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Tableaux for non-normal public announcement logic

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Abstract. This paper presents a tableau calculus for two semantic interpretations of public announcements over monotone neighbourhood models: the intersection and the subset semantics, developed by Ma and Sano. We show that, without employing reduction axioms, both calculi are sound and complete with respect to their corresponding semantic interpretations and, moreover, we establish that the satisfiability problem of this public announcement extensions is NP-complete in both cases. The tableau calculi has been implemented in Lotrecscheme.

1 Introduction

Public announcement logic (**PAL**; [7,21]) studies the effect of the most basic communicative action on the knowledge of epistemic logic agents (**EL**; [12,6]), and it has served as the basis for the study of more complex announcements [3] and other forms of epistemic changes [27,25]. Under the standard **EL** semantic model, relational models, **PAL** relies on a natural interpretation of what the public announcement of a formula φ does: it eliminates those epistemic possibilities that do not satisfy φ. Despite its simplicity, **PAL** has proved to be a fruitful field for interesting research, as the characterisation of successful formulas (those that are still true after being truthfully announced: [28,14]), the characterisation of schematic validities [13] and many others [24].

However, relational models are not the unique structures for interpreting **EL** formulas, and recently there have been approaches that, using the so called *minimal* or *neighborhood* models [23,18,19,4], have studied not only epistemic phenomena but also their dynamics [31,26,17,30]. The set of **EL** validities under neighborhood models is smaller than that under relational models, so the agent's knowledge has less 'built-in' properties, which allows a finer representation of epistemic notions and their dynamics without resorting to 'syntactic' awareness models [5].

In [17], the authors presented two ways of updating (monotone) neighborhood models and thus of representing public announcements: one intersecting the current neighborhoods with the new information ($\cap$-*semantics*, already proposed in [31]), and another preserving only those neighborhoods which are subsets of the new information ($\subseteq$-*semantics*). The two updates behave differently, as their provided sound and complete axiom systems show. The present work continues the study of such updates, first, by extending the tableau system for monotone neighborhood models of [15] with rules for dealing with its public announcement extensions, and second, by showing how the satisfiability problem is NP-complete for both the intersection and the subset semantics.

2 Preliminaries

This section recalls some basic concepts from [17]. We work on the single agent case, but the results obtained can be easily extended to multi-agent scenarios.

Throughout this paper, let Prop be a countable set of atomic propositions. The language $\mathcal{L}_{\mathbf{EL}}$ extends the classical propositional language with formulas of the form $\Box\varphi$, read as "the agent knows that φ". Formally,

$$\varphi \ ::= \ p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi$$

with $p \in \mathsf{Prop}$. Other propositional connectives ($\vee$, $\rightarrow$ and $\leftrightarrow$) are defined as usual. The dual of $\Box$ is defined as $\Diamond\varphi := \neg\Box\neg\varphi$.

A *monotone neighborhood frame* is a pair $\mathcal{F} = (W, \tau)$ where $W \neq \emptyset$ is the domain, a set of possible worlds, and $\tau : W \rightarrow \wp(\wp(W))$ is a neighborhood function satisfying the following monotonicity condition: for all $w \in W$ and all $X, Y \subseteq W$, $X \in \tau(w)$ and $X \subseteq Y$ implies $Y \in \tau(w)$. A *monotone neighborhood model* (*MNM*) $\mathcal{M} = (\mathcal{F}, V)$ is a monotone neighborhood frame $\mathcal{F}$ together with a valuation function $V : \mathsf{Prop} \rightarrow \wp(W)$. Given a $\mathcal{M} = (W, \tau, V)$ and a $\mathcal{L}_{\mathbf{EL}}$-formula φ, the notion of φ being true at a state w in the model $\mathcal{M}$ (written $\mathcal{M}, w \models \varphi$) is defined inductively as follows:

$$\begin{array}{ll} \mathcal{M}, w \models p \text{ iff } w \in V(p), & \mathcal{M}, w \models \varphi \wedge \psi \text{ iff } \mathcal{M}, w \models \varphi \text{ and } \mathcal{M}, w \models \psi, \\ \mathcal{M}, w \models \neg\varphi \text{ iff } \mathcal{M}, w \not\models \varphi, & \mathcal{M}, w \models \Box\varphi \text{ iff } [\![\varphi]\!]_{\mathcal{M}} \in \tau(w). \end{array}$$

where $[\![\varphi]\!]_{\mathcal{M}} := \{u \in W \mid \mathcal{M}, u \models \varphi\}$ is the truth set of φ in $\mathcal{M}$. Since $\mathcal{M}$ is a *MNM*, the satisfaction clause for $\Box$ can be equivalently rewritten as follows:

$$\mathcal{M}, w \models \Box\varphi \text{ iff } X \subseteq [\![\varphi]\!]_{\mathcal{M}} \text{ for some } X \in \tau(w).$$

The language $\mathcal{L}_{\mathbf{PAL}}$ extends $\mathcal{L}_{\mathbf{EL}}$ with the public announcement operator $[\varphi]$, allowing the construction of formulas of the form $[\varphi]\psi$, read as "ψ is true after the public announcement of φ". (Define $\langle\varphi\rangle\psi := \neg[\varphi]\neg\psi$.) For the semantic interpretation, we recall the intersection and subset semantics of [17].

Definition 1. *Let $\mathcal{M} = (W, \tau, V)$ be a MNM. For any non-empty $U \subseteq W$, define the function $V^U : \mathsf{Prop} \rightarrow U$ by $V^U(p) := V(p) \cap U$ for each $p \in \mathsf{Prop}$.*

- *The* intersection submodel *of $\mathcal{M}$ induced by U, $\mathcal{M}^{\cap U} = (U, \tau^{\cap U}, V^U)$, is given by $\tau^{\cap U}(u) := \{P \cap U \mid P \in \tau(u)\}$, for every $u \in U$.*
- *The* subset submodel *of $\mathcal{M}$ induced by U, $\mathcal{M}^{\subseteq U} = (U, \tau^{\subseteq U}, V^U)$, is given by $\tau^{\subseteq U}(u) := \{P \in \tau(u) \mid P \subseteq U\}$, for every $u \in U$.*

If $\mathcal{M}$ is monotone, then so are $\mathcal{M}^{\cap U}$ and $\mathcal{M}^{\subseteq U}$, as shown in [17].

Given a *MNM* $\mathcal{M} = (W, \tau, V)$, formulas φ, ψ in $\mathcal{L}_{\mathbf{PAL}}$, the notion of a formula being true at a state of a model extends that for formulas in $\mathcal{L}_{\mathbf{EL}}$ with the following clauses:

- $\mathcal{M}, w \models_{\cap} [\varphi]\psi$ iff $\mathcal{M}, w \models_{\cap} \varphi$ implies $\mathcal{M}^{\cap\varphi}, w \models_{\cap} \psi$,
- $\mathcal{M}, w \models_{\subseteq} [\varphi]\psi$ iff $\mathcal{M}, w \models_{\subseteq} \varphi$ implies $\mathcal{M}^{\subseteq\varphi}, w \models_{\subseteq} \psi$;

where $\mathcal{M}^{\cap\varphi}$ abbreviates $\mathcal{M}^{\cap[\![\varphi]\!]_{\mathcal{M}}}$ and $\mathcal{M}^{\subseteq\varphi}$ abbreviates $\mathcal{M}^{\subseteq[\![\varphi]\!]_{\mathcal{M}}}$. If we use the symbol $* \in \{\cap, \subseteq\}$ to denote either semantics then, from $\langle\varphi\rangle\psi$'s definition,

– $\mathcal{M}, w \models_* \langle\varphi\rangle\psi$ iff $\mathcal{M}, w \models_* \varphi$ and $\mathcal{M}^{*\varphi}, w \models_* \psi$.

