Scaling and anisotropic heterogeneities of ocean SST images from satellite data

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Oceanic fields display a large variability over large temporal and spatial scales. One way to characterize such variability, borrowed from the field of turbulence, is to consider scaling regimes and multi-scaling properties.

He we use 2D power spectral analysis as well as 2D structure functions $<|X(M)-X(N)|^q>=F(q,d(M,N))$, between two points M and N belonging to the region of interest. By performing statistics with respect to the distance d(M,N), one may extract the scaling property of the 2D field, for a range of distances $L_{min}<d<L_{max}$, of the form $F(q,d)=d^{\zeta(q)}$. This approach can be used even for irregular images (having missing values due to cloud coverage) or for part of images in order to estimate the statistical heterogeneity of different zones of a given image.

In the framework of the French CNRS/IMECO project, we consider MODIS Aqua SST images, in France (English Channel versus Bay of Biscay) and in Chile (Eastern Boundary Upwelling System). We illustrate the use of the 2D structure function analysis for different parts of these images and also different times. Scaling ranges and also scaling exponents are compared. To take into account the anisotropy of some of these zones, an anisotropic version of the 2D structure functions is also used.

Objectives

Characterize the multiscale complexity of satellite images.

In the framework of scaling approach (developed mainly for time series), consider **scaling and multifractal theories** and methods in 2D.

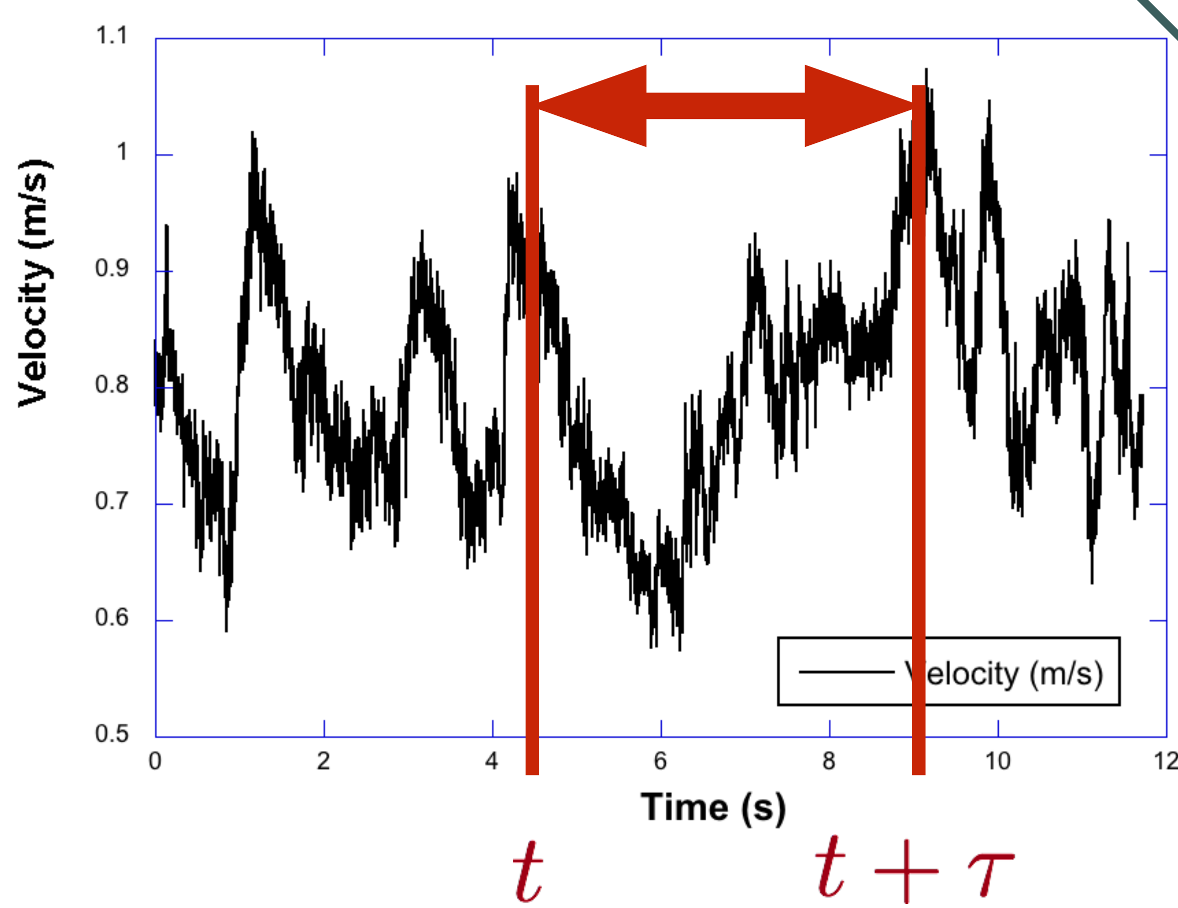
Use 2D structure functions, for two points belonging to images; make statistics over all **orientations** and **distances**.

Illustration with SST satellite images.

