Linear response for spiking neuronal networks with unbounded memory

Bruno Cessac, Rodrigo Cofré, Ignacio Ampuero

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Bruno Cessac

Biovision Team, INRIA Sophia Antipolis,France.

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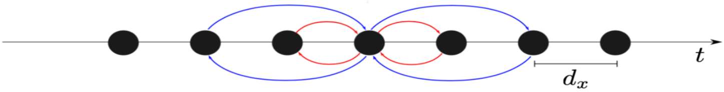
with
Rodrigo Cofré, CIMFAV, Facultad de Ingeniería, Universidad de Valparaíso, Valparaíso, Chile. and
Ignacio Ampuero, Departamento de Informática, Universidad Técnica Federico Santa María, Valparaíso, Chile

Spontaneous spiking activity



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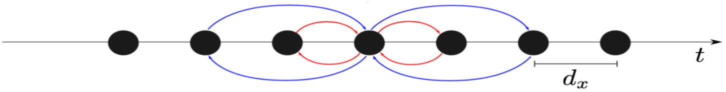

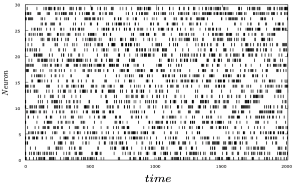

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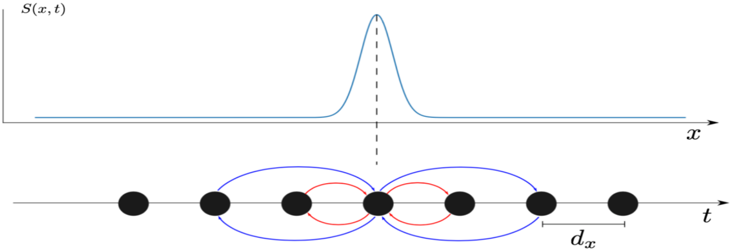
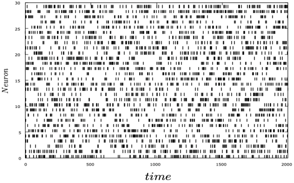


$\Rightarrow$



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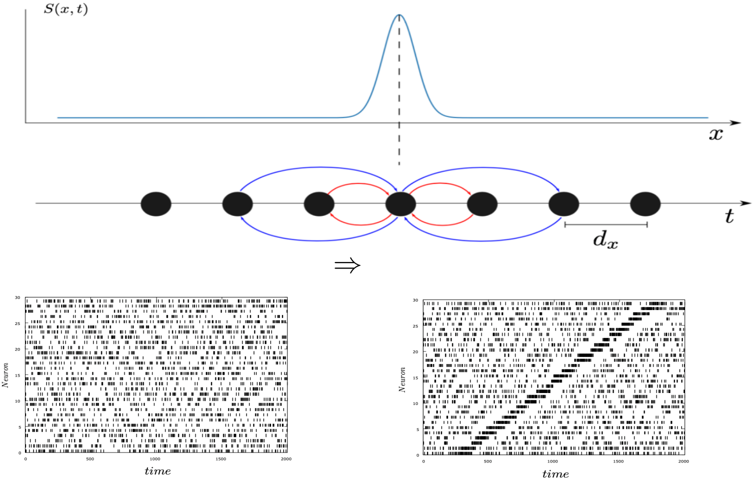

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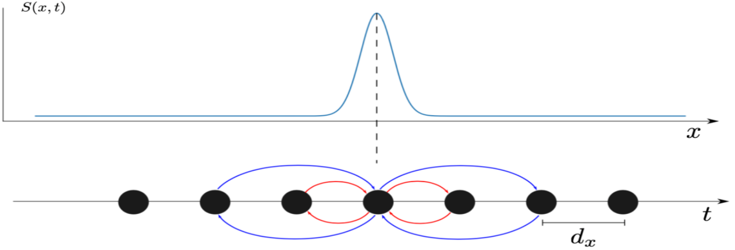

