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THE PREDICTIVE POWER OF THE YIELD CURVE: THE PORTUGUESE CASE

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ABSTRACT

This work project studies the historical relationship between the yield curve and real economic activity in Portugal, comparing results with Germany and Spain. Controlling for other indicators, on average, each percentage point increase in the Portuguese yield spread was associated with a 0.6 pp. increase in real growth over the subsequent year. In general, a longer maturity short-term rate is preferable in Portugal, similarly to Spain. To forecast recessions, as expected, the lower the slope of the yield curve, the higher the probability of a downturn. As in Spain, an expanded model is more effective for Portugal, whilst for Germany the univariate setup was already relatively accurate. These conclusions could be useful in Risk Management or in the improvement of a Portuguese leading economic indicator.

Keywords: Yield Curve, Term Spread, Real Economic Growth, Recessions.

JEL Classification Number: C22, E43, E44.

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1. Introduction

This work project studies the historical predictive power of the yield curve for Portugal in the context of the Eurozone. I find that each percentage point increase in the Portuguese yield spread was on average associated with a 0.6 p.p. increase in real growth over the subsequent year. I control for past growth and leading economic indicators (the variation of the PSI 20, M2 Money Supply, Employed Population, and Consumer Confidence and Industrial Order Indexes) to find the independent predictive content of the Portuguese yield curve. Regarding recession (defined as at least two successive quarters of negative real growth) forecasting, when the Portuguese yield spread was at its mean, the estimated probability of recession over the following semester was around 11%. As expected, the lower the slope, the higher the probability of recession. These conclusions are compared with the German and Spanish cases.

Since the 1980s, among leading economic indicators (like stocks, interest rates, monetary aggregates, for instance) the yield curve emerged as arguably the most popular. The well-established measure of the yield curve slope, the difference between the government bond long- and short-term benchmark rates (also known as the "term spread" or "term premium"), is used in this study. This simple definition was shown by Estrella and Mishkin (1996, 1998), Moneta (2003), or Chinn and Kucko (2015) to be a very effective forecaster of real economic activity, even outperforming more complex approaches. These authors found that steep curves predict robust economic growth while a flat or inverted curve anticipates economic underperformance, results that are corroborated in this work project.

Where this study diverges from previous papers is in the choice of the short-term rate: I conclude that, while for Germany both the 3-month and 2-year rates yield relatively robust results, for Portugal (and Spain) it is more appropriate to use the 2-year (1-year) as the short-term rate. For all models, the 10-year rate is assumed as the long-term benchmark. Additionally, whilst for Germany the multivariate models corroborate the relevant results of the single linear regression for both spreads, for Portugal (and Spain) the multivariate models are needed to reach a consistent conclusion. This relationship can be visually confirmed in Graph 1 (Data Appendix), which displays this spread and recessions for the U.S. and the analysed countries.

Concerning recession forecasting, for Portugal and Germany I obtain intuitive results: lower spreads are linked with higher probability of recession. The less significant result for Spain is hypothesized as being related with a “Japanification of the Eurozone”. If this becomes a reality in the future, a weakening/reversal of the term spread’s significance is possible, as some authors have pointed out. Furthermore, in the Portuguese and Spanish cases expanding the simple probabilistic model with a recession lag and short-term volatility is preferable; for Germany just using the term premium is already fairly accurate.

Finally, for Portugal, I propose some practical applications in Risk Management, with the use of the 10Y-2Y spread in Factor Models to estimate portfolio volatility, or in the use of this variable to improve a leading economic indicator.

Section 2 reviews the main findings regarding the relation between the yield curve and expected economic performance, and the motivation is provided. In section 3, the determinants of the long-term relative to the short-term interest rates are examined. Section 4 describes the data, and the empirical tests are implemented to gauge the significance of the yield spread in forecasting real economic growth. In Section 5, the exercise is replicated, but in a non-linear framework using a recession dummy. Section 6 concludes, presents the limitations of the used models, and provides some guidelines for future research.

2. Literature Review

The theoretical and empirical relationship between of the yield curve and real economic activity has been documented since the 1980s. Authors like Harvey (1988, 1989), Stock and

Watson (1989), Estrella and Hardouvelis (1991), among others, found consistent evidence for the United States that a steep curve predicts robust economic growth; a flat curve, slowing growth, and an inverted curve, a recession. The latter introduced what would become the most popular measure of the yield curve slope: the spread (difference) between the 10-year Treasury note and the 3-month Treasury bill rates. Based on their work, the New York FED created a page with the probability of recession for the American economy centred around this indicator[1].

Following research focused on whether this relation held up in countries other than the US. Plosser and Rouwenhorst (1994) concluded that the term structure has significant independent predictive power for structural economic growth in the U.S., Germany, and the U.K. Harvey (1991), Davis and Henry (1994), Estrella and Mishkin (1997), and Estrella, et al. (2003) studied other OECD countries, finding evidence that the yield spread generally does a relatively good job in forecasting real economic activity. More recently, Chinn and Kucko (2015) argued that there is still predictive content in the yield curve, though it has deteriorated.

Regarding the out-of-sample behaviour, results are generally more mixed. Haubrich and Dombrosky (1996) and Estrella and Mishkin (1998) found the spread to be an efficient out-of-sample forecaster of four-quarter economic growth for the U.S., though the former authors concluded that parameter estimates are unstable over time. Dueker (1997) added that, among major leading indicators, the yield spread is a relatively good recession predictor. Davis and Fagan (1997) and Dotsey (1998) remarked that the spread provides information on future output and inflation, though its effectiveness has declined over more recent periods. Similarly, Chinn and Kucko (2015) obtained relatively poor out-of-sample forecasting results, which was argued as being related with the lack of variability of the macroeconomic data during the studied period.

[1] For more details, see the Federal Reserve of New York's page: "The Yield Curve as a Leading Indicator."

While the simpler models require the yield curve slope as the only independent variable, following research used supplementary regressors to “clean” the effect of the studied indicator. In the prediction of real economic growth, for example, Hamilton and Kim (2002) proposed adding lagged GDP growth. Additionally, Chinn and Kucko (2015) suggested building a Factor Model to explore if the yield spread contains independent information about future growth, when leading economic indicators are included in the equation. In general, these authors concluded that expanding the univariate model is beneficial for both the overall fit and significance of the yield curve.

Regarding the recession forecast, Wright (2006) concluded that the short rate strengthens the in-sample results. However, by constructing a no arbitrage term structure model, Cieslak and Povala (2016) found that volatility of short rates, rather than its level, predicts economic activity independently of the term spread. Dueker (1997) and Moneta (2003) also proposed adding a recession lag to solve serial autocorrelation of the error term.

The subject of the yield curve’s predictive power has recently become somewhat controversial, with some questioning its validity as a leading indicator. Its main critics have questioned the efficacy of this variable, pointing out that the inverted curve has “predicted 9 out of the last 5 American recessions”. Common arguments for this deterioration are the possibility of a “self-fulfilling prophecy”, the improved credibility of Central Banks, or the fact that this relationship might only be truly accentuated for the U.S. (The Economist, 2018, 2019). Therefore, it is valuable to understand to which extent the yield curve still holds forecasting power, especially for countries other than the U.S. (where most literature focuses on).

The work project will use as reference the recent NBER paper, “The Predictive Power of the Yield Curve across Countries and Time” (Chinn M., Kucko K., 2015). This paper was chosen because it substantiates very thoroughly the theoretical background, introducing Factor analysis to study a more representative and diverse dataset of countries.

As explained by the authors, the creation of the euro in 1999 led to more integrated European financial markets, increasing real economic links. At the time of the paper's analysis, however, the European Monetary Union had not experienced a substantial downturn. In the words of the authors, "*there is (was) little opportunity to test the predictive power of the yield curve in this (that) context*". Furthermore, the lack of observations for some European countries lead to their omission. With the 2012 Sovereign Debt crisis and subsequent recovery, there is now a larger and more representative sample, making the study of the linkages between interest rates and output in Eurozone countries more relevant.

Additionally, because research on this subject is very intertwined, it is always valuable to apply these theories and models to other countries in other time periods. The analysis will be dedicated to Portugal, comparing its results with a similar European country that was not studied in the paper (Spain), and a reference Economy: Germany. The main objective is to determine whether the established conclusions still apply, especially in a smaller economy like Portugal, where there is a lack of extensive and up-to-date research on the topic.

3. Theoretical Background

3.1. The Yield Curve

The yield curve represents the constant annual interest rate correspondent to the quoted price of a government bond i.e., it is the average annualized benefit/return of "borrowing money to the government", over different maturities. The interest rates of government bonds usually have the lowest yields among financial assets (sometimes even negative) because they are perceived as the lowest-risk investments and in some cases, virtually risk-free. More recently, interest rates have been at historic lows throughout Europe, North American and Asia. The low-interest-rate-environment was mainly created by Central Banks, which used both conventional and unconventional monetary policy to stimulate their economies, in the aftermath of the Great Recession and Eurozone Sovereign Debt Crisis. Because interest rates tend to be more persistent, they usually hold less information than spreads. The latter are, therefore, more commonly used to assess the macroeconomic and financial conjuncture (the yield curve spread, the TED spread, or the yield differential between sovereign bonds represent some examples).

Because future spot interest rates are unknown, yields need to be estimated from the implied interest rates in government bond prices for different residual maturities, which are observable in the markets. The European Central Bank, for example, uses the Svensson model to estimate the Eurozone yield curve[2]. Whereas the shorter end of the curve is mainly determined by monetary policy and cyclical expectations, the longer end tends to be more impacted by demand and supply, reflecting structural economic prospects (like fiscal policy and long-term inflation expectations). While often upward sloping, sometimes the yield curve can invert, if short-term interest rates exceed long-term rates. But why should a "normal yield curve" be upward sloping?

3.2. The Financial Argument for a Positive Slope – Liquidity Premium Theory

Liquidity is a measure of how easy/fast it is to sell an asset without lowering its price, i.e., the "easiness" of converting it to cash. The Liquidity Premium/Preference Theory (LPT) is directly connected to interest rate risk. If rates increase, bond prices will go down, because the existing lower-yielding assets are less attractive to investors. To induce an investor to buy these lower-return bonds, their price should fall up until the point in which its return is equivalent to the newer higher-coupon bonds'. LPT argues that, if rates go up, bond investors prefer short-dated over long-dated securities, because they will be able to reinvest their money faster,

[2] For more details, see European Central Bank: "Euro area Yield Curves."

holding the undervalued security for a shorter period, without taking a loss. Adding the fact that there are more risks in the long-term, like default or inflation risk, this theory concludes that longer-term Bonds should have a premium to compensate investors.