Method

In turbulence, use of structure functions since the 1940s (Kolmogorov 1941)
Applied to time series

$$\Delta X_\tau = |X(t+\tau) - X(t)| \qquad f(\tau) =< \Delta X_\tau^2 >$$

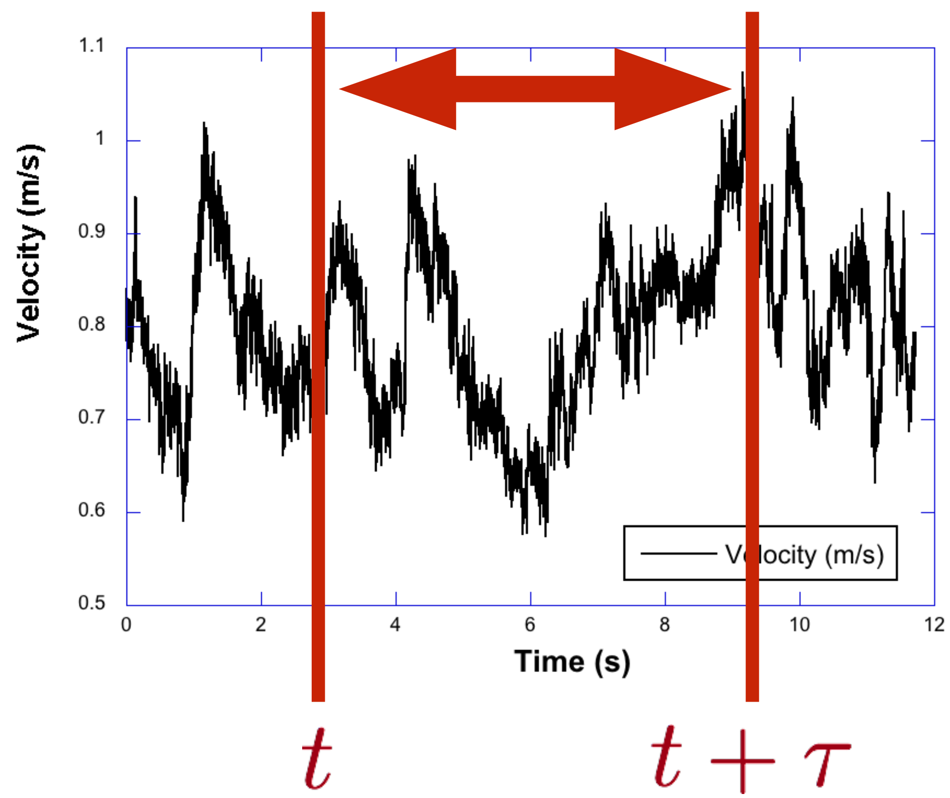


<X> means « statistical average of X

Characterizes the scale dependence of fluctuating fields. Applies to non-stationary series with stationary increments (often the case in geosciences)

Method

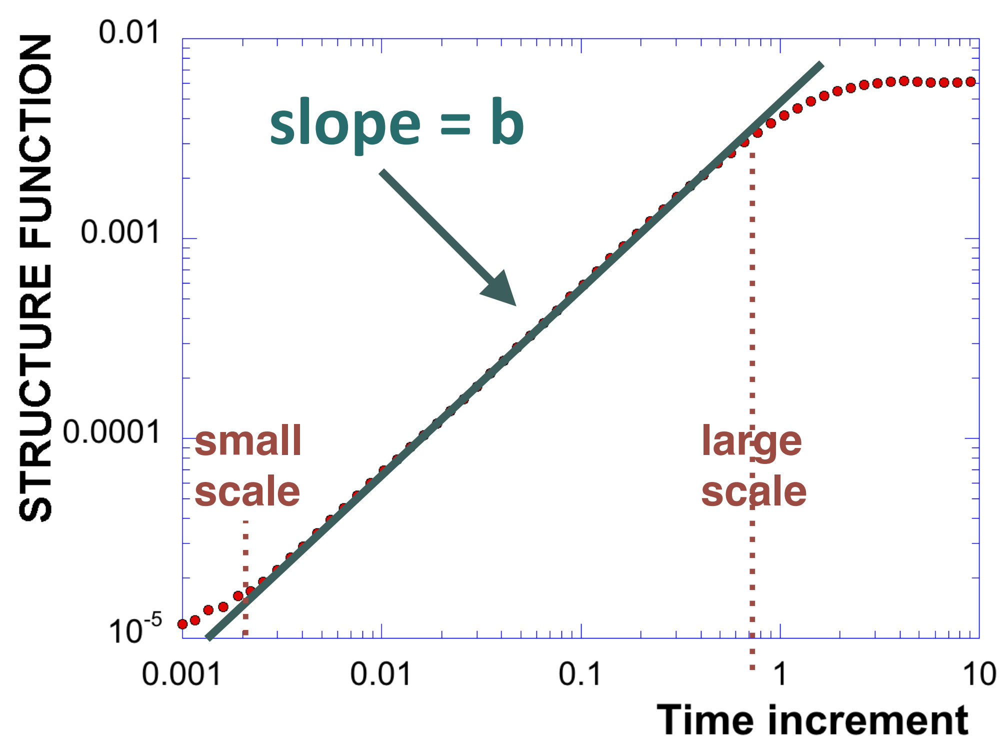
$$\Delta X_\tau = |X(t+\tau) - X(t)| \qquad f(\tau) =< \Delta X_\tau^2 >$$



