Advances in Youla-Kucera parametrization: A Review

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ABSTRACT

Youla-Kucera (YK) parametrization was formulated decades ago for obtaining the set of controllers stabilizing a linear plant. This fundamental result of control theory has been used to develop theoretical tools solving many control problems ranging from stable controller switching, closed-loop identification, robust control, disturbance rejection, adaptive control to fault tolerant control. This paper collects the recent work and classifies them according to the use of YK parametrization, Dual YK parametrization or both, providing the latest advances with main applications in different control fields. A final discussion gives some insights on the future trends in the field.

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Contents

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1. Introduction

The origin of the YK parameterization is in the 70s, Youla Youla, Bongiorno, and Jabr (1976a); Youla, Jabr, and Bongiorno (1976b) and Kucera Kučera (1975) developed the scientific basis. They proposed a parameterization that provides all linear stabilizing controllers for a given Linear Time-Invariant (LTI) plant in a feedback control loop (see Fig. 1). All stabilizing controllers are parametrized based on the transfer function called YK parameter Q, leading to a control form $K(Q)$. The parameter $Q(s)$ is the one guaranteeing the stability. Similarly, its dual theory (also known as the dual YK parametrization) provides all the linear plants stabilized by a given controller. The class of all the plant stabilized by a controller depends on the transfer function called dual YK parameter S, so $G(S)$. This parameter could represent any plant variations. Hence, this useful way of parametrizing either plants, controllers or both is employed to solve many control issues.

According to the control objectives, three main configurations can be targeted:

- **Controller parametrization** allows stable controller reconfiguration when some change occurs. It is also widely used in disturbance and noise rejection control. A number of successful applications can be found in the last two decades, being the most used approach in different control fields Tay, Mareels, and Moore (1997).
- **Plant parametrization** is employed to solve the problem of closed-loop identification. Some successful implementation can be found in Plug & Play control where the dynamics of new sensors or actuators are identified on real-time without system disconnectionNiemann (2003).
- **Simultaneous control and plant parametrization** provides a new control structure that changes according to new identified dynamics on the plant. This principle is mainly used in fault tolerant control and adaptive control Bendtsen and Trangbaek (2012).

The first survey paper gathering YK and dual YK parametrizations is Anderson (1998). It presented 20 years of theoretical development from its origin in the late 70s up to 1998, with special emphasis in Q design for H_∞ and H_2 solutions; closed-loop identification; and some preliminary results in nonlinear systems and disturbance rejection. More recently, a review on the use of YK parametrization in the specific problem of disturbance and noise rejection is provided in Landau (2018). However, this useful tool has given other successful applications in the last two decades. This paper collects the recent work and classify them according to the use of Q, S or both in different practical control systems, covering controller reconfiguration, noise rejection, dynamics identification, adaptive control, fault tolerant control and Plug&Play (P&P) control. The aim is to give an insight of the actual state of YK parametrization and the future trends.

The rest of the paper is organized as follows: in Section 2 fundamentals and mathematical basis of YK and dual YK parametrizations are presented. Section 3 presents the use of the YK parameter Q as a technique of controller reconfiguration for both LTI and Linear Parameter-Variant (LPV) systems. Disturbance and noise rejection control structure based on the YK parameter Q is explained in Section 4. In Section 5, the use of dual YK parameter in closed-loop identification is presented. The combination of both YK parameter and dual YK parameter in adaptive control field is detailed in Section 6. The fault tolerant control scheme based on YK framework is detailed in Section 7. In Section 8, the use of YK parametrizations in Plug & Play control and Multi Model Adaptive Control (MMAC) is briefly reviewed. Finally, some challenging issues about the use of YK parametrization are discussed in the last section.

2. Fundamentals on YK parametrization

Let's consider a Single-Input-Single-Output (SISO) stable plant $G(s)$ connected to a given controller $K(s)$ in a stable feedback loop depicted as in Fig. 1. Closed-loop transfer function $CL(s)$ from reference r to output y is in the following equation:

$$CL(s) = \frac{K(s)G(s)}{1 + K(s)G(s)} \tag{1}$$

The transfer function from the reference $r(s)$ to the controller output $u(s)$ yields:

$$Q(s) = \frac{K(s)}{1 + K(s)G(s)} \tag{2}$$

if $Q(s)$ and $G(s)$ are known, the controller $K(s)$ can be expressed as:

$$K(s) = \frac{Q(s)}{1 - G(s)Q(s)} \tag{3}$$

If $K(s)$ is stabilizing $G(s)$, $Q(s)$ is stable and proper. Reciprocally, if $Q(s)$ is stable and proper, it is easy to demonstrate that $K(s)$ stabilizes $G(s)$ using Eq. 3.

Thus, stabilizing controllers can be parametrized in terms of the set of all stable proper functions $Q(s)$ for a given plant $G(s)$.

Furthermore, the closed-loop transfer function $CL(s)$ in Eq. 1 becomes linear in terms of $Q(s)$ and $G(s)$ which is not the case with $K(s)$.

$$CL(s) = G(s)Q(s) \tag{4}$$

From those two results (3,4), the concept of controllers parametrization appeared for the first time in Newton, Gould, and Kaiser (1957) applied to scalar only known and stable plants.

Youla Youla et al. (1976a, 1976b) and Kucera Kučera (1975) explained simultaneously how the initial idea of controller parametrization can be extended to cover Multi-Inputs-Multi-Outputs (MIMO) plants and that are not necessarily stable. A set of all stabilizing controllers for a given plant is characterized using the so called YK parameter $Q(s)$.

Conversely, a dual concept is proposed by leading the same reasoning in Eqs. (2) and (3) and use the fact that $K(s)$ and $G(s)$ are commutative. The set of all plants stabilized by a given stabilizing controller is characterized using the so called dual YK parameter $S(s)$ Niemann (2003).
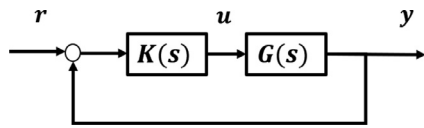
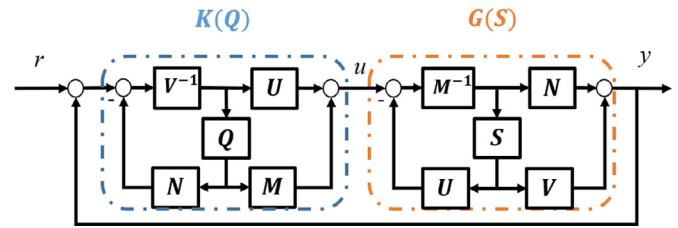**Fig. 1.** Feedback control loop block diagram.

Fig. 2. $K(Q)$ is the set of all controllers stabilizing the plant $G = MN^{-1}$. $G(S)$ is the set of all the plants stabilized by the controller $K = UV^{-1}$.

2.1. Coprime factorization

Both YK and dual YK parametrization are based on the doubly coprime factorization Desoer, Liu, Murray, and Saeks (1980); Vidyasagar (1985) to reduce algebraic complexity in computing $Q(s)$ and $S(s)$.

The plant model and controller matrix transfer functions are factorized as a product of a stable transfer function matrix and a transfer function matrix with a stable inverse with no common unstable zeros as follows:

$$G = NM^{-1} = \tilde{M}^{-1}\tilde{N} \tag{5}$$

$$K = UV^{-1} = \tilde{V}^{-1}\tilde{U} \tag{6}$$

These coprime factors are calculated following the double *Bezout identity* Pommaret and Quadrat (1998) as follows:

$$\begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} \begin{bmatrix} M & U \\ N & V \end{bmatrix} = \begin{bmatrix} M & U \\ N & V \end{bmatrix} \begin{bmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{7}$$

2.2. Q-Parametrization

The YK parametrization Q describes how the set of all stabilizing controllers for a given plant $G(s)$ can be characterized from knowing a controller $K(s)$ stabilizing the given plant $G(s)$.

