Decoupled models for vehicle dynamics and estimation of coupling terms

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Sliding Modes Observers for Vehicle Dynamics and Variable Structure Automatic Systems (SMO-VSAS)

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Abstract—This paper will show how to handle modeling of vehicles to get efficient and good Sliding Mode Observers. A car model with 16 DoF is decomposed for partial state observation with SMO. This decomposition method may lead, in VSAS, to good estimates of different kind variables and inputs (subsystems states, environment and ground variables, interfaces variables, connections and constraint variables). Robust Sliding Mode Observers and unknown input estimations are developed.

I. INTRODUCTION

[1] In literature, many studies deal with vehicle modeling [1][2][3]. Vehicle dynamics can be represented by approximate models which are either too much simplified to be realistic or complex and too much wide involving big simulation softwares. In nature they have a variable structure (VSAS) but in literature, their properties are never detailed nor their passivity property emphasized. This kind of systems, like in general VSAS, are composed with many passively coupled subsystems: wheels, motor and braking control system, suspensions, steering, more and more inboard and embedded electronics. There are several non linear parts in VSAS, which are coupled. These coupling may be time varying and non stationary. Approximations have to be made carefully regarding to the desired application, see eg [4].

In our previous works a good nominal vehicle model with 16 DOF have been validated in a simulator, we developed for a French car type (Peugeot 406), [2]. Several interesting applications was successful and have been evaluated by use of this simulator before actual results [5]. We have also considered this modeling for estimation of unknown inputs [6], interaction parameters and exchanges of VSAS with environment [7]. This approach has been used successfully also for heavy vehicles [8].

In this paper the car model is revisited as VSAS and structured for estimation of inputs and diagnosis. We split the model in five subsystems [9] and then show and justify the rationale behind the successful splitting to get simple and efficient partial (sub) models. The subsystems and the overall system obey the passivity property. This feature, like in Bond Graphs modeling emphasize the energy flow in VSAS and exchanges between the system parts and also with the environment. After the structure and model analysis, we consider estimation of the partial states for diagnosis and motion control in the vehicle.

Robust estimations are necessary to be able to obtain good evaluation of the VSAS driving situation at each time instant.

II. VEHICLE MODEL PARAMETRIZATION

A. The Nominal global model

The Nominal model of a car vehicle with uncertainties has been developed assuming the car body rigid and pneumatic contact permanent and reduced to one point for each wheel (see eg [7] [9] [10]). It is composed by 4 non linear equations: one for the mechanical dynamics of the vehicle (equation (1)), one for interface torques and forces (vehicle inputs equation (2)), one for the contact with environment (equation (3)) and the last one for the environment normal reactions (equation(4)). τ is the input vector composed by torques and forces applied to the vehicle.

$$\tau = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + V(q,\dot{q}) + G(q) + \eta_o(t,q,\dot{q}) \quad (1)$$

$$\tau = \Gamma_e + \xi J(x_1)^T F \quad (2)$$

$$\dot{F} = f(\alpha, \lambda, q, F_N) + e(t) \quad (3)$$

$$F_N = h(l_f, l_r, h, g, \dot{v}_x, \dot{v}_y, q, x_{road}, \beta, \gamma) \quad (4)$$

The generalized coordinates vector $q \in R^{16}$ is defined as

$$q^T = [x, y, z, \theta_z, \theta_y, \theta_x, q_{31}, q_{32}, q_{33}, q_{34}, \delta_3, \delta_4, \varphi_1, \varphi_2, \varphi_3, \varphi_4]$$

where x, y, and z represent displacements. Angles of roll, pitch and yaw are θ_x, θ_y et θ_z respectively. The suspensions elongations are noted q_{3i}: ($i = 1..4$). δ_i: stands for the steering angles. φ_i: are angles of wheels rotations ($i = 1..4$.). $\dot{q}, \ddot{q} \in R^{16}$ are respectively velocities and corresponding accelerations. The state vector is $x^T = (x_1^T, x_2^T) = (q^T, \dot{q}^T)$. The random part in forces $e(t)$ is due to neglected and uncertain dynamics in wheels contact. the variable ξ is equal to unity when the corresponding wheel is in contact with the ground and zero if not.

Equation (1) describes the dynamics of the vehicle excited by the external forces of equation (2) which stands as interface with environment. $\xi J(x_1)^T F$ represents the environment reactions and control inputs are in $U = M(x_1)^{-1}\Gamma_e$. The $\eta_o(t, q, \dot{q})$ represent external perturbations, uncertainties and neglected dynamics.

The gravity term is $G(q)$. $V(q, \dot{q}) = \xi(K_v\dot{q} + K_p q)$ are Suspensions forces (with respectively damping and stiffness matrices K_v, K_p); We can have also in $V(q, \dot{q})$ Coulomb friction which contain relay terms depending on velocities and positions (non linear frictions).

Equation (4) gives the environment reaction forces to vehicle posture. The latter forces produce the wheel-ground contact forces (3).