The following **scaling property** is very classical in turbulence (and many geophysical fields, temperature, pressure, humidity, …)

$$f(\tau) =< \Delta X_\tau^2 >= C \times \tau^b$$

b is estimated by a log-log plot of $f(\tau)$ versus τ



Method

$$\Delta X_\tau = |X(t+\tau) - X(t)|$$

$$f_q(\tau) =< (\Delta X_\tau)^q > \quad q > 0$$

Generalization to moments of order ***q***

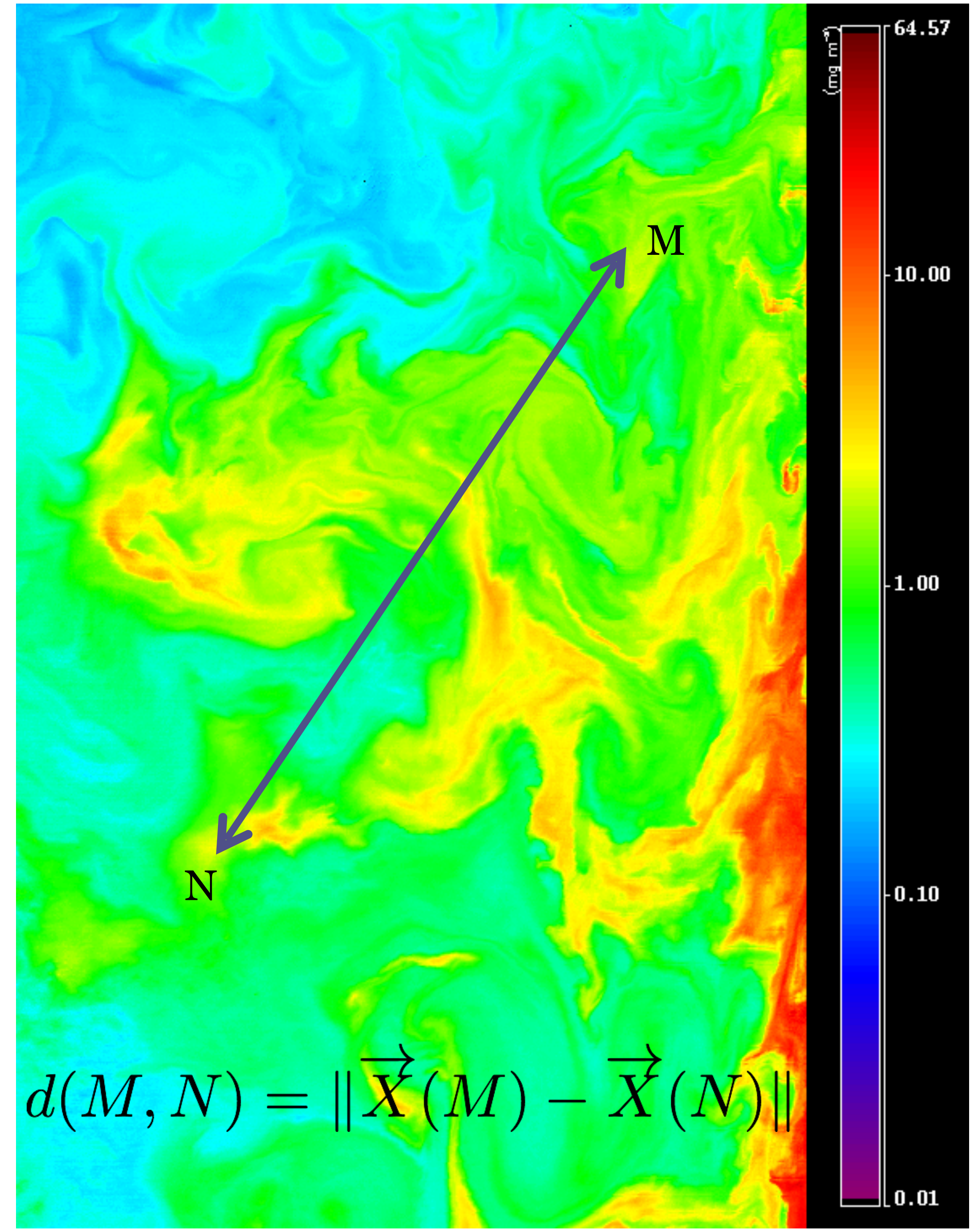
The larger *q*, the larger the fluctuations characterized

Example: *q*=1 medium fluctuations, *q*=4 large fluctuations

Method

Can be done also in 2D

Images: **2D structure functions**
also called « **variogram** » in the field of geosciences (only for moment q=2) (Matheron, 1963): geostatistics, soil studies, geomorphology.
Chase (1992), Lucazeau and Hurtez (1997), Western et al. (1998)



Distance dependence:

$$f_q(d) = < |H(M) - H(N)|^q >$$

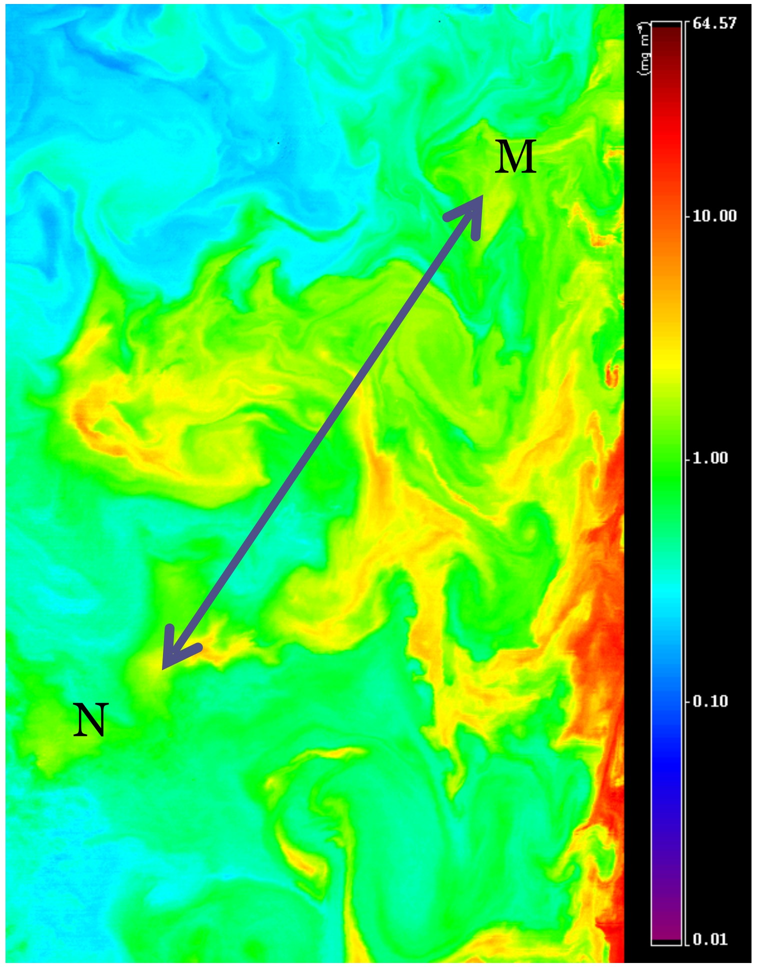
In case of **scaling property:**

$$f_q(d) = < |H(M) - H(N)|^q > = c_q \times d(M,N)^{\zeta(q)}$$

Multifractal scaling exponent

Method

Can be done also in 2D



Advantages:

- Can be used for **irregular** 2D fields
- Characterizes many scales and many intensities in the same time
- Global statistical characterization of the roughness of the data

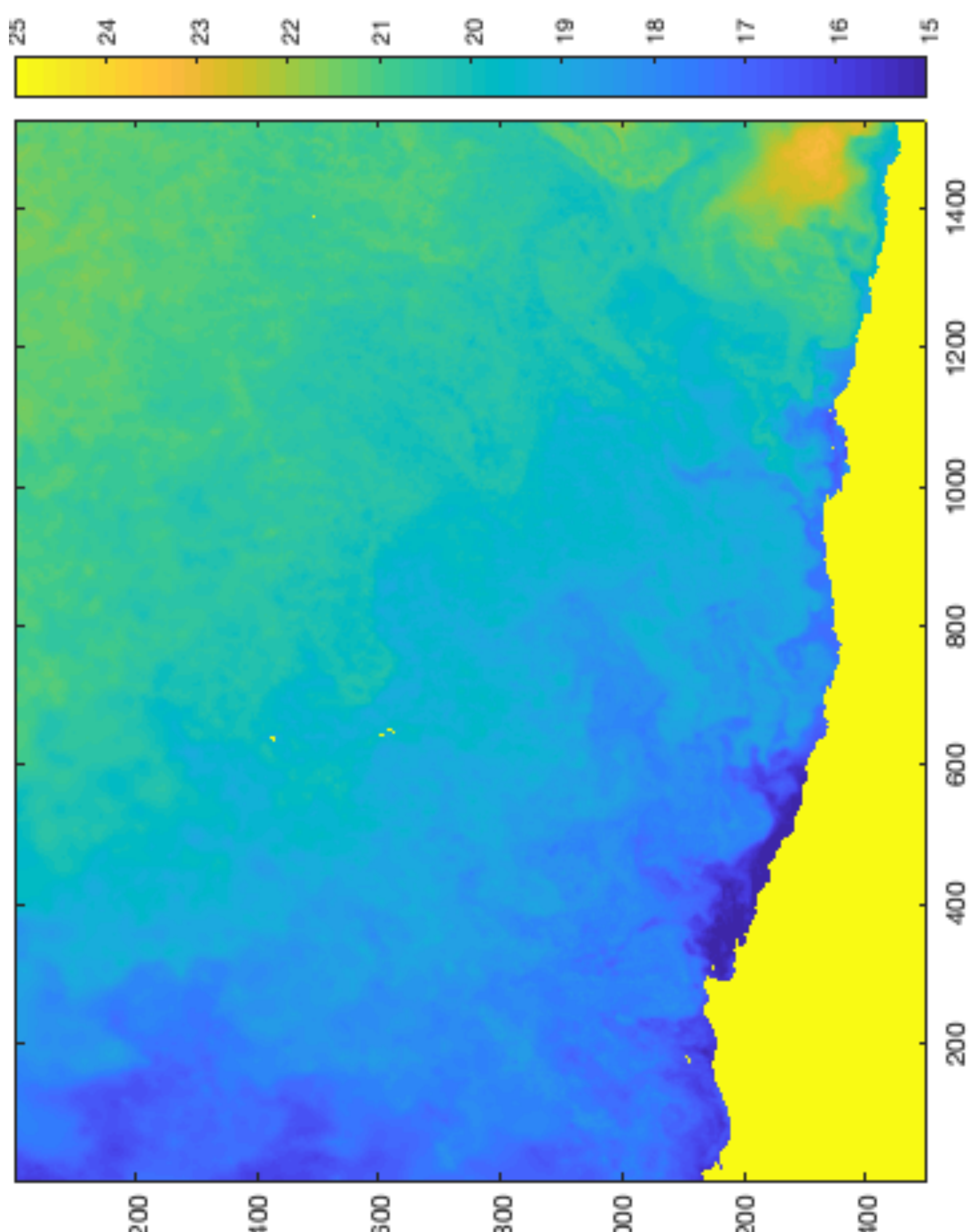
Limits and methodology to overcome this limit:

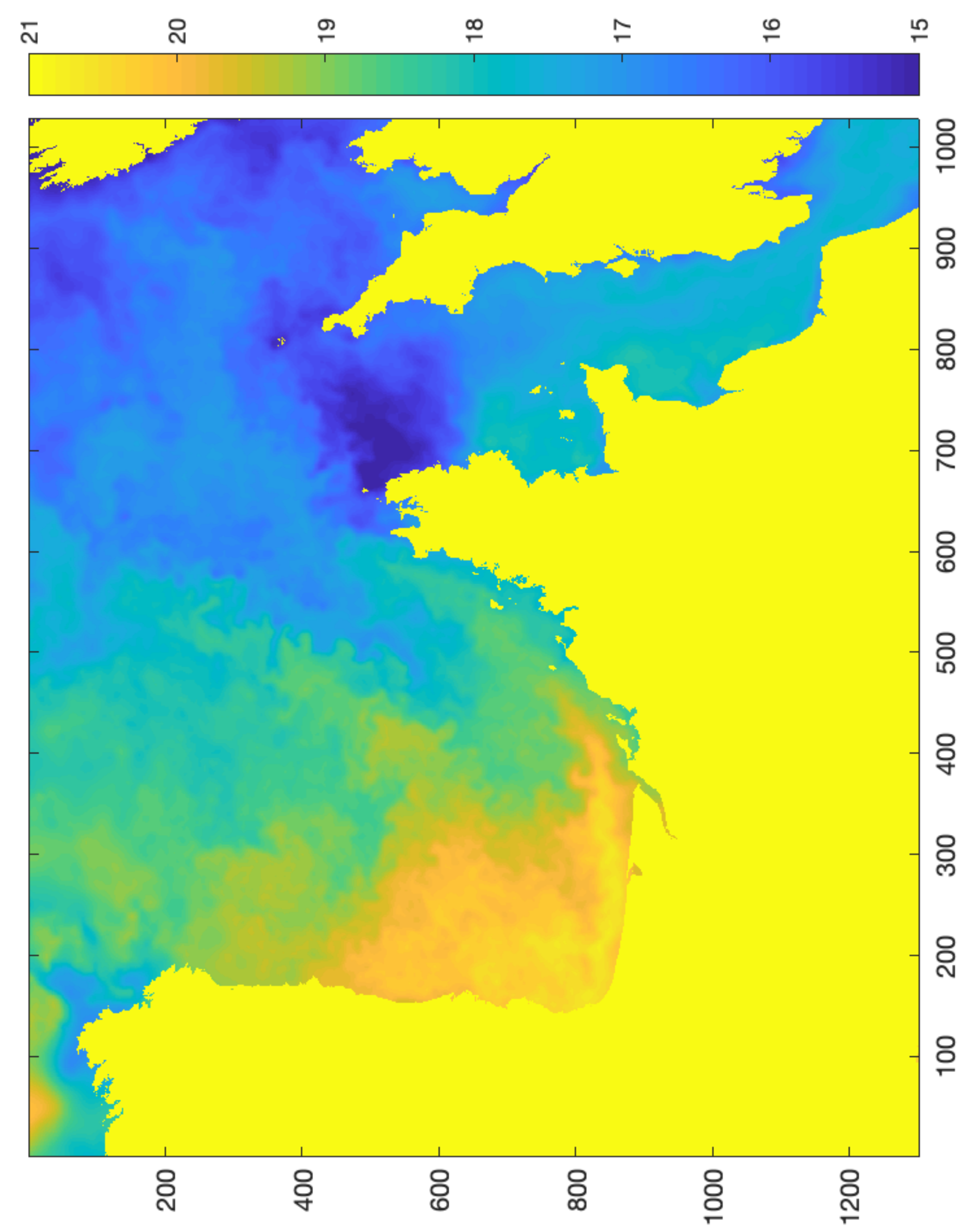
- for 1000 x 1000 images, 10^6 points, 10^{12} couple of points: can be too large for computations;
- Choice of **100 millions of couple of points**, chosen to have enough statistics for each distance: (i) random choice of a point M in the domain; (ii) random direction; (iii) choice of the distance for the other point N (power law statistics for the distance);
- Tested for convergence of statistical moments

Images considered

We consider here SST images as methodological examples.

These are from **MODIS Aqua satellite**, and represent (left) an Eastern Boundary Upwelling Systems (EBUS) near the coast of Chile (-275°E -> 290°E and 40.125°S -> 24.875°S), and (right) the French Bay of Biscay of the Atlantic Ocean and the Channel (350°E -> 3°E and 41.875°N -> 52.125°N). Resolution: 4 km. Below the scale is in pixels