3.3. The Economic Arguments – Expectations Theory and Central Banks' Policy

The Expectations Theory Hypothesis (ETH) postulates that a positive slopping yield curve reflects higher expected interest rates. One possible way of formalizing the pure ETH is:

$$f_{t:t+k} = [(1 + r_{o:t+k})^{t+k}/(1 + r_{0:t})^{t}]^{1/k} - 1, \quad (1)$$

where $f_{t:t+k}$ represents the forward rate from period *t* to *t*+*k*, i.e., the expected spot rate at time *t*, with maturity *k*; $r_{0:t}$ and $r_{o:t+k}$ represent spot rates at time 0, with maturity *t* and *t*+*k*, respectively. From equation (1), one concludes that if $r_{o:t+k}$ increases and/or $r_{0:t}$ decreases i.e., if the yield curve becomes (more) positively sloped, then $f_{t:t+k}$ will be higher. This means that a positive slope implies higher expected spot rates. Chinn and Kucko (2015) also decompose this relationship as:

$$i_{t:t+k} = \frac{(i_{t:t+1} + i^{e}_{t+1:t+2} + \cdots + i^{e}_{t+k-1:t+k})}{k} + l^{k}_{t}, \quad (2)$$

where $i_{t:t+k}$ is the interest rate on a bond of maturity *k* at time *t*; $i^{e}_{t+j:t+j+1}$ is the one-period forward rate at time *t* and l^{k}_{t} is that bond's liquidity premium. Equation (2) agglutinates the arithmetic approximation of the Expectations Hypothesis (first-term) with the Liquidity Premium Theory (second term). This definition establishes that the yield on a longer-term bond is the average of the expected one-period interest rates until maturity. As explained above, $l^{k}_{t} > 0$, and rising with maturity (*k*), which implies that, keeping short-term interest rates (at least) constant over time, the yield curve will slope upward. However, if the longer-term rate is lower than the short rate, i.e., if the yield curve is inverted, then the expected short rates must be lower than the homologous spot rate.

The question that follows is: "Why should the steepness of yield curve be associated with economic activity?". According to Estrella and Mishkin (1996, 1997), expected interest rates are related with expectations of monetary policy, inflation, and real growth.

Firstly, inflation tends to be positively correlated to economic activity, which means the expected inflation should contain information about future growth. Recurring to the Fisher Equation, defined as $i = r + \pi^e$, a positive slope is indicative of higher expected inflation, as expected rates are higher. Following the same logic, a lower slope should indicate expected disinflation and slower growth. Kozicki (1997) points out that the yield spread is an effective predictor of inflation at moderate horizons, albeit less accurate than the level of yields.

On the other hand, if investors expect higher economic activity in the future, they should require higher compensation for longer-term bonds, as they anticipate their other investments to perform well (government bonds' return is the opportunity cost of capital). Alternatively, if there is a perceived short-to-mid-run risk in the economy that outweighs that of holding long-term bonds (like a recession) investors will become more risk-averse, wanting to "lock" their money for a longer period. The demand for longer-term bonds will thus increase, decreasing its return, and ultimately flattening or even inverting the curve. This inherently reflects financial markets' expectation that the Central Bank will respond to the slowdown and probable decrease in the demand for credit. As the expansion cycle ends, the monetary authority will likely cut interest rates to stimulate the economy, switching from monetary tightening to easing, and "re-balancing" the yield curve. Essentially, the inverted yield curve reflects the financial markets' belief that the economy will be (much) worse in the short- than in the medium-/long-term.

Another scenario is a Central Bank induced slowdown. This hypothesis posits that an economic slowdown can be precipitated by monetary tightening. The increase in interest rates should have a higher impact on short rates than on long-term rates, as explained in Wu (2003), leading to a downward-sloping term structure which will discourage consumption and investment. Estrella and Mishkin (1997) added that the Central Bank's credibility affects the extent of the flattening of the yield curve in response to a change in the policy/target rate.

These ideas are corroborated by several authors, and the more recent study by Kurmann and Otrok (2013) puts together all the pieces: they find evidence that steep yield curves, mainly due to fluctuations in short-term rates, generally predict future economic growth; that Central Banks respond aggressively to inflation and that consequently, monetary policy plays a central role in establishing the bridge between macroeconomic and term structure dynamics.

4. The Significance of the Yield Curve in Predicting Real Economic Growth

4.1. Data Description

The analysis will be essentially dedicated to Portugal, a relatively small economy, comparing its results with a medium-sized (and similar) economy, Spain, and Germany, the reference country. This decision stemmed from trying to do a comprehensive analysis of the Portuguese case in the context of the Eurozone, which inherently complements the reference paper. Furthermore, this sample of three countries was considered to be a fairly accurate representation of the Eurozone. Graph 2 shows that the average GDP *per capita* and Debt-to-GDP ratios among these countries are similar to that of the Euro Area. One challenge that arose was the frequency of the GDP data (quarterly) and, inherently, the size of the time sample. Moreover, the data will range from the first quarter of 1995 until the fourth quarter of 2019, which corresponds to the longest common sample size (100 quarters) among the three countries.

Unlike Chinn and Kucko (2015), that used Industrial Production as the main representation of economic activity, real GDP was chosen. Following the prevailing research that relates interest rates and output, this measure was considered the broadest indicator of the economy activity. But regardless of the possible advantages and disadvantages of choosing one or the other, as referred by the authors, Industrial Production tends to follow GDP closely, which should lead to the same conclusions.

The real Economic growth and yield curve slope were computed as follows:

$$y_{t:t+k} = \frac{4}{k} * (\ln Y_{t+k} - \ln Y_t),$$

$$Spread_t = i_t^{10Y} - i_t^{ST},$$

where Y_{t+k} is the real GDP in quarter $t+k$ (adjusted for seasonal and calendar effects), $y_{t:t+k}$ is the annualized real GDP growth over the next k quarters, and the spread is computed as the difference between the quarterly moving averages of the 10-year and the 3-month/2-year rates[3]. The data sources are presented in the Data Appendix.

4.2. Portuguese Univariate Model

The first estimated equation for Portugal assessed the significance of yield spread in predicting real activity:

$$y_{t:t+k} = \alpha + \beta Spread_t + \varepsilon_{t+k}. \tag{3}$$

In short, the yield curve slope is observed at time t, and based on that, the annual real growth is predicted k quarters ahead. This means that the number of observations will decrease by k. This model was examined with k equal to 1, 2, 4 and 8 (growth over a quarter, semester, one- and two-year horizon). Following previous researchers, all inference will be made using Newey-West heteroskedasticity and serial autocorrelation robust standard errors. This happens because growth measures are drawing from common observations and because economic growth rates tend to be persistent, making the error term serially correlated[4].

[3] For some intermediate periods, because there was no data on the 2Y rate, linear interpolation was used.
[4] Following Hamilton and Kim (2002) that also used quarterly data, all models were corrected for the maximum AR process found in the error term (12 lags), as a conservative approach.

Unit root tests were also conducted to assess whether the spreads and economic growth rates are Stationary. The results of the Augmented Dickey-Fuller test are presented in Table 1 (Data Appendix), suggesting that economic growth rates and the spread changes are stationary.

The first results, presented in Table 2, are somewhat mixed. They seem to contradict each other: the estimated spread coefficient was positive when using the 2-year rate, and negative when using the 3-month. As Chinn and Kucko (2015) explained for Japan, the negative coefficient of the term premium could be associated with aggressive monetary policy to stimulate the economy (zero or negative interest rates and quantitative easing). When the target rate hits the zero-lower-bound and the monetary authority starts a quantitative easing program, long-term interest rates face a downward pressure. If the resulting narrowing of the yield curve stimulates the economy, this relation might be reversed. With a potential "Japanification of Europe" (Financial Times, 2019), it is possible that in the future this might become the norm. However, one should not focus too much on these results since the simple linear regression is very limited in terms of exogeneity.

4.3. Including Lagged Growth Variables

To solve the likely endogeneity in the behaviour of the term spread and output growth, and because current and lagged rates of growth may be useful in GDP forecasting, Hamilton and Kim (2002) introduced quarterly lagged growth in the equation. Henceforth:

$$y_{t:t+k} = \alpha + \beta_1 Spread_t + \beta_2 y_{t-1:t} + \beta_3 y_{t-2:t-1} + \beta_4 y_{t-3:t-2} + \beta_5 y_{t-4:t-3} + \varepsilon_{t+k}, \quad (4)$$

where $y_{t-j-1:t-j}$ is quarterly real GDP growth beginning in quarter *t-j-1*.

Table 3 shows more encouraging estimation results. Controlling for previous growth, the estimated parameters using the 10Y-2Y spread are positive, relevant for 1, 2 and 4 quarters ahead, and just slightly smaller than the estimated coefficients without including lagged real

GDP growth[5]. Thus, the yield spread provides information beyond that contained in lagged growth rates for the Portuguese economy, especially in the short/medium term. On average, *ceteris paribus*, for each percentage point increase in the yield spread, the Portuguese real growth increased by around 0.62 percentage points over the subsequent year.

Interestingly, when using the 10Y/3m pair, the estimated spread coefficient became insignificant. At the time of the writing of this thesis the only reliable dataset on the Portuguese 3-month rate came from the OECD database. According to the OECD, these short rates are based on the three-month money market rates (rates at which banks lend to each other). However, they standardise this measure as "money market rate" or "treasury bill rate". While these can be analogous in the U.S., where the FED targets overnight rates, and in the Eurozone as whole[6], for a smaller economy with a higher (country-specific) risk premium like Portugal the context might be different. The yield on a liquid bond might contain more independent information than the Central-Bank-controlled money market rate, which will be more persistent and will have too short of a duration to catch the short-term credit/economic trends. According to the N.Y. FED, the usefulness of the 3-month rate as an indicator of market expectations can be blurred by its "*fairly direct control by the Federal Reserve*". It argues that the 2-year rate can be an efficient substitute, reflecting both monetary policy and cyclical growth expectations (in the context of the Central Bank's transmission mechanism and credibility).

Furthermore, the correlation between these rates seems to be time-varying: it is 97% in the first half, but just 36% in the second. This is visible in Graph 3. They moved in tandem for most of the time but diverged between The Great Recession and The Sovereign Debt Crisis: it is possible that the 3-month rate was a reflex of the ECB easing, disregarding the increased Portuguese-specific risk (which was priced in the 10-year) .

[5] Only the 1 quarter model showed some sensitivity to the number of Newey-West lags.
[6] Using aggregate Euro area data, Moneta (2003) found that the 10Y-3m spread was the most effective.

The last question to be tackled is why one should not include the growth lags corresponding to the respective time-step. Albeit a valid exercise, it would probably lead to the same conclusions: the annualized lagged quarterly growth rates are fairly correlated with the yield spread and with semi-annual, annual, and biennial annualized growth. But most importantly, the spread is less correlated with lagged quarterly growth rates than with other time intervals. Together with preserving more degrees of freedom, the adopted method most likely led to a more efficient inference exercise. This is illustrated in Table 4.

4.4. Factor Model

Following Chinn and Kucko (2015), the final step was introducing leading economic indicators into the equation to assess whether the yield curve continued to have independent predictive power. The chosen leading indicators were an adaption from The Conference Board's "Leading Economic Index" for Spain and Germany (there was no specific index for Portugal). When exact matches were not found or when the dataset was too short, the closest series was used. The inputs were the quarterly evolution of the following variables: the Stock Index (PSI 20), the Money Supply (measured as the nominal contribution to Eurozone M2), the Employed population, the OECD's Consumer Confidence Index and the Industrial Order Books Survey.

To represent these indicators, a statistical construct was built by applying Principal Component Analysis. The factor was defined as the first principle component. The advantage of this method is that the information of several variables is synthesized in just one "Factor". This is beneficial for the statistical inference of the spread in two ways: it "cleans" the marginal effect of the yield curve and preserves the degrees of freedom of the estimation[7]. Henceforth:

$$y_{t:t+k} = \alpha + \beta_1 Spread_t + \beta_2 Factor_t + \varepsilon_{t+k}. \quad (5)$$

[7] For more information on PCA, refer to the Technical Appendix.