B. Mechanical Model Properties

Then we have the state space representation of our VSAS (see [9])

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x_1, x_2) + M^{-1}\xi J^T F + U - \eta(x_1, x_2)$$
$$y = h(x)$$
$$\tau = \Gamma_e + \xi J(x_1)^T F, \qquad U = M^{-1}\Gamma_e \qquad (5)$$
$$\dot{F} = f(\alpha, \lambda, q, F_N) + e(t)$$
$$F_N = h(l_f, l_r, h, g, \dot{v}_x, \dot{v}_y, q, x_{road}, \beta, \gamma)$$

$f(x_1, x_2) = -M(x_1)^{-1}(C(x_1, x_2)x_2 + V(x_1, x_2) + G(q))$, is the nominal dynamics assuming normal driving situation. $\eta(x_1, x_2) = M(x_1)^{-1}\eta_o(t, x_1, x_2)$ are the model uncertainties, input perturbations and neglected dynamics in VSAS.

The mechanical model part (1) is passive and has several interesting properties, which are well known and extensively used in robotics. We note that the system (1-4) can be cast in the Equivalent Passive Feedback Scheme (EPFS) shown in figure (1)
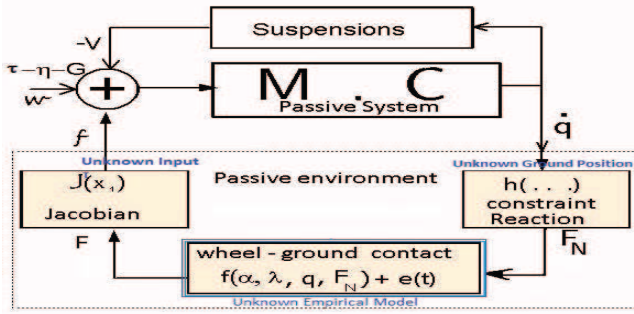

Figure 1.   Vehicle dynamics in a Passive Feedback Equivalent Scheme.

Properties: *Matrices M and C are such as (see [11][2][12]):*

1) *The inertia matrix $M(q)$, of dimensions 16×16, is Symmetric Positive Definite(SPD).*
2) *$N = \dot{M}(q) - 2C(q, \dot{q})$ is skew symmetric, i.e $\forall v \in R^n$, $v^T N v = 0$ or $N = -N^T$.*
3) *$C(q, \dot{q})\epsilon = C(q, v)v - \Pi(q, v)\epsilon$, with $\epsilon = v - \dot{q}$ and $\Pi(q, v) = \frac{\partial}{\partial x_2}\{C(q, \dot{q})\dot{q}\}_{\dot{q}=v}$.*
4) *The input torque $w = \tau - \eta_0 - G(q)$ is composed by two parts one from actuators (driver) and a second from environment reaction and perturbations.*
5) *All the perturbation terms η_i (coupling, neglected dynamics), are bounded: $\exists k_i > 0, i = 1, ..5$, such and as $|\eta_i| < k_i \; \forall t \in R$*

C. Coupled sub models

The VSAS model (1) is then split in 5 equations corresponding respectively to chassis translations, chassis rotations, Suspensions elongations, wheel steering and wheel rotations, with as positions q_1, q_2, q_3, q_4 and q_5. The fastest motions

are wheel rotations and the slowest ones are the chassis translations and rotations with wheels steering.

Owing to the structure of his matrices $M(x_1)$ and $C(x_1, x_2)$, we can split the model (1) in five parts as follows (see for details [13], [4]) with $q^T = \left[q_1^T, q_2^T, q_3^T, q_4^T, q_5^T\right]$.

$$q_1^T = [x, y, z] \quad q_2^T = [\theta_z, \theta_y, \theta_x] \quad q_4^T = [\delta_3, \delta_4]$$
$$q_3^T = [q_{31}, q_{32}, q_{33}, q_{34}] \qquad q_5^T = [\varphi_1, \varphi_2, \varphi_3, \varphi_4] \quad (6)$$

The 16 Degrees of Freedom model (eq(1)) is rewritten:

$$
\begin{bmatrix} F_T \\ F_R \\ F_S \\ U_4 \\ U_5 \end{bmatrix} =
\begin{bmatrix}
\bar{M}_{1,1} & \bar{M}_{1,2} & \bar{M}_{1,3} & 0 & 0 \\
\bar{M}_{2,1} & \bar{M}_{2,2} & \bar{M}_{2,3} & \bar{M}_{2,4} & \bar{M}_{2,5} \\
\bar{M}_{3,1} & \bar{M}_{3,2} & \bar{M}_{3,3} & 0 & 0 \\
0 & \bar{M}_{4,2} & 0 & \bar{M}_{4,4} & 0 \\
0 & \bar{M}_{5,2} & 0 & 0 & \bar{M}_{5,5}
\end{bmatrix}
\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \end{bmatrix} +
$$

$$
+ \begin{bmatrix}
0 & \bar{C}_{12} & \bar{C}_{13} & 0 & 0 \\
0 & \bar{C}_{22} & \bar{C}_{23} & \bar{C}_{24} & \bar{C}_{25} \\
0 & \bar{C}_{32} & \bar{C}_{33} & 0 & 0 \\
0 & \bar{C}_{42} & 0 & 0 & \bar{C}_{45} \\
0 & \bar{C}_{52} & 0 & \bar{C}_{54} & 0
\end{bmatrix}
\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} +
\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{bmatrix} +
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}
\tag{7}
$$

F_T, F_R and F_S are forces or torques corresponding to translation, rotations of the chassis and excitation of suspensions respectively. They come from $J(x_1)^T F$. U_4 is the steering control and U_5 stands for brake and motor torques. The terms V_i contain the remaining part of equation (1) accounting for frictions, suspensions stiffness, damping and gravity.