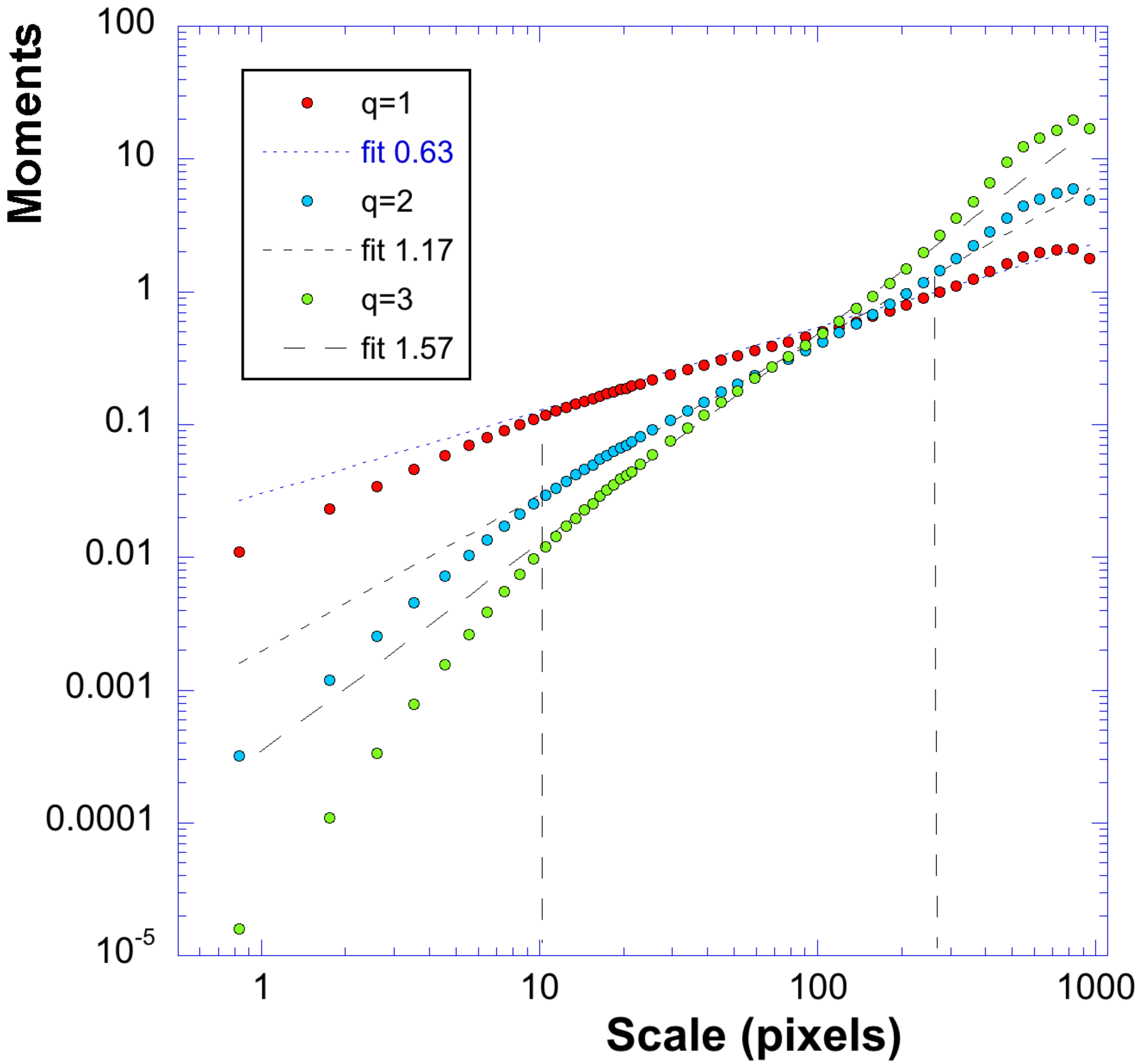




Preliminary results

For an image of size 1000 x 1000 pixels, there are 10^12 couple of points. This statistical approach is thus time consuming, but provides robust statistics.

Below one example performed on image SST2 (Bay of Biscay): statistical moments versus scale, where the scale is here the distance between points M and N



The scaling exponents are the slopes of the scaling curves estimated over the range between the two vertical dotted lines.

Preliminary results

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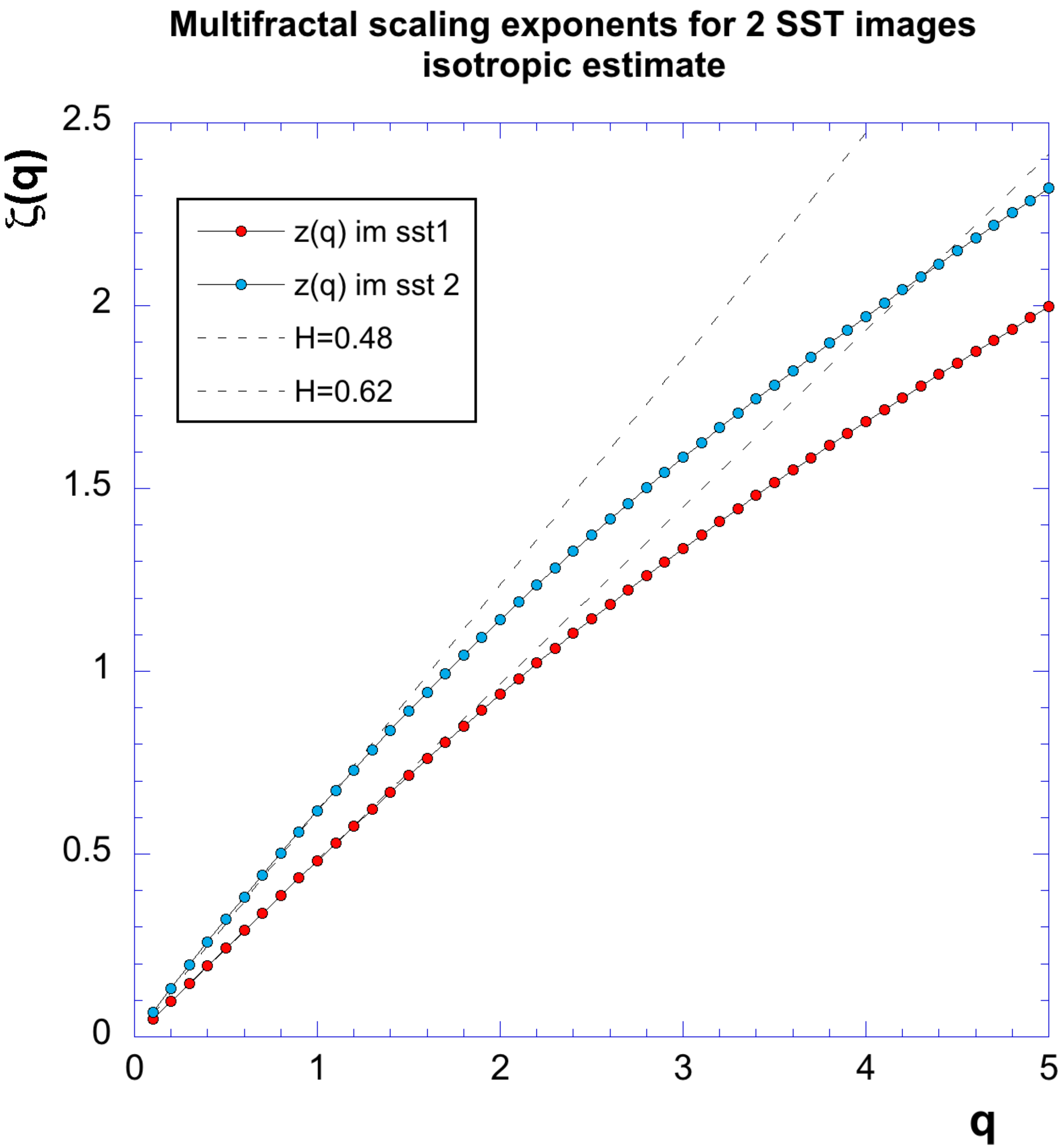