The subscript $* \in \{\cap, \subseteq\}$ will be dropped from $\models_*$ when its meaining is clear from the context. A sound and complete axiomatization for $\mathcal{L}_{\mathbf{PAL}}$ w.r.t. the provided semantics under *MNM*s can be found in [17]. The purpose of this paper is to develop tableau systems for both logics. The following proposition is a generalization of the monotonicity of $\Box$ under *MNM*s ($[\![\varphi]\!]_\mathcal{M} \subseteq [\![\psi]\!]_\mathcal{M}$ implies $[\![\Box\varphi]\!]_\mathcal{M} \subseteq [\![\Box\psi]\!]_\mathcal{M}$) to the public announcements extensions and it will be key for providing $\Box$'s rules for both intersection and subset semantics.

Proposition 1. *Let ρ_i $(1 \leq i \leq n)$, θ_j $(1 \leq j \leq m)$ and φ be $\mathcal{L}_{\mathbf{PAL}}$-formulas and $\mathcal{M} = (W, \tau, V)$ be a MNM.*

(i) $[\![[\rho_1]\cdots[\rho_n]\varphi]\!]_\mathcal{M} \subseteq [\![[\theta_1]\cdots[\theta_m]\psi]\!]_\mathcal{M}$ *implies* $[\![\Box\varphi]\!]_{\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}} \subseteq [\![\Box\psi]\!]_{\mathcal{M}^{\cap\theta_1;\cdots;\cap\theta_m}}$
(ii) $[\![\langle\rho_1\rangle\cdots\langle\rho_n\rangle\varphi]\!]_\mathcal{M} \subseteq [\![\langle\theta_1\rangle\cdots\langle\theta_m\rangle\psi]\!]_\mathcal{M}$ *implies* $[\![\Box\varphi]\!]_{\mathcal{M}^{\subseteq\rho_1;\cdots;\subseteq\rho_n}} \subseteq [\![\Box\psi]\!]_{\mathcal{M}^{\subseteq\theta_1;\cdots;\subseteq\theta_m}}$

Proof. For (i), assume $[\![[\rho_1]\cdots[\rho_n]\varphi]\!]_\mathcal{M} \subseteq [\![[\theta_1]\cdots[\theta_m]\psi]\!]_\mathcal{M}$. Now fix any $w \in W$ with $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}, w \models_\cap \Box\varphi$. By semantic interpretation, there is $X \in \tau^{\cap\rho_1;\cdots;\cap\rho_n}(w)$ s.t. $X \subseteq [\![\varphi]\!]_{\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}}$; then, by the definition of $\tau^{\cap\rho_1;\cdots;\cap\rho_n}(w)$, there is $Y \in \tau(w)$ s.t. $(Y \cap [\![\rho_1]\!]_\mathcal{M} \cap \cdots \cap [\![\rho_n]\!]_{\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_{n-1}}}) \subseteq [\![\varphi]\!]_{\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}}$, i.e., $Y \subseteq [\![[\rho_1]\cdots[\rho_n]\varphi]\!]_\mathcal{M}$ and hence, by assumption, $Y \subseteq [\![[\theta_1]\cdots[\theta_m]\psi]\!]_\mathcal{M}$. Thus, $Y \subseteq [\![\psi]\!]_{\mathcal{M}^{\cap\theta_1;\cdots;\cap\theta_m}}$ for $Y \in \tau^{\cap\theta_1;\cdots;\cap\theta_m}(w)$ so $\mathcal{M}^{\cap\theta_1;\cdots;\cap\theta_m}, w \models_\cap \Box\psi$, as needed.

For (ii), assume $[\![\langle\rho_1\rangle\cdots\langle\rho_n\rangle\varphi]\!]_\mathcal{M} \subseteq [\![\langle\theta_1\rangle\cdots\langle\theta_m\rangle\psi]\!]_\mathcal{M}$. Now fix any $w \in W$ with $\mathcal{M}^{\subseteq\rho_1;\cdots;\subseteq\rho_n}, w \models_\subseteq \Box\varphi$. Then there is $X \in \tau^{\subseteq\rho_1;\cdots;\subseteq\rho_n}(w)$ s.t. $X \subseteq [\![\varphi]\!]_{\mathcal{M}^{\subseteq\rho_1;\cdots;\subseteq\rho_n}}$ and, by definition of $\tau^{\subseteq\rho_1;\cdots;\subseteq\rho_n}(w)$, both $X \in \tau(w)$ and $X \subseteq ([\![\rho_1]\!]_\mathcal{M} \cap \cdots \cap [\![\rho_n]\!]_{\mathcal{M}^{\subseteq\rho_1;\cdots;\cap\rho_{n-1}}} \cap [\![\varphi]\!]_{\mathcal{M}^{\subseteq\rho_1;\cdots;\cap\rho_n}})$, i.e., $X \subseteq [\![\langle\rho_1\rangle\cdots\langle\rho_n\rangle\varphi]\!]_\mathcal{M}$ and hence, by assumption, $X \subseteq [\![\langle\theta_1\rangle\cdots\langle\theta_m\rangle\psi]\!]_\mathcal{M}$. Thus, $X \in \tau^{\subseteq\theta_1;\cdots;\subseteq\theta_m}(w)$ and $X \subseteq [\![\psi]\!]_{\mathcal{M}^{\subseteq\theta_1;\cdots;\subseteq\theta_m}}$ so $\mathcal{M}^{\subseteq\theta_1;\cdots;\subseteq\theta_m}, w \models_\subseteq \Box\psi$, as needed.

3 Tableaux for non-normal monotone (static) epistemic logic

There are several works on tableau calculus of non-normal modal logic. Kripke [16] proposed a calculus based on Kripke semantics which allow the notion of *normal world*, and [8] constructed a uniform framework for tableau calculi for neighborhood semantics employing labels for both a states and set of states. More recently, Indrzejczak [15] avoided the label for set of states while presenting tableau calculi for several non-normal logics over neighborhood semantics.

As a prelude to our contribution, here we recall the tableau method for non-normal monotone modal logic of Indrzejczak [15], of which our proposal is an extension, as well as the argument for soundness and completeness. Then we recall why the satisfiability problem for non-normal monotone modal logic is NP-complete [29].

$$\frac{(\sigma : \varphi \wedge \psi)}{(\sigma : \varphi)(\sigma : \psi)}\ (\wedge) \qquad \frac{(\sigma : \neg(\varphi \wedge \psi))}{(\sigma : \neg\varphi) \mid (\sigma : \neg\psi)}\ (\neg\wedge) \qquad \frac{(\sigma : \neg\neg\varphi)}{(\sigma : \varphi)}\ (\neg\neg) \qquad \frac{(\sigma : \Box\varphi)(\sigma : \neg\Box\psi)}{(\sigma_{new} : \varphi)(\sigma_{new} : \neg\psi)}\ (\Box)$$

Fig. 1. Tableau rules for non-normal monotone logic [15]

The terms in the tableau rules (Figure 1), of the form $(\sigma : \varphi)$, indicate that formula φ is true in state (prefix) σ. Rules $(\wedge)$, $(\neg\wedge)$ and $(\neg\neg)$ correspond to propositional reasoning, and rule $(\Box)$ is the *prefix generating rule*. There are two general constraints on the construction of tableaus: (1) The prefix generating rule is never applied twice to the same premise on the same branch; (2) A formula is never added to a tableau branch where it already occurs.

As usual, a tableau is *saturated* when no more rules that satisfy the constraints can be applied. A branch is *saturated* if it belongs to a saturated tableau, and it is *closed* if it contains formulas $(\sigma : \varphi)$ and $(\sigma : \neg\varphi)$ for some σ and φ (otherwise, the branch is *open*). A tableau is *closed* if all its branches are closed, and it is *open* if at least one of its branches is open.

Rule $(\Box)$ might surprise readers familiar with tableaux for normal modal logic, but it states a straightforward fact: if both $\Box\varphi$ and $\neg\Box\psi$ hold in a world σ, then while $\Box\varphi$ imposes the existence of a neighborhood in $\tau(\sigma)$ containing only φ-worlds, $\neg\Box\psi$ imposes a $\neg\psi$-world in every neighborhood in $\tau(\sigma)$. The world σ_{new} denotes exactly that.