Lemma 1. Let a plant $G = MN^{-1}$, with N and M coprime and stable, be stabilised by a controller $K = UV^{-1}$, with U and V coprime and stable. Then the set of all stabilizing controllers for G is given as a function of a stable filter YK parameter Q with appropriate dimensions (see Tay et al. (1997)):

$$K = \left\{ K(Q) = (U + MQ)(V + NQ)^{-1} \right\}$$
$$= \left\{ K(Q) = (\tilde{V} + Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M}) \right\} \tag{8}$$

2.3. S-Parametrization

The dual-YK parametrization S describes how the set of all the plants stabilized by a given controller $K(s)$ can be characterized from knowing a plant $G(s)$ stabilized by the given controller $K(s)$.

Lemma 2. Let a plant $G = NM^{-1}$, with N and M coprime and stable, be stabilized by a controller (in positive feedback loop) $K = UV^{-1}$, with U an V coprime and stable. Then the set of all the plants stabilized by K is given as:

$$G = \left\{ G(S) = (N + SV)(M + SU)^{-1} \right\}$$
$$= \left\{ G(S) = (\tilde{M} + \tilde{U}S)^{-1}(\tilde{N} + \tilde{V}S) \right\} \tag{9}$$

2.4. (Q,S)-parametrization

Finally, let's consider the connection between both parametrizations. The parametrized controller $K(Q)$ described in Eq. 8 is connected to the parametrized plant $G(S)$ described in Eq. 9. The resulting closed loop is in Fig. 2.

Lemma 3. The stability of $[G(S), K(Q)]$ is equivalent to the stability of the positive closed-loop $[Q, S]$ Bendtsen and Trangbaek (2012).

Hence, this useful linear way of parametrizing either controllers, plants or both is employed to solve many control issues. The rest of the paper reviews the control applications developed in the last two decades by using Q, S or both.

3. Q-based controller reconfiguration

The main motivation behind controller reconfiguration is performance enhancement while controlling complex systems with multiple control objectives in dynamical environments. Achieving different performance specifications (i.e desired bandwidth, time response, robustness against modeling errors) can be hard while using a single controller. The design process inevitably requires compromises or trade-offs between various conflicting performance objectives. A common solution is designing a controller for each objective, and appropriately switching/interpolating among them in order to accommodate the changing operating conditions, reaching a satisfactory performance level and guarantying overall system stability.

The concept of controller reconfiguration by switching/interpolation different controllers is tackled in the literature Liberzon (2003) using different techniques as self-scheduled approach Packard (1994), ad-hoc technique Rugh and Shamma (2000) or bumpless transfer Zaccarian and Teel (2005).

Controller reconfiguration using YK parametrization is done by mapping a set of linear stabilizing controllers onto a Q-based controller. Main advantages of using YK parametrization in controllers switching/interpolation are: 1) It allows stable switching between open-loop unstable controllers Stoustrup and Niemann (1997); 2) Switched/interpolated controllers can be designed and tuned separately using different techniques (H_∞, LQR, PID) Tay et al. (1997); 3) The closed-loop stability is guaranteed under arbitrary interpolation/switching between different stabilizing controllers, since it is always possible to find a common *Lyapunov* function while using YK parametrization satisfying stability conditions under arbitrary switching signal Hespanha and Morse (2002).

The first application of YK parametrization in controller reconfiguration was in the 80s. Two controllers $K1$ and $K2$ were designed to ensure reference tracking and disturbance minimization respectively Moore, Xia, and Glover (1986). Both controllers were mapped in a single controller structure using two YK parameters $Q1$ and $Q2$.

After that, the use of YK parameter Q in controller reconfiguration evolved from interpolating two controllers to mapping many controllers and switching arbitrary between them, covering both LTI and LPV frameworks.

In this section, YK Q-based controller reconfiguration in both LTI and LPV frameworks are reviewed. Different applications are highlighted showing the transfer from theory to practice in real time experimentations.

3.1. Control implementation

In LTI control framework, a fixed LTI plant is considered $G = NM^{-1}$. Several LTI controllers K_i are designed separately, all of them stabilizing the plant G and each controller fulfilling a specific control objective. The set of controllers K_i are mapped in a multi-controller architecture shown in Fig. 3 using YK parameter Q
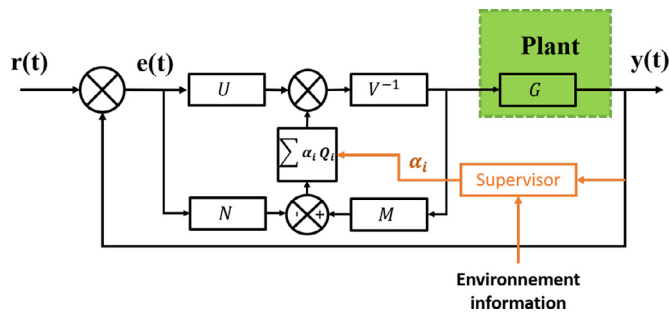
Fig. 3. Q-based multi-controller scheme.

as follow :

$$K(\alpha_i Q_i) = (U + M \sum_{i=1}^{i=n} \alpha_i Q_i)(V + N \sum_{i=1}^{i=n} \alpha_i Q_i)^{-1} \qquad (10)$$

Each parameter Q_i is computed to connect the controller K_i to the nominal controller $K = UV^{-1}$:

$$Q_i = (\tilde{U}_i V - \tilde{V}_i U) \qquad (11)$$

The interpolation signal $\alpha_i \in [0, 1]$ represents the fraction of the controller K_i that is activated at each instant. α_i is computed online by a supervisor that orchestrates the switching between the pre-designed controllers with respect to a performance metric. It is computed using the system output and the current operating conditions described by the environment information.

The goal of the Q-based multi-controller is not only keeping closed-loop stability while switching/interpolation between controllers, but also satisfying a desired level of performance at each instance.

In LPV framework, a LPV plant is assumed $G(\theta)$, where θ is a scheduling parameter within a predefined range $\theta \in [\theta_{\min}, \theta_{\max}]$, with a set of critical design points $\theta_i, i = 1, ., n$.

In many cases, a LPV plant model is a result of a linearisation technique applied to a non linear system, and θ is tied to the system physics (i.e. speed-dependent on lateral vehicle dynamics using the bicycle model, where the parameter θ is the vehicle speed and its inverse). In each critical design point $\theta = \theta_i$, a controller K_i stabilizing $G(\theta_i)$ is designed to satisfy specific performance criteria, the controller K_i is thus named local controller.

Using YK Q parameter, local controllers can be interpolated in a control scheme that satisfies: 1) stability of the closed-loop including the nonlinear plant (linearized into LPV system) under fast changes of the system parameter θ; 2) recovering the local controllers K_i in the critical design points; 3) covering a broad class of local controllers as in LTI framework, including those with multiple inputs/outputs, open-loop unstable dynamics, and controllers with different orders and designed by different techniques (H_∞, PID, LQR).

Q-based multi-controller structure for LPV plant is designed following the steps: first, a nominal controller $K_0(\theta)$ is designed to stabilize the LPV plant over the range $[\theta_{\min}, \theta_{\max}]$ without any performance specification (not necessarily LPV controller). Then, local controllers $K_i(\theta_i)$ can be designed to achieve high performance in critical operating points. Once controllers are designed, they need to be decomposed to LPV coprime factors satisfying a LPV double Bezout identity as the one in Rasmussen and Chang (2010):

$$G(\theta) = N(\theta)M(\theta)^{-1} = \tilde{M}(\theta)^{-1}\tilde{N}(\theta) \qquad (12)$$

$$K_0(\theta) = U_0(\theta)V_0(\theta)^{-1} = \tilde{V}_0(\theta)^{-1}\tilde{U}_0(\theta) \qquad (13)$$

$$K_i = U_i V_i^{-1} = \tilde{V}_i^{-1}\tilde{U}_i \qquad (14)$$

The set of all controllers K_i assumed stabilizing the LTI plants $G(\theta_i)$; are mapped in a Q-based controller $K(\theta) = F_l(J_K(\theta), Q(\theta))$, where F_l is a lower fractional transformation of the interconnection system $J_K(\theta)$ and $Q(\theta)$ (see Tay et al. (1997) for details).