The scaling exponents estimated for images SST1 and SST2.

These curves are nonlinear and convex, as expected. The behaviours are similar, but the first moment H is larger for case SST2. This means that the corresponding field is smoother: SST field of Chile is smoother than French Bay of Biscay.

The convexity of the curves indicates that the fields are multifractal, i.e. a multiscale intermittent behaviour.

Preliminary conclusions

Here we tested a simple generalization of the classical structure function framework which is classically used for the characterization of time series **intermittency and multifractal** properties.

The generalization is to do this in 2D, over a whole image, and all **directions** and all **scales**.

Two points M and N are chosen randomly in an image and the multiscale heterogeneity of the field studied is characterized using statistical moments of increments of the field, where the increment is the distance between the two points.

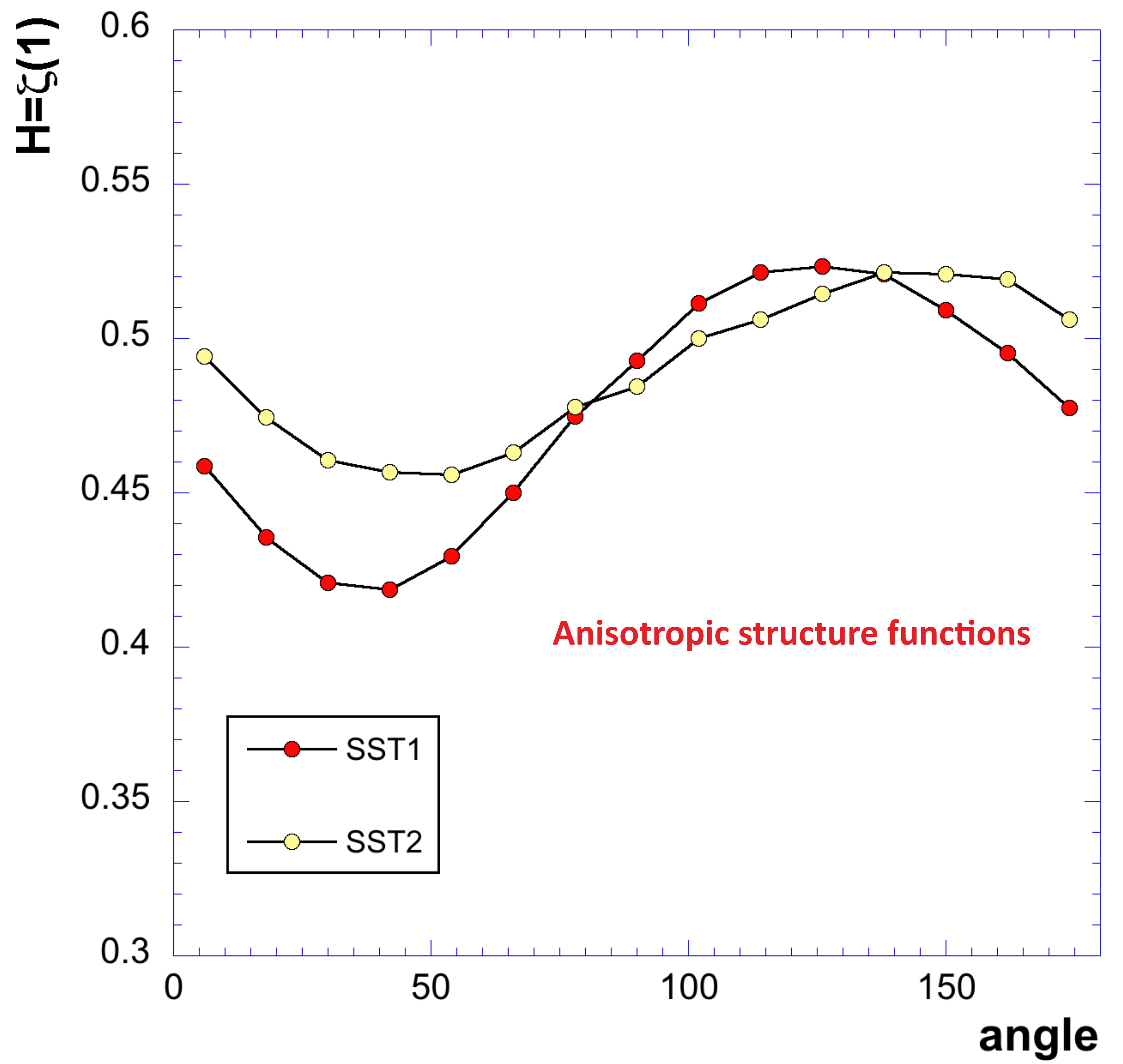
We obtained scaling properties, and scale invariant exponents are extracted over this range of scales. These exponents are used to compare intermittency properties of different images. For example we find similar intermittency (same concave shape of the exponents) but smoother behavior for Chile waters compared to Bay of Biscay.