3.1 Soundness and Completeness

Definition 2. *Given a branch Θ,* Prefix(Θ) *is the set of all its prefixes. We say that Θ is* faithful *to a MNM $\mathcal{M} = (W, \tau, V)$ if there is a mapping f :* Prefix$(\Theta) \to W$ *such that $(\sigma : \varphi) \in \Theta$ implies $\mathcal{M}, f(\sigma) \models \varphi$ for all $\sigma \in$* Prefix(Θ).

Lemma 1. *Let Θ be any branch of a tableau and $\mathcal{M} = (W, \tau, V)$ a MNM. If Θ is faithful to $\mathcal{M}$, and a tableau rule is applied to it, then it produces at least one extension Θ' such that Θ' is faithful to $\mathcal{M}$.*

For the proof, see Appendix A.1.

Theorem 1 (Soundness). *Given any formula φ, if there is a closed tableau for $(\sigma_{initial} : \neg\varphi)$, then φ is valid in the class of all MNMs.*

Proof. We show the contrapositive. Suppose that $\neg\varphi$ is satisfiable, i.e., there is a *MNM* $\mathcal{M} = (W, \tau, V)$ and a $w \in W$ s.t. $\mathcal{M}, w \not\models \varphi$. Then the initial tableau $\Theta = \{(\sigma_{initial} : \neg\varphi)\}$ is faithful to $\mathcal{M}$ and hence, by Lemma 1, only faithful tableau to *MNM* will be produced. A faithful branch cannot be closed. Hence $(\sigma_{initial} : \neg\varphi)$ can have no closed tableau.

Lemma 2. *Given an open saturated branch Θ, define the model $\mathcal{M}^\Theta = (W^\Theta, \tau^\Theta, V^\Theta)$ as $W^\Theta :=$* Prefix(Θ), *$V^\Theta(p) := \{\sigma \in W^\Theta \mid (\sigma : p) \in \Theta\}$ and, for every $\sigma \in W^\Theta$,*

$$X \in \tau^\Theta(\sigma) \quad \textit{iff} \quad \textit{there is } \varphi \textit{ s.t.} (\sigma : \Box\varphi) \in \Theta \textit{ and } \{\sigma' \in W^\Theta \mid (\sigma' : \varphi) \in \Theta\} \subseteq X$$

Then, for all formulas φ and all prefix σ, (i) *$(\sigma : \varphi) \in \Theta$ implies $\mathcal{M}^\Theta, \sigma \models \varphi$ and* (ii) *$(\sigma : \neg\varphi) \in \Theta$ implies $\mathcal{M}^\Theta, \sigma \not\models \varphi$.*

Note that τ^Θ is clearly monotone and thus, if Θ is non-empty, $\mathcal{M}^\Theta$ is a *MNM*. For the proof, see Appendix A.2.

Theorem 2 (Completeness). *Given any formula φ, if there is an open saturated tableau for $(\sigma_{initial} : \varphi)$, then φ is satisfiable in a MNM.*

Proof. If there is an open saturated branch Θ containing $(\sigma_{initial} : \varphi)$, Lemma 2 yields $\mathcal{M}^\Theta, \sigma_{initial} \models \varphi$ so φ is satisfiable in a *MNM*.

3.2 Complexity

Normal modal logics as **K** and **KT** are PSPACE-complete, and negative introspection $\neg\Box p \rightarrow \Box\neg\Box p$ makes any modal logics between **K** and **S4** NP-complete [11]. Tableau systems for such logics have been given in [10].

The satisfiability problem (deciding whether a given φ is satisfiable) for non-normal monotone modal logic is NP-complete [29]. A known method is to build a tableau from $\{(\sigma_{initial} : \varphi)\}$; at each step, the process adds non-deterministically a term of the form $(\sigma : \psi)$ with σ is a symbol and ψ is a subformula or a negation of a subformula of φ.

Proposition 2. *When executing the tableau method from $\{(\sigma_{initial} : \varphi)\}$, the number of terms $(\sigma : \psi)$ that can be added is polynomial in the length of φ.*

Proof. As ψ is a subformula or a negation of a subformula of φ, the number of possible ψ is linear in the size of φ. The number of possible world symbols σ is polynomial in the size of φ, as they are created only for pairs of the form $\Box\psi_1$, $\neg\Box\psi_2$. Thus, the number of such σ is bounded by $|\varphi|^2$, and hence the number of possible terms $(\sigma : \psi)$ is bounded by $|\varphi|^3$.

Corollary 1. *The satisfiability problem in non-normal monotone modal logic is NP-complete.*

Proof. NP-hardness comes from the fact that the satisfiability problem for classical propositional logic is polynomially reducible to the satisfiability problem for non-normal monotone modal logic. Now let us figure out why it is in NP. In the non-deterministic algorithm shown below, the size of Θ is polynomial in the length of φ (Proposition 2). Testing that Θ is saturated or non-deterministically applying a rule can be implemented in polynomial time in the size of Θ; then, these operations are polynomial in the length of φ. As we add a term to Θ at each iteration of the **while** loop, there are at most a polynomial number of iterations. Therefore, the tableau method can be implemented in polynomial time on a non-deterministic machine.

```
procedure sat(φ)
    Θ := {(σ_initial : φ)}
    while Θ is not saturated
        Θ := result of the (non-deterministic) application of a rule on Θ
        if Θ is closed then reject
    accept
```

4 Tableaux for non-normal public annoucement logics

Tableaux for public announcements for normal modal logic already appeared in [2], where the tableau formalism needed to represent the information of accessibility relation. Since we are concerned with non-normal modal logic characterized by neighborhood models, our tableau calculus will not introduce any formalism for accessibility relation. In this sense, our work is not a trivial generalization of [2]. For non-normal monotone modal logic, this section adapts the tableau method of Section 3 to deal with public announcements under both the $\cap$- and the $\subseteq$-semantics. Here, terms in the tableau rules can be either

– of the form $(\sigma :_L \varphi)$ with σ a world symbol, L a list of announced formulas (ϵ is the empty list) and φ a formula, indicating that σ survives the successive announcements of the elements of L and afterwards it satisfies φ, or
– of the form $(\sigma :_L \times)$, indicating that σ does not survive successive announcements of the elements of L.

Figure 2 shows the tableau rules for non-normal public annoucement logics. We define the rule set for the $\cap$-semantics as all the common rules plus ($\Box^{\cap}$), while the rule set for the $\subseteq$-semantics as all the common rules plus ($\Box^{\subseteq}$).

Common rules: $\dfrac{(\sigma :_L p)}{(\sigma :_\epsilon p)}\ (\downarrow \epsilon)\quad \dfrac{(\sigma :_L \neg p)}{(\sigma :_\epsilon \neg p)}\ (\downarrow \epsilon\neg)\quad \dfrac{(\sigma :_{L;\varphi} \times)}{(\sigma :_L \neg\varphi) \mid (\sigma :_L \times)}\ (\times \mathit{Back})$

$$\dfrac{(\sigma :_L \varphi \land \psi)}{(\sigma :_L \varphi)(\sigma :_L \psi)}\ (\land)\quad \dfrac{(\sigma :_L \neg(\varphi \land \psi))}{(\sigma :_L \neg\varphi) \mid (\sigma :_L \neg\psi)}\ (\neg\land)\quad \dfrac{(\sigma :_L \neg\neg\varphi)}{(\sigma :_L \varphi)}\ (\neg\neg)$$

$$\dfrac{(\sigma :_{L;\varphi} \psi)}{(\sigma :_L \varphi)}\ (\mathit{Back})\quad \dfrac{(\sigma :_L [\varphi]\psi)}{(\sigma :_L \neg\varphi) \mid (\sigma :_{L;\varphi} \psi)}\ ([\cdot])\quad \dfrac{(\sigma :_L \neg[\varphi]\psi)}{(\sigma :_{L;\varphi} \neg\psi)}\ (\neg[\cdot])$$