Due to data limitations, the sample size for Portugal was reduced to 85 observations. Looking at Figure 1 below, when using the 2-year rate the coefficients for the yield curve were close to the model without the factor and are significant for all time horizons (the weakest being for *k*=1). The largest registered difference happened for quarterly and biennial prediction, where the estimated spread parameter changed from 0.45 and 0.34 to around 0.36 and 0.39, respectively (notice however that they were not statistically significant before). [8]

Portugal

		Constant	Spread	Factor	R-Squared	DW
10Y - 3m	k=1	**0.014**	-0.146	**0.011**	0.34	2.04
		2.54	-0.89	5.01		
	k=2	**0.014**	-0.164	**0.009**	0.37	1.07
		2.36	-0.9	4.31		
	k=4	**0.013**	-0.157	**0.007**	0.31	0.54
		2.41	-0.94	4.09		
	k=8	**0.009**	-0.045	**0.006**	0.24	0.28
		2.26	-0.38	3.54		
10Y - 2Y	k=1	0.004	**0.364**	**0.012**	0.35	2.11
		1.2	2.23	7.8		
	k=2	0.001	**0.556**	**0.01**	0.42	1.18
		0.2	3.52	7.61		
	k=4	-0.001	**0.644**	**0.007**	0.43	0.65
		-0.21	5.11	5.93		
	k=8	0.002	**0.389**	**0.005**	0.32	0.34
		0.71	2.31	3.73		

Notes: t statistics below OLS estimates, using Newey-West Robust Standard Errors for a maximum of 12 lags. **Bold** entries indicate significance at 5%.

Figure 1 – Factor Model Results for Portugal

As expected, the Factor is statistically and economically significant for all periods. Results suggest the yield curve slope contains information that is independent from other leading economic indicators[9]. The results were the same for detrended and demeaned values of

[8] The statistical significance of the Spread using the 3-month rate once again disappeared.
[9] Except for the 1-quarter, the significance did not show any sensitivity to the number of Newey-West lags.

economic growth. Therefore, on average, *ceteris paribus*, for each pp. increase in the yield spread, the Portuguese growth increased by around 0.64 pp. over the subsequent year.

4.5. The German and Spanish Cases

In this section, the analysis is expanded to Spain and Germany. Because the Spanish Central Bank did not provide data for the 2-year rate, the 1-year rate was used. Regardless, the correlation between the 1Y and 3Y is 0.98, and so it is likely (but not necessary) that the 2-year provides roughly the same information than the two others (the German 1-year is also very correlated with the 2-year). The results are presented in Tables 5, 6 and 7.

For both countries, unlike Portugal, the 3-month and the 1/2-year rates tended to move more in tandem. The German yield spread already showed a strong and significant relation with output in the single linear regression. For Spain, the coefficients were negative and, for the most part, insignificant, signs of the discussed "Japanification". Including past growth corroborates the results for Germany: both spreads contain information of future growth that is independent from past growth. For Spain, the relation reversed for the 10Y-1Y specification, becoming positive and significant for an annual and biennial forecast. Like in Portugal, in Spain the 10Y/3m pair had no significance.

Regarding the Principle Component Analysis, the Factor was more relevant for Spain (and Portugal) than for Germany. However, the overall results do not improve as much as in the Portuguese case: this might be related with the chosen components, which were an adaptation for Portugal of these countries' individual indexes. Nonetheless, especially in the longer-term, the German 10Y-3m and the Spanish 10Y-1Y show relatively strong results. Additionally, the marginal impact of the yield curve slope is higher than in Portugal. One interesting point is that in all models for Germany the 10Y/3m pair works better for k = 4 and 8, whilst the 10Y-2Y tends to be more effective in the shorter-term.

In sum, for Germany both pairs work relatively well, and the multivariate models corroborate the results of the univariate setup; for Spain, the 10Y-1Y is more effective, but the multivariate models are needed to reach a significant conclusion. Like in Portugal, this might be possibly related with these countries having more idiosyncratic risk, meaning that there is more volatility arising from correlated “exogenous” factors. When these are omitted, their impact is absorbed by the yield spread, making it less relevant. In practice this might imply that for Portugal one should analyse the yield curve together with other leading economic indicators to make assertions on future growth as effective as possible. Additionally, there is evidence that suggests that for Portugal the 10Y-2Y spread can be suitable for Factor Analysis, a popular technique in Risk Management to estimate portfolio volatility, for instance.

5. The Significance of the Yield Curve in Forecasting Recessions

5.1. The Portuguese Case

Thus far, the analysis has been conducted in the OLS context with a continuous dependent variable. Moving to a nonlinear framework, the yield spread was tested as a predictor of recessions. For that end, probabilistic models were employed, defining “recession”, a specific representation of real economic activity, as a binary dependent variable[10].

Wright (2006) and Chinn and Kucko (2015) found evidence that including the shorter-term rate improves in-sample forecasts, but the latter concluded that its inclusion often led to a decrease in the significance of the yield spread. On the other hand, Cieslak and Povala (2016), concluded that volatility of short rates, rather than its level, predicts economic activity independently of the term spread.

[10] For more information on these models, refer to the Technical Appendix.

Therefore, three models were estimated. The first one is a univariate setup, regressing the recession dummy on the 10Y-2Y spread[11]. To isolate its effect from changes in the 2-year rate, the recession/yield curve specification was augmented in the second model. The third model also specified the "short-term" volatility, measured by the quarterly standard deviation of 2-year rates. Henceforth, the estimated models were:

$$Pr(R_{t+k} = 1) = \Phi(\alpha + \beta Spread_t), \tag{6}$$

$$Pr(R_{t+k} = 1) = \Phi(\alpha + \beta_1 Spread_t + \beta_2 2Y), \tag{7}$$

$$Pr(R_{t+k} = 1) = \Phi(\alpha + \beta_1 Spread_t + \beta_2 STVol), \tag{8}$$

where t is the current time period and k is the forecast period; $\Phi(.)$ is the standard normal cumulative distribution function. R_t equals 1 if the economy is in a recession in quarter t, and 0 otherwise. The models were estimated for k equal to 1, 2 and 4. In terms of the recession definition, previous papers used the NBER measure for the U.S. Since there is no comparable official entity in Europe, the most common rule was adopted: at least two consecutive quarters of negative real GDP growth. Tables 8 and 9 display the first results. The pseudo R-squared statistic was shown as an indicator of goodness of fit. It compares the log-likelihood of the normal model (unconstrained model) with that of a constrained (model with $\beta_i = 0$), and hence, it does not penalise for increased model size:

$$pseudo - R^2 = 1 - \frac{L_u}{L_c}$$

As before, like Estrella and Rodrigues (1998) and Moneta (2003) pointed out, because of the overlap in forecast horizons, the errors should suffer from serial autocorrelation. To correct this bias, the Newey-West robust errors were again used. As expected, in the univariate model the estimated spread coefficients are significant and negative: as the yield curve slope decreases, the probability of recession increases. When the Portuguese yield curve spread was at its mean

[11] Once again, the 10Y-3m Spread showed little Economic and Statistical Significance.

(1.4%), the estimated probability of recession over the subsequent semester was around 11%; when it was flat (0%), this probability increased to 28%; when it was inverted (-1%), the probability jumped to 44%. Like Chinn and Kucko (2015) found for non-US countries, due to the relatively high correlation (-0.7), including short-term rate is not beneficial.

On the other hand, there is some evidence that short-term volatility does contain some information independent from the term spread, especially when using a quarterly and semi-annual prediction interval. The drop of significance of the slope is not as high (though it is not significant), the short-term volatility is statistically significant and there is a bigger improvement in goodness of fit. Furthermore, as expected, the marginal effect is positive: the higher the short-term interest rate volatility, the higher the probability of recession is.

In general, two important conclusions from these models can de drawn: movements in the Portuguese yield curve are mainly generated by short-term rates, like Kurmann and Otrok (2013) pointed out, and as Cieslak and Povala (2016) found, their volatility provides more independent information than their level about future recessions. However, the possible impact of omitting an autoregressive process on the estimated parameters still has not been directly dealt with. Moreover, following Dueker (1997) and Moneta (2003), a recession lag was added:

$$Pr(R_{t+k} = 1) = \Phi(\alpha + \beta_1 Spread_t + \beta_2 STVol_t + \beta_2 R_t). \quad (9)$$

The results are presented in Table 10. Like Moneta (2003) found for the Eurozone, there is evidence that in Portugal a recession is very likely in a period preceded by a recessionary quarter. There is an increased quality of fit, though this measure is sensible to the inclusion of additional regressors, making the comparison not very pertinent.

Rather, one can analyse the Accuracy, Sensitivity and Specificity, indicators commonly used in Data Science and Machine Learning. For that, three models were selected: the “Simple Model” (univariate model); one with the addition of short-term volatility, the “Volatility

Model” and another also adding a recession lag, the “Autoregressive Model”. The Hosmer–Lemeshow test suggested that there is no evidence that these models are not a good fit (does not reject the null hypothesis).

Accuracy is the ratio between correct predictions and total observations:

$$Accuracy = \frac{True\ Positives\ \ + True\ Negatives}{True\ Positives + True\ Negatives + False\ Positives\ \ + False\ Negatives}$$

In other words, Accuracy will gauge the percentage of times that the model was right in predicting the state of the economy (recession/no recession).

Sensitivity measures the ratio between correct positive predictions and actual positives:

$$Sensitivity = \frac{True\ Positives\ (TP)}{True\ Positives + False\ Negatives\ (FN)}$$

In other words, Sensitivity will measure the proportion of recessions that the model correctly predicted.

Specificity measures the ratio between correct negative predictions and actual negatives:

$$Specificity = \frac{True\ Negatives\ (TN)}{True\ Negatives + False\ Positives\ (FP)}$$

In other words, Specificity will evaluate the proportion of non-recessions the model correctly predicted.

It was considered that a recession is forecasted if the estimated probability is greater than 13.13%. This (un)conditional threshold is the result of the ratio between the number of recessions and the total number of periods. While the default threshold is 0.5, there are not enough predictions above that value, making it relatively redundant. The Area Under the Curve (AUROC) was also included. It gauges how well the model is at distinguishing recessions from non-recessions, given the threshold. The results are presented in Figure 2 below.

Time Step	1 Quarter			1 Semester			1 Year		
Model	**i)**	**ii)**	**iii)**	**i)**	**ii)**	**iii)**	**i)**	**ii)**	**iii)**
TP	7	10	11	7	9	8	7	6	10
TN	53	71	81	54	71	72	49	65	60
FP	33	15	4	31	14	12	34	18	22
FN	6	3	2	6	4	5	6	7	3
Observations	99	99	98	98	98	97	96	96	95
Accuracy	61%	82%	94%	62%	82%	83%	58%	74%	74%
Sensitivity	54%	77%	85%	54%	69%	62%	54%	46%	77%
Specificity	62%	83%	95%	64%	84%	86%	59%	78%	73%
AUROC	65%	89%	94%	69%	86%	83%	61%	77%	82%

Figure 2: Accuracy, Sensitivity and Specificity for the three estimated probit models

In general, the predicative power seems to be stronger in the shorter-term. Overall, the yield spread does a relatively good job at predicting recessions and non-recessions, but it clearly beneficiates from adding short-term volatility. On the other hand, the addition of a recession lag is only truly beneficial for the very short-term. The yield spread alone performs better for a semesterly lag, though the best performing multivariate models are for a quarterly forecast. The simple model's fitted values for a semesterly forecast are plotted below in Figure 3.