The interconnection system $J_K(\theta)$ includes the nominal controller $K_0(\theta)$ stabilizing the LPV plant over the range of θ:

$$J_K(\theta) = \begin{bmatrix} U_0(\theta)V_0(\theta)^{-1} & \tilde{V}_0(\theta) \\ V_0(\theta)^{-1} & V_0(\theta)^{-1}N(\theta) \end{bmatrix} \qquad (15)$$

and the YK parameter Q is computed to recover the local controllers K_i in the design point $\theta = \theta_i$ using a polytopic system $Q(\theta)$ formed by Q_i and a selected weighting function $\alpha = f(\theta)$ such as:

$$Q_i = (\tilde{U}_i V_0(\theta_i) - \tilde{V}_i U_0(\theta_i)) \qquad (16)$$

The stability of the Q-based LPV controller is proved using a common Lyaponov function all over the range of the parameter θ Xie and Eisaka (2004).

An important consequence of using the YK LPV framework is that LTI design techniques can be adopted for LPV plants too.

3.2. Applications

YK Q-based LTI controllers switching/interpolation has been used in different applications requiring multiple control objectives.

Related to LTI, the intrinsic conflict between performance and robustness in standard feedback loop is tackled in Hespanha and Morse (2002), a Q-based YK controller is used to control the roll angle of an aircraft. Based on a supervisor(Hespanha, Morse, 1996), the YK controller switches between a high performance controller when the measured angle is not noisy, to a robust controller when there are model uncertainties and external disturbances.

In Cifdaloz, Rodriguez, and Anderies (2008) Q-based controller is used to control two classes of complex systems: Irrigation systems and hypersonic vehicles with flexible dynamics. Both classes are generally described by hyperbolic partial differential equations. The Q-based controller switches between different LTI controllers to accommodate time-domain specifications (i.e. peak value of control signal, overshoot). In Trangbaek and Bendtsen (2009); Trangbaek, Stoustrup, and Bendtsen (2008) a method for introducing new components in the control loop in a stable way is presented using the Q-based controller. The controller reconfiguration is illustrated on a livestock stable climate system, where a new temperature measurement becomes available during system operation and a new controller is added to the existent control loop. In Johansen, Kallesøe, Bendtsen, and Andersen (2016) a sequential stepwise commissioning controller for a steam boiler is developed, the Q-based controller switches gradually between two stabilizing controllers with different objectives, the primary one is to keep the water level within max and min bounds, and the second control objective is to keep the feed water flow steady. In Navas and Milanés (2019); Navas, Milanés, and Nashashibi (2016) the Q-parametrization is used in the field of Intelligent Transportation Systems (ITS), the proposed control structure ensure stable switching between Cooperative Adaptive Cruise Control system (CACC) and ACC when the communication between vehicles fails. Another application in ITS is provided in Mahtout, Navas, Gonzalez, Milanés, and Nashashibi (2018), a lateral control structure is proposed to deal with different initial lateral error by switching between two controllers with different objectives.

The use of YK parametrization Q in LPV plants is illustrated in different LPV systems. In Xie and Eisaka (2004) a parameter varying mass-spring-damper system with varying stifness is controlled using Q-based controller. In Rasmussen and Chang (2010) a nonlinear MIMO plant modeling a quadruple tank system controlling the water height in four tanks using two valves; the

plant changes from minimum phase to a non minimum-phase with respect to the operating points (valves values). The Q-based LPV controller switches between a MIMO PID controller designed in the minimum phase operating point to an H_∞ controller in the non-minimum phase operating point. The Q-based controller shows optimal performance in both operating conditions. In Blanchini, Casagrande, Miani, and Viaro (2010) the LPV Q-based controller is used in fixed pole assignment application, the Q-based controller switches among different controllers to locate the closed-loop poles always at the same place independently of the varying parameter. Thus, the LPV closed-loop achieves the same performance in the range of the varying parameter without loosing stability. In Bianchi and Peña (2011) a Q-based LPV controller is designed to control a simulated missile autopilot, the system is modeled using a LPV plant with four design points. The LPV Q-based controller achieves higher performance compared to a classical gain scheduling LPV controller.

3.3. Discussion

The use of the Q-based parametrization in both frameworks LTI and LPV allows performance enhancement of the system while keeping closed-loop stability. This strong property has been proven through different experimental applications. However, the depicted Q-based controllers implementation does not guarantee the closed-loop performance during the switching instant, which remains a practical implementation problem. In order to avoid chattering while switching between two controllers, a minimum dwell-time or a hysteresis is usually included in the switching signal Landau, Lozano, M'Saad, and Karimi (2011). However, when using YK parametrization, the switching can be arbitrary without affecting the stability, enhancing the transient behavior by using minimum dwell-time. Xie et al., Xie (2019) considers average dwell time when switching between linear controllers using YK parametrization. Different switching compensators Q are designed using different techniques to satisfy a given dwell time, comparing their transient behavior. Promising results are presented in a simulation example. In Friedrich and Buss (2019b) the enhancement of the transition behavior is analysed with respect to the choice of the full range LPV initial controller.

4. Q-based noise rejection and vibration control

The problem of noise rejection and vibration control is widely studied in control systems. Such imperfections in control structures are often due to hardware problems (i.e. signal acquisition in sensors, delays in actuators) or environmental disturbances (i.e. external vibrations).

Disturbance and noise rejection have been tackled using different techniques: Pole placement Carmona and Alvarado (2000), output sensitivity shaping Langer and Landau (1999), H_∞ Liebich et al. (2016) and linear matrix inequality (LMI) approach Yucelen, Sadahalli, and Pourboghrat (2010).

YK parametrization is used in noise rejection and vibration control through Q-based controller adaptation. Since the plant is assumed known and constant with eventual small variations, vibrations and noise can be handled just by Q modification.

The first application of YK parametrization in this control field was in 1992 where a feedback controller that can achieve: closed-loop stability; a unit step response and a known sinusoidal disturbance rejection Francis and Tannenbaum (1992) was designed. Authors used the relation between the sensitivity function of the closed-loop and YK parameter Q to design the required parametrized controller. After that, the design of YK parameter Q evolved from offline to online and adaptive design, covering feedback and feedforward implementations.
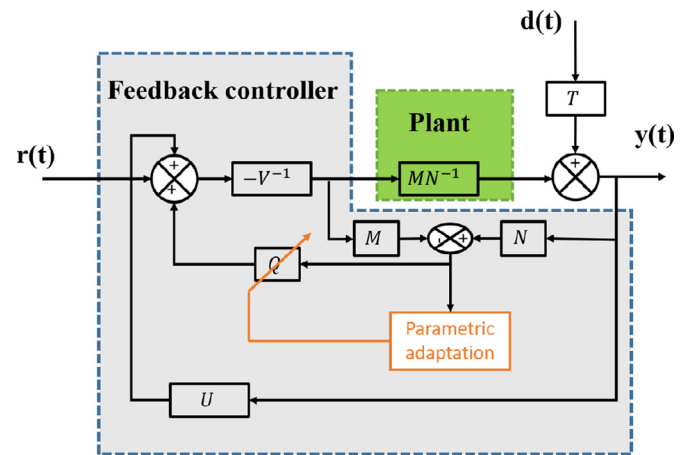

Fig. 4. YK feedback controller scheme: the disturbance propagates through the plant T (so called primary path) and the compensation is done through the plant (MN^{-1}) driven by a feedback compensator Tomizuka (2008).