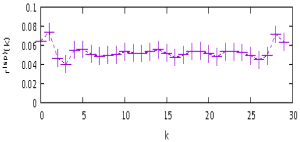

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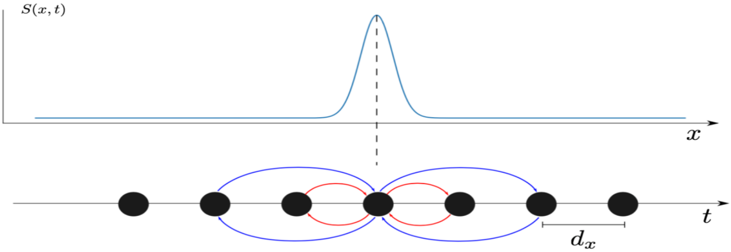

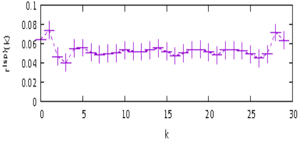

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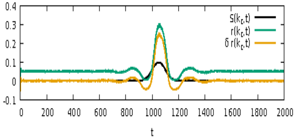
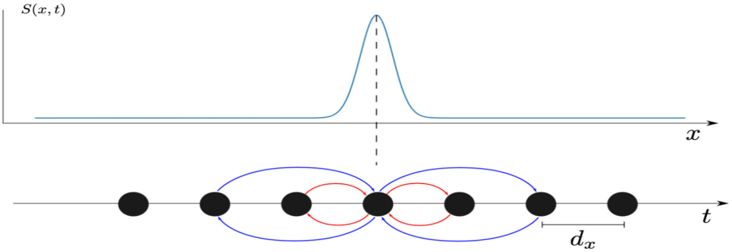
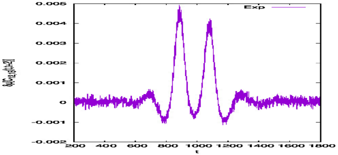


Sample averaging (non stationary)

Spontaneous spiking activity





Variation in the correlation $C_{12,15}(t, t+2)$

Correlation variations are triggered by the stimulus, but they are constrained by the network dynamics.

Question

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How to compute the spatio-temporal response of a neuronal network to a time dependent stimulus ?

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Can we compute the time variation of spike-observables as a function of the stimulus and spontaneous dynamics ?

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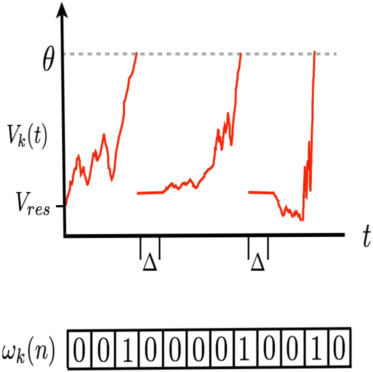
submitted to J. Math. Neuro, arXiv:1704.05344

From spiking neurons dynamics to linear response

An Integrate and Fire neural network model with unbounded memory

R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013



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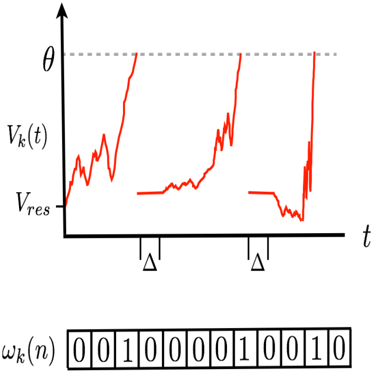

Spikes

- Voltage dynamics is time-continuous.
- Spikes are time-discrete events (time resolution $\delta > 0$).

$$t_k^{(l)} \in [n\delta, (n+1)\delta[$$
$$\Rightarrow$$
$$\omega_k(n) = 1$$

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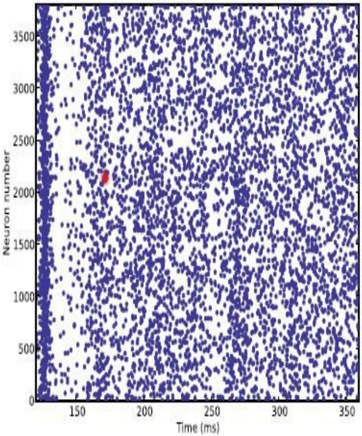