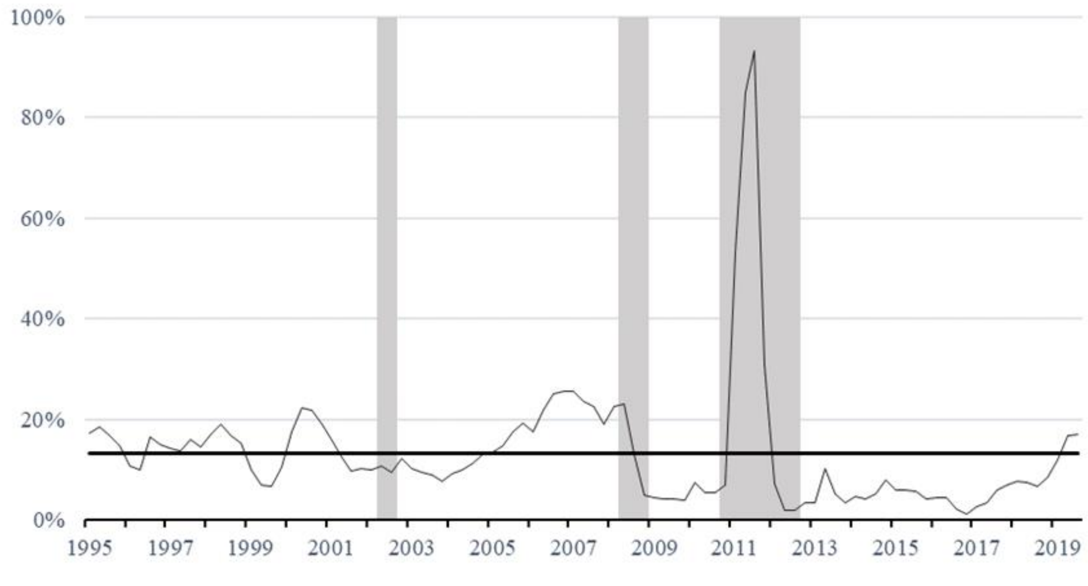

Figure 3: Probability of Recession in the Next Semester for Portugal (Simple Model)

The historical forecasted probabilities of recession for Portugal obtained from the other two models are presented in Graph 4. All the models seem to anticipate the 2008 crisis. Regarding the 2012 Sovereign Debt Crisis, the models are slower to signal a recession, but the estimated probability of the recession rises to 100% quite dramatically in the modified models before the height of the crisis. No model predicts very well the 2002 recession that arose from the "Dot-Com Bubble" (the simple model forecasted a recession too early).

5.2. Comparison with Germany and Spain

The results for Germany and Spain are presented in Tables 11, 12, 13 as well as in Graphs 5 and 6. For Germany, as before, all time-horizons are economically and statistically relevant in the univariate model using the 10Y-3m. When the German yield curve spread was at its mean (2.6%), the estimated probability of recession over the subsequent semester was around 9%; when it was flat (0%) or inverted (-1%), this probability increased to 42% and 60%, respectively. Adding variables does not seem so beneficial as in Portugal: short-term volatility does not have enough variability, while the 3-month rate removes the significance from the spread, not improving the Accuracy that much (as Graph 5 shows). Regarding the Spanish case, when at its mean (1.7%), the yield spread was associated with a semesterly forecasted recession probability of 12% (notice that the estimated coefficients had no significance, for the most part).

Likewise to Portugal, for Spain the use of short-term volatility and a recession lag together with the yield spread helps to forecast recessions, though it reduces the studied variable's significance. Notice however that Spanish recessions tend to be more persistent. For Portugal (and Spain), an expanded probit model thus appears to be preferable than using just the term premium, while for Germany the univariate specification is already fairly accurate. Enhancing these models with Factor Analysis might be a valuable additional tool to understand if the improvement of the fit and significance of the yield spread can "coexist" in this context.

6. Conclusion, Limitations & Directions for Further Research

This work project explored the historical importance of the yield curve in forecasting economic growth and recessions, focusing on Portugal and contextualizing the results with Spain and Germany's. Using the longest common data series available (1995-2019), in-sample results overall confirm what theory suggests: the slope of the yield spread has significant predictive power when forecasting real activity.

When studying the relationship with future real growth, I find evidence that both specifications (10Y-2Y and 10Y-3m) are relatively efficient for Germany, whilst for Portugal and Spain a longer maturity short-term rate is preferable. Specifically for Portugal, the yield spread measured with the 2-year rate provides additional information beyond lagged growth figures or leading economic indicators. Controlling for other indicators, on average, each percentage point increase in the Portuguese yield spread was associated with a 0.6 pp. increase in growth over the subsequent year. The insignificance of the 10Y/3m pair was hypothesised as possibly being related with the ECB's direct influence over short-term rates (like the 3-month or money market rates). While for Germany the multivariate models corroborate the relevant results of the single linear regression for both spreads, for Portugal (and Spain) the multivariate models using the 10Y-2Y (10Y-1Y) spread are needed to reach a consistent conclusion. In practice this implies that in Portugal one should analyse the yield curve together with other leading indicators to make assertions on future growth as effective as possible.

In the second part of the analysis, recurring to a non-linear probabilistic model, the significance of the term premium as a predictor of recessions was assessed. For Portugal, when the slope was at its mean, the estimated probability of recession over the subsequent semester was around 11% (lower than the unconditional threshold of 13%). Once again, the univariate model results are stronger for Germany. Similarly to Spain, adding short-term volatility and a recession lag together with the yield spread in Portugal helps to forecast recessions. Notice however that the Spanish results, for the most part, are not economically and statistically significant. The appearance of a less intuitive coefficient was hypothesized as possibly being related with a "Japanification of the Eurozone", which might imply a future weakening/reversal of the term spread's significance. Nevertheless, the simple probit models predicted reasonably well the 2008 and 2012 crises.

In general, the yield curve slope possesses forecasting power in Portugal, though it is not a "simple story": its relationship with real output is not as direct as it was found for Germany, or as researchers have verified for the United States. This phenomenon, also present in Spain, is perhaps related to the higher idiosyncratic risk: recent history has suggested that a European/American downturn is a sufficient but not necessary condition for a Portuguese (and Spanish) downturn. These results can be a starting point for future research, with potential implications in Risk Management of Portuguese institutions or in the improvement of a leading economic indicator, though it is valuable to recognize some limitations of the presented models.

The first one is the sample size. Whereas reliable U.S. or Germany data is obtainable since the 1970s, the same availability for Portugal is not verified (though it is not a "small" sample *per se*). This constrains the analysis in some ways: it limits the significance exercises, it hinders a possible out-of-sample analysis using sub-samples, and it exacerbates the impact of outliers. Notice however that the yield spread has shown worse out-of-sample forecasting power in more recent studies. Moreover, some authors have suggested that there is a trade-off between the chosen time sample and the significance of this variable. That is why it is important to review the predictive power of the yield curve periodically for different countries.

The second is the different monetary regimes of the analysed countries. In theory, the process of the creation of the EMU in 1991 and consequent loss of monetary sovereignty should imply that the relation between interest rates and domestic output could have changed over time. And in this sense the smaller sample is an advantage: in 1995 the convergence criteria had already been established. Moreover, even though the Euro would only be introduced in 1999, the “level-playing field” had already been established, making the analysis coherent.

The third one is the own definition of “yield curve slope”. There are several ways of computing this indicator, and, whereas for some periods and countries one might be better, in other contexts it might not. An illustration of this issue is the use of the 2-year instead of the typical 3-month rate in Portugal as the short-term proxy. While it was not the first time that this was done, economic intuition and rationale was prioritized (this decision was justified by the existence of a Portuguese-specific risk that is not reflected in the 3-month so efficiently).

Additionally, some have recently argued that "the yield curve inversion does not predict a recession, it causes it.", i.e., that there is some reverse causality between output and the yield curve. While there is no robust statistical evidence that supports this theory, it can be a valid point: as the curve flattens, it hurts the profitability of financial institutions, reducing access to capital, and overall liquidity of the economy. Seeing the past predictability of this indicator in previous crises, markets can get scared and begin a sell-off (like in late 2018), which can slow down the economy that is already hurt by the low liquidity. However, by construction, the yield curve ends up relating more to a market "reaction" to Central Bank and government policies: it gauges expectations. Thus, an inverted yield curve is a "symptom" that the Economic fundamentals are weaker, rather than the cause of said weakness.

Finally, I encourage studies to be conducted to understand to which extent these conclusions remain valid in the future. Interesting avenues to move forward might be enhancing this study with out-of-sample analysis or incorporating Factor Analysis in recession forecasting.

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Data Appendix

Data for this work project came from the following Sources:

Portugal:

3-month rate: OECD database

10Y/2Y rates: Banco de Portugal BPStat

Real GDP and Factor Model Indicators: Banco de Portugal BPStat/Statistics Portugal

Germany:

3-month rate: OECD database

10Y/3Y/2Y/1Y rates: Bundesbank Statistics

Real GDP and Factor Model Indicators: Bundesbank Statistics/Statistisches Bundesamt

Spain:

3-month rate: OECD database

10Y/3Y/1Y rates: Banco de España Statistics

Real GDP and Factor Model Indicators: Banco de España Statistics/Instituto Nacional de Estadística

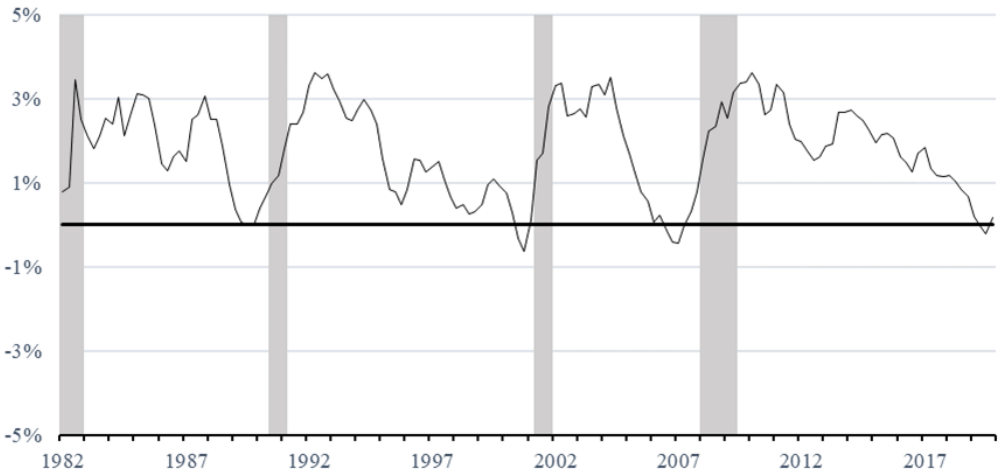
US:

Real GDP and 10Y/3-month rates: FRED database

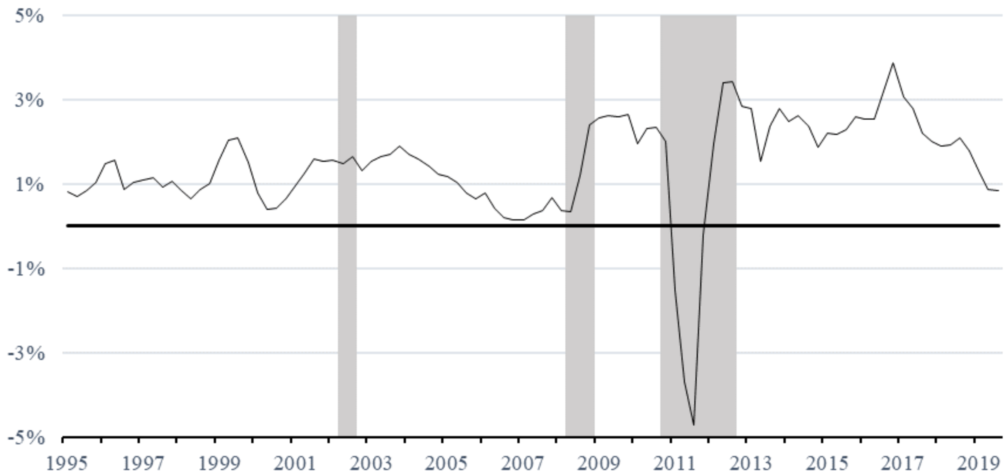
Where OECD indicates Organization for Economic Cooperation and Development and FRED stands for Federal Reserve of St. Louis Economic Database.