In this section, the advantages of using YK parametrization both in adaptive feedback configuration as well as in adaptive feedforward compensation schemes is reviewed. A non exhaustive list of recent developments and applications is presented.

4.1. Control implementation

During the last two decades, the use of YK parameter Q in noise and vibration rejection evolved from rejecting a known signal with a given frequency to variable known and unknown spectral characterics. Different implementations of the YK parameter Q are proposed according to the disturbance type Landau (2018).

When a non-correlated measurement with the disturbance signal is available, adaptive feedback controller (see Fig. 4) is used. If the disturbance structure is a priori known (i.e. single frequency signal) the adaptation is direct, otherwise it is indirect.

In direct feedback adaptation, the objective is to directly estimate YK parameter Q which minimizes the disturbance effect without affecting closed-loop stability. This is possible since the disturbance structure is known, thus allowing to define the order of Q. According to Martinez and Alma (2012) two coefficients in the polynomial Q are enough to characterize the frequency of an unknown sinusoidal disturbance. Parameters of the Q filter are adjusted online by minimizing an adaptation error describing the mismatch between the response of the theoretical system model and the disturbed measured response. Error minimization requires using Parametric Adaptation Algorithm (PAA) Doumiati, Martinez, Sename, Dugard, and Lechner (2017).

In indirect feedback adaptation, the objective of noise and vibration attenuation is reached in two steps: 1) identify the disturbance structure by building a disturbance observer (DO); and 2) design a feedback controller achieving different attenuation levels in the frequency domain (not necessarily total rejection as in direct adaptation), by shaping in real time the output sensitivity function to a desired one, assigning an amount of attenuation for each narrow-band disturbance. The updated controller is computed by solving the Bezout equation, the use of YK parametrization allows to reduce the computation load since it simplifies the optimized error equation Airimiţoaie and Landau (2013). A comparative experimental evaluation of rejection of unknown narrow band disturbances on an active vibration control test bench is presented in Landau, Silva, Airimitoaie, Buche, and (NOE) (2013). It compares seven approaches. Interestingly, all the approaches can be interpreted as Youla-Kucera parametrization of a special kind. The performances of the various approaches are evaluated using relevant
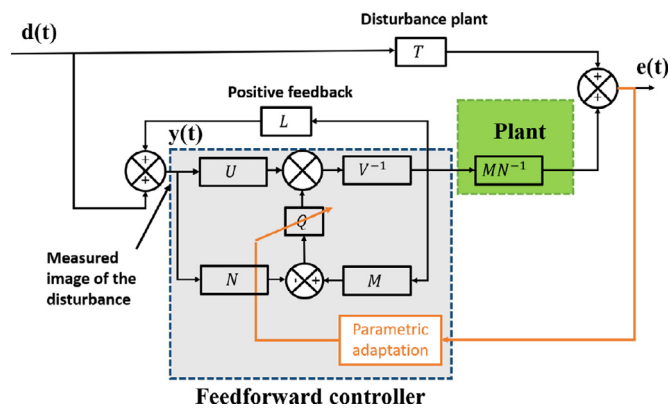
Fig. 5. YK feedforward controller scheme: the disturbance d(t) propagates through T (so called primary path) and its effect is compensated through the plant (MN^{-1}) driven by a feedforward compensator. The input to the feedforward compensator is the sum of the image of the disturbance and of the internal positive feedback. Landau et al. (2019a) .

practical criteria including steady state and transient performance, robustness with respect to low damped complex zeros close to the attenuation zone, robustness with respect to noise and complexity of the various approaches. This benchmark provides useful insights for choosing the best approach according to given constrains.

When a correlation exists between the disturbance signal and measurement, adaptive feedforward controller is used to compensate broad-band disturbances Elliott and Nelson (1993).

The main problem in adaptive feedforward control is the internal positive feedback between the compensation system and the reference source that can cause instabilities (see Fig. 5) while adapting the controller parameters. The use of the YK parametrization allows the separation between the stabilization of the internal positive feedback problem and the optimization of the feedforward controller parameters minimizing the residual noise Zeng and De Callafon (2006). The feedforward controller can be designed either using direct adaptive configuration or indirect one; In indirect configuration, the controller is estimated over a certain time horizon. Then, the YK parameter Q is deduced to switch from the nominal controller to the updated one (see Section 3). In direct configuration, a tuning algorithm is developed to define design adaptive Finite Impulse Response (FIR) or Infinite Impulse Response (IIR) compensators as a YK parameter Q and update directly the controller that minimizes the residual noise Landau (2018).

The main difference between the use of feedback and feedforward implementations relies on the availability of a measurable signal correlated with the noise or the vibration. Also, adaptive feedback implementation is more dedicated to single and multiple narrow-band disturbances rejection. While adaptive feedforward implementation is particularly dedicated to the attenuation of broad-band noise with unknown and time-varying characteristics Landau, Airimitoaie, Meléndez, and Dugard (2019a).

4.2. Applications

The use of YK parametrization Q in adaptive noise and vibration rejection context covered different applications requiring high control precision and low noise sensitivity as: wafer scanning in semiconductors Chen, Jiang, and Tomizuka (2015), data storage systems (reading/writing) Chen and Tomizuka (2013); Martinez and Alma (2012); Wu, Zhang, Chen, and Wang (2018), mechatronicsTomizuka (2008), active suspension systems Doumiati et al. (2017); Landau, Constantinescu, and Rey (2005), and biochemistry Valentinotti et al. (2003) where the regulation problem is to maximize the biomass productivity in the fed-batch

fermentation of a specie of yeasts and the cell growth is considered as an unstable disturbance rejected by the YK parameter Q. In Luca, Rodriguez-Ayerbe, and Dumur (2011) the feedback YK noise rejection controller is extended to control a LPV plant.

Those are encouraging experimental results showing how YK can be used for dealing with the different types of disturbance one can find in different applications. Table 1 presents an overview with the different practical applications, including details on the disturbance type and YK controller design.

4.3. Discussion

The use of YK parametrization in disturbance rejection has been compared to different robust techniques Landau and Meléndez (2017),Landau, Meléndez, Airimitoaie, and Dugard (2019b),Doumiati et al. (2017). Experimentation proved its efficiency against different disturbance types (known/unknown, narrow/broad band).

Current challenge is to ensure noise rejection while important variations in the plant model occur. When designing an adaptive controller to cancel an unknown narrow band disturbance, the uncertainties linked to the plant can lead to instability. This problem is addressed in the literature using the dual YK parameter to model the plant uncertainties Kinney and de Callafon (2009). However, this solution needs to satisfy stability conditions which cannot be fulfilled for all systems especially when the uncertainty is large. A recent work Vau and Landau (2019) showed that this difficulty can be overcome by increasing the order of the filter Q. This solution is called YK overparametrization.

Another possible improvement is to use dual YK parametrization in the identification of disturbance path instead of using autoregressive methods Meléndez, Landau, Dugard, and Buche (2017).

5. S-Based closed-loop identification

This section explains how dual YK parameterization has been used to cope with some of the problems related to closed-loop identification of system dynamics.

System dynamics identification is an essential step for any control design. Initially, identification techniques were developed to carry out open-loop identification. However, in practice many plants cannot be easily operated in open loop for carrying an identification protocol for different reasons (i.e. open loop unstable, integrator behaviour). Additionally, some controllers may be already in place without the option to be disconnected to the plant, being the close loop identification the only way to obtain a better model for either designing a new controller or re-tunning the exiting one in an adaptive way. Several theoretical tools for closed loop identification are proposed the literature Forssell and Ljung (1999); Landau and Karimi (1997). In this section the use of dual YK parametrization as an identification tool is highlighted.