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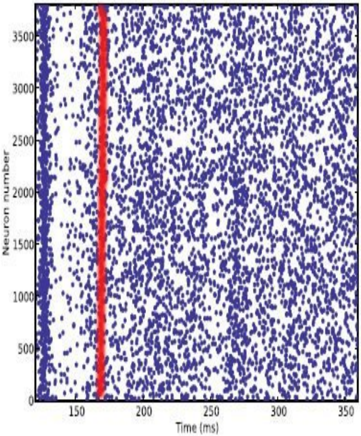

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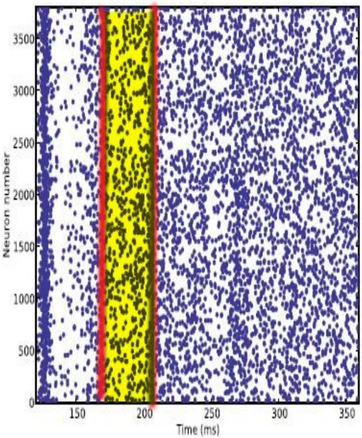

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- Spike block ω_m^n.

A conductance-based Integrate and Fire model

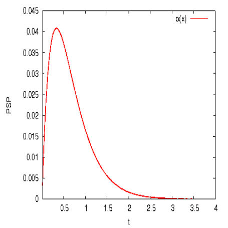
M. Rudolph, A. Destexhe, Neural Comput. 2006, (GIF model)
R.Cofré, B. Cessac, Chaos, Solitons and Fractals, 2013

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} = -g_{L,k}(V_k - E_L)$$

$$-\sum_j g_{kj}(t,\omega)(V_k - E_j)$$

Synapses



$$\alpha_{kj}(t) = \frac{t}{\tau} e^{-\frac{t}{\tau_{kj}}} H(t),$$

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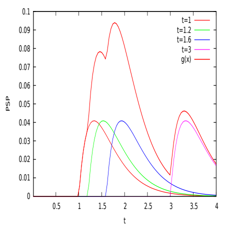

$$g_{kj}(t) = g_{kj}(t_j) + G_{kj}\alpha_{kj}(t - t_j)$$

$$t > t_j$$

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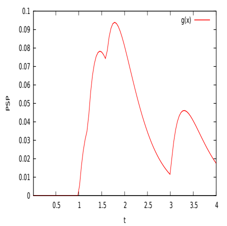

$$g_{kj}(t) = G_{kj} \sum_{n \geq 0} \alpha_{kj}(t - t_j^{(n)})$$

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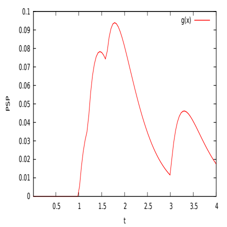

$$g_{kj}(t,\omega) = G_{kj} \sum_{n \geq 0} \alpha_{kj}(t - n\delta)\omega_j(n)$$

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$$+S_k(t) + \sigma_B \xi_k(t)$$

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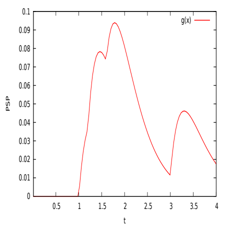

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A conductance-based Integrate and Fire model

Sub-threshold dynamics:

$$C_k \frac{dV_k}{dt} + g_k\,(\,t,\omega\,)\,V_k = i_k(t,\omega).$$

$$W_{kj} \stackrel{\mathrm{def}}{=} G_{kj} E_j$$

$$\alpha_{kj}(t,\omega) = \sum_{n \geq 0} \alpha_{kj}(t - n\delta)\omega_j(n)$$

$$i_k(t,\omega) = g_{L,k} E_L + \sum_j W_{kj}\alpha_{kj}(t,\omega) + S_k(t) + \sigma_B \xi_k(t)$$

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- Linear in V.
- Spike history-dependent.

