.t.} \quad \pi_w - \pi_v + p_a T \leq u_a \quad \forall a = (v, w) \in A \tag{2b}$$

$$\pi_w - \pi_v + p_a T \geq l_a \quad \forall a = (v, w) \in A \tag{2c}$$

$$p_a \in \mathbb{Z} \quad \forall a \in A \tag{2d}$$

Solution approaches to tackle the PESP include satisfiability solving (Großmann et al., 2012), the modulo simplex method by Nachtigall (1998), banch-and-cut of the MIP (Liebchen,

2006) and mixtures of these (Borndörfer et al., 2020). When the event network is a tree, then the PESP is polynomially solvable (Liebchen, 2006). Lindner and Reisch (2020) extend this result by giving pseudo-polynomial-time dynamic programming algorithms if the event network has bounded tree- or branchwidth. Liebchen (2008) put the PESP into practice and computed a cyclic timetable for the Berlin underground. In general, cyclic timetabling is most common for long-term planning of passenger trains.

# 4 Train Path Assignment Problem

In Section 2, we pointed out that a single slot can be constructed by applying a flow algorithm. If multiple slots are constructed simultaneously such that no two of them occupy the same infrastructure at the same time, this problem generalizes to the multi-commodity flow problem (Caprara et al., 2002). In general, this problem arises in long-term planning of both freight and passenger trains and we denote the problem of assigning a slot to each of maximally many trains $\mathcal{R}$ the *Train Path Assignment Problem (TPAP)*. Let $q_r$ incidate whether or not a train $r$ is assigned to a slot, $\mathbf{F}_r$ the incidence matrix of the train $r$ and $\mathbf{C}$ the conflict matrix between arcs occupying the same infrastructure at the same time. Then, the TPAP can be modelled in the following MIP.

$$
\begin{aligned}
\textbf{MIP-}\mathit{TPAP}\ \text{Maximize} \quad & \sum_{r\in\mathcal{R}} q_r && && \text{(3a)}\\
\text{s.t.} \quad & \mathbf{F}_r y_r = q_r I_r && \forall\, r \in \mathcal{R} && \text{(3b)}\\
& \mathbf{C}\mathbf{y} \leq \mathbf{1} && && \text{(3c)}\\
& q_r \in \{0,1\} && \forall\, r \in \mathcal{R} && \text{(3d)}\\
& y_r = (((x,t)_i)_{i=0}^{n})_r && \forall\, r \in \mathcal{R} && \text{(3e)}\\
& (y_i)_r \in \{0,1\} && \forall\, (y_i)_r = ((x,t)_i)_r\ \forall\, r \in \mathcal{R} && \text{(3f)}
\end{aligned}
$$

A variant is that the number of trains is fixed and the traveling times of the slots are minimized (Caprara, 2015; Zhang et al., 2019). Even et al. (1975) proved that the multi-commodity flow problem is NP-complete even for two commodities. This complexity result holds for both variants. Therefore, studies that solve TPAP for traveling time minimization, schedule at most several hundred of trains, as Zhang et al. (2019) have pointed out. On the other hand, Nachtigall and Opitz (2014) use column generation to solve the TPAP for maximizing the capacity utilization of freight trains in the east of Germany. Reisch et al. (2020) extend this work by a heuristic column general approach and solve the TPAP for capacity maximization for all freight trains in Germany, that is, more than 5000. Zhang et al. (2019) close the gap to periodic timetabling by incorporating constraints to generate a cyclic timetable in the TPAP model.

# 5 Timetable Robustness

So far, we have seen models and solution approaches to schedule as many trains as possible or to find schedules with minimal traveling times. A third objective in railway timetabling is the robustness against minor delays that occur stochastically in railway operations. That is, given a distribution of minor disturbances, the sum of expected delays is to be minimized. The means to achieve this goal are buffer times between train trips on the one hand and time supplements in the timetable of a train, on the other hand. When buffer times are sufficiently large, minor delays of a train trip will not be propagated to the consecutive trip whereas supplements enable a train to compensate for delays that have occured already.

