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## ► To cite this version:

Christelle Caillouet, Martin Heusse, Franck Rousseau. Optimisation de la capacité des réseaux LoRa. CORES 2020 – 5ème Rencontres Francophones sur la Conception de Protocoles, l'Évaluation de Performance et l'Expérimentation des Réseaux de Communication, Sep 2020, Lyon, France. hal-02877138

**HAL Id: hal-02877138**

**<https://hal.inria.fr/hal-02877138>**

Submitted on 22 Jun 2020

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# Optimisation de la capacité des réseaux LoRa

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Nous proposons un modèle théorique pour maximiser la capacité d'un réseau LoRaWAN en termes de nombre de nœuds servis, lorsqu'ils ont tous le même processus de génération de trafic. Le modèle alloue de manière optimale le facteur d'étalement (SF) aux nœuds afin d'optimiser l'atténuation et les collisions. Nous utilisons un modèle de propagation considérant le canal de Rayleigh, et nous prenons en compte l'effet de capture et l'orthogonalité imparfaite des SF tout en garantissant une probabilité de succès de transmission à chaque nœud servi. Les résultats numériques montrent l'efficacité de notre politique d'allocation des SF. Notre cadre quantifie également la capacité maximale des réseaux à une cellule et le gain induit par l'ajout d'antennes sur la zone couverte. Enfin, nous évaluons l'impact de la capture physique et de l'orthogonalité imparfaite des SF sur l'allocation des SF et les performances du réseau.

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## 1 Introduction

The growth of the Internet of Things (IoT) has opened up new challenges in recent years in the establishment of efficient networks, such as Low Power Wide Area Networks (LPWANs). These radios are cheap and able to send and receive short messages over very long ranges with very low power consumption [GR17]. LoRaWAN technology has recently established itself in the LPWAN market. It uses LoRa physical layer based on CSS, Chirp Spread Spectrum modulation, and a simple access method based on ALOHA. There are several spreading factors (SF) to choose from, which allows to trade data rate for range. The capacity of a cell is the number of nodes that a single gateway can handle before losses due to contention or attenuation reach unacceptable levels. Many papers address this question [GR17, BRVA16] to analytically determine the capacity of a LoRa cell. In this work, we present a framework for optimally allocating the spreading factors (SF) in order to maximize the number of served nodes. We consider a realistic propagation model with physical capture that may arise at the gateways, and express the potential interferers of each node considering intra-SF and inter-SF conditions for collisions [BKL17]. Transmissions occurring with the same SF may collide if they happen simultaneously, except if one signal is significantly stronger than the other. The latter is called "capture effect", in which case the gateway can still decode the stronger transmission. The SFs are assumed to be quasi-orthogonal. The imperfect orthogonality of the SF has been studied and the SINR threshold for concurrent transmissions in different SFs have been quantified [GG15, WKGER18]. If two frames with different SF collide, both succeed if they are not significantly stronger than each other.

We develop a linear program to optimally allocate the spreading factors of the LoRaWAN nodes and guarantee a given reception success probability of the frames at the gateway, taking into account intra-SF and inter-SF collisions.

## 2 Model

**Propagation model** All nodes transmit at power  $P_{tx} = 14$  dBm. The signal power at the gateway depends on the distance and Rayleigh fading. The transmission power is attenuated depending on the distance between the transmitter and the gateway :  $P_{rx} = P_{tx} * g(d_i)$ , where  $g(d_i)$  is the path-loss attenuation function based on the Okumura Hata model in a suburban environment with an antenna height of 15 m. Considering a Rayleigh channel, the received signal power is affected by a random variable which follows an exponential distribution with unit mean. The success probability of an isolated frame reception at distance  $d_i$  is :  $\mathbf{H}_i = \mathbf{e}^{-\frac{Nq_f}{P_{rx}}}$  where  $N$  is the thermal noise for a 125 kHz-wide band and  $q_f$  is the minimum SNR for the

corresponding spreading factor  $f$ . We consider the Rayleigh channel property to estimate the success probability of frame reception. If  $H_i \geq \beta$  for SF  $f$ , then the node located at distance  $d_i$  can use the corresponding SF. In case of several gateways,  $i$  can be allocated SF  $f$  if this condition holds for at least one gateway.