Dynamics integration

$$\Gamma_k(t_1,t,\omega) = e^{-\frac{1}{C_k}\int_{t_1 \vee \tau_k(t,\omega)}^{t} g_k(\,u,\omega\,)\,du}$$

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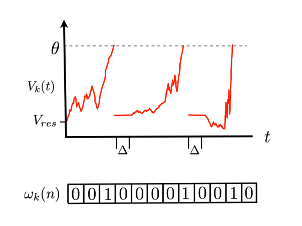

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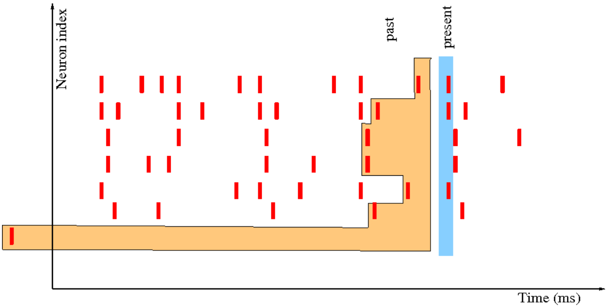
A conductance-based Integrate and Fire model

Variable length Markov chain with unbounded memory

$$P_n\left[\omega(n)\,\middle|\,\omega_{-\infty}^{n-1}\right] \equiv \Pi\left(\omega(n), \frac{\theta - V_k^{(det)}(n-1,\omega)}{\sigma_k(n-1,\omega)}\right)$$



Markov chains and Gibbs distributions

Markov chain: $P\left[\,\omega(n)\;\middle|\;\omega_{n-D}^{n-1}\,\right] > 0 \Rightarrow \exists$ a unique, invariant, ergodic distribution μ.

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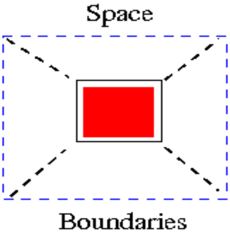

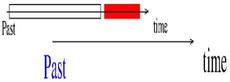

Gibbs distribution

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- Non equ. stat. phys. Onsager theory.

$$P[\omega] = \frac{1}{Z} e^{-\beta H\{\omega\}}$$

$$H\{\omega\} = \sum_\alpha \lambda_\alpha X_\alpha\{\omega\}$$

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$X_{\alpha}(\omega) =$ Product of spike events

Hammersley, Clifford, 1971

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- Markov chains - finite memory.

$$P[\omega] = \frac{1}{Z} e^{-\beta H\{\omega\}}$$

$$H\{\omega\} = \sum_\alpha \lambda_\alpha X_\alpha\{\omega\}$$

$$X_\alpha(\omega) = \text{Product of spike events}$$

Hammersley, Clifford, 1971

Gibbs distribution

- Equilibrium stat. phys. (Max. Entropy Principle).
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- Ergodic theory.
- Markov chains - finite memory.

- Chains with complete connections - infinite memory (Left Interval Specification).

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O. Onicescu and G. Mihoc. CRAS Paris, 1935

R. Fernandez, G. Maillard, A. Le Ny, J.R. Chazottes, ...

Response to stimuli

Bruno Cessac, Ignacio Ampuero, Rodrigo Cofré, submitted to J. Math. Neuro, arXiv:1704.05344

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History dependence, observable

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History dependence, observable, network dynamics

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Spontaneous correlation between observable and network dynamics

An illustrative example

The simplest example of the gIF model is the ... discrete time leaky-integrate and fire model

$$V_k(n+1) = \gamma\, V_k(n) + \sum_j W_{kj}\omega_j(n) + I_0 + S_k(t) + \sigma_B \xi_k(n), \quad \text{if } V_k(n) < \theta$$

Linear response

$$\delta\mu^{(1)}\left[\,f(n)\,\right] \sim -\sum_{k=1}^{N}\sum_{m=1}^{D+1}\sum_{l=0}^{\frac{1}{\nu_k}} \gamma^{l}\,\mathcal{K}_{km}^{(1)}\,S_k(n-m-l)$$

$$\mathcal{K}_{k,m}^{(1)} = \mathcal{C}^{(sp)}\left[\,f(m,\cdot),\zeta_k(0,.)\,\right]$$

$$\zeta_k(r-1,\omega) = \frac{\mathcal{H}_k^{(1)}(r,\omega)}{\sigma_k(r-1,\omega)}$$

Firing rates

Computations have been done using the INRIA Pranas software, B. Cessac et al, Frontiers in Neuroinformatics, Vol 11, page 49, (2017).