Graph 1: Yield Curve Slope and Recessions

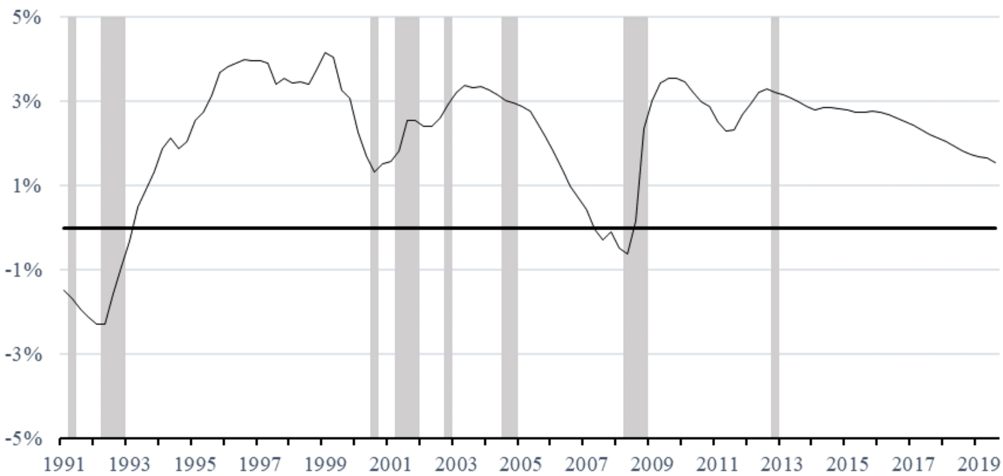
1.1. Historical Recessions and Yield Spread (10Y-3m) for the U.S.

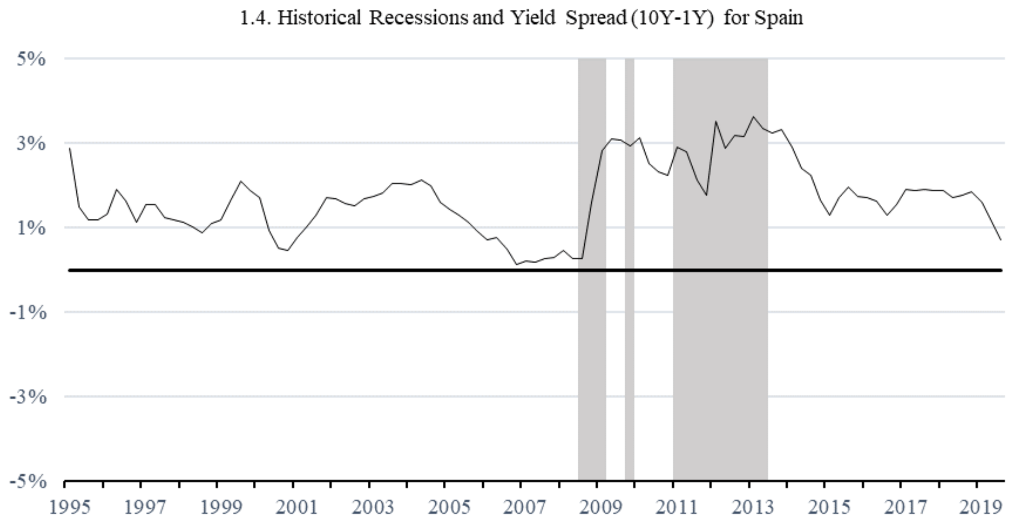


1.2. Historical Recessions and Yield Spread (10Y-2Y) for Portugal

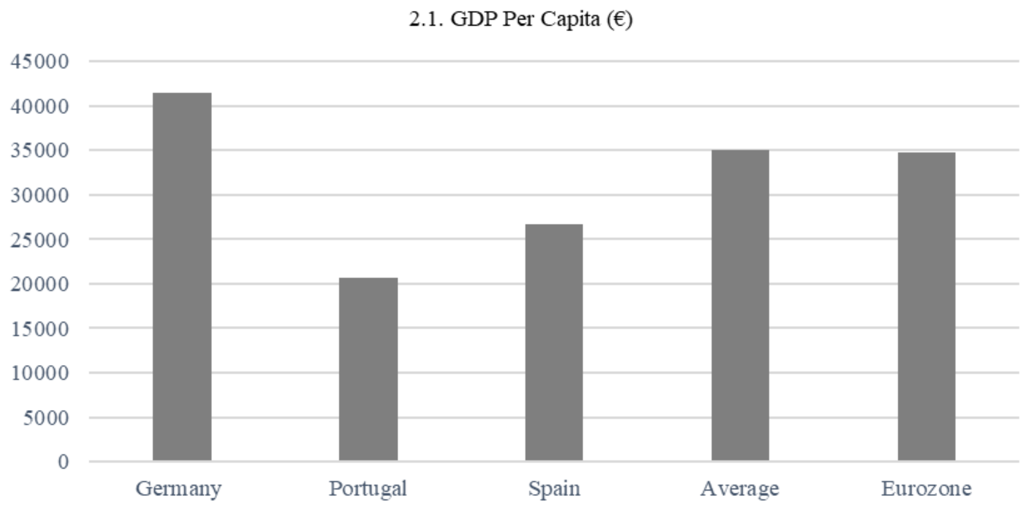


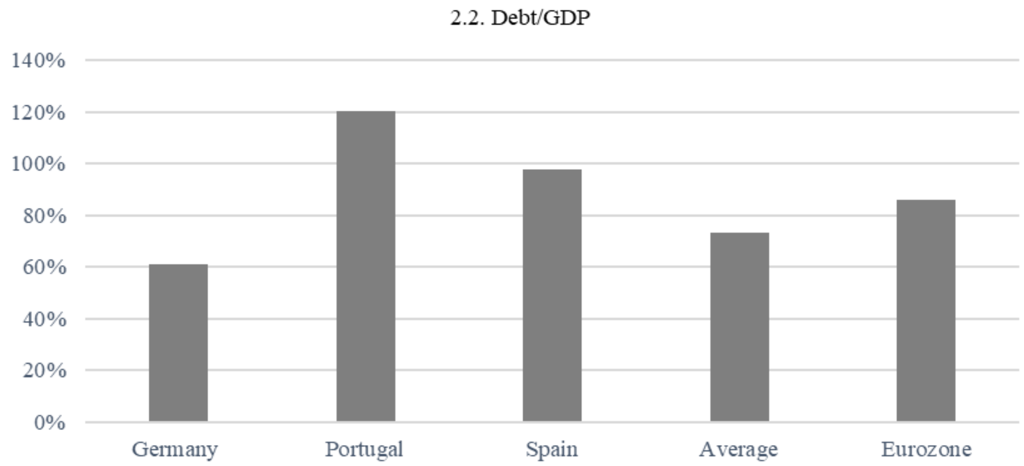
1.3. Historical Recessions and Yield Spread (10Y-3m) for Germany



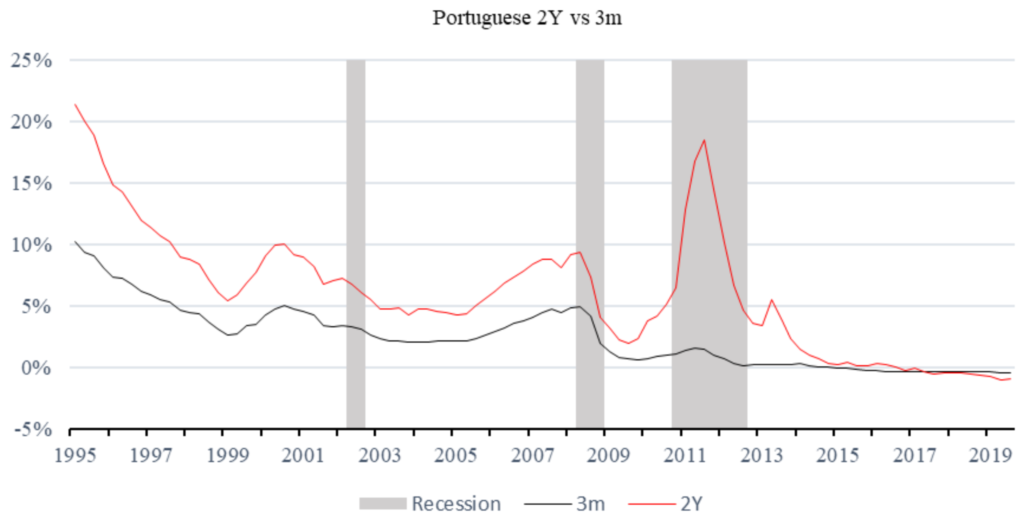


Graph 2: Gross Domestic Product *per capita* and Debt-to-GDP ratio in the Eurozone