**Physical capture** LoRaWAN medium access control scheme can be well approximated by un-slotted ALOHA. A collision may occur between two simultaneous LoRa frames in the same frequency and using the same SF. In reality, colliding transmissions may still be received due to the capture effect. The frame with the highest power can be decoded if the received power at the gateway is at least 6 dB more than the other frame (*i.e.* 4 times stronger) [GR17]. The capture effect can be modeled as follows, considering the average received signal power at the gateway. Let  $C_{ij}$  be a parameter indicating if the LoRa frames of nodes  $i$  and  $j$  using the same SF  $f$  can collide. By definition :  $C_{ij}^f = 1$  if  $P_{rx}^i - P_{rx}^j \leq 6$ , and 0 otherwise. In case there are several gateways, two colliding node transmissions  $i$  and  $j$  in the same SF are lost if the power difference is less than 6 dB at all the gateways. Otherwise, there exists at least one gateway that is capable of decoding  $i$ 's frame while  $j$  is transmitting too.

**Imperfect SF orthogonality** If the transmitting nodes use different SF, then each packet can be demodulated if the difference between the received power is greater than the SINR (Signal-to-Interference-Plus-Noise-Ratio) threshold of each SF (cf Table 1 in [GG15]). We define parameter  $I_{ij}^{ff'}$  for inter-SF potential collisions between SF  $f$  and  $f'$ .  $I_{ij}^{ff'} = 1$  if  $P_{rx}^i - P_{rx}^j \leq SINR_{ff'}$ , 0 otherwise, where  $SINR_{ff'}$  is the value of the required SINR when the transmitting node  $i$  use SF  $f$  and the other simultaneous transmission is done using SF  $f' \neq f$ . In the multiple gateways case,  $I_{ij}^{ff'} = 1$  if all the gateways receive  $P_{rx}^i - P_{rx}^j \leq SINR_{ff'}$ .

**Success probability** Given the total number of potential interferers  $N_i = \sum_{j \neq i} C_{ij}^f + \sum_{f' \neq f} \sum_{j \neq i} I_{ij}^{ff'}$  of node  $i$  using SF  $f$ , we have to ensure that none of these interferers starts a transmission within  $2T^f$  to avoid overlap, where  $T^f$  is the transmission air time at SF  $f$ . The probability of successful transmission for node  $i$  thus equals  $\Pr(\mathbf{i}) = e^{(-2T^f \lambda N_i)}$  where  $\lambda$  is the traffic intensity. We seek to optimally allocate the SF and maximize the number of nodes supported by the network so that the transmission success probability of each served node is greater than a threshold  $\gamma$ .

### 3 Optimal SF allocation

Let  $y_i^f$  be a binary variable stating that spreading factor  $f$  has been assigned to sensor  $i \in I$ , with  $f \in \{7, \dots, 12\}$ . We say that a node  $i \in I$  is *served* if  $\sum_{f \in \{7, \dots, 12\}} y_i^f = 1$ . The optimal framework for SF allocation is given by the following integer program :

$$\max \sum_{i \in I} \sum_{f \in \{7, \dots, 12\}} \omega_i y_i^f \quad (1)$$

$$\sum_{f \in \{7, \dots, 12\}} y_i^f \leq 1 \quad \forall i \in I \quad (2)$$

$$\max_{gateways} H_i \geq \beta y_i^f \quad \forall i \in I, f \in \{7, \dots, 12\} \quad (3)$$

$$e^{-2\lambda \sum_{f \in \{7, \dots, 12\}} T^f y_i^f (1 + \sum_{j \neq i} C_{ij}^f y_j^f + \sum_{f' \neq f} \sum_{j \neq i} I_{ij}^{ff'} y_j^{f'})} \geq \gamma \quad \forall i \in I \quad (4)$$