Stochastically occuring delays can be modelled by adding the amount $\zeta$ of delay to the minimum traveling times from $x$ to $x'$ in the TPAP model or to the lower bounds on the activities $l$ in the PESP model. The vector $\zeta$ is referred to as a *scenario*. The scenario $\hat{\zeta} \in Z$ without any delays is called the *nominal scenario*. In (strict) robust optimization, the generated timetable needs to be feasible for every possible scenario $\zeta$ in a set $Z$ of plausible delay scenarios (Goerigk and Schöbel, 2010). Let $F$ be the constraints, $x$ be the variable and $f$ the objective. Then, in its most general form, a (strict) robust optimization problem reads as follows.

$$
\begin{aligned}
\textit{strict}\textbf{-robustness}\ \text{Minimize}\ & f(x) && && \text{(4a)}\\
\text{s.t.}\ & F(x,\zeta) \leq 0 && \forall \zeta \in Z && \text{(4b)}
\end{aligned}
$$

Since this modelling is very conservative, Fischetti and Monaci (2009) introduce the concept of light robustness for train timetabling where exceedances $\gamma_i$ of a constraint $i$ are minimized in the objective function. Let $z^*$ be the optimal value of the nominal problem and $\delta$ a parameter restricting the deviation from the optimal solution value of the nominal scenario. Then, the light robustness problem reads as follows.

$$
\begin{aligned}
\textit{light}\textbf{-robustness}\ \text{Minimize}\ & \sum \gamma_i && && && \text{(5a)}\\
\text{s.t.}\ & F(x,\hat{\zeta}) && \leq 0 && && \text{(5b)}\\
& f(x,\hat{\zeta}) && \leq (1+\delta)z^* && && \text{(5c)}\\
& F_i(x,\zeta) && \leq \gamma_i && \forall i\ \forall \zeta \in Z && \text{(5d)}\\
& \gamma && \geq 0 && && \text{(5e)}
\end{aligned}
$$

Schöbel and Kratz (2009) apply the robust optimization to the aperiodic timetabling in an bi-criteria approach compromising robustness and travelling times. Furthermore, there are studies where the robust timetable is not generated but merely modified. For instance, Maróti (2017) applies stochastic programming to find an optimal allocation of buffer and supplement times of a given reference timetable and applies it to a 1-hour timetable of the whole Netherlands Railways (NS).

Finally, there is a number of approaches that consist of evaluations of the robustness of a modified timetable only. Such approaches include both simulation (Middelkoop and Bouwman, 2000), and analytical computations (Huisman and Boucherie, 2001) to derive the expected amount of propagated delays in a timetable with respect to a distribution of occurring delays. Reisch and Kliewer (2020) close the gap to robust timetable modification by introducing blackbox optimization rules for these evaluation approaches with the aim of adjusting the timetable such that the new one improves the objective of minimal delays.

# 6 Conclusion

In this state of the art overview, we considered different aspects of computer-aided railway timetabling. We presented the notion of a slot which is a timetable for a single train and that it can be computed in polynomial time. Furthermore, we stated the difference between periodic and aperiodic timetables and showed that the periodic problem modelled as the PESP, is NP-complete. Likewise, the complexity of scheduling multiple trains on a limited infrastructure, modelled as the TPAP, is NP-complete. Finally, we presented approaches that optimize railway timetables with respect to minimizing the sum of expected delays.

## References


Borndörfer, R., Lindner, N., and Roth, S. (2020). A concurrent approach to the periodic event scheduling problem. *Journal of Rail Transport Planning & Management*, (15):100175.

Caprara, A. (2015). Timetabling and assignment problems in railway planning and integer multicommodity flow. *Networks*, 66(1):1–10.