1) Dynamics of the chassis Σ_1: From the global system we keep the two first equations (7) for translations and rotations.

$$F_1 = M_1 \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + C_1 \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \nu_1$$

$$F_1 = \begin{bmatrix} F_T \\ F_R \end{bmatrix}; \quad \nu_1 = \begin{bmatrix} \eta_c^1 \\ \eta_c^2 \end{bmatrix}; \quad J_1^T = \begin{bmatrix} J_{1,1} & J_{1,2} \\ J_{2,1} & J_{2,2} \end{bmatrix}$$

$$\eta_1^c = \bar{M}_{1,3}\ddot{q}_3 + \bar{C}_{13}\dot{q}_3 + \eta_1 \tag{8}$$
$$\eta_2^c = \bar{M}_{2,3}\ddot{q}_3 + \bar{M}_{2,4}\ddot{q}_4 + \bar{M}_{2,5}\ddot{q}_5 + \bar{C}_{23}\dot{q}_3 +$$
$$+ \bar{C}_{24}\dot{q}_4 + \bar{C}_{25}\dot{q}_5 + \eta_2 \tag{9}$$

- $M_1 = \begin{bmatrix} \bar{M}_{1,1} & \bar{M}_{1,2} \\ \bar{M}_{2,1} & \bar{M}_{2,2} \end{bmatrix}$ is SPD , $C_1 = \begin{bmatrix} 0 & \bar{C}_{12} \\ 0 & \bar{C}_{22} \end{bmatrix}$ (composed by the first elements of $C(q, \dot{q})$) is such that $N_1 = \dot{M}_1 - 2C_1$ is a skew symmetric matrix (traducing the passivity property of the subsystem).
- $J_1^T = J^T(x_{11}) \in R^{(6 \times 12)}$ is the Jacobian matrix reduced to the system Σ_1 and F_1 the corresponding forces vector.
- ν_1 represents the coupling terms du to dynamics of the other subsystems. These coupling terms affect a passive (sub) system. This is the key property of the proposed parametrization and the system decomposition in simpler subsystems. It highlights the interest of coupling terms coming from passive subsystems.

By using $x_{11} = (q_1, q_2)$ and $x_{12} = (\dot{q}_1, \dot{q}_2)$, an equivalent state space representation can be written:

$$\dot{x}_{11} = x_{12}$$
$$\dot{x}_{12} = M_1^{-1}(J_1^T F_1 - C_1 x_{12} - V_{12}(x_{11}, x_{12}) - \nu_1) \tag{10}$$
$$y_1 = h(x_{11}, x_{12})$$

2) Suspensions Dynamics Σ_2: From the global equations we take the third one (equ 11).

$$F_S = \bar{M}_{33}\ddot{q}_3 + \bar{C}_{33}\dot{q}_3 + V_3(q,\dot{q}) + \eta_c^3 \qquad (11)$$

$$\eta_c^3 = \bar{M}_{31}\ddot{q}_1 + \bar{M}_{32}\ddot{q}_2 + \bar{C}_{32}\dot{q}_2 + \eta_3 \qquad (12)$$

Let $x_2 = (x_{21}, x_{22}) = (q_3, \dot{q}_3)$, the state space representation of (Σ_2) is then:

$$\dot{x}_{21} = x_{22}$$
$$\dot{x}_{22} = \bar{M}_{3,3}^{-1}(J_2^T F_2 - \bar{C}_{33}x_{22} - V_3(x_{21}, x_{22}) - \nu_2) \qquad (13)$$
$$y_2 = h(x_2)$$

with $\nu_2 = [\eta_c^3 - J_2^T F_2)]$ and $F_S = J_2^T F_2$

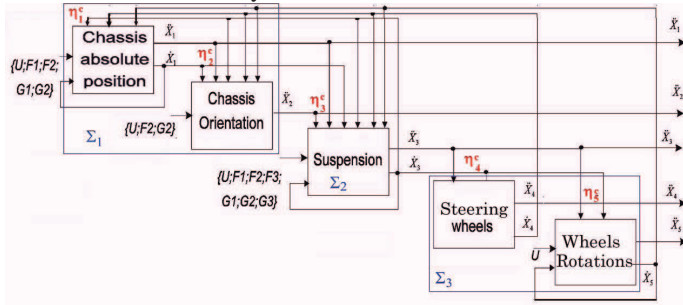This subsystem can also be shown to be passive ($N_2 = \dot{M}_2 - 2C_2$ is a skew symmetric matrix).

Figure 2.   Five Passive sub Models for Robust Sliding Mode Observers

3) Wheels dynamics Σ_3: The fastest dynamic in the vehicle model is the one of equation (7) is for wheels steering and rotations. The fourth equation of the model (1) stands for the steering front wheels with as inputs U_4. The motor / brake torque U_5 is applied to the two rear wheels.

$$\begin{bmatrix} U_4 \\ U_5 \end{bmatrix} = \begin{bmatrix} \bar{M}_{4,4} & 0 \\ 0 & \bar{M}_{5,5} \end{bmatrix} \begin{bmatrix} \ddot{q}_4 \\ \ddot{q}_5 \end{bmatrix} + \begin{bmatrix} 0 & \bar{C}_{4,5} \\ \bar{C}_{5,4} & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_4 \\ \dot{q}_5 \end{bmatrix}$$
$$+ \begin{bmatrix} V_4(\dot{q}_4, \ddot{q}_4) \\ V_5(\dot{q}_5, \ddot{q}_5) \end{bmatrix} + \begin{bmatrix} \eta_c^4 \\ \eta_c^5 \end{bmatrix} \qquad (14)$$
$$q_4^T = [\delta_3, \delta_4]; \quad and \quad q_5^T = [\varphi_1, \varphi_2, \varphi_3, \varphi_4]$$

$$\eta_c^4 = \bar{M}_{42}\ddot{q}_2 + \bar{C}_{42}\dot{q}_2 + \eta_4 \qquad (15)$$

$$\eta_c^5 = \bar{M}_{52}\ddot{q}_2 + \bar{C}_{52}\dot{q}_2 + \eta_5 \qquad (16)$$