In order to clarify the use of dual YK parametrization in closed loop identification, subsection 5.1 compares a general open-loop identification setup with a general closed-loop identification setup, so advantages of using the second one can be highlighted. Once advantages and disadvantages of this general closed-loop identification method are clear, origin and evolution of this dual YK-based identification scheme are explained in subsection 5.2. Finally, some practical applications and concluding remarks are provided in subsections 5.3 and 5.4 respectively.

5.1. Preliminaries

First, let's consider an open-loop identification case where a control input *u* and measurement noise *n* are assumed to be non-correlated. Some control input *u* can be applied to the system *G*,

Table 1
Applications on the use of YK in noise rejection and vibration control.

Noise	Controller	Applications
No information is available on real time	Feedback controller using a DO (Indirect adapation)	Data storage system Wu et al. (2018): - Multiple narrow band disturbance. - Controller is computed online to filter the observed disturbance. - Q is deduced as a weighted notch filter to switch to the final controller.
	Feedback controller with PAA (Direct adaptation) - YK parameter Q is computed by the PAA to mininize the noise effect.	Acoustic duct Amara et al. (1999): - Single varying sinusoidal disturbance. - Q parameter is a FIR with 2 parameters. Active suspension Landau et al. (2005): - Single varying narrow band disturbance. Blu Ray Disc Martinez and Alma (2012): - Repetitive narrow band disturbance. Road profiler Doumiati et al. (2017): - The road profile is modeled as a disturbance with time varying charachteristics acting on the vehicle dynamics. - The estimated Q polynome represents the road profile
Correlated measurement is available on real time	Feedforward controller using DO (Indirect adaptation)	Astronomical Telescope Tang, Qi, and Yang (2018): - Low frequency large-magnitude narrow-band disturbances. - Different Q designed offline are compared experimentally.
	Feedforward controller with PAA (Direct adaptation)	Duct active noise test-bench Landau et al. (2019a): - Broad band disturbance. - Comparaison between IIR and FIR YK parameter Q is provided. - FIR Q filter is more robust.

obtaining the corresponding output y with noise n:

$$y = Gu + n \qquad (17)$$

If measurements u and y are available, many open-loop identification schemes (Auto Regressive model with eXternal inputs ARX Karaboyas and Kalouptsidis (1991), PBSIDopt van Wingerden (2012) ...) can be used to find cross-correlation with u, and estimate G_i

$$\Phi_{yu} = G\Phi_{uu} + \Phi_{nu} \qquad (18)$$

where $\Phi_{nu} = 0$ as n and u are independent.

Second, let's consider the case where the loop is closed with a controller K, where reference signal r and output noise n are non-correlated. Eq. 17 remains, but Φ_{nu} is not zero, as n is feedback through the controller K affecting the control signal u. Cross-correlation expression results:

$$\Phi_{yu} = G\Phi_{uu} - (1 + K^*G^*)^{-1}\Phi_{nn} \qquad (19)$$

where the superscript $*$ denotes complex conjugation on the $j\omega$ axis.

The identification process becomes complex. Even if Φ_{nn} is really small, Eq. 18 denotes that G should be stable, what could not be the case. As a solution, one could seek to estimate the closed-loop transfer function from the reference signal r to y: $P = KG(1 + KG)^{-1}$. Once an estimation of the CL function is obtained $\hat{P}$, an estimation of the real system $\hat{G}$ would be:

$$\hat{G} = \frac{\hat{P}}{K(1 - \hat{P})} \qquad (20)$$

but problems could occur if K has some unstables poles/zeros; the estimation could result again unstable.

It is then logical to disconnect the plant in order to carry out an open-loop identification. But there will be cases in which this is not possible: The plant is unstable, disconnecting the plant supposes a great economic cost, the feedback controller is embedded

in the system, or an online estimation of the system is needed for controller improvement. Between open-loop and closed-loop identification methods, it is well-known that for model-based control design, closed-loop identification gives better performance Hjalmarsson, Gevers, and Bruyne (1996); but one needs to deal with its associated difficulties: Linear matrix inequality (LMI) feasibility Sznaier and Mazzaro (2003), linear fractional dependence with respect to measured variables Salcedo and Martinez (2008) or linear-deterministic subspace selection Santos, Ramos, and de Carvalho (2007) are some examples picturing these difficulties.

5.2. Control implementation

The dual YK-parameterization appeared as a solution to suppress closed-loop identification difficulties Hansen, Franklin, and Kosut (1989); Schrama (1991) (called Hansen scheme). Specifically, given a LTI initial model and a controller, the key idea was to identify the dual YK S instead of G. Interestingly, the identification of S was a standard open-loop identification problem, so a closed-loop problem is transformed into an open-loop-like problem. Several analysis demonstrated how the obtained model with the Hansen scheme is superior than an open-loop identification solution for subsequent control design Gevers, Ljung, and Van den Hof (2001) Douma, Van den Hof, and Bosgra (2003).

The original LTI Hansen scheme has been modified and extended to deal with more complex systems:

- Related to LTI, De Bruyne, Anderson, and Linard (1998) presented a modification able to tune the order of the resulting model given by the Hansen scheme. The idea is to have control over the possible order explosion when the model is employed in any adaptive control law; Tay et al. (1997) extended the scheme when connected to a controller with the YK filter Q. Robust stabilization results connecting $K(Q)$ and $G(S)$ are here useful for unbiased identification of S when a YK parameter Q

is applied; Sekunda, Niemann, Poulsen, and Santos (2015) modified the original identification scheme in order to avoid the use of indirect excitation signals in reference and feedforward inputs. It imposes any desired excitation signal for the identification of S without affecting those.

- The Hansen scheme has been also extended to LPV systems Bendtsen and Trangbaek (2014) Trangbaek and Bendtsen (2010). Interesting results are obtained in terms of stability, and doubly coprime factors based on these scheduling parameters of the LPV system.
- Related to system with a changing number of inputs/outputs, Bendtsen, Trangbaek, and Stoustrup (2011) modified the Hansen scheme in order to deal with new measurements that become available during online operation. New dynamics related to new sensors are identified by the dual YK parameter S.
- A non-linear initial model G_0 connected to a stabilizing controller K_0 is considered in Linard and Anderson (1996) Linard and Anderson (1997).

5.3. Applications

Closed-loop identification of a piezoelectrically controlled gas bearing using the modified version of the original Hansen scheme in Sekunda et al. (2015) was presented in Sekunda, Niemann, Poulsen, and Santos (2018). It highlights the need of closed-loop identification since the system is unstable without feedback control.

Related to LPV, simulation results of coupled dynamics identification in heat distribution systems with different valve settings are in Trangbaek and Bendtsen (2010). Corresponding experimental results are in Trangbaek (2009). Specifically, strong coupling dynamics in the network are identified after a consumption increment. On the other hand, experimental results related to low-speed longitudinal vehicle dynamics are presented in Navas, Milanés, and Nashashibi (2017).

Concerning the identification of new sensors added to a system, simulation results with a livestock stable ventilation system were presented in Bendtsen et al. (2011). A new temperature sensor is added in order to regulate better the temperature of the stable. The proper identification of the sensor allows the correct adaptation of the corresponding temperature control system.

5.4. Discussion

The dual YK parametrization S is used to cope with closed-loop identification limitations in an open-loop identification framework. This solution plays a key role when there are system variations and controller could not be disconnected due to stability, physical or economical reasons. This type of identification is also important when controller needs to be adapted online to a new situation in the system.

Current challenges are associated to degree reduction of the resulting identified model for its use in iterative adaptive controllers; its application to system with non-linear dynamics (initial work can be found in Linard and Anderson (1996) Linard and Anderson (1997)); and extension of the modified Hansen scheme solving the excitation signals problem and the approach adding sensors/actuators to LPV systems.