For $\cap$-semantics: $\dfrac{(\sigma :_L \Box\varphi)(\sigma :_{L'} \neg\Box\psi)}{(\sigma_{new} :_L \varphi)(\sigma_{new} :_{L'} \neg\psi) \mid (\sigma_{new} :_L \times)(\sigma_{new} :_{L'} \neg\psi)}\ (\Box^{\cap})$

For $\subseteq$-semantics: $\dfrac{(\sigma :_L \Box\varphi)(\sigma :_{L'} \neg\Box\psi)}{(\sigma_{new} :_{L'} \neg\psi)(\sigma_{new} :_L \varphi) \mid (\sigma_{new} :_{L'} \times)(\sigma_{new} :_L \varphi)}\ (\Box^{\subseteq})$

Fig. 2. Tableau rules for handling public announcements

Rules ($\land$), ($\neg\land$) and ($\neg\neg$) deal with propositional reasoning. Rules ($\downarrow \epsilon$), ($\downarrow \epsilon\neg$) indicate that valuations do not change after a sequence of announcements. Rule ($\neg[\cdot]$) states that if $\neg[\varphi]\psi$ holds in σ after a sequence of announcements L then $\neg\psi$ must hold in σ after the sequence of announcements $L;\varphi$. Rule ($[\cdot]$) states that if $[\varphi]\psi$ holds in σ after a sequence of announcements L, then either φ fails in σ after a sequence of announcements L or else ψ holds in σ after the sequence of announcements $L;\varphi$. Rule (*Back*) deals with a world surviving a sequence of announcements, and rule ($\times$*Back*) deals with a world not surviving it.

The rule of ($\Box^{\cap}$) is a rewriting of the first item of Proposition 1 into the rule of tableau calculus. For simplicity, let us assume that $L \equiv \rho;\rho'$ and $L' \equiv \theta$. By taking the contrapositive implication of Proposition 1.(i), we obtain the following rule:

$$\dfrac{(\sigma :_{\rho;\rho'} \Box\varphi)(\sigma :_\theta \neg\Box\psi)}{(\sigma_{new} :_\epsilon [\rho][\rho']\varphi)(\sigma_{new} :_\epsilon \neg[\theta]\psi)}$$

While $(\sigma_{new} :_\epsilon \neg[\theta]\psi)$ generates $(\sigma_{new} :_\theta \neg\psi)$ by the rule ($\neg[\cdot]$), we have two cases for expanding $(\sigma_{new} :_\epsilon [\rho][\rho']\varphi)$. First, assume that σ_{new} survives after the successive updates of ρ and ρ'. Then, we may add $(\sigma_{new} :_{\rho;\rho'} \varphi)$ to the branch. Second, suppose that σ_{new} does not survive after the successive updates of ρ and ρ'. Then, we add $(\sigma_{new} :_{\rho;\rho'} \times)$ to the branch. This also explains the soundness of ($\Box^{\cap}$) for $\cap$-semantics.

Rule ($\Box^{\subseteq}$) can also be explained in terms of the second item of Proposition 1. Let L and L' as above. By taking the contrapositive implication of Proposition 1.(ii) and rewriting the diamond $\langle\gamma\rangle$ in terms of the dual $[\gamma]$, we obtain the following:

$$\frac{(\sigma :_{\rho;\rho'} \Box\varphi)(\sigma :_{\theta} \neg\Box\psi)}{(\sigma_{new} :_{\epsilon} \neg[\rho][\rho']\neg\varphi)(\sigma_{new} :_{\epsilon} [\theta]\neg\psi)}$$

By a procedure similar to the used for ($\Box^{\cap}$) we can justify the rule ($\Box^{\subseteq}$).

As before, there are two constraints on the construction of tableaus: A prefix generating rule is never applied twice to the same premise on the same branch; A formula is never added to a tableau branch where it already occurs. The notions of *saturated tableau* and *saturated branch* are as before. In order to deal with terms of the form $(\sigma :_L \times)$, the notion of closed branch is extended as follows: a branch of a tableau is *closed* when (1) it contains terms $(\sigma :_L \varphi)$ and $(\sigma :_L \neg\varphi)$ for some σ, L and φ, or (2) it contains $(\sigma :_{\epsilon} \times)$ for some σ; otherwise, the branch is called *open*. The notions of *closed* and *open tableau* are defined as before.

4.1 Soundness

We start with the $\cap$-semantics. As before, given a branch Θ, Prefix(Θ) denotes the set of all prefixes in Θ.

Definition 3. *Given a branch Θ and a MNM $\mathcal{M} = (W, \tau, V)$, Θ is* faithful *to $\mathcal{M}$ if there is a mapping f :* Prefix(Θ) $\to W$ *such that, for all $\sigma \in$* Prefix(Θ)*,*

- $(\sigma :_{\psi_1;\cdots;\psi_n} \varphi) \in \Theta$ *implies* $\mathcal{M}^{\cap\psi_1;\cdots;\cap\psi_n}, f(\sigma) \models \varphi$*, and*
- $(\sigma :_{\psi_1;\cdots;\psi_n} \times) \in \Theta$ *implies that $f(\sigma)$ is not in $\mathcal{M}^{\cap\psi_1;\cdots;\cap\psi_n}$'s domain.*

Lemma 3. *Let Θ be any branch of a tableau and $\mathcal{M} = (W, \tau, V)$ a MNM. If Θ is faithful to $\mathcal{M}$, and a tableau rule is applied to it, then it produces at least one extension Θ' such that Θ' is faithful to $\mathcal{M}$.*

Proof. We only show the case for rule ($\Box^{\cap}$). For the cases of rules ($\downarrow \epsilon$), ($[\cdot]$), ($\times$*Back*), (*Back*), see Appendix A.3. Throughout this proof, let $L \equiv \rho_1;\cdots;\rho_n$. Let $L' \equiv \theta_1;\cdots;\theta_m$ in the rule ($\Box^{\cap}$) of Table 2. Since $(\sigma :_L \Box\varphi), (\sigma :_{L'} \neg\Box\psi) \in \Theta$, there is an f s.t. $f(\sigma) \in [\![\Box\varphi]\!]_{\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}}$ and $f(\sigma) \notin [\![\Box\psi]\!]_{\mathcal{M}^{\cap\theta_1;\cdots;\cap\theta_m}}$. Thus, $[\![\Box\varphi]\!]_{\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}} \nsubseteq [\![\Box\psi]\!]_{\mathcal{M}^{\cap\theta_1;\cdots;\cap\theta_m}}$ and hence, by Proposition 1, $[\![[\rho_1]\cdots[\rho_n]\varphi]\!]_{\mathcal{M}} \nsubseteq [\![[\theta_1]\cdots[\theta_m]\psi]\!]_{\mathcal{M}}$: there is u in $\mathcal{M}$ such that $u \in [\![[\rho_1]\cdots[\rho_n]\varphi]\!]_{\mathcal{M}}$ but $u \notin [\![[\theta_1]\cdots[\theta_m]\psi]\!]_{\mathcal{M}}$. From the latter it follows that u survives the successive intersection updates of $\theta_1, \ldots, \theta_n$ but $\mathcal{M}^{\cap\theta_1;\cdots;\cap\theta_m}, u \not\models \psi$. From the former, suppose (1) u is in the domain of $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}$; then $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}, u \models \varphi$ and we can take $\Theta' := \Theta \cup \{(\sigma_{new} :_L \varphi), (\sigma_{new} :_{L'} \neg\psi)\}$ and extend the original f into f' : Prefix(Θ') $\to W$ by defining $f'(\sigma_{new}) := u$. It follows that $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}, f(\sigma_{new}) \models \varphi$ and $\mathcal{M}^{\cap\theta_1;\cdots;\cap\theta_m}, f(\sigma_{new}) \not\models \psi$, and so Θ' is faithful to $\mathcal{M}$. Otherwise, (2) u is not in the domain of $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}$, an a similar argument shows that $\Theta' = \Theta \cup \{(\sigma_{new} :_L \times), (\sigma_{new} :_{L'} \neg\psi)\}$ is faithful to $\mathcal{M}$.