By choosing $x_{31} = (q_4^T, q_5^T)^T$ and $x_{32} = \dot{x}_{31}$, the equivalent state space representation can be written:

$$\dot{x}_{31} = x_{32}$$
$$\dot{x}_{32} = \bar{M}_3^{-1}(U_{45} - C_3 x_{32} - V_{45}(x_{31}, x_{32}) - \nu_3) \qquad (17)$$
$$y_3 = h(x_{31}, x_{32})$$

with $\nu_3 = \begin{bmatrix} \eta_c^4 \\ \eta_c^5 \end{bmatrix}$ and $U_{45} = \begin{bmatrix} U_4 \\ U_5 \end{bmatrix}$

Then in conclusion, the vehicle can be presented as subsystems Σ_1, Σ_2 and Σ_3 corresponding respectively to chassis translations and rotations, Suspensions elongations, wheel steering and rotations (see figure 2). It is worthwhile to note that until now there are no approximations when considering the 5 equations. All the subsystems can be shown to fulfill the previously presented Passive Feedback Equivalent Scheme (PEFS). Approximations will be made when neglecting the coupling terms η_c^i. In the previous expressions, we remark

that splitting the model is helpful, when using reduced models, to identify what is neglected regard to our proposed nominal model with 16 DoF. This is emphasized by the PEFS which show us that the main system properties are preserved and what is neglected do not change stability properties. η_c^i are coupling terms du to connections with the other PEFS sub systems. We can verify that these terms are bounded such and as $|\eta_c^i| < k_i \ \forall t$.

4) Evaluation of the coupling terms in simulation: To evaluate the level of the coupling terms some simulation have been realized. We use in simulations, sinusoidal steering (with a period near to 5s) and a velocity near to 5m/s during 6 seconds and then decreasing (see figure 3).
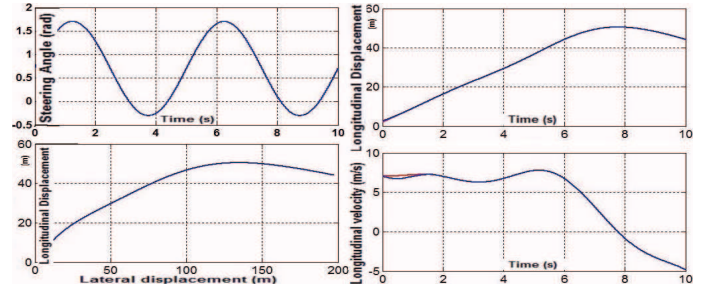

Figure 3.   Vehicle behavior for a sinusoidal steering

Figure (4) show the 3 components of the coupling terms η_c^1 left column and η_c^2 on the right column. We can remark that they are very small and negligible compared to the inputs F_t and F_r, drawn bellow in figure (5).
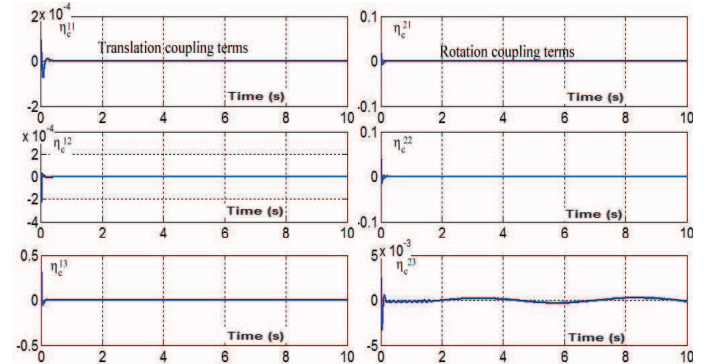

Figure 4.   The components of the coupling terms η_c^1 left and η_c^2 right

The inputs F_t and F_r (figure 5) are located at the same place as the coupling terms, which are then matched perturbations. Figures (6) show the 4 components of the coupling terms η_c^3
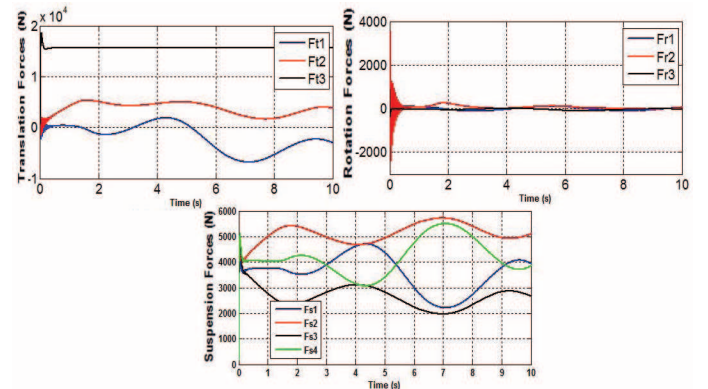