Objective (1) maximizes the number of served nodes in the network by maximizing  $\sum_i \sum_f y_i^f$ . We associate a weight parameter  $\omega_i$  to give a priority in the SF allocation. If a node can be served, then we want to smallest possible SF while meeting the global requirements in terms of transmission probability. We choose a weight that decreases with the spreading factor in order to encourage small SF :  $\omega_i = (1 - H_i)$  since probability  $H_i$  increases with the SF. At most one SF  $f$  can be assigned to a node  $i \in I$  (Constraints (2)) if it meets the signal strength reception condition for at least one gateway (with probability H) (Constraints (3)), and if the success probability among concurrent transmissions is greater than  $\gamma$  for all the nodes (Constraints (4)). Since we do not know what spreading factor the nodes will use, these constraints are non linear. We

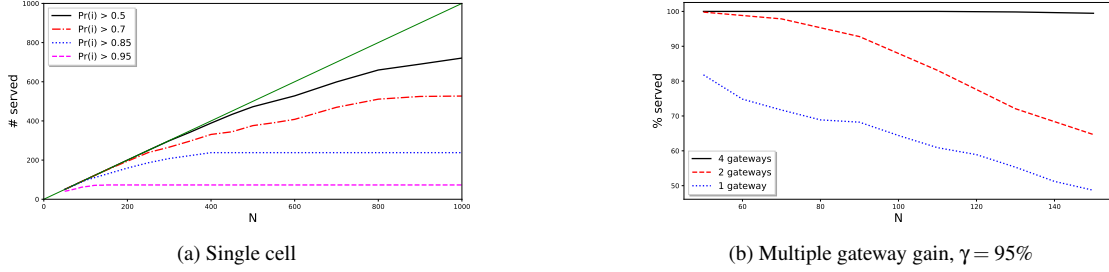


FIGURE 1: Capacity analysis.

linearize them using the log function and *big M* parameter and replace the set of constraints (4) by the following linear constraints defined for each  $i \in I$  and  $f \in \{7, \dots, 12\}$  :

$$T^f \left( 1 + \sum_{j \neq i} C_{ij}^f y_j^f + \sum_{f' \neq f} \sum_{j \neq i} I_{ij}^{ff'} y_j^{f'} \right) \leq -\frac{\log(\gamma)}{2\lambda} + M(1 - y_i^f), \forall i \in I, f \in \{7, \dots, 12\} \quad (5)$$

## 4 Results

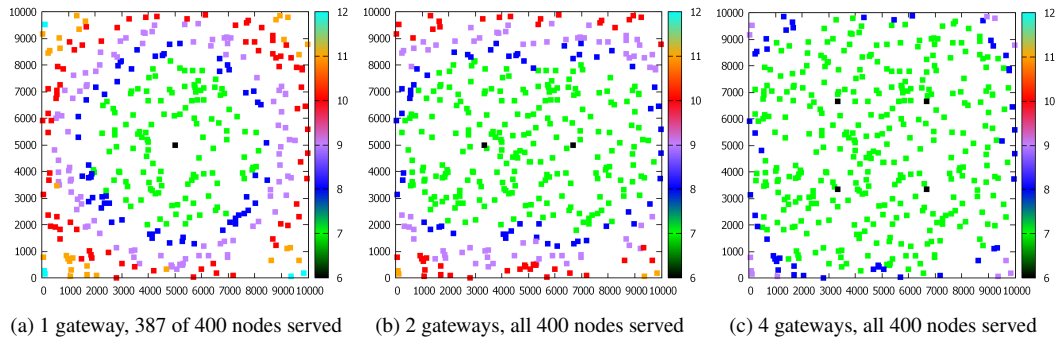
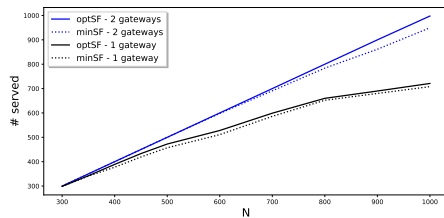
We consider an  $10\text{km} \times 10\text{km}$  square area with one, two, or four regularly placed gateways, cf. Figure 2. We randomly deploy  $N$  nodes (from 50 to 1000). We consider a network with a single application, in which case all nodes produce the same traffic. We compute  $\lambda$  as the maximum intensity for any node at their authorized duty cycle limit, and using 59 B frames. It corresponds to 2.47 s of airtime, thus  $\lambda = \frac{1}{747s}$ . SNR threshold and airtime values for all SF are respectively  $\{-6, -9, -12, -15, -17.5, -20\}$  (dB) and  $\{102, 184, 328, 616, 1315, 2466\}$  (ms). The model is implemented in Java and solved using IBM CPLEX solver 12.8 on an Intel Core i7-5500U CPU, 2.40 GHz, 32 Gb RAM computer under Linux Fedora operating system. The resolution time limit of CPLEX has been set to 1h.