6. (Q,S)-based adaptive control

As already stated, the linearity of Q within the closed-loop function facilitates optimization over the class of all stabilizing controllers. Every single controller could be augmented with Q. In this section, this Q is seen as a stable filter that can be optimized/adapted offline or online in order to improve the system

performance depending on the desired response. Notice that this differs from the solution in Section 3, where controllers for different system responses are designed a-priori, switching between them depending on any intelligent algorithm that considers the system environment.

6.1. Control implementation

There are many different ways of designing the corresponding Q to adapt the controller to the specific system situation. This section summarizes some of them highlighting in which situations each solution should be employed. Q, S or both are employed depending if an identification process would be needed to correctly modify the corresponding controller.

Offline optimization of Q for multi-objective controllers is the most basic approach in Q-based adaptive control. The idea is to design a controller in the class of all stabilizing controllers. Different control performance objectives can be set in order to optimize Q. These performance requirements can be described in time or frequency domain. System norms in the frequency domain is directly related to $\mathcal{H}_\infty$ and $\mathcal{H}_2$ concepts. For instance, penalization of the energy of the tracking error and control energy are examples of $\mathcal{H}_2$ control; while penalizing the maximum tracking error subject to control limits is an example of $\mathcal{H}_\infty$ control. Specifically, in Moore and Tay (1989), loop recovery was achieved by augmenting the original $\mathcal{H}_2$ controller with the additional filter Q. They showed how full or partial loop recovery may be obtained depending if minimum or non-minimum phase plants are considered. Improvements over standard loop recovery techniques were obtained.

Next step would be an online modification of Q without any identification algorithm. This is the idea presented in Wang, Mareels, and Moore (1991). Q's optimization process is based on root-mean-square signals measures. A state-space relationship between a nominal plant with disturbances and an observer-based feedback controller K(Q) is obtained. The order of Q should be fixed depending on the application. A steepest descent algorithm is used to obtain the parameter values of the predefined YK parameter Q, so the error is minimized from the disturbances on the system. However, the method is valid when the uncertainty is limited but unknown.

An identification algorithm is needed when there is a large model-plant mistmach. This identification algorithm is directly related to the dual YK parametrization in three different methods: Iterated, nested and indirect adaptive control designs.

For iterated and nested solutions Tay et al. (1997), K(Q) is seen as a controller where Q changes online, as well as G(S) is seen as a nominal plant with an augmentation related to unmodeled dynamics. The process is as follows: First, a nominal controller K is designed for a nominal plant G. Plant-model mistmatch is identified through the dual YK parameter S, and then the augmented controller Q is designed to optimally control S to some performance criteria.

For iterative control design, unmodeled dynamics represented by S are identified by using the Hansen scheme with Q inclusion presented in Section 5. It avoids bias problems in the identification process. S is used in an iterative manner for finding the Q that improves the performance criteria. Iteration is needed as the value of S is not initially reliable or due to new deficiencies in the model. In each iteration the order of the controller increases as S includes the applied Q, followed by a control update step.

For nested control design, successive S are identified on the residual mistmatch between model and plant. An external signal needs to be injected in order to identify the new S. In each step the model of the system is updated. This new model is then taken into consideration for obtaining a new Q, until the performance criteria is fulfilled. This kind of structure is practical when a plant is de-

scribed by different recursive fractional forms Yadav, Voulgaris, and Salapaka (2003).

Although, iterated and nested solutions look sufficient for any system uncertainty, algorithms are conceived for a time-invariant Q property. In order to deal with a time-variant Q, an adaptive version of nested control was proposed in Tay et al. (1989). A fixed structure of Q is created. Parameters in Q are the changing ones, depending on the model-plant mistmach identified by S. As Q varies with time, the unbiased identification provided in the iterated solution is no longer available. External excitation signals are needed in order to identify S, and this could compromise the control performance. For solving so, two different solutions were targeted: two time scales in the adaptive algorithm was proposed in Wang (1991), a faster one for the identification of S, and a much slower for the adaptation of Q; and Q augmentation with a filter in the frequency of the excitation signals needed for the identification of S in Tay et al. (1997).

Finally, structural changes are also considered in connection with both YK and dual YK in Niemann (2006a). It is demonstrated how it is possible to introduce new sensors/actuators into the system, and use them in the YK parameterization to keep a given performance. The stability of the CL system is still affine in Q even if new sensors or actuators are added. This work is the basis of the P&P project, let's see how this explained in the last section of the paper.

6.2. Applications

This section describes some illustrative examples of (Q,S)-based adaptive control solutions.

Teo and Tay (1995) used direct Q adaptive control in a hard disk servo system to minimize the maximum position error signal, which is the deviation of the read/write head from the center of the track.

Simulation results of the direct adaptive-Q controller were presented in Tay and Moore (1991) to illustrate their performance enhancement capabilities when disturbances appear on the system. Part of these results were previously validated in a 55th order aircraft model with a $\mathcal{H}_2$ controller design with Q augmentations for achieving resonance suppression Tay et al. (1997).

Experimental results are in Bendtsen, Trangbaek, and Stoustrup (2013) for laboratory-scale model of a district heating system. In the district heating system, as consumers are not happy with the variable supply rate, differential pressure sensors are added to examine the problem. That revealed a performance problem, so control capabilities are added to another pump, improving the initial $\mathcal{H}_2$ controller through the corresponding augmented Q.

6.3. Discussion

This section reviewed offline and online YK-based methods when system variations exist. Current challenges are associated to the non-linear extension of the YK parameterization; order reduction of the identified S to avoid degree explosion in nested/iterated applications; integration of intelligent control systems as fuzzy control, genetic algorithm or neuronal networks in this adaptive frameworks; and extension of P&P to a more general control structure, because it is only based in observer-based feedback controllers.

7. (Q,S)-based fault tolerant control

The key idea of using Fault Tolerant Control (FTC) strategy is to keep the closed-loop system stable while possibly accepting a reduced performance when critical faults occur in the system (i.e. loss of sensors and/ or actuators). FTC architectures are mainly
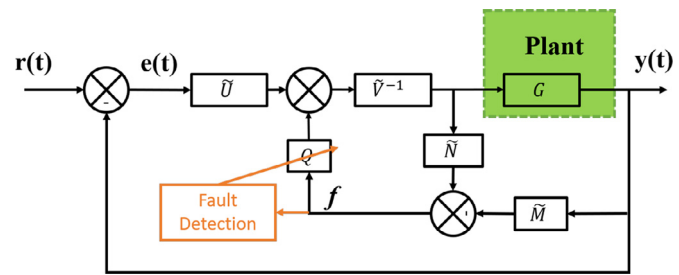

Fig. 6. Q-based fault tolerant control scheme.

based on fault detection and controller reconfiguration techniques Zhang and Jiang (2008).

Both YK parameter Q and dual YK parameter S have been used in different FTC schemes to handle controller reconfiguration and fault diagnosis respectively.

7.1. Control implementation

Different FTC schemes only based on YK parameter Q are proposed in the literature Niemann and Stoustrup (2002). In that case, Q is applied to the controller reconfiguration, while the fault detection part is developed based on some coprime factors. In the FTC scheme depicted in Fig. 6, f is the fault diagnosis signal calculated as:

$$f = \tilde{N}u - \tilde{M}y \qquad (21)$$

When there is no fault, the plant has the same response as the nominal model, which yields to $f = 0$ and the system is controlled by the nominal controller $K = \tilde{v}^{-1}\tilde{u}$ providing adequate performance. Otherwise, the residual signal f is analysed to detect the current fault Ding et al. (2009); Zhou and Ren (2001), and then design the suitable YK parameter Q that modifies the controller in the loop in order to manage the current fault.