Future work

- Compare 2D structure functions and singularity exponents for a same image;
- Compare the multiscale properties of satellite images versus model outputs, for a same regional domain;

- Consider **anisotropic structure functions exponents: the scaling is considered verbs the direction of the vector MN.**

  This helps to consider anisotropic scaling processes with different exponents associated with the direction of the vector composing the couple of points M and N. The figure here shows an example using the **two SST images**, for the first moment. We see a smooth evolution of the angle with respect to the direction of the vector.



It is the end…

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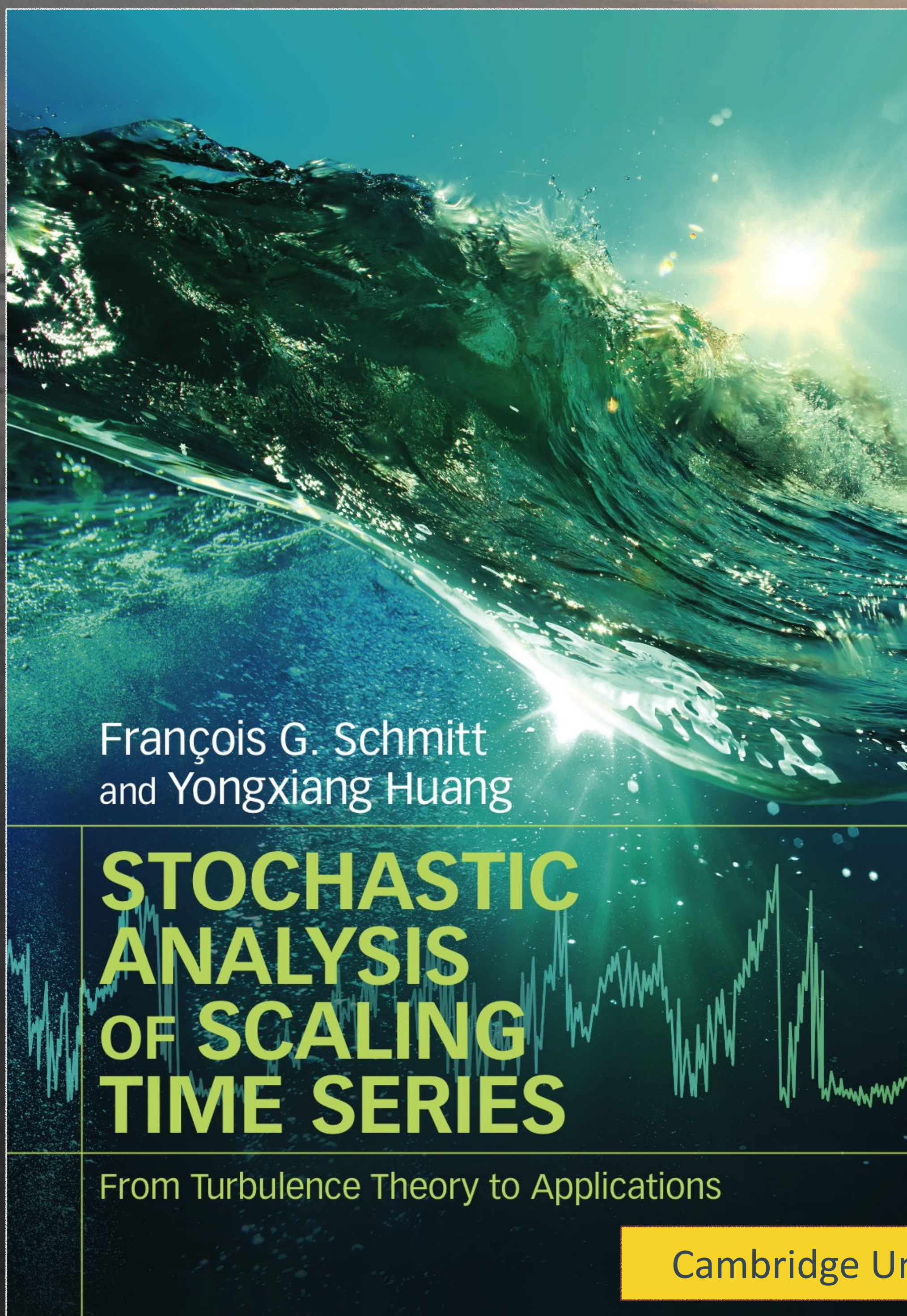

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