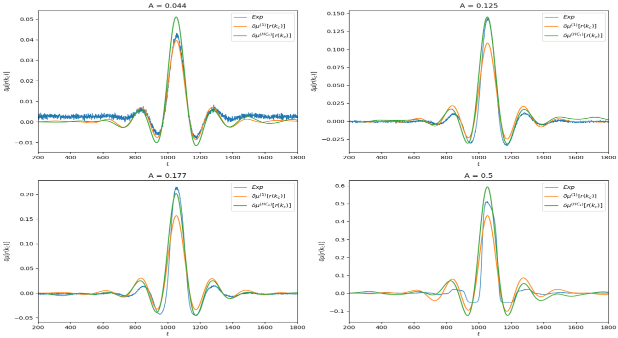

Figure: Linear response of $f(\omega, n) = \omega_{k_c}(n)$ for different values of stimulus amplitude A. From `https://arxiv.org/abs/1704.05344`

Pairwise correlations

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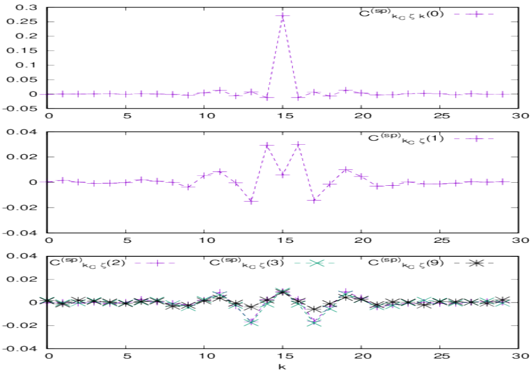

Figure: Correlation functions corresponding to the firing rate of the neuron $k_c = \frac{N}{2}$ as a function of the neuron index k (abscissa), for different values of the time delay m. From https://arxiv.org/abs/1704.05344

Higher order observable

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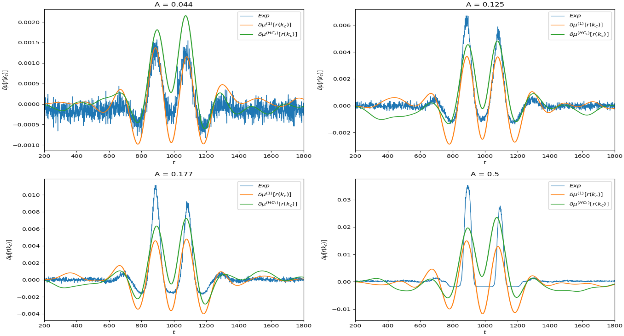

Figure: Linear response of the observable $f(n, \omega) = \omega_{k_c - 2}(n-3)\omega_{k_c}(n)$.

Range of validity

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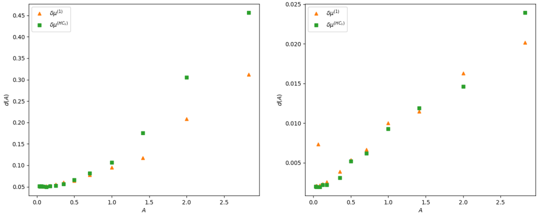

Figure: L^2 distance between the curves $\delta\mu^{(1)}[f(n)]$, $\delta\mu^{(HC_1)}[f(n)]$ and the empirical curve, as a function of the stimulus amplitude A. Left panel show distance between rate curves and right panel distance between pairwise observable with delay.

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    Which ones are relevant ? $\Rightarrow$ Requires a method to reduce the "dimensionality of the potential"

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    - Similar to Volterra-expansion.
    - Higher order terms computed by D. Ruelle (Nonlinearity, 11(1):518, 1998.) for hyperbolic dynamical systems $\Rightarrow$ Complex expansion hard to handle from data.
    - Sigmoid are "hard" to approximate with Taylor expansions.

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