Another scheme based on YK parameter Q is proposed in Niemann and Stoustrup (2005b). A FTC architecture is designed to tolerate three possible multiplicative faults. Three Q filters are designed offline according to each fault, when a fault occurs the controller switches smoothly from the nominal one to the fault tolerant one by activating the adequate parameter Q. The extension of this architecture where the number of faults can be changed is proposed in Niemann and Stoustrup (2005a).

Q-based FTC architectures handle systems with additive faults, and the controller is directly changed with Q, without consideration of S, and without affecting the closed-loop stability. When it comes to parametric faults S plays a key role, obtaining the value of Q that makes stable the control loop.

In (Q, S) based FTC structure the dual YK parameter S is used in the fault diagnosis part. When a fault appears in a system, a nonzero S results. If S is unstable, the fault makes the closed-loop system unstable. Then, controller reconfiguration needs to be carried out to recover stability. This reconfiguration is done through Q. A connection with different additive and parametric faults is in Niemann and Stoustrup (2005a) where the optimization of Q is done offline, so only fault diagnosis will be needed in order to choose the proper value of Q. This method is restricted to closed-loop systems with a fault. The fault diagnosis method based on dual YK is extended in Niemann (2006b) to deal with open-loop systems and closed-loop systems with a feedback controller different from the nominal one. The latter is important in fault tolerant control, as fault diagnosis should be running after the first fault has been detected and the controller has been reconfigured. A novel fault tolerant control based control structure to deal with different operational modes (i.e. Start-up or safe mode, normal, full perfor-

mance, reduced performance and closed-down modes) is proposed in Niemann (2012). Fault detection based on dual YK parametrization determines which mode is applied through the corresponding Q.

7.2. Applications

In the last two decades YK and dual YK based FTC controllers have been used in different applications. In Niemann and Stoustrup (2005a) a servo control system with tacho gain fault that leads to instability is successfully handled using (Q,S)-based FTC scheme. Another approach using an observer based controller is proposed in Stoustrup (2009a). The method is applied to a drag racing vehicle control based on a set of observers parametrized with Q, where each observer responds to one or more faults. In Hua, Ding, and Shardt (2018) a fault tolerant control structure is proposed without requiring any model information or identification procedure. It is applied in weakly stochastic environments using a learning of the YK-parameter Q, the effectiveness of the proposed method is demonstrated in simulation of a DC motor. In Li, Luo, Ding, Yang, and Peng (2019) YK parametrization is applied in the case of a non-linear faulty system. A fault detection scheme is investigated to estimate and detect the stability performance degradation. To recover the system performance, a performance-based FTC strategy is proposed. Specifically, a generalization of the YK parametrization to non-linear systems is carried out by replacing right and left coprime factors by stable image representation and stable kernel representation respectively. An initial controller K is designed to ensure the system stability, and Q is added to enlarge the stability performance margin in the case of fault. There will be 2 different thresholds one activating the Q, and other one reconfiguring the initial K in order to ensure stability. An example with a T-S fuzzy controller as a nominal controller is also given in the paper and applied to a simulation example with a faulty three tank-system.

7.3. Discussion

This section reviewed both Q-based an (Q,S)-based FTC architectures, handling different fault types (additive, multiplicative or parametric). Current challenges are associated to system performance enhancement and multi objective control in FTC scheme. The goal is not only to properly control the plant when the fault occurs and avoid instability but also to optimize the closed-loop response for both LTI and LPV plants, an initial implementation is proposed in Yin, Luo, and Ding (2013), where the performance of a LTI plant is automatically optimized and the controller is adapted to the faulty situation.

8. (Q,S)-based P&P and multi model adaptive control

P&P investigates control problems for complex systems with a modular structure. Contrary to the FTC field, P&P control tackled the problem of control loop reconfiguration when a new component (i.e. sensor, actuator) is added. It is based on two stages: 1) detection and identification of the additional components to the system; 2) controller reconfiguration to exploit new sensors or actuators to keep the closed-loop stability and improve the overall performance Stoustrup (2009b).

YK framework is exploited in P&P control as follows: 1) dual YK parameter S is used in the identification part, the added component is usually identified within the closed-loop through the Hansen scheme, since some plants are not permitted to operate in open loop; 2) YK parameter Q is used to modify smoothly the current controller to maintain the system stable and achieve optimal performance. The general theory is explained in Bendtsen et al. (2013).

The applicability of P&P control using YK framework is illustrated on different practical systems. In Michelsen and Trangbaek (2009) a supermarket refrigeration system with the possibility of adding display cases is controlled with a P&P controller based on YK framework, showing the potential of the approach in maintaining stability and performance of the closed-loop. In Trangbaek (2009) two experimental applications of P&P control are presented: a laboratory-scale model of a district heating system, where the control task is to maintain constant differential pressure between the two valves in the heating system. The system has an additional differential pressure sensor, the proposed P&P controller used the added information and improved the pressure regulation.

The second application is a livestock stable climate control system where a real-life experiment is reported, the initial temperature control loop contained a temperature sensor, but the livestock stable is not completely airtight due to cracks in the walls, an additional temperature sensor is added in the cracks area, the P&P controller is extended version of the initial controller taking into account the two temperature information with a variable weighting factor since it is not trivial to attend zero steady state error on both measurements using one actuator.

YK is also employed in the area of MMAC. MMAC is a supervisor that chooses the proper controller among pre-designed candidates controllers once more information is known about the plant. Controllers are designed based on a predefined set of linear models. Once the closer model in the set is known, the switching is direct Anderson, Brinsmead, Liberzon, and Stephen Morse (2001). In Bendtsen and Trangbaek (2012) a MMAC approach using YK framework is proposed to deal with systems subject to significant uncertainties, as noise correlation problem in closed-loop is suppressed by employing the dual YK parametrization. A LPV simulation example with a total of five predefined linear-quadratic-regulator controllers is provided; the closer model in the set to the real system is chosen, switching to the corresponding controller through the correct Q. Finally, as in MMAC the switching is based on the closer model in a predefined set, the CL stability when switching controllers with a real plant is not guaranteed. This situation is analysed in Trangbaek (2011).

9. Discussion

This sections presents some final remarks of the different YK applications accordingly to the research institution where they took place. They are temporally cited to understand the scientific work evolution in the field, providing some insights about the future research directions.

The YK-based applications are mainly developed in four different institutions: Technical University of Danemark, Aalborg University, Grenoble University and University of Berkeley California. Fig. 7 shows a timeline evolution of YK research in the last two decades classified by application type in columns. Different color marks are used for research institutions.