Caprara, A., Fischetti, M., Guida, P., Monaci, M., Sacco, G., and Toth, P. (2002). Solution of real-world train timetabling problems. *Proceedings of the 34th Annual Hawaii International Conference on System Sciences*.

Dahms, F., Pöhle, D., Frank, A.-L., and Kühn, S. (2019). Transforming automatic scheduling in a working application for a railway infrastructure manager. Rail Norrköping Conference.

Even, S., Itai, A., and Shamir, A. (1975). On the complexity of time table and multi-commodity flow problems. In *16th Annual Symposium on Foundations of Computer Science (sfcs 1975)*, pages 184–193.

Fischetti, M. and Monaci, M. (2009). *Light Robustness*, page 61–84. Springer-Verlag, Berlin, Heidelberg.

Ford, L. R. and Fulkerson, D. R. (1956). Maximal flow through a network. *Canadian Journal of Mathematics*, 8:399–404.

Goerigk, M. and Schöbel, A. (2010). An Empirical Analysis of Robustness Concepts for Timetabling. In Erlebach, T. and Lübbecke, M., editors, *10th Workshop on Algorithmic Approaches for Transportation Modelling, Optimization, and Systems (ATMOS'10)*, volume 14 of *OpenAccess Series in Informatics (OASIcs)*, pages 100–113, Dagstuhl, Germany. Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik.

Großmann, P., Hölldobler, S., Manthey, N., Nachtigall, K., Opitz, J., and Steinke, P. (2012). Solving periodic event scheduling problems with sat. In Jiang, H., Ding, W., Ali, M., and Wu, X., editors, *Advanced Research in Applied Artificial Intelligence*, pages 166–175, Berlin, Heidelberg. Springer Berlin Heidelberg.

Hansen, I. and Pachl, J. (2014). *Railway Timetabling Operations. Analysis - Modelling - Optimisation - Simulation - Performance Evaluation.*

Huisman, T. and Boucherie, R. J. (2001). Running times on railway sections with heterogeneous train traffic. *Transportation Research Part B: Methodological*, 35(3):271 – 292.

Liebchen, C. (2006). *Periodic Timetable Optimization in Public Transport*. dissertation.de.

Liebchen, C. (2008). The first optimized railway timetable in practice. *Transportation Science*, 42(4):420–435.

Lindner, N. and Reisch, J. (2020). Parameterized complexity of periodic timetabling. Technical Report 20-15, ZIB, Takustr. 7, 14195 Berlin.

Maróti, G. (2017). A branch-and-bound approach for robust railway timetabling. *Public Transport*, 9(1-2):73–94.

Middelkoop, D. and Bouwman, M. (2000). Train network simulator for support of network wide planning of infrastructure and timetables. *Advances in Transport*, 7:267–276.

Nachtigall, K. (1998). *Periodic Network Optimization and Fixed Interval Timetables*. Habilitation thesis, Universität Hildesheim.

Nachtigall, K. and Opitz, J. (2014). Modelling and solving a train path assignment model. Proceedings of the International Conference on Operations Research, Aachen.

Odijk, M. (1994). *Construction of Periodic Timetables: A Cutting Plane Algorithm*. in: Technical Report, TU Delft.

Reisch, J., Großmann, P., Pöhle, D., and Kliewer, N. (2020). Conflict resolving - a local search algorithm for solving large scale conflict graphs in freight railway timetabling. Refubium.

Reisch, J. and Kliewer, N. (2020). Black-box optimization in railway simulations. In Neufeld, J. S., Buscher, U., Lasch, R., Möst, D., and Schönberger, J., editors, *Operations Research Proceedings 2019*, pages 717–723, Cham. Springer International Publishing.

Schöbel, A. and Kratz, A. (2009). *A Bicriteria Approach for Robust Timetabling*, pages 119–144. Springer Berlin Heidelberg, Berlin, Heidelberg.