Figure 5.   The inputs F_t and F_r of the 2 first blocks for sinusoidal steering

appearing in the suspension block as perturbations. We remark also that they are very small and negligible compared to the inputs $F_{s_{1..4}} = J_2^T F_2$, drawn in the bottom of figure (5).
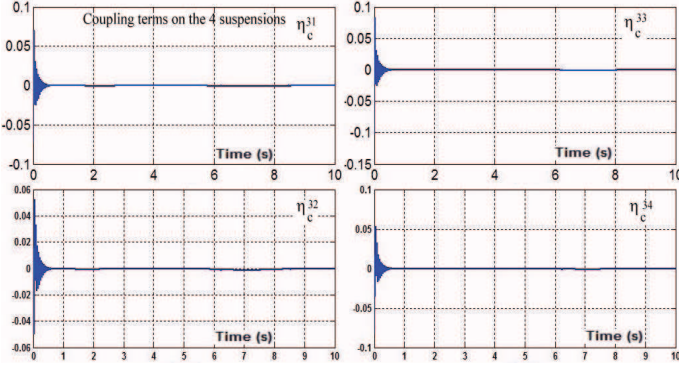

Figure 6. The VSAS suspension Coupling terms for sinusoidal steering

III. OBSERVERS DESIGN

The Sliding Mode Observers (SMO) technique is an attractive approach for its robustness and second for its finite time convergence feature. The latter is interesting to avoid use of costy sensors in complex systems. As we show in this work, to be able to estimate the unknown input forces and then adherence and road characteristics, we can use several steps of observations and estimations. Using partial state observers to get good and robust estimation of the vehicle state, using the previously presented blocks and splitting, we can consider estimation of the remaining variables in the process dynamics. This is done by filtering and estimating what have been considered as perturbations in the previous steps. This allowed us to avoid observability problems by using robust and cheap SMOs instead of sensors in a procedural estimation approach. In what follows, for the proposed observers, we consider the input forces unknown but slowly time varying. The assumption $\dot{F} \approx 0$ means that the changes in the forces are small in the mean as for example only one change with significant amplitude but during an intervall which is not too short. Adding a linearized model arround some operating point can enhance the estimations quality.

A. First Order Sliding Mode Observers

1) Observer for the chassis Dynamics Σ_1:: Estimations of the nominal functions $\hat{M}(q)$, $\hat{C}(q,\dot{q})$, $\hat{V}(q,\dot{q})$, $\hat{G}(q)$ are assumed known. If not, some intermediate values can be considered to develop the following proposed observers. Let us note the states estimation errors $\tilde{x}_{ij} = \hat{x}_{ij} - x_{ij}$

The proposed observer for the chassis dynamics is:

$$\dot{\hat{x}}_{11} = \hat{x}_{12} - \Lambda_{11} sign(\tilde{x}_{11}) \tag{18}$$

$$\dot{\hat{x}}_{12} = \hat{M}_1^{-1}(J_1^T \hat{F}_1 - \hat{C}_1(\hat{x}_{11}, \hat{x}_{12})\hat{x}_{12} - \hat{V}_{12}) - \Lambda_{12} sign(\tilde{x}_{11})$$

$$\dot{\hat{F}}_1 = -P\Lambda_{13} sign(\tilde{x}_{11}) \tag{19}$$

The observation error dynamics is (see eq10 and 18):

$$\dot{\tilde{x}}_{11} = \tilde{x}_{12} - \Lambda_{11} sign(\tilde{x}_{11}) \tag{20}$$

$$\dot{\tilde{x}}_{12} = \zeta_1 - M_1(x_{11})^{-1} J_1^T(x_{11})\tilde{F}_1 - \Lambda_{12} sign(\tilde{x}_{11}) \tag{21}$$

$$\dot{\tilde{F}}_1 = -P\Lambda_{13} sign(\tilde{x}_{11}) \tag{22}$$

$$\tilde{M}_1^{-1} = \hat{M}_1^{-1} - M_1^{-1}; \tilde{C}_1 = \hat{C}_1 - C_1; \tilde{V}_{12} = \hat{V}_{12} - V_{12}$$

with ζ_1 as matched perturbation $\zeta_1 = M_1(x_{11})^{-1}(C_1(x_1,x_2)x_2 - \hat{C}_1(\hat{x}_1,\hat{x}_2)\hat{x}_2 + \tilde{V}_{12}(\hat{x}_1,\hat{x}_2) + \nu_1) + \tilde{M}_{11}^{-1}(x_{11})(\hat{C}_1(\hat{x}_1,\hat{x}_2)\hat{x}_2 + \hat{V}_{12}(\hat{x}_1,\hat{x}_2) - J_1^T(x_{11})\hat{F}_1)$;

Estimation errors on forces are: $\tilde{F}_1 = \hat{F}_1 - F_1$ with ζ_1 all neglected terms and remaining modeling and coupling errors. These can be assumed bounded owing to fact that all involved terms are either estimates or come from a passive mechanical part of the system and $|\nu_1| < \kappa_0 \ \forall \ t \in \mathbb{R}_+$.

The Lyapunov function $V_1 = \frac{1}{2}\tilde{x}_{11}^T \tilde{x}_{11}$, help to show that the sliding surface $\tilde{x}_{11} = 0$ is attractive if we choose λ_{12}^i such as $|\tilde{x}_{12}^i| < \lambda_{12}^i$ for $i = 1,..,3$.