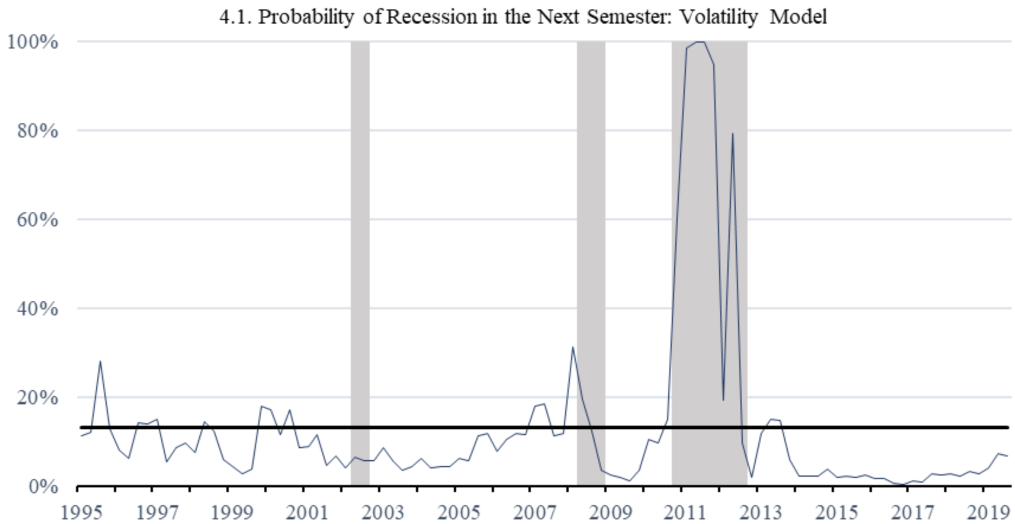


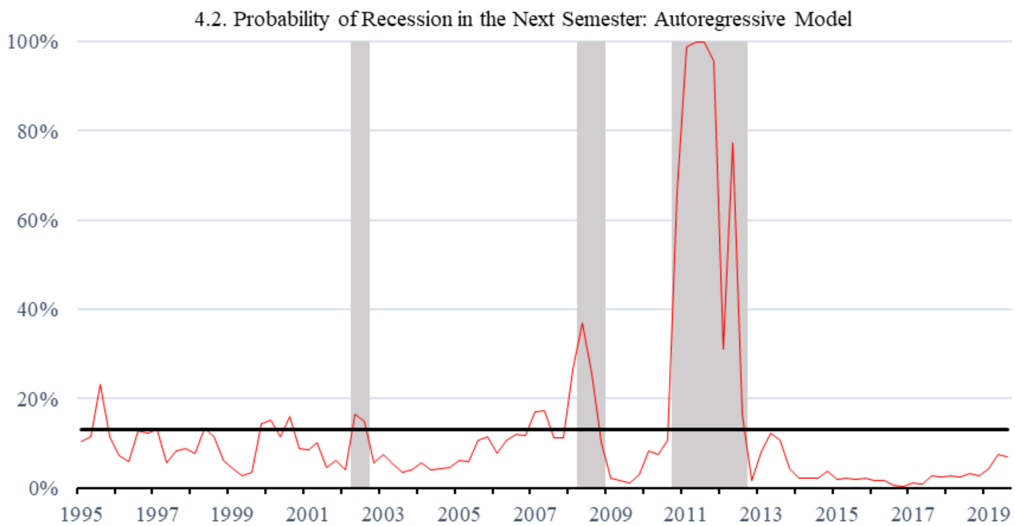


Graph 3: Portuguese 2-year vs 3-month rate

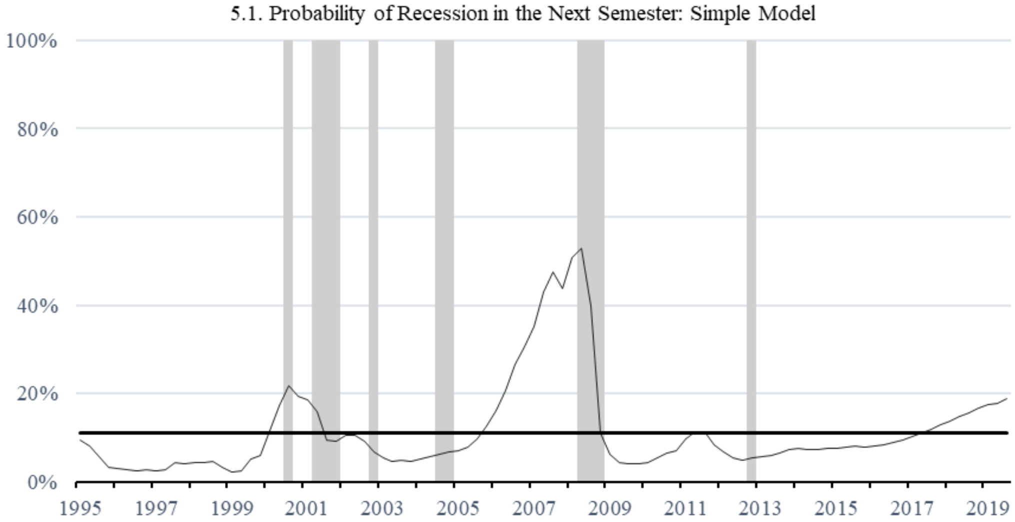


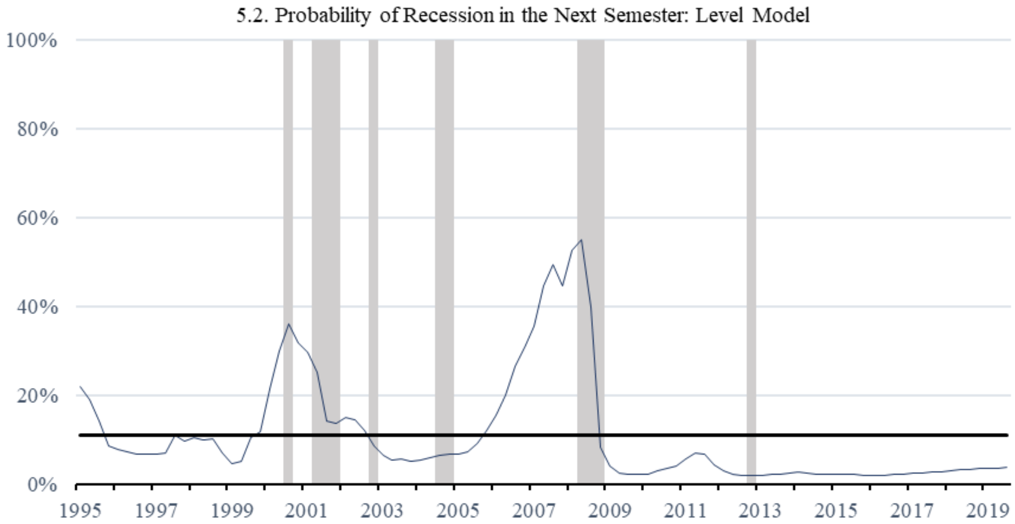
Graph 4: Probability of Recession in the Next Semester for Portugal

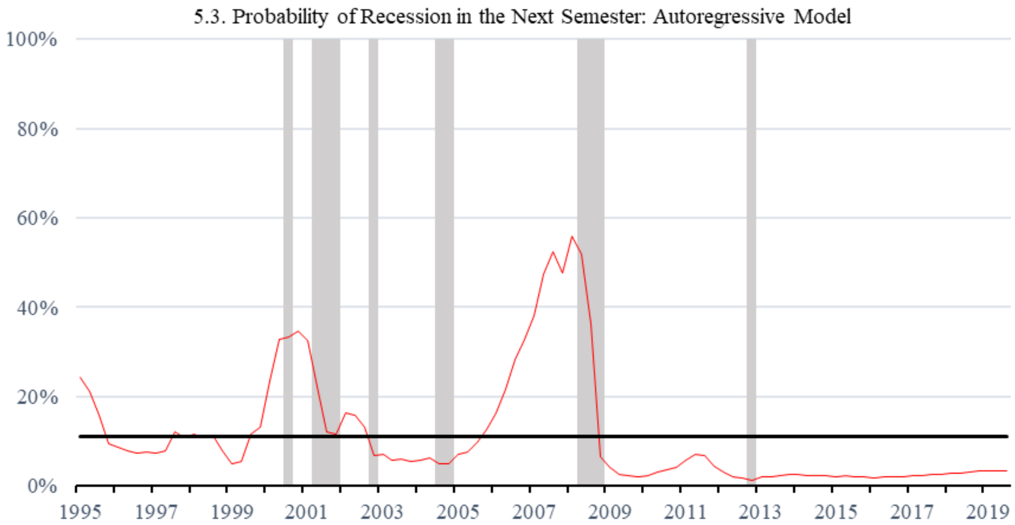




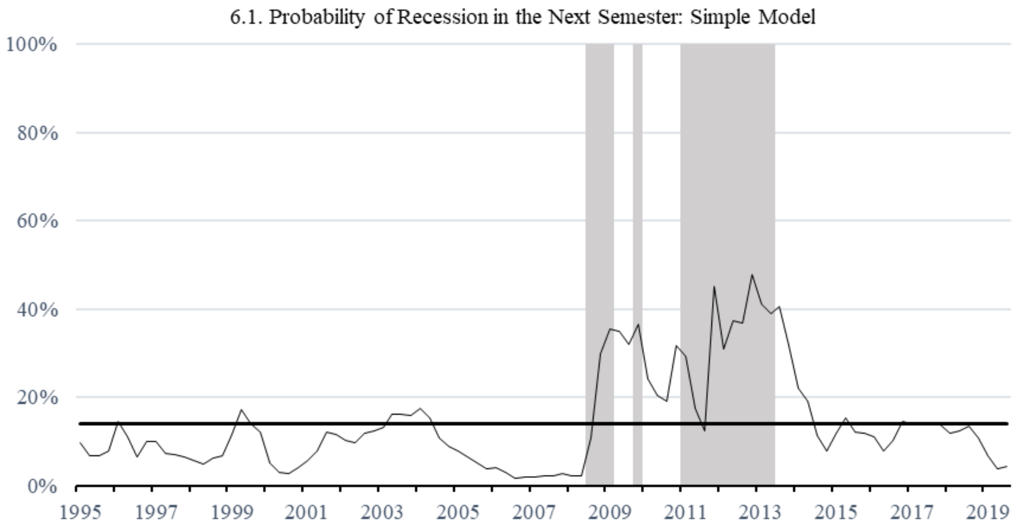
Graph 5: Probability of Recession in the Next Semester for Germany

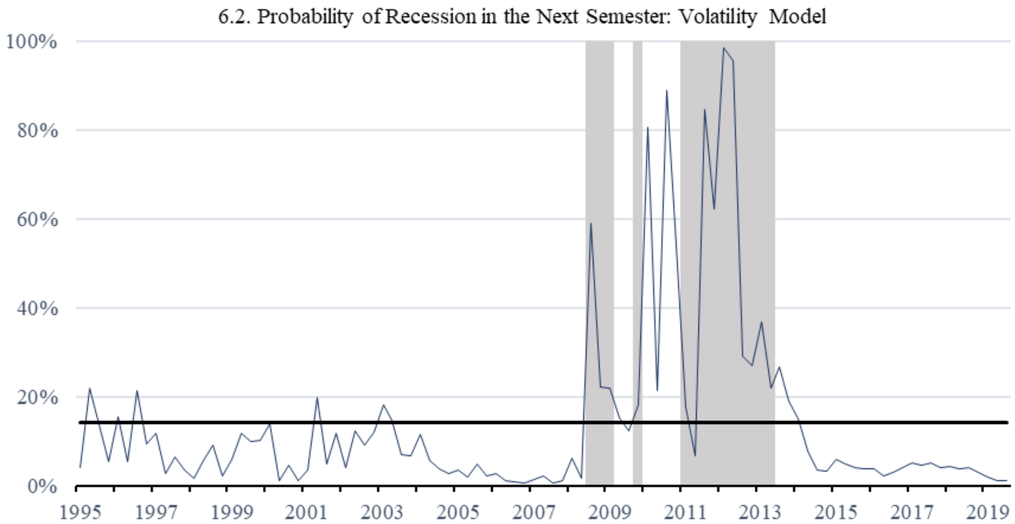






Graph 6: Probability of Recession in the Next Semester for Spain





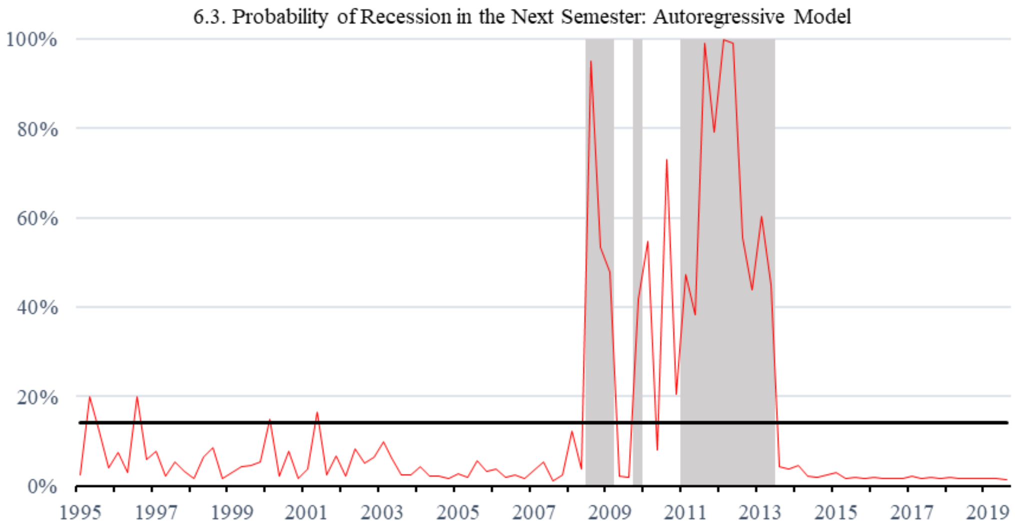


Table 1: Augmented Dickey-Fuller Test for Stationarity for the Portuguese Data

. dfuller YC_Slope, drift lags(3)