Disturbance and noise rejection applications are in the first column of the timeline in Fig. 7. Research in the area has evolved to deal with disturbances and noise with increasingly complex characteristics, showing experimental results in high-precision systems (i.e. semiconductor manufacturing, active suspension and hard disk drive). In 1999 an experimentation rejecting a single sinusoidal disturbance in an acoustic duct using an adaptive YK parameter Q is described in Amara, Kabamba, and Ulsoy (1999). In 2005, the YK based control structure evolved to cancel a single narrow band vibration in an active suspension Landau et al. (2005). After that, a repetitive and multiple narrow band disturbance were handled in Doumiati et al. (2017); Martinez and Alma (2012). And
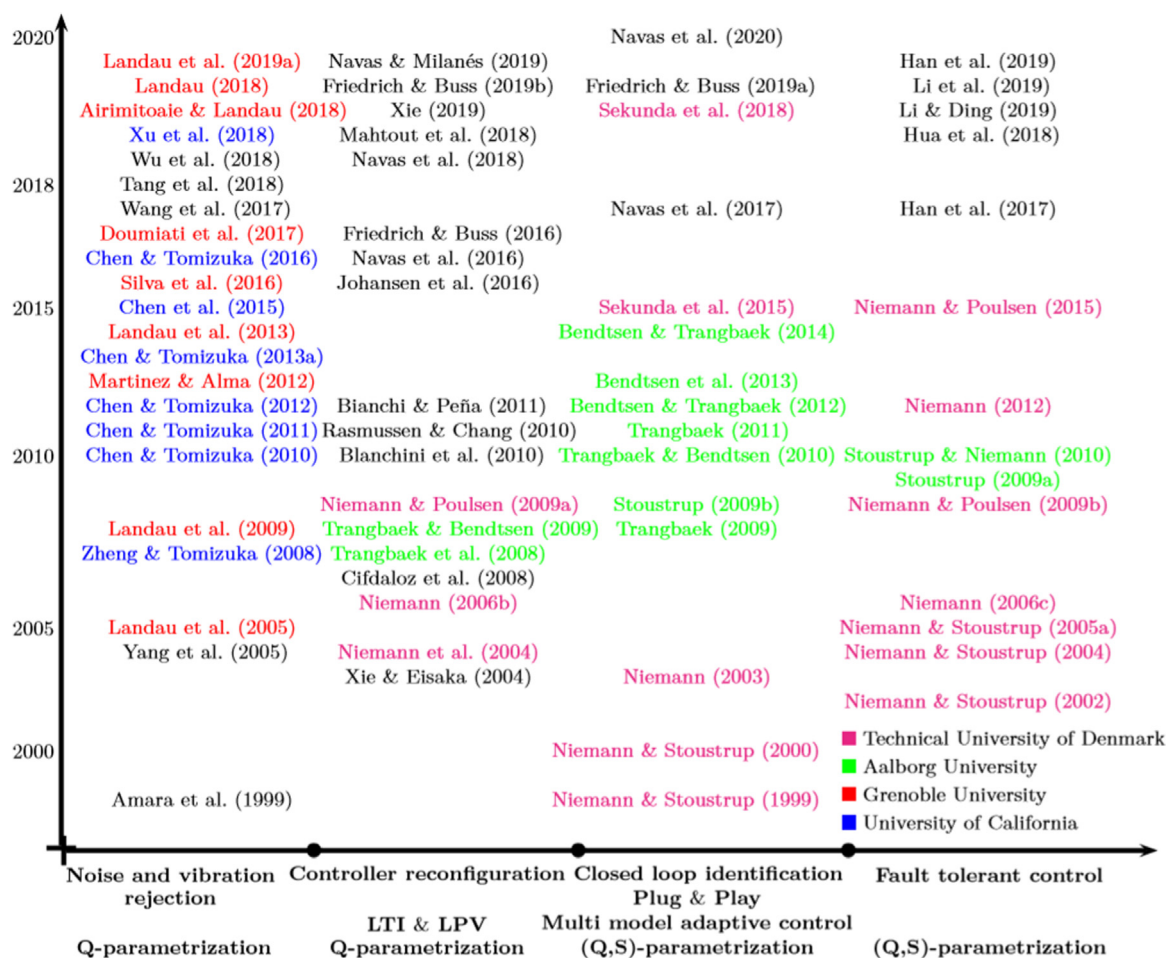
Fig. 7. Timeline with important applications of YK parametrization *Q* and dual YK parametrizartion *S*.

lately, in 2019 YK based control structure is proposed to handle a broad band noise signal Landau et al. (2019a). Research in this field are mainly developed by two teams in the University of California and University of Grenoble (blue and red marks respectively in Fig. 7). The practical importance of the use of YK parameter in adaptive disturbance and noise rejection have formed a rich research field and new challenges and applications remain to be explored considering multiple frequency noise signal, and large variations in the plant model by including the dual YK parametrization.

YK-based controller reconfiguration is in the second column of the timeline in Fig. 7. From 2000 to 2010, papers reported results on YK based controller reconfiguration for LTI systems. In 2004, Technical University of Denmark (marked in pink in Fig. 7) studied the implementation of multi-variable controller based on the YK parametrization, in order to handle the start-up and the shut down of multi-variable systems Niemann, Stoustrup, and Abrahamsen (2004). In 2006, they extended the controller architecture to handle systems with additional sensors and/or actuators in stable way, proposing different implementation structures Niemann and Poulsen (2009). Those results were used in fault tolerant control field. In 2008, YK based controller reconfiguration technique was used to implement a multi-objective controller accommodating the controller to time-domaine specification with illustrative simulation examples Cifdaloz et al. (2008). In 2009, Alborg University (marked in green in Fig. 7) presented an experimentation of controller reconfiguration based on YK parametrization handling the online introduction of a new sensor in the control loop, pro-

viding stability guarantees. In 2010, simulation results on using YK parametrization in controller interpolation of LPV system were provided in Bianchi and Peña (2011); Rasmussen and Chang (2010), extending the initial development on LPV coprime factors studied in 2004 Xie and Eisaka (2004). In 2018 and 2019, developments have been carried out handling controller reconfiguration on ITS systems by the team RITS at INRIA. YK parameter is used to accommodate both longitudinal and lateral behavior of an autonomous vehicle to environment specifications, showing promising experimental results. Latest research on using YK parametrization for controller reconfiguration are more focused on studying the transient behavior while switching between LTI controllers Friedrich and Buss (2019a); Xie (2019).

YK and dual YK based P&P and MMAC control applications are in the third column of the timeline in Fig. 7. Initial research were conducted in 2000, by the Technical University of Denmark (marked in pink in Fig. 7) introducing dual YK based closed loop identification Niemann (2003). After that, using the contemporary results on controller reconfiguration and closed loop identification, University of Aalborg (marked in green in Fig. 7), through its project P&P control developed a novel concept for process distributed control, which allows the control system to self reconfigure once an instrumental change is introduced Stoustrup (2009b). In 2015, a new dual YK based identification scheme is proposed in order to avoid the use of indirect excitation signals in the reference and feedforward inputs modifying the one proposed in 2003 Sekunda et al. (2015) by the Technical University of Denmark, and in 2018 they showed experimental results on using the proposed

scheme. In 2019, experimental results are shown using YK based MMAC in ITS systems Navas, Milans, Flores, and Nashashibi (2020).

YK and dual YK based FTC control applications are in the forth column of the timeline in Fig. 7. Collaboration between Technical University of Denmark and Alborg University significantly contributed on the development of a fault tolerant controller based on both YK and dual YK parametrizations. They proposed first a passive FTC controller Niemann and Stoustrup (2005b), then an active FTC controller Niemann and Poulsen (2006). In 2012, Technical University of Denmark proposed a model-based FTC controller Niemann (2012). In 2015, they extended the active FTC to cover sampled data systems. In 2017, the concept of fault tolerant margin for nonlinear systems is introduced, it plays a key role in detecting system performance degradation caused by some faults Han, Yang, Li, and Ding (2017). In 2019, this concept is used in a novel FTC scheme by applying the Takagi - Sugeno fuzzy dynamic modeling technique in designing YK parameter Q Han, Yang, Li, and Ding (2019). In 2018, a YK based FTC control scheme is introduced using reinforcement learning technique to adapt the YK parameter when some faults occur. As shown, the latest results on YK based FTC are more oriented on using intelligent control techniques to design the adequate YK parameter Q dealing with nonlinear systems.

10. Conclusion

This paper reviewed the use of YK and dual YK parametrizations in different control fields. Different experimental control applications developed during the last 20 years are reported showing the efficiency of YK framework in controlling complex systems. Some challenging issues for potential applications in the use of YK parametrization are listed below:

- Integration of intelligent control system as fuzzy control, model predictive control, genetic algorithm or neuronal networks.
- Order explosion of Q and S which is a real practical issue.
- Limitation of the use of the scalar factor γ regulating the switch and its frequency.
- Maximum level of disturbance that could be rejected when employing YK, and comparison to classical robust control.
- Combination of YK-based fault tolerant control and Plug & Play to a more general control structure.

Declaration of Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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