Theorem 3. *Given any formula φ and any list $L \equiv \rho_1;\cdots;\rho_n$, if there is a closed tableau for $(\sigma_{initial} :_L \neg\varphi)$, then φ is valid in $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}$ for all MNMs $\mathcal{M}$.*

Proof. We show the contrapositive. Suppose that there is a *MNM* $\mathcal{M} = (W, \tau, V)$ and a $w \in W$ such that $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}, w \not\models \varphi$. Then the initial tableau $\Theta = \{(\sigma_{initial} :_L \neg\varphi)\}$ is faithful to $\mathcal{M}$ and hence, by Lemma 3, only faithful tableau to *MNM* will be produced. A faithful branch cannot be closed. Hence $(\sigma_{initial} :_L \neg\varphi)$ can have no closed tableau.

Now, for the $\subseteq$-semantics, we have the following.

Lemma 4. *Let Θ be any branch of a tableau and $\mathcal{M} = (W, \tau, V)$ a MNM. If Θ is faithful to $\mathcal{M}$, and a tableau rule is applied to it, then it produces at least one extension Θ' such that Θ' is faithful to $\mathcal{M}$.*

Proof. We only show the case for the rule $(\Box^{\subseteq})$. Let $L \equiv \rho_1; \cdots; \rho_n$ and $L' \equiv \theta_1; \cdots; \theta_m$ in the rule $(\Box^{\subseteq})$ of Table 2. Since $(\sigma :_L \Box\varphi), (\sigma :_{L'} \neg\Box\psi) \in \Theta$, there is an f s.t. $f(\sigma) \in [\![\Box\varphi]\!]_{\mathcal{M}^{\subseteq\rho_1;\cdots;\subseteq\rho_n}}$ and $f(\sigma) \notin [\![\Box\psi]\!]_{\mathcal{M}^{\subseteq\theta_1;\cdots;\subseteq\theta_m}}$. Thus, $[\![\Box\varphi]\!]_{\mathcal{M}^{\subseteq\rho_1;\cdots;\subseteq\rho_n}} \not\subseteq [\![\Box\psi]\!]_{\mathcal{M}^{\subseteq\theta_1;\cdots;\subseteq\theta_m}}$ and hence, by Proposition 1, $[\![\langle\rho_1\rangle \cdots \langle\rho_n\rangle\varphi]\!]_{\mathcal{M}} \not\subseteq [\![\langle\theta_1\rangle \cdots \langle\theta_m\rangle\psi]\!]_{\mathcal{M}}$. Then, there is u in $\mathcal{M}$ such that $u \in [\![\langle\rho_1\rangle \cdots \langle\rho_n\rangle\varphi]\!]_{\mathcal{M}}$ but $u \notin [\![\langle\theta_1\rangle \cdots \langle\theta_m\rangle\psi]\!]_{\mathcal{M}}$. From the former it follows that u survives the successive subset updates of $\rho_1, \ldots, \rho_n$ and $\mathcal{M}^{\subseteq\rho_1;\cdots;\subseteq\rho_n}, u \models \varphi$. From the latter, suppose (1) u is in the domain of $\mathcal{M}^{\subseteq\theta_1;\cdots;\subseteq\theta_m}$; then $\mathcal{M}^{\subseteq\theta_1;\cdots;\subseteq\theta_m}, u \models \neg\psi$ and we can take $\Theta' := \Theta \cup \{(\sigma_{new} :_{L'} \neg\psi), (\sigma_{new} :_L \varphi)\}$ and extend the original f into $f' : \mathrm{Prefix}(\Theta') \to W$ by defining $f'(\sigma_{new}) := u$. It follows that $\mathcal{M}^{\subseteq\theta_1;\cdots;\subseteq\theta_m}, f(\sigma_{new}) \not\models \psi$ and $\mathcal{M}^{\subseteq\rho_1;\cdots;\subseteq\rho_n}, f(\sigma_{new}) \models \varphi$, and so Θ' is faithful to $\mathcal{M}$. Otherwise, (2) u is not in the domain of $\mathcal{M}^{\subseteq\theta_1;\cdots;\subseteq\theta_m}$, and a similar argument shows that $\Theta' := \Theta \cup \{(\sigma_{new} :_{L'} \times), (\sigma_{new} :_L \varphi)\}$ is faithful to $\mathcal{M}$.

Theorem 4. *Given any formula φ and any list $L \equiv \rho_1; \cdots; \rho_n$, if there is a closed tableau for $(\sigma_{initial} :_L \varphi)$, then φ is valid in $\mathcal{M}^{\subseteq\rho_1;\cdots;\subseteq\rho_n}$ for all MNMs $\mathcal{M}$.*

4.2 Completeness

We start with the $\cap$-semantics. Define the function $\mathrm{len} : \mathcal{L}_{\mathbf{PAL}} \cup \{\times, L\} \to \mathbb{N}$ as

$$\begin{array}{lll} \mathrm{len}(\times) := 1, & \mathrm{len}(\neg\varphi) := \mathrm{len}(\varphi) + 1, & \mathrm{len}(\varphi \wedge \psi) := \mathrm{len}(\varphi) + \mathrm{len}(\psi) + 1, \\ \mathrm{len}(p) := 1, & \mathrm{len}(\Box\varphi) := \mathrm{len}(\varphi) + 1, & \mathrm{len}([\varphi]\psi) := \mathrm{len}(\varphi) + \mathrm{len}(\psi) + 1, \\ \multicolumn{3}{c}{\mathrm{len}(L) := \mathrm{len}(\varphi_1) + \cdots + \mathrm{len}(\varphi_n) \qquad \text{for } L \equiv \varphi_1; \cdots; \varphi_n.} \end{array}$$

Lemma 5. *Given an open saturated branch Θ, define the model $\mathcal{M}^\Theta = (W^\Theta, \tau^\Theta, V^\Theta)$ as $W^\Theta := \mathrm{Prefix}(\Theta)$, $V^\Theta(p) := \{\sigma \in W^\Theta \mid (\sigma :_\epsilon p) \in \Theta\}$ and, for every $\sigma \in W^\Theta$, $X \in \tau^\Theta(\sigma)$ iff there are φ and L such that*

$$(\sigma :_L \Box\varphi) \in \Theta \text{ and } \{\sigma' \in W^\Theta \mid (\sigma' :_L \times) \in \Theta \text{ or } (\sigma' :_L \varphi) \in \Theta\} \subseteq X.$$

Then, for all lists $L = \rho_1; \cdots; \rho_n$ and all formulas φ,

(i) $(\sigma :_L \varphi) \in \Theta$ *implies* $(\mathcal{M}^\Theta)^{\cap\rho_1;\cdots\cap\rho_n}, \sigma \models \varphi$
(ii) $(\sigma :_L \neg\varphi) \in \Theta$ *implies* $(\mathcal{M}^\Theta)^{\cap\rho_1;\cdots\cap\rho_n}, \sigma \not\models \varphi$
(iii) $(\sigma :_L \times) \in \Theta$ *implies σ is not in the domain of* $(\mathcal{M}^\Theta)^{\cap\rho_1;\cdots\cap\rho_n}$.

Proof. All of (i), (ii) and (iii) are proved by simultaneous induction on $\mathrm{len}(*) + \mathrm{len}(L)$, where $*$ is a formula φ or $\times$. We show the cases (i) and (ii) for $\Box\gamma$. In Appendix A.4, the reader can find arguments for case (iii) fully and the cases for φ of the form p, $[\psi]\gamma$.