Augmented Dickey-Fuller test for unit root Number of obs = 102

	Test Statistic	Z(t) has t-distribution: 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-3.484	-2.365	-1.661	-1.290

p-value for Z(t) = 0.0004

. dfuller YC_Slope_3m, drift lags(1)

Augmented Dickey-Fuller test for unit root Number of obs = 104

	Test Statistic	Z(t) has t-distribution: 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-2.614	-2.364	-1.660	-1.290

p-value for Z(t) = 0.0052

. dfuller GDP_Growth1, drift lags(6)

Augmented Dickey-Fuller test for unit root Number of obs = 92

	Test Statistic	Z(t) has t-distribution: 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-2.150	-2.372	-1.663	-1.292

p-value for Z(t) = 0.0172

. dfuller GDP_Growth2, drift lags(10)

Augmented Dickey-Fuller test for unit root Number of obs = 87

	Test Statistic	Z(t) has t-distribution: 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-1.790	-2.377	-1.665	-1.293

p-value for Z(t) = 0.0388

. dfuller GDP_Growth4, drift lags(8)

Augmented Dickey-Fuller test for unit root Number of obs = 87

	Test Statistic	Z(t) has t-distribution: 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-1.806	-2.376	-1.665	-1.293

p-value for Z(t) = 0.0374

. dfuller GDP_Growth8, drift lags(8)

Augmented Dickey-Fuller test for unit root Number of obs = 83

	Test Statistic	Z(t) has t-distribution: 1% Critical Value	5% Critical Value	10% Critical Value
Z(t)	-1.740	-2.379	-1.666	-1.293

p-value for Z(t) = 0.0430

Table 2: Predicting Future Real GDP Growth using the Yield Spread for Portugal

$$y_{t:t+k} = \alpha + \beta Spread_t + \varepsilon_{t+k}$$

Portugal	10Y - 3m		10Y - 2Y	
	Constant	**Spread**	Constant	**Spread**
k=1	**0.028**	**-0.51**	0.008	0.453
	3.28	-2.79	0.85	1.8
k=2	**0.027**	**-0.482**	0.006	**0.591**
	3.13	-2.56	0.61	2.66
k=4	**0.025**	**-0.425**	0.005	**0.624**
	2.95	-2.32	0.54	2.85
k=8	**0.021**	-0.293	0.009	0.335
	2.56	-1.91	1.12	1.93

Notes: t statistics below OLS estimates, using Newey-West Robust Standard Errors for a maximum of 12 lags. **Bold** entries indicate significance at 5%.

Table 3: Predicting Future Real GDP Growth using the Yield Spread and Lagged Real GDP Growth for Portugal

$$y_{t:t+k} = \alpha + \beta_1 Spread_t + \beta_2 y_{t-1:t} + \beta_3 y_{t-2:t-1} + \beta_4 y_{t-3:t-2} + \beta_5 y_{t-4:t-3} + \varepsilon_{t+k}$$

Portugal		Constant	**Spread**	1st Lag	2nd Lag	3rd Lag	4th Lag
10Y - 3m	k=1	0.013	-0.231	0.208	0.235	0.094	-0.045
		1.58	-1.17	1.45	1.87	1.15	-0.46
	k=2	0.014	-0.238	**0.255**	**0.205**	0.042	-0.071
		1.84	-1.24	3.66	2.95	0.52	-0.95
	k=4	**0.015**	-0.24	**0.232**	0.122	-0.016	-0.015
		2.1	-1.22	3.63	1.84	-0.29	-0.24
	k=8	0.011	-0.115	**0.152**	0.092	0.034	0.02
		1.95	-0.71	3.14	1.53	0.62	0.35
10Y - 2Y	k=1	-0.002	**0.439**	0.207	0.264	**0.147**	0.026
		-0.53	3.44	1.62	1.93	2.26	0.31
	k=2	-0.003	**0.552**	**0.247**	**0.233**	0.099	0.001
		-0.89	4.06	3.82	3.67	1.58	0.15
	k=4	-0.003	**0.62**	**0.219**	**0.148**	0.043	0.07
		-0.72	4.53	3.37	2.2	0.98	1.64
	k=8	0.002	0.374	**0.14**	0.103	0.067	0.067
		0.5	1.97	2.14	1.96	1.6	1.42

Notes: t statistics below OLS estimates, using Newey-West Robust Standard Errors for a maximum of 12 lags. **Bold** entries indicate significance at 5%.

Table 4: Correlation Matrices using Lagged Quarterly and Lagged Annual GDP Real Growth to Explain Annual GDP Growth for Portugal

	y4	L. y1	L2. y1	L3. y1	L4. y1	YC_Slope
y4	1.0000					
y1						
L1.	0.4824	1.0000				
L2.	0.4000	0.4135	1.0000			
L3.	0.2939	0.4280	0.4136	1.0000		
L4.	0.2744	0.3486	0.4299	0.4125	1.0000	
YC_Slope	0.3603	0.1309	0.0303	-0.0458	-0.1097	1.0000

	y4	L. y4	L2. y4	L3. y4	L4. y4	YC_Slope
y4						
--.	1.0000					
L1.	0.9147	1.0000				
L2.	0.7956	0.9156	1.0000			
L3.	0.6414	0.7967	0.9155	1.0000		
L4.	0.4864	0.6432	0.7977	0.9159	1.0000	
YC_Slope	0.3603	0.3118	0.2219	0.1053	0.0017	1.0000

Table 5: Predicting Future Real GDP Growth using the Yield Spread for Germany and Spain

Germany

	10Y - 3m			**10Y - 2Y**	
	Constant	**Spread**		Constant	**Spread**
k=1	-0.012	1.018	k=1	-0.021	**1.524**
	-0.79	1.83		-1.56	2.68
k=2	-0.017	**1.189**	k=2	-0.022	**1.533**
	-1.16	2.25		-1.76	2.97
k=4	-0.018	**1.244**	k=4	-0.018	**1.378**
	-1.8	3.28		-1.67	3.07
k=8	**-0.012**	**0.989**	k=8	-0.007	**0.926**
	-3.55	7.54		-0.73	2.42

Spain

	10Y - 3m			**10Y - 1Y**	
	Constant	**Spread**		Constant	**Spread**
k=1	**0.035**	-0.76	k=1	**0.04**	**-1.137**
	2.97	-1.63		3.43	-2.05
k=2	**0.0344**	-0.71	k=2	**0.037**	-0.952
	2.78	-1.45		2.79	-1.5
k=4	**0.033**	-0.602	k=4	**0.034**	-0.732
	2.55	-1.2		2.37	-1.08
k=8	**0.029**	-0.408	k=8	**0.028**	-0.384
	2.31	-0.87		1.9	-0.54

Notes: t statistics below OLS estimates, using Newey-West Robust Standard Errors for a maximum of 12 lags. **Bold** entries indicate significance at 5%.

Table 6: Predicting Future Real GDP Growth using the Yield Spread and Lagged Real GDP

Germany

		Constant	**Spread**	1st Lag	2nd Lag	3rd Lag	4th Lag
10Y - 3m	k=1	-0.019	**1.093**	0.145	0.148	0.052	0.029
		-1.56	2.44	1.41	1.87	0.87	0.35
	k=2	-0.02	**1.19**	0.132	**0.103**	0.046	-0.02
		-1.78	2.74	1.92	2.09	1.15	-0.25
	k=4	**-0.018**	**1.201**	0.065	0.0308	-0.002	-0.028
		-2.07	3.5	1.41	0.63	-0.04	-0.57
	k=8	-0.008	**0.928**	-0.052	0.0344	-0.038	**-0.053**
		-1.94	7.91	-1.46	1.55	-1.61	-2.08
10Y - 2Y	k=1	-0.018	**1.295**	0.131	0.095	-0.0288	-0.0575
		-1.4	2.32	1.24	1.01	-0.46	-0.56
	k=2	-0.019	**1.429**	0.116	0.043	-0.042	-0.114
		-1.52	2.45	1.48	0.65	-0.86	-1.12
	k=4	-0.019	**1.526**	0.042	-0.032	-0.094	**-0.125**
		-1.69	2.97	0.74	-0.52	-1.73	-1.99
	k=8	-0.012	**1.341**	**0.079**	**-0.09**	**-0.11**	**-0.128**
		-1.97	5.63	2.07	-3.63	-5.16	-4.15

Spain

		Constant	**Spread**	1st Lag	2nd Lag	3rd Lag	4th Lag
10Y - 3m	k=1	-0.00304	0.232	**0.778**	-0.0727	**0.274**	-0.043
		-0.39	0.75	7.57	-0.41	2.95	-0.38
	k=2	-0.00293	0.271	**0.634**	0.0786	0.183	-0.004
		-0.3	0.69	6.24	0.51	1.77	-0.04
	k=4	-0.00128	0.281	**0.576**	0.0748	**0.15**	0.0001
		-0.11	0.62	4.39	0.7	2.12	0
	k=8	-0.00145	0.39	**0.427**	0.0914	0.112	0.066
		-0.12	0.84	3.12	1.11	1.92	0.5
10Y - 1Y	k=1	-0.013	0.721	**0.766**	-0.0429	**0.293**	0.0145
		-1.46	1.76	7.26	-0.27	3.43	0.12
	k=2	-0.0203	1.091	**0.611**	0.138	**0.216**	0.103
		-1.73	1.98	6.71	1.12	2.2	0.93
	k=4	-0.0216	**1.231**	**0.549**	0.144	**0.189**	0.127
		-1.8	2.15	4.65	1.91	3.02	1.04
	k=8	-0.0255	**1.529**	**0.394**	**0.175**	**0.151**	0.22
		-1.9	2.39	3.34	2.66	3.01	1.7

Notes: t statistics below OLS estimates, using Newey-West Robust Standard Errors for a maximum of 12 lags. **Bold** entries indicate significance at 5%.

Table 7: Factor Model Results

Germany

		Constant	**Spread**	Factor	R-Squared	DW
10Y - 3m	k=1	0.003	0.436	**0.014**	0.4	2.06
		0.39	1.4	5.05		
	k=2	-0.006	0.818	**0.01**	0.47	1.29
		-0.53	1.83	6.39		
	k=4	-0.0135	**1.113**	**0.006**	0.53	0.69
		-1.52	3.2	3.52		
	k=8	**-0.014**	**1.092**	0.001	0.52	0.29
		-4.3	6.73	0.11		
10Y - 2Y	k=1	-0.005	**0.815**	**0.014**	0.41	2.11
		-0.65	2.73	4.51		
	k=2	-0.009	**1.006**	**0.011**	0.46	1.35
		-1.08	2.76	6.69		
	k=4	-0.0012	**1.096**	**0.007**	0.43	0.7
		-1.16	2.68	3.83		
	k=8	-0.007	0.92	0.002	0.25	0.26
		-0.58	1.71	0.93		

Spain

		Constant	**Spread**	Factor	R-Squared	DW
10Y - 3m	k=1	0.014	0.229	**0.015**	0.47	1.19
		1.22	0.42	4.93		
	k=2	0.015	0.196	**0.013**	0.41	0.83
		1.25	0.37	5.41		
	k=4	0.014	0.241	**0.012**	0.36	0.54
		1.19	0.48	6.12		
	k=8	0.011	0.37	**0.01**	0.29	0.39
		1.07	0.94	8.74		
10Y - 1Y	k=1	0.005	0.752	**0.016**	0.49	1.38
		0.4	1.04	5.04		
	k=2	0.002	0.908	**0.016**	0.46	1.05
		0.18	1.24	5.41		
	k=4	0	1.044	**0.015**	0.43	0.81
		0	1.6	6.28		
	k=8	-0.005	**1.32**	**0.014**	0.41	0.73
		-0.48	2.5	8.75		

Notes: t statistics below OLS estimates, using Newey-West Robust Standard Errors for a maximum of 12 lags. **Bold** entries indicate significance at 5%.