**Network performances** Figure 1a depicts the number of served nodes in function of  $N$ , the number of deployed nodes in the cell. All nodes can be served for small-sized networks except for very high required success probability  $\gamma$ . The number of served nodes increases with  $N$  until it reaches a maximum value corresponding to the maximum cell capacity : 73 nodes in a cell with  $\gamma = 95\%$  with  $N \geq 150$ , 238 nodes for  $\gamma = 85\%$  and  $N \geq 400$ , 527 nodes for  $\gamma = 70\%$  and  $N \geq 900$ , and more than 720 nodes for  $\gamma = 50\%$  since the limit has not been reached for  $N = 1000$ .

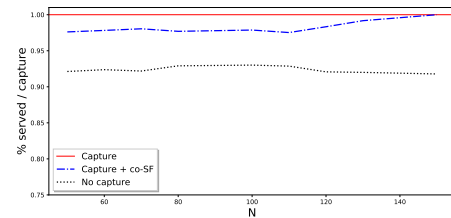
Increasing the number of gateways in the network provides a capacity gain quantified in Figure 1b. The gain is at least 20% for  $\gamma = 95\%$  when we double the gateways in the area, given the same set of deployed nodes. And for  $\gamma = 50\%$ , even with 2 gateways, 100% of nodes can be served for all values of  $N \in [50, 1000]$ , which gives already a gain of 30% for  $N = 1000$  compared to the single cell capacity.

**SF allocation** We evaluate our SF allocation policy balancing attenuation and collision by comparing it to the distance-based SF allocation. In this case, the allocation depends on the distance to the gateways and the propagation model. The smallest SF achieving  $H \geq \beta$  is allocated to the node. In case of multiple gateways, the selected SF is the minimum SF among all the reachable gateways. For sparse cells, our SF allocation (optSF) provides the same capacity as the distance-base SF allocation (minSF). However for dense cells, our policy gives better results in terms of number of served nodes (Figure 3a). Figure 2 depicts the optimal SF allocation for  $N = 400$  and  $\gamma = 50\%$ . Around the SF boundaries, some nodes may receive SF  $f + 1$  while being closer to nodes at SF  $f$ , highlighting the fact that the SF boundaries should be carefully optimized.

**Impact of physical capture and SF orthogonality** We compare two policies : (i) SF allocation without capture effect where all nodes sharing an SF interfere, and (ii) SF allocation with physical capture as defined in Section 2. The capacity increases when considering the capture effect. We quantify this gain in Figure 3b for  $\gamma = 95\%$ . The number of served nodes for each policy is normalized against case 2 (called "Capture"). We see that without physical capture ("No capture"), the capacity loss is around 8% for all values of  $N$

FIGURE 2: Optimal SF allocation for a network of  $N = 400$ ,  $\beta = 66\%$ ,  $\gamma = 50\%$ .

(a) Comparison between SF allocation policies.



(b) Effects of physical capture and SF orthogonality.

FIGURE 3

corresponding to 6 nodes that cannot be served. The imperfect SF orthogonality affects nodes located near the gateways (using SF7) and those far away (using SF11 or 12). This affects the capacity when the nodes too close to the gateway cannot be served (see Figure 2a). The capacity with imperfect SF orthogonality ("Capture + co-SF" on Figure 3b) is lower than the one without (labeled "Capture"). However, imperfect SF orthogonality does not impact the maximum capacity (Figure 3b).

## 5 Conclusion

In this paper, we present an optimal framework for the SF allocation problem in order to maximize the number of served nodes in a LoRaWAN network while ensuring a transmission success probability to the nodes taking into account the physical capture and imperfect SF orthogonality. The simulation results show the effectiveness of our strategy both in terms of deployment and computation cost. Following this work, we would like to validate this allocation by simulation and experimentally to quantify the benefits of proper SF allocation in terms on packet delivery ratio in a realistic environment.

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