After a finite time t_{01}, we will get in average $\tilde{x}_{11} = 0$ and $\dot{\tilde{x}}_{11} = 0$. We obtain a reduced dynamic for the estimation error:

$$\dot{\tilde{x}}_{12} = \zeta_1 - M_1(x_{11})^{-1} J_1^T(x_{11})\tilde{F}_1 - \Lambda_{12}\Lambda_{11}^{-1}\tilde{x}_{12} \tag{23}$$

$$\dot{\tilde{F}} = -P\Lambda_{13}\Lambda_{11}^{-1}\tilde{x}_{12} \tag{24}$$

For the second step of the convergence proof, consider $V_2(\tilde{x}_{12}, \tilde{F}_1) = \frac{1}{2}\tilde{x}_{12}^T \tilde{x}_{12} + \frac{1}{2}\tilde{F}_1^T P^{-1}\tilde{F}_1$ then $\dot{V}_2(\tilde{x}_{12}, \tilde{F}_1)$ becomes if we let $\Lambda_{13} = \left(M_1(x_{11})^{-1} J_1^T(x_{11})\right)\Lambda_{11}$

$$\dot{V}_2 = -\tilde{x}_{12}^T \Lambda_{12}\Lambda_{11}^{-1}\tilde{x}_{12} - \tilde{x}_{12}^T \zeta_1$$

Now as previously choose λ_{11}^i and λ_{12}^i (the diagonal elements of the gain matrices Λ_{11}, Λ_{12}) large enough and $\Lambda_{13} = \left(M_1(x_{11}) J_1^T(x_{11})\right)\Lambda_{11}$. Then convergence of $(\hat{x}_{11}^i, \hat{x}_{21}^i)$ toward (x_{11}^i, x_{21}^i) is obtained and estimation errors on forces are bounded.

2) Observer for Suspensions dynamics Σ_2:: We assume that the wheels are always in contact with the ground ($\xi = 1$) and note $\tilde{x}_{21}^i = \hat{x}_{21}^i - x_{21}^i$ and $\tilde{x}_{22}^i = \hat{x}_{22}^i - x_{22}^i$ the estimation errors and $\tilde{F}_{i2} = \hat{F}_{i2} - F_{i2}$ force estimation error. The proposed observer, for each wheel suspension, is:

$$\dot{\hat{x}}_{21} = \hat{x}_{22} - \lambda_{21} sign(\tilde{x}_{21}) \tag{25}$$

$$\dot{\hat{x}}_{22} = \hat{M}_{33}^{-1}(J_2^T \hat{F}_2 - \hat{C}_{33}(\hat{x}_{21}, \hat{x}_{22})\hat{x}_{22} - \hat{V}_3(\hat{x}_{21}, \hat{x}_{22})) - \lambda_{22}^i sign(\tilde{x}_{21}) \tag{26}$$

$$\dot{\hat{F}}_{i2} = -P\lambda_{23}^i sign(\tilde{x}_{21}) \tag{27}$$

The observation error dynamics is (25-13):

$$\dot{\tilde{x}}_{21} = \tilde{x}_{22} - \lambda_{21} sign(\tilde{x}_{21}) \tag{28}$$

$$\dot{\tilde{x}}_{22} = -\zeta_2 + M_{33}^{-1}(J_2^T(x_{21})\tilde{F}_2 - K_v\tilde{x}_{22}) - \lambda_{22} sign \tilde{x}_{21} \tag{29}$$

$$\dot{\tilde{F}}_{i2} = -P\lambda_{23} sign(\tilde{x}_{21}) \tag{30}$$

with $\zeta_2 = M_{33}^{-1}(C_{33}(x_{21}, x_{22})x_2 - C_{33}(\hat{x}_{21}, \hat{x}_{22})\hat{x}_2 + \tilde{G} + \nu_2) + \bar{M}_{33}^{-1}((K_p\hat{x}_{21}^i + C_{33}(\hat{x}_{21}, \hat{x}_{22})\hat{x}_2 + \hat{G})$

Like the previous case we choose $V_1 = \frac{1}{2}(\tilde{x}_{21}^T\tilde{x}_{21})$ and show that $\hat{x}_{12}$ converges to x_{12} in finite time t_{02} if we ensure that $\forall t > 0$ that $|\tilde{x}_{22}| < \lambda_{21}$. Then we deduce the reduced average dynamics $\tilde{x}_{22} = \lambda_{21}sign(\tilde{x}_{12})$

$$\dot{\tilde{x}}_{22} = -\zeta_2 - \bar{M}_{33}^{-1}(K_v + \lambda_{22}\lambda_{21}^{-1})\tilde{x}_{22} + M_{33}^{-1}J^T(x_{21})\hat{F}_2 \tag{31}$$

$$\dot{\tilde{F}}_2 = -P\lambda_{23}\lambda_{21}^{-1}\tilde{x}_{21} \tag{32}$$

Let $V_2 = \frac{1}{2}\tilde{x}_{22}^T\bar{M}_{33}(x_{21})\tilde{x}_{22} + \frac{1}{2}\tilde{F}_2^TP^{-1}\tilde{F}_2$, its derivative $\dot{V}_2$ becomes, if we take $\lambda_{23} = J^T(x_{21})\lambda_{12}$,

$$\dot{V}_2 = -\tilde{x}_2^T(K_v + \lambda_{23}\lambda_{21}^{-1})\tilde{x}_{22}^i - \tilde{x}_2^T\bar{M}_{2,2}\zeta_2$$

We can conclude as previously that if we choose λ_{21} and λ_{22} (the diagonal elements of the gain matrices Λ_{21} et Λ_{22}) large enough and $\Lambda_{23} = J^T\Lambda_{21}$ then convergence of $(\hat{x}_{21}, \hat{x}_{22})$ toward (x_{21}, x_{22}) is obtained and estimation errors on forces $\tilde{F}_2$ remains only bounded.