Table 8: Simple Probit Model Performance for Portugal

$$Pr(R_{t+k} = 1) = \Phi(\alpha + \beta Spread_t)$$

Portugal			
	Constant	**Spread (10Y-2Y)**	pseudo R2
k=1	**-0.684**	**-35.87**	0.113
	-2.31	-2.81	
k=2	**-0.59**	**-43.98**	0.149
	-2.03	-3.12	
k=4	**-0.688**	**-33.57**	0.099
	-2.73	-2.14	

Notes: t statistics below Probit estimates, using Newey-West Robust Standard Errors for a maximum of 4 lags. **Bold** entries indicate significance at 5%.

Table 9: Probit Model Adding the Short-Term and the Short-Term Volatility for Portugal

$$Pr(R_{t+k} = 1) = \Phi(\alpha + \beta_1 Spread_t + \beta_2 2y_t),$$

$$Pr(R_{t+k} = 1) = \Phi(\alpha + \beta_1 Spread_t + \beta_2 STVol_t).$$

Portugal					
		Constant	**Spread (10Y-2Y)**	X	pseudo R2
X = 2Y	k=1	**-2.07**	-0.209	19.96	0.23
		-2.49	-0.01	1.74	
	k=2	**-1.69**	-13.96	15.81	0.22
		-2.4	-0.56	1.61	
	k=4	-0.967	-25.45	4.088	0.11
		-1.62	-0.93	0.49	
X = ST Vol.	k=1	**-1.638**	-48.68	**385.6**	0.47
		-3.47	-1.41	-2.47	
	k=2	**-1.188**	-44.69	**205.5**	0.33
		-2.5	-1.76	-2.65	
	k=4	**-1.195**	-23.71	123.8	0.18
		-3	-1.29	-1.72	

Notes: t statistics below Probit estimates, using Newey-West Robust Standard Errors for a maximum of 4 lags. **Bold** entries indicate significance at 5%.

Table 10: Probit Model Adding the Lagged Recession for Portugal

$$Pr(R_{t+k} = 1) = \Phi(\alpha + \beta_1 Spread_t + \beta_2 STVol_t + \beta_2 R_t).$$

Portugal

	Constant	**Spread (10Y-2Y)**	ST Vol.	Lag	pseudo R2
k=1	**-1.647**	-65.85	**322.9**	**1.959**	0.614
	-3.25	-1.73	2.06	3.13	
k=2	**-1.155**	-45.07	**166.3**	0.583	0.345
	-3.06	-1.79	2.21	0.96	
k=4	**-1.252**	-30.65	**235.5**	-1.597	0.22
	-3.7	-1.82	2.31	-1.28	

Notes: t statistics below Probit estimates, using Newey-West Robust Standard Errors for a maximum of 4 lags. **Bold** entries indicate significance at 5%.

Table 11: Simple Probit Model Performance for Germany and Spain

Germany			
	Constant	**Spread (10Y-3m)**	pseudo R2
k=1	-0.276	**-40.88**	0.11
	-0.58	-2.25	
k=2	-0.197	**-44.19**	0.12
	-0.47	-2.72	
k=4	-0.279	**-39.35**	0.1
	-0.68	-2.48	

Spain			
	Constant	**Spread (10Y-1Y)**	pseudo R2
k=1	**-2.884**	**90.74**	0.25
	-2.89	2.17	
k=2	**-2.177**	58.35	0.12
	-2.27	1.34	
k=4	**-1.729**	36.66	0.05
	-2.21	0.97	

Notes: t statistics below Probit estimates, using Newey-West Robust Standard Errors for a maximum of 4 lags. **Bold** entries indicate significance at 5%.

Table 12: Probit Model Adding the Short-Term Rate and the Short-Term Volatility

Germany					
		Constant	**Spread (10Y-3m)**	X	pseudo R2
X = 3m	k=1	**-1.239**	-24.4	**22.79**	0.16
		-2.31	-1.45	2.21	
	k=2	**-1.282**	-25.7	**25.15**	0.19
		-2.49	-1.75	2.23	
	k=4	**-1.529**	-19.32	**28.72**	0.17
		-2.71	-1.28	2.43	
X = ST Vol.	k=1	-0.295	**-40.85**	48.96	0.11
		-0.62	-2.25	0.13	
	k=2	-0.174	**-44.22**	-57.72	0.12
		-0.38	-2.72	-0.12	
	k=4	-0.336	**-39.26**	135.3	0.1
		-0.84	-2.51	0.32	

		Spain			
		Constant	**Spread (10Y-1Y)**	X	pseudo R2
X = 1Y	k=1	**-3.402**	**102.3**	10.72	0.27
		-2.85	2.23	1.41	
	k=2	**-2.66**	68.85	10.14	0.15
		-2.34	1.46	1.3	
	k=4	**-2.05**	43.57	6.849	0.06
		-2.16	1.05	0.92	
X = ST Vol.	k=1	**-3.272**	**87.12**	**253.2**	0.35
		-3.24	2.09	2.47	
	k=2	**-2.797**	54.14	**384.9**	0.33
		-2.88	1.21	4.76	
	k=4	**-2.163**	22.77	**386**	0.27
		-2.34	0.57	4.36	

Notes: t statistics below Probit estimates, using Newey-West Robust Standard Errors for a maximum of 4 lags. **Bold** entries indicate significance at 5%.

Table 13: Probit Model Adding the Lagged recession

	Germany				
	Constant	**Spread (10Y-3m)**	3m	Lag	pseudo R2
k=1	**-1.474**	-20.96	20.66	**1.124**	0.25
	-1.99	-1.12	1.48	2.44	
k=2	-1.318	-25.43	**27.57**	-0.161	0.19
	-1.85	-1.45	2	-0.29	
k=4	**-1.589**	-18.63	**31.24**	-0.0874	0.18
	-2.09	-1.08	2.09	-0.16	

	Spain				
	Constant	**Spread (10Y-1Y)**	ST Vol.	Lag	pseudo R2
k=1	**-2.453**	21.98	200.6	**2.171**	0.54
	-4.01	0.73	1.63	3.55	
k=2	**-2.273**	5.377	**379.7**	**1.713**	0.45
	-4.21	0.2	2.88	2.77	
k=4	**-1.82**	-9.268	**362.5**	**1.176**	0.33
	-3.99	-0.36	3.24	2.06	

Notes: t statistics below Probit estimates, using Newey-West Robust Standard Errors for a maximum of 4 lags. **Bold** entries indicate significance at 5%.

Technical Appendix

Appendix 1: Probabilistic (Probit) regression

Let

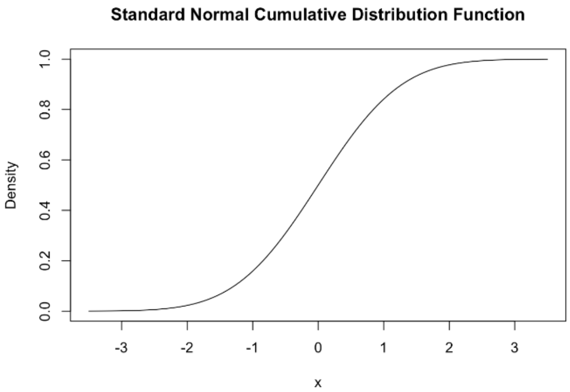
$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k \qquad (10.1)$$

Where X_n , n = 1,..., k are the explanatory variables and β_i, i = 1,…, k are parameters to be estimated. The probit model estimates the probability of $Y = 1$, i.e.,

$$P\ (Y = 1 | X_1, X_2, \dots, X_k) = \Phi(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k) \qquad (10.2)$$

Where $\Phi(.)$ is the standard normal cumulative distribution function (thus ensuring a probability between 0 and 1), i.e.:

$$\Phi(\ t\ ) = \frac{1}{\sqrt{2}\pi} \int_{-\infty}^{x} e^{-\frac{t^2}{2}}\, dt \qquad (10.3)$$



Source: Econometrics with R

The method to estimate the logit model is Maximum Likelihood Estimation (MLE). The Likelihood (L) function is the probability of observing the sample:

$$L = \prod_{i=1}^{N} (p_i)^{y_i} (1 - p_i)^{1-y_i} \qquad (10.4)$$

Where $p_i$is the function in (1.1), and $y_i = 1\ or\ 0$, depending on the chosen criteria. The coefficients β_i are the numbers that maximize the log likelihood function:

$$\ln(L) = \sum_{i=1}^{N} y_i ln(p_i) + (1 - y_i) ln(1 - p_i) \qquad (10.5)$$

Appendix 2: Principle Component Analysis

Let x_{ij} be the set of *T* independent variables, with *n* observations, composing matrix *X* and Z be the matrix containing the *n* observations of the *T* standardized independent variables:

$$Z = \begin{bmatrix} z_{11} & \cdots & z_{1T} \\ \vdots & \ddots & \vdots \\ z_{n1} & \cdots & z_{nT} \end{bmatrix} \tag{11.1}$$

Where $z_{ij} = \frac{x_{ij} - \overline{X}_j}{\sigma_j}$, $i = 1,\ldots,n \; and \; j = 1, \ldots, T$.

Λ will be the Variance-Covariance Matrix of Z:

$$\Lambda = \mathrm{Z^T Z} \tag{11.2}$$

Let *A* be a square matrix. Let *v* be a vector and λ a scalar that satisfies $Av = \lambda v$, then λ is called eigenvalue associated with eigenvector *v* of *A*. The eigen decomposition of Z^TZ is thus:

$$\Lambda = \mathrm{PDP^{-1}} = \sum_{i=1}^{T} \lambda_i \, v_i {v_i}^{\mathrm{T}} \tag{11.3}$$

$$v_i v_j = \delta_{ij} \; for \; all \; i,j \; (orthonormality)$$

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_T > 0$$

Where *P* is the matrix of eigenvectors and *D* is the diagonal matrix with eigenvalues on the diagonal and values of zero everywhere else. The eigenvectors v_i are also known as the principal components. The sorted eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_T$ will be associated with correspondent columns of the eigenvectors in *P*. The sorted matrix of eigenvectors *P* will be called *P**. Finally, *Z** will be the centered/standardized version of *X* but now each observation is a combination of the original variables, where the weights are determined by the eigenvector:

$$Z^* = ZP^* \quad (11.4)$$

Where $Var(Z_j{}^*) = \lambda_j, j = 1, \ldots, T$, and where the proportion of variance explained by the *jth* principal component(s) is given by: $\frac{\lambda_j}{\lambda_1 + \lambda_2 + \ldots + \lambda_T}$.