Serafini, P. and Ukovich, W. (1989). *A Mathematical Model for Periodic Scheduling Problems*. in: SIAM Journal on Discrete Mathematics, 2, pp. 550-581.

Zhang, Y., Peng, Q., Yao, Y., Zhang, X., and Zhou, X. (2019). Solving cyclic train timetabling problem through model reformulation: Extended time-space network construct and alternating direction method of multipliers methods. *Transportation Research Part B: Methodological*, 128:344 – 379.

**Diskussionsbeiträge - Fachbereich Wirtschaftswissenschaft - Freie Universität Berlin**
**Discussion Paper - School of Business & Economics - Freie Universität Berlin**

2020 erschienen:

| | |
|---|---|
| 2020/1 | ASSEYER, Andreas: Wholesale price discrimination with regulatory asymmetry<br>*Economics* |
| 2020/2 | JESSEN, Robin und Johannes KÖNIG: Hours Risk and Wage Risk: Repercussions over the Life-Cycle<br>*Economics* |
| 2020/3 | ASSEYER, Andreas: Collusion and Delegation under Information Control<br>*Economics* |
| 2020/4 | ADAM, Marc Christopher: Liquidating Bankers' Acceptances: International Crisis, Doctrinal Conflict and American Exceptionalism in the Federal Reserve 1913-1932<br>*Economics* |
| 2020/5 | BÖNKE, Timm; Rick GLAUBITZ; Konstantin GÖBLER; Astrid HARNACK; Astrid PAPE und Miriam WETTER: Die Entwicklung und Prognose von Lebenserwerbseinkommen in Deutschland<br>*Economics* |
| 2020/6 | HUNDSDOERFER, Jochen und Eva MATTHAEI: Gender Discriminatory Taxes, Fairness Perception, and Labor Supply<br>*FACTS* |
| 2020/7 | ZHU, Junbing und Theocharis GRIGORIADIS: Chinese Dialects, Revolutionary War & Economic Performance<br>*Economics* |
| 2020/8 | POLUGODINA, Maria und Theocharis GRIGORIADIS: East Prussia 2.0: Persistent Regions, Rising Nations<br>*Economics* |
| 2020/9 | DOMBI, Akos; Theocharis GRIGORIADIS und Junbing ZHU: Antiquity and capitalism: the finance-growth perspective<br>*Economics* |
| 2020/10 | AHMED LAHSEN, Amina; Alan T. PIPER und Ida-Anna THIELE: Kim Jiyoung, Born 1982, and the Labour Market: Overeducation, Gender, Income and Life Satisfaction. Panel evidence from Korea<br>*Economics* |

2020/11 COLEMAN, Winnie und Dieter NAUTZ: The Credibility of the ECB's Inflation Target in times of Corona: New Evidence from an Online Survey
*Economics*

2020/12 SYDOW, Jörg; Georg SCHREYÖGG und Jochen KOCH: Current interest in the theory of organizational path dependence : a short update on the occasion of the 2019 AMR Decade Award
*Management*

2020/13 DIECKELMANN, Daniel: Cross-Border Lending and the International Transmission of Banking Crises
*Economics*

2020/14 WOLF, Tobias: Welfare while working: How does the life satisfaction approach help to explain job search behavior?
*Economics*

2020/15 PIPER, Alan: Temps dip deeper: temporary employment and the midlife nadir in human well-being
*Economics*

2020/16 OLSEN, Nils: A literature overview on scheduling electric vehicles in public transport and location planning of the charging infrastructure
*Information Systems*

2020/17 WINKELMANN, Lars und Wenying YAO: Cojump Anchoring
*Economics*

2020/18 MARTÍNEZ-HERNÁNDEZ, Catalina: Disentangling the effects of multidimensional monetary policy on inflation and inflation expectations in the euro area
*Economics*

2020/19 DIEGEL, Max und Dieter NAUTZ: The role of long-term inflation expectations for the transmission of monetary policy shocks
*Economics*