3) Observer for Wheels Dynamics Σ_3: For each wheel i, the proposed observer is as follows:

$$\dot{\hat{x}}_{31}^i = \hat{x}_{32}^i - \lambda_{31}^i sign(\hat{x}_{31}^i - x_{31}^i) \tag{33}$$

$$\dot{\hat{x}}_{32}^i = J^{-1}(\tau - r\hat{F}_{3i}) - \lambda_{32}^i sign(\hat{x}_{31}^i - x_{31}^i) \tag{34}$$

$$\dot{\hat{F}}_{3i} = -P\lambda_{33}^i sign(\hat{x}_{31}^i - x_{31}^i) \tag{35}$$

The torque τ is assumed known. This observer can be easily extended to estimate the torque by adding an equation defining the drive line producing the torque. Observation error dynamics is then:

$$\dot{\tilde{x}}_{31}^i = \tilde{x}_{32}^i - \lambda_{31}^i sign(\tilde{x}_{31}^i) \tag{36}$$

$$\dot{\tilde{x}}_{32}^i = -rJ^{-1}\tilde{F}_{3i} - \lambda_{32}^i sign(\tilde{x}_{31}^i) + \eta_c^{3i} \tag{37}$$

$$\dot{\tilde{F}}_{3i} = -P\lambda_{33}^i sign(\tilde{x}_{31}^i) \tag{38}$$

with $\tilde{x}_{31}^i = \hat{x}_{31}^i - x_{31}^i$, $\tilde{x}_{32}^i = \hat{x}_{32}^i - x_{32}^i$ the errors on estimations of states x_3^i and forces $\tilde{F}_{3i} = \hat{F}_{3i} - F_{3i}$.

We can prove the convergence in finite time (t_{03}) of states estimates $\hat{x}_{31}^i$ and bounded of forces estimation by using the Lyapunov functions $V_1 = \frac{1}{2}(\tilde{x}_{11}^i)^2$, and $V_2 = \frac{1}{2}(\tilde{x}_{21}^i)^2 + \frac{1}{P}(\tilde{F}_{i1})^2$.

Let us note $\xi^T = [\tilde{x}_{21}^i, v_c^{1i}]$ then $\dot{V}_2$ become upper bounded

$$\dot{V}_2 = -\xi^TQ\xi + (v_c^{1i})^2 = -\xi^T\begin{bmatrix} \frac{\lambda_{21}^i}{\lambda_1^i} & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}\xi + (v_c^{1i})^2$$

$$\dot{V}_2 \le -\lambda_{inf}\{Q\}\|\xi\|^2 + \|v_c^{1i}\|^2$$

$$\dot{V}_2 \le -\lambda_{inf}\{Q\}\|\tilde{x}_{21}^i\|^2 + (1 - \lambda_{inf}\{Q\})\|v_c^{1i}\|^2$$

then $\tilde{x}_{21}^i$ converges to 0 if λ_{21}^i is such as $\lambda_{inf}\{Q\}$ is large enough. λ_{11}^i and λ_{21}^i large enough give convergence of $(\hat{x}_{11}^i, \hat{x}_{21}^i)$ toward (x_{11}^i, x_{21}^i) and errors $\tilde{F}_{i1}$remain bounded.

B. High Order Sliding Mode Observers

In this part we will use a High Order Sliding Mode observers (HOSM See [14]) to deduce our estimations. The HOSM dynamics converge in finite time.

1) Observer for the chassis Dynamics Σ_1: The proposed observer is the following (with $\tilde{x}_{11} = \hat{x}_{11} - x_{11}$):

$$\dot{\hat{x}}_{11} = v_{11} = \hat{x}_{12} - \lambda_{11}|\hat{x}_{11} - x_{11}|^{\frac{2}{3}}sign(\hat{x}_{11} - x_{11}) \tag{39}$$

$$\dot{\hat{x}}_{12} = -\zeta_{12} - \lambda_{12}|\hat{x}_{12} - v_{11}|^{\frac{1}{2}}sign(\hat{x}_{12} - v_{11}) \tag{40}$$

with $\zeta_{12} = \hat{M}_{11}^{-1}(\hat{C}_1(\hat{x}_1, \hat{x}_2)\hat{x}_2 + \hat{G} - J_1^T\hat{F})$

IV. SIMULATION RESULTS

The simulator previously developed by our staff (SimK106N, available on demand to first author) is used in order to test and validate the proposed observers and our approach of model splitting and developing partial state estimators. The used parameters and environment characteristics have been validated in a previous work in collaboration with the LCPC [2][7][5].

The simulation results presented are obtained for a driving with sinusoidal steering command of 20 deg amplitude. The results are good for the First Order Sliding Mode Observers (see figure 7) and also for HOSM based observers (see figure 8); The two kind of observers may give very good results by adjusting the gain parameters.

V. CONCLUSION

In this paper, we have proposed efficient and robust observers allowing to estimate states and unknown inputs (torques or forces). The model formulation has been done such that the passivity property is preserved. This copes well for SMO approach and avoid chattering problems.