Let $\varphi \equiv \Box\gamma$. For (i), assume $(\sigma :_{\rho_1;\cdots;\rho_n} \Box\gamma) \in \Theta$; we show that $[\![\gamma]\!]_{(\mathcal{M}^\Theta)^{\cap\rho_1;\cdots;\cap\rho_n}} \in (\tau^\Theta)^{\cap\rho_1;\cdots;\cap\rho_n}(\sigma)$ or, equivalently, $[\![[\rho_1]\cdots[\rho_n]\gamma]\!]_{\mathcal{M}^\Theta} \in \tau^\Theta(\sigma)$. It suffices to show both

- $(\sigma :_L \Box\gamma) \in \Theta$,
- $\{\sigma' \in W^\Theta \mid (\sigma' :_L \times) \in \Theta \text{ or } (\sigma' :_L \gamma) \in \Theta\} \subseteq [\![[\rho_1]\cdots[\rho_n]\gamma]\!]_{\mathcal{M}^\Theta}$.

The first is the assumption; the second holds by induction hypothesis. For (ii), assume $(\sigma :_L \neg\Box\gamma) \in \Theta$; we show that $[\![\gamma]\!]_{(\mathcal{M}^\Theta)^{\cap\rho_1;\cdots;\cap\rho_n}} \notin (\tau^\Theta)^{\cap\rho_1;\cdots;\cap\rho_n}(\sigma)$ or, equivalently, $[\![[\rho_1]\cdots[\rho_n]\gamma]\!]_{\mathcal{M}^\Theta} \notin \tau^\Theta(\sigma)$, i.e., for all φ and L',

$$(\sigma :_{L'} \Box\varphi) \in \Theta \text{ implies } \{\sigma' \in W^\Theta \mid (\sigma' :_{L'} \times) \in \Theta \text{ or } (\sigma' :_{L'} \varphi) \in \Theta\} \nsubseteq [\![[\rho_1]\cdots[\rho_n]\gamma]\!]_{\mathcal{M}^\Theta}$$

Thus, take any φ and L' such that $(\sigma :_{L'} \Box\varphi) \in \Theta$. By the saturatedness of Θ and rule $(\Box^\cap)$ we obtain, for some fresh σ_{new}, either

$$(\sigma_{new} :_{L'} \varphi), (\sigma_{new} :_L \neg\gamma) \in \Theta \text{ or } (\sigma_{new} :_{L'} \times), (\sigma_{new} :_L \neg\gamma) \in \Theta.$$

In either case, it follows from $(\sigma_{new} :_L \neg\gamma) \in \Theta$ and induction hypothesis that γ is false at σ_{new} in $(\mathcal{M}^\Theta)^{\cap\rho_1;\cdots;\cap\rho_n}$, which is equivalent to $\mathcal{M}^\Theta, \sigma_{new} \not\models [\rho_1]\cdots[\rho_n]\gamma$. This finishes establishing our goal; $\{\sigma' \in W^\Theta \mid (\sigma' :_{L'} \times) \in \Theta \text{ or } (\sigma' :_{L'} \varphi) \in \Theta\} \nsubseteq [\![[\rho_1]\cdots[\rho_n]\gamma]\!]_{\mathcal{M}^\Theta}$.

Theorem 5. *Given any formula φ, if there is an open saturated tableau for $(\sigma_{initial} :_\epsilon \varphi)$, then φ is satisfiable in the class of all MNMs for intersection semantics.*

Proof. By assumption, there is an open saturated branch Θ containing $(\sigma_{initial} :_\epsilon \varphi)$. By Lemma 5, $\mathcal{M}^\Theta, \sigma_{initial} \models \varphi$, which implies the satisfiability of φ in the class of all *MNM*s for intersection semantics.

Now, let us move to the $\subseteq$-semantics.

Lemma 6. *Given an open saturated branch Θ, define the model $\mathcal{M}^\Theta = (W^\Theta, \tau^\Theta, V^\Theta)$ as in Lemma 5 except that, for every $\sigma \in W^\Theta$, $X \in \tau^\Theta(\sigma)$ iff*

$$(\sigma :_L \Box\varphi) \in \Theta \textit{ and } \{\sigma' \in W^\Theta \mid (\sigma' :_L \varphi) \in \Theta\} \subseteq X \quad \textit{for some } \varphi \textit{ and } L.$$

Then, for all lists $L = \rho_1; \cdots; \rho_n$ and all formulas φ,

(i) $(\sigma :_L \varphi) \in \Theta$ *implies* $(\mathcal{M}^\Theta)^{\subseteq\rho_1;\cdots;\subseteq\rho_n}, \sigma \models \varphi$
(ii) $(\sigma :_L \neg\varphi) \in \Theta$ *implies* $(\mathcal{M}^\Theta)^{\subseteq\rho_1;\cdots;\subseteq\rho_n}, \sigma \not\models \varphi$
(iii) $(\sigma :_L \times) \in \Theta$ *implies σ is not in the domain of* $(\mathcal{M}^\Theta)^{\subseteq\rho_1;\cdots;\subseteq\rho_n}$.

Proof. All of (i), (ii) and (iii) are proved by simultaneous induction on $\mathrm{len}(*) + \mathrm{len}(L)$, where $*$ is a formula φ or $\times$. We show cases (i) and (ii) for φ of the form $\Box\gamma$.

For (i), assume $(\sigma :_L \Box\gamma) \in \Theta$; we show that $(\mathcal{M}^\Theta)^{\subseteq\rho_1;\cdots;\subseteq\rho_n}, \sigma \models \Box\gamma$, i.e., $[\![\gamma]\!]_{(\mathcal{M}^\Theta)^{\subseteq\rho_1;\cdots;\subseteq\rho_n}} \in (\tau^\Theta)^{\subseteq\rho_1;\cdots;\subseteq\rho_n}(\sigma)$ or, equivalently, $[\![\langle\rho_1\rangle\cdots\langle\rho_n\rangle\gamma]\!]_{\mathcal{M}} \in \tau^\Theta(\sigma)$. It suffices to show

$$(\sigma :_L \Box\gamma) \in \Theta \text{ and } \{\sigma' \in W^\Theta \mid (\sigma' :_L \gamma) \in \Theta\} \subseteq [\![\langle\rho_1\rangle\cdots\langle\rho_n\rangle\gamma]\!]_{\mathcal{M}^\Theta}$$

The first conjunct is the assumption. For the second conjunct, suppose $(\sigma' :_L \gamma) \in \Theta$; we show that $\mathcal{M}^\Theta, \sigma' \models \langle \rho_1 \rangle \cdots \langle \rho_n \rangle \gamma$, i.e.,

$$\mathcal{M}^\Theta, \sigma' \models \rho_1, \ \ \ldots, \ \ (\mathcal{M}^\Theta)^{\subseteq \rho_1; \cdots ; \subseteq \rho_{n-1}}, \sigma' \models \rho_n \quad \text{and} \quad (\mathcal{M}^\Theta)^{\subseteq \rho_1; \cdots ; \subseteq \rho_n}, \sigma' \models \gamma.$$

These can be derived from $(\sigma' :_L \gamma) \in \Theta$, the rule (*Back*) and induction hypothesis. Therefore, $\mathcal{M}^\Theta, \sigma' \models \langle \rho_1 \rangle \cdots \langle \rho_n \rangle \gamma$, as required.

For (ii), assume $(\sigma :_L \neg \Box \gamma) \in \Theta$; we show that $(\mathcal{M}^\Theta)^{\subseteq \rho_1; \cdots ; \subseteq \rho_n}, \sigma \not\models \Box \gamma$, i.e., $[\![\langle \rho_1 \rangle \cdots \langle \rho_n \rangle \gamma]\!]_{\mathcal{M}} \notin \tau^\Theta(\sigma)$. It suffices to show that, for all L' and all φ,

$$(\sigma :_{L'} \Box \varphi) \in \Theta \quad \text{implies} \quad \{\sigma' \in W^\Theta \mid (\sigma' :_{L'} \varphi) \in \Theta\} \nsubseteq [\![\langle \rho_1 \rangle \cdots \langle \rho_n \rangle \gamma]\!]_{\mathcal{M}^\Theta}.$$

Thus, take any L' and φ such that $(\sigma :_{L'} \Box \varphi) \in \Theta$. Since $(\sigma :_{L'} \Box \varphi), (\sigma :_L \neg \Box \gamma) \in \Theta$, Θ's saturatedness and rule $(\Box^{\subseteq})$ imply, for some fresh σ_{new}, either

$$(\sigma_{new} :_L \neg \gamma), (\sigma_{new} :_{L'} \varphi) \in \Theta \text{ or } (\sigma_{new} :_L \times), (\sigma_{new} :_{L'} \varphi) \in \Theta$$

In either case, it follows from induction hypothesis that either σ_{new} is not in $(\mathcal{M}^\Theta)^{\subseteq \rho_1; \cdots ; \subseteq \rho_n}$, or else $(\mathcal{M}^\Theta)^{\subseteq \rho_1; \cdots ; \subseteq \rho_n}, \sigma_{new} \not\models \gamma$, which is equivalent with $\mathcal{M}^\Theta, \sigma_{new} \not\models \langle \rho_1 \rangle \cdots \langle \rho_n \rangle \gamma$. This finishes establishing our goal; $\{\sigma' \in W^\Theta \mid (\sigma' :_{L'} \varphi) \in \Theta\} \nsubseteq [\![\langle \rho_1 \rangle \cdots \langle \rho_n \rangle \gamma]\!]_{\mathcal{M}^\Theta}$.

Theorem 6. *Given any formula φ, if there is an open saturated tableau for $(\sigma_{initial} :_\epsilon \varphi)$, then φ is satisfiable in the class of all MNMs for subset semantics.*

Proof. Similar to Theorem 5, using Lemma 6 instead.

4.3 Termination and complexity

The same argument works for both semantics. In order to check φ's satisfiability, start the tableau method from the set of terms $\{(\sigma_{initial} :_\epsilon \varphi)\}$ where $\sigma_{initial}$ is the initial symbol. At each step, add non-deterministically at least one term of the form $(\sigma :_L *)$ where σ is a symbol, L is a list of subformulas or negation subformulas of φ and $*$ is a subformula or a negation of a subformula of φ or the symbol $\times$.

Proposition 3. *When executing the tableau method from $\{(\sigma_{initial} :_\epsilon \varphi)\}$, the number of terms $\{(\sigma :_L *)\}$ that can be added is polynomial in the length of φ.*

Proof. As $*$ is a subformula or a negation of a subformula of φ or the symbol $\times$, the number of possible $*$ is linear in the size of φ. The number of possible L is linear in the size of φ since each entry corresponds to an occurrence of an operator $[\psi]$ in φ. The number of possible σ is polynomial in the size of φ since new world symbols σ are created for 4-tuple of subformulas of the form $\Box \psi_1, \neg \Box \psi_2$. Thus, the number of possible terms $(\sigma : \psi)$ is bounded by a polynomial in $|\varphi|$.

Corollary 2. *The satisfiability problem in non-normal monotone public announcement logic is NP-complete.*

Proof. The proof is similar to the proof of Corollary 1 except that we use Proposition 3 instead of Proposition 2 and that we start with $\Theta := \{(\sigma_{initial} :_\epsilon \varphi)\}$ instead of $\Theta := \{(\sigma_{initial} : \varphi)\}$.

4.4 Implementation

We implemented the tableau method for both ∩-semantics and ⊆-semantics in Lotrec-scheme [22]. The tool and the files for logics are available, respectively, at:

http://people.irisa.fr/Francois.Schwarzentruber/lotrecscheme/
http://people.irisa.fr/Francois.Schwarzentruber/publications/ICLA2015/

Appendix A.5 shows an output of Lotrecscheme.

5 Conclusion

We develop tableau system for both intersection and subset **PAL** based on monotone modal logic. Here we present some problems for future work.

- We may generalize our tableau systems to the general dynamic epistemic logic setting. Intersection **DEL** is already proposed in [31] and subset **DEL** is also proposed in [17]. Our idea for developing tableau system for **PAL**s is to take finite sequences of public announcements into consider. In the **DEL** setting, we may consider histories of actions in the action model. Thus we may develop the tableau rules for operations as it is done in [1] for the **DEL** extension of modal logic **K**.
- It is well-known that modal formulas corresponds to conditions on neighborhood frames ([19]). Thus we may consider how tableau systems can be developed for extensions of monotone modal logic with additional modal axioms, and then consider their dynamics extensions. The problem is to take those special frame conditions into account in the tableau rules for modal operations.
- As the satisfiability problems for both intersection and subset **PAL** are in NP, they are reducible to the satisfiability problem for classical propositional logic [20]. We aim at finding elegant reductions for obtaining efficient solvers for both intersection and subset **PAL**.

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A Appendix

A.1 Proof of Lemma 1

Proof. We work only with ($\Box$). Assume Θ is faithful to $\mathcal{M}$; applying ($\Box$) to $(\sigma : \Box\varphi)$ and $(\sigma : \neg\Box\psi)$ in Θ yields $\Theta' := \Theta \cup \{(\sigma_{new} : \varphi), (\sigma_{new} : \neg\psi)\}$. Since $\{(\sigma : \Box\varphi), (\sigma : \neg\Box\psi)\} \subseteq \Theta$, the assumption implies both $\mathcal{M}, f(\sigma) \models \Box\varphi$ and $\mathcal{M}, f(\sigma) \models \neg\Box\psi$; then $f(\sigma) \notin [\![\Box\varphi \to \Box\psi]\!]_{\mathcal{M}} \neq W$ and hence $[\![\varphi \to \psi]\!]_{\mathcal{M}} \neq W$, so there is $v \in W$ s.t. $v \in [\![\varphi]\!]_{\mathcal{M}}$ and $v \notin [\![\psi]\!]_{\mathcal{M}}$. Now, since Θ is faithful to $\mathcal{M}$, there is f s.t. $\mathcal{M}, f(\sigma) \models \gamma$ for all $(\sigma : \gamma) \in \Theta$. The function $f' : \text{Prefix}(\Theta') \to W$, extending f by defining $f'(\sigma_{new}) := v$ (and thus yielding $\mathcal{M}, f'(\sigma_{new}) \models \varphi$, $\mathcal{M}, f'(\sigma_{new}) \models \neg\psi$), is a witness showing that Θ' is faithful to $\mathcal{M}$.

A.2 Proof of Lemma 2

Proof. Both (i) and (ii) are proved by simultaneous induction on φ. We only check the cases where φ is atomic and of the form $\Box\psi$. First, if φ is an atom p, (i) is immediate from the definition of V^{Θ}. For (ii), assume $(\sigma : \neg p) \in \Theta$; since Θ is open, $(\sigma : p) \notin \Theta$, and hence it follows from V^{Θ}'s definition that $\mathcal{M}^{\Theta}, \sigma \not\models p$.

Second, suppose φ is $\Box\psi$. For (i), assume $(\sigma : \Box\psi) \in \Theta$. In order to show $[\![\psi]\!]_{\mathcal{M}^{\Theta}} \in \tau^{\Theta}(\sigma)$, our candidate for a witness of $[\![\psi]\!]_{\mathcal{M}^{\Theta}} \in \tau^{\Theta}(\sigma)$ is, of course, ψ. Thus, it suffices to show that $\{\sigma' \in W^{\Theta} \mid (\sigma' : \psi) \in \Theta\} \subseteq [\![\psi]\!]_{\mathcal{M}^{\Theta}}$, so suppose $(\sigma' : \psi) \in \Theta$; by induction hypothesis, we obtain $\sigma' \in [\![\psi]\!]_{\mathcal{M}^{\Theta}}$.