These observers obey to the first kind assuming that input forces and torques are constant or slowly time varying ($\dot{F} \simeq 0$). The robustness of the sliding mode observer versus uncertainties on model parameters is an important feature. First and Second Order Sliding Mode Observers have been developed and their performance evaluated. These observer are illustrated by simulation results to show effectiveness of their performance. These results validate the proposed observers and our approach of model splitting and developing partial state estimators. .

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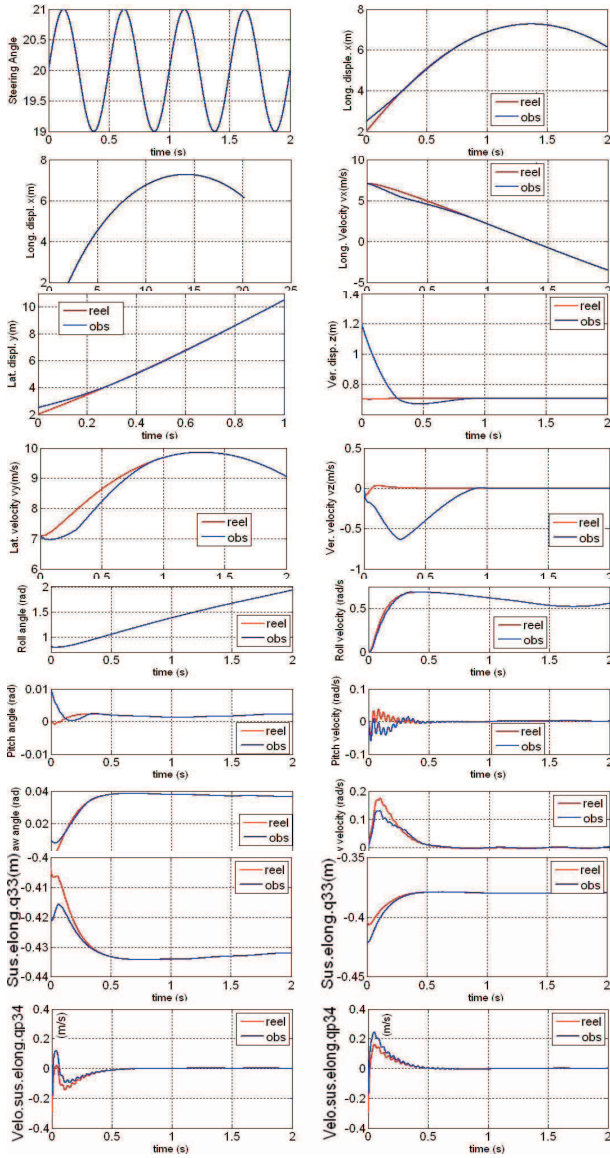
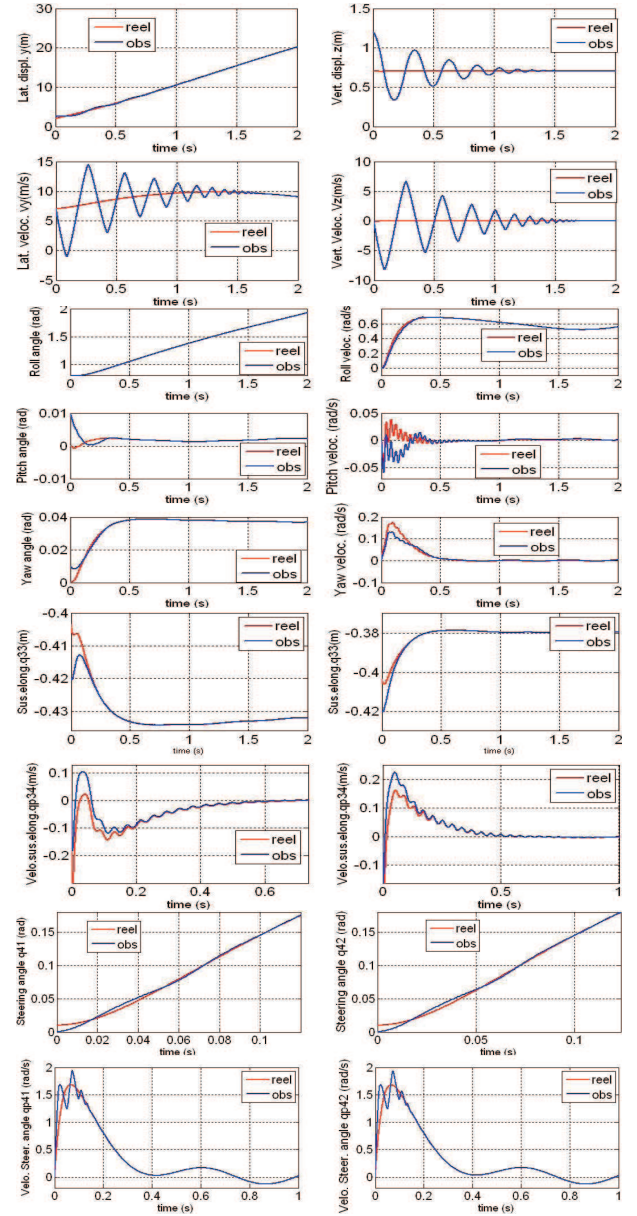Figure 7. Results of the Partial state Observer FOSM

Figure 8. Results of the HOSM Observers

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