For (ii), suppose $(\sigma : \neg\Box\psi) \in \Theta$; we show that $[\![\psi]\!]_{\mathcal{M}^{\Theta}} \notin \tau^{\Theta}(\sigma)$, i.e., for all formulas γ, $(\sigma : \Box\gamma) \in \Theta$ implies $\{\sigma' \in W^{\Theta} \mid (\sigma' : \gamma) \in \Theta\} \not\subseteq [\![\psi]\!]_{\mathcal{M}^{\Theta}}$. So take any γ such that $(\sigma : \Box\gamma) \in \Theta$. Since Θ is saturated, it follows from the rule ($\Box$) that there is a prefix $\sigma_{new} \in W^{\Theta}$ such that $(\sigma_{new} : \gamma), (\sigma_{new} : \neg\psi) \in \Theta$. Then $\sigma_{new} \in \{\sigma' \in W^{\Theta} \mid (\sigma' : \gamma) \in \Theta\}$ but, by induction hypothesis, $\sigma_{new} \notin [\![\psi]\!]_{\mathcal{M}^{\Theta}}$.

A.3 Proof of Lemma 3

Here we provide arguments for the remaining cases in the proof of Lemma 3.

($\downarrow \epsilon$): Since $(\sigma :_L p) \in \Theta$, we obtain $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}, f(\sigma) \models p$ so $\mathcal{M}, f(\sigma) \models p$. Hence, $\Theta \cup \{ (\sigma :_\epsilon p) \}$ is faithful to $\mathcal{M}$.

($[\cdot]$): Since $(\sigma :_L [\varphi]\psi) \in \Theta$, we obtain $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}, f(\sigma) \models [\varphi]\psi$. Thus, either $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}, f(\sigma) \not\models \varphi$ or else $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n;\cap\varphi}, f(\sigma) \models \psi$, so either $\Theta \cup \{ (\sigma :_L \neg\varphi) \}$ or else $\Theta \cup \{ (\sigma :_{L;\varphi} \psi) \}$ is faithful to $\mathcal{M}$.

($\times$*Back*): Since $(\sigma :_{L;\varphi} \times) \in \Theta$, $f(\sigma)$ is not in the domain of $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n;\cap\varphi}$. If $f(\sigma)$ is in the domain of $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}$, then φ fails at $f(\sigma)$ in $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}$, so $\Theta \cup \{(\sigma :_L \neg\varphi)\}$ is faithful to $\mathcal{M}$. Otherwise, $f(\sigma)$ is not in the domain of $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}$, so $\Theta \cup \{(\sigma :_L \times)\}$ is faithful to $\mathcal{M}$.

(*Back*): Since $(\sigma :_{L;\varphi} \psi) \in \Theta$, we obtain $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n;\cap\varphi}, f(\sigma) \models \psi$, which implies $\mathcal{M}^{\cap\rho_1;\cdots;\cap\rho_n}, f(\sigma) \models \varphi$. Hence, $\Theta \cup \{(\sigma :_L \varphi)\}$ is faithful to $\mathcal{M}$.

A.4 Remaining Proof of Lemma 5

Here we show case (iii) fully and the cases for φ of the form p, $[\psi]\gamma$ of Lemma 5.

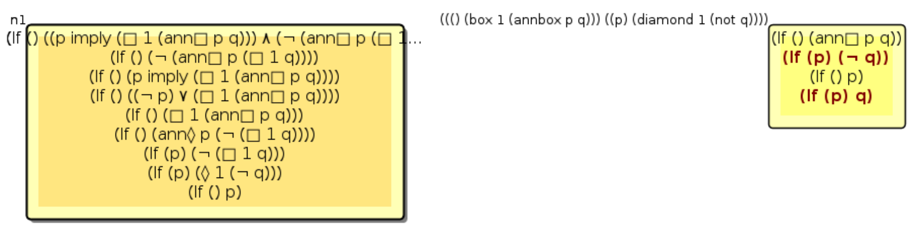
First consider the case (iii). If L is empty, the statement of (iii) becomes vacuously true since Θ is open. Otherwise, $L \equiv \rho_1; \cdots; \rho_n$, and the saturatedness of Θ and the rule ($\times Back$) imply either $(\sigma :_{\rho_1;\cdots;\rho_{n-1}} \neg\rho_n) \in \Theta$ or else $(\sigma :_{\rho_1;\cdots;\rho_{n-1}} \times) \in \Theta$. By induction hypothesis, either $(\mathcal{M}^{\Theta})^{\cap\rho_1;\cdots\cap\rho_{n-1}}, \sigma \not\models \rho_n$ or else σ is not in $(\mathcal{M}^{\Theta})^{\cap\rho_1;\cdots\cap\rho_{n-1}}$. In both cases, σ is not in $(\mathcal{M}^{\Theta})^{\cap\rho_1;\cdots\cap\rho_n}$.

Second, let $\varphi \equiv p$. For (i), suppose $(\sigma :_L p) \in \Theta$; since Θ is saturated, rule ($\downarrow \epsilon$) implies $(\sigma :_\epsilon p) \in \Theta$ so, by definition, $\sigma \in V^{\Theta}(p)$. Moreover, rule (*Back*) and induction hypothesis imply that σ is in $(\mathcal{M}^{\Theta})^{\cap\rho_1;\cdots\cap\rho_n}$; hence, $(\mathcal{M}^{\Theta})^{\cap\rho_1;\cdots\cap\rho_n}, \sigma \models p$. For (ii), use a similar argument now with ($\downarrow \epsilon\neg$) and (*Back*).

Third, let $\varphi \equiv [\psi]\gamma$. For (i), suppose $(\sigma :_{\rho_1;\cdots;\rho_n} [\psi]\gamma) \in \Theta$ and, further, that $(\mathcal{M}^{\Theta})^{\cap\rho_1;\cdots\cap\rho_n}, \sigma \models \psi$; we show $(\mathcal{M}^{\Theta})^{\cap\rho_1;\cdots\cap\rho_n;\cap\psi}, \sigma \models \gamma$. Since Θ is saturated, rules ($[\cdot]$) and (Back) imply either $(\sigma :_L \neg\psi) \in \Theta$ or else both $(\sigma :_L \psi) \in \Theta$ and $(\sigma :_{L;\psi} \gamma) \in \Theta$. But from assumption and induction hypothesis, $(\sigma :_L \neg\psi) \notin \Theta$ and thus $(\sigma :_L \psi) \in \Theta$ and $(\sigma :_{L;\psi} \gamma) \in \Theta$. Then, again by induction hypothesis, $(\mathcal{M}^{\Theta})^{\cap\rho_1;\cdots\cap\rho_n;\cap\psi}, \sigma \models \gamma$. For (ii), suppose $(\sigma :_{\rho_1;\cdots;\rho_n} \neg[\psi]\gamma) \in \Theta$. Since Θ is saturated, rule ($\neg[\cdot]$) implies both $(\sigma :_L \psi) \in \Theta$ and $(\sigma :_{L;\psi} \neg\gamma) \in \Theta$. By induction hypothesis, both $(\mathcal{M}^{\Theta})^{\cap\rho_1;\cdots\cap\rho_n}, \sigma \models \psi$ and $(\mathcal{M}^{\Theta})^{\cap\rho_1;\cdots\cap\rho_n;\cap\psi}, \sigma \not\models \gamma$ so $(\mathcal{M}^{\Theta})^{\cap\rho_1;\cdots\cap\rho_n}, \sigma \not\models [\psi]\gamma$.

A.5 Execution of the tableau method

When we run Lotrecscheme with the tableau method for intersection semantics for the formula $(p \to \Box[p]q) \wedge \neg[p]\Box q$ we obtain the following closed branch at some point:



The branch contains two world symbols (that are the two nodes above). As the node `n1` contains `lf () p` means that the term $(n1 :_\epsilon p)$ is in the current branch.