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Multivariate α-Stable Models in OFDM-Based IoT Networks with Interference From a Poisson Spatial Field of Interferers

Malcolm Egan[1] and Laurent Clavier[2]

[1]Univ. Lyon, INSA Lyon, INRIA, CITI, France
[2]IMT Lille Douai, France

Abstract

The uncoordinated nature of IoT networks makes interference management a challenging problem. Motivated by NB-IoT and SCMA protocols, we study the interference statistics of a Poisson spatial field of IoT interferers exploiting OFDM. We show for a sufficiently large number of subcarriers that the interference statistics are well-approximated by a sub-Gaussian α-stable random vector with a non-isotropic underlying Gaussian random vector. This result forms a basis to improve detection and decoding algorithms at the receiver.

Keywords: α-stable, interference, IoT, NB-IoT, SCMA

1. Introduction

A key feature of IoT wireless networks is a lack of coordination between devices. In contrast to mobile cellular networks, with a high level of coordination, only limited interference mitigation strategies are available. In particular, transmissions are typically only constrained by duty cycle guidelines or the time to listen in carrier-sense multiple access (CSMA) protocols [1]. As such, devices must adapt to interference locally via signal processing tailored to the interference statistics [2].

Motivated by the need for appropriate statistical models to perform signal processing, there have recently been a number of studies of the interference statistics in the context of IoT communications. One key observation, obtained from an experimental campaign in Aalborg, Denmark, is that the interference power on each subband is heavy tailed[1], with the probability of large interference power significantly higher than for Gaussian models. This experimental observation is consistent with theoretical analysis of interference arising from a Poisson spatial field of interferers [3, 4] and variations including the Poisson-Poisson clustered interferers in [5]. In particular, α-stable models have been shown to be a good approximation of the statistics for the interference amplitude when guard zones are sufficiently small and the network radius is sufficiently large [6, 5].

Despite this progress, focusing on the interference on a single subband, there has been limited work investigating interference in the context of OFDM-based systems. In the context of the IoT, OFDM is exploited in narrowband IoT (NB-IoT). In [7, 8, 2], we have studied the scenario where interfering devices randomly select a subset of bands and developed accurate approximations of the interference statistics exploiting copula models. In [9], we developed a multivariate α-stable model under a Gaussian assumption on the product of fading and the baseband emission. However, in both of these papers, the interferers are assumed to have the same subcarrier spacing as the desired link, which is not necessarily the case in NB-IoT.

In this paper, we study the interference statistics arising from a Poisson spatial field of interferers in the scenario where the desired transmitter exploits OFDM with a different subcarrier spacing to the interfering devices. As a consequence, the signal on each subband—as observed by the receiver—is affected by each OFDM symbol of the interfering devices. This introduces non-trivial statistical dependence between the signal observed on each subband at the receiver, even if each interferer transmits on all subcarriers.

Under the assumption of interferers adopting PSK modulation, we show that the resulting interference is well approximated by a multivariate α-stable model when each interferers transmits on all of their subcarriers. In particular, the interference model is sub-Gaussian α-stable with an underlying non-isotropic Gaussian random vector. In contrast, when the interferers use the same subcarrier spacing as the desired transmitter, the interference is sub-Gaussian α-stable with an isotropic Gaussian random vector [8].

2. System Model

Consider a desired system with receiver at the origin exploiting OFDM with a symbol period T; that is, the subcarriers have a bandwidth of Δf satisfying $\frac{1}{\Delta f} = T$. The desired system coexists with a network of interfering devices with locations forming a homogeneous Poisson point process Φ with intensity λ. Devices in the interfering network also transmit using OFDM with a total of L subcarriers and subcarrier spacing $\Delta f'$, which in general does not satisfy the OFDM orthogonality conditions with the symbol period utilized by the desired receiver.

We assume that the symbol period of the interfering system, $T' = \frac{1}{\Delta f'}$, satisfies $T' > T$. As such, at most two interfering symbols can be present in each time period of duration T. We also assume that each interferer utilizes M-PSK with average power P, with equally likely symbols.

In continuous time, the interference observed by the desired receiver at the origin due to the l-th subcarrier of the interfering system is then given by

$$Y_l(t) = \sum_{k \in \Phi} r_k^{-\eta/2} h_{k,l} \Bigg[\sqrt{\frac{P}{2}} \sqrt{\frac{2}{T}} \cos\left(2\pi(f_c + l\Delta f')t + \theta_{k,l} + \phi_{k,l}\right) u(D_k - t) \\ \sqrt{\frac{P}{2}} \sqrt{\frac{2}{T}} \cos\left(2\pi(f_c + l\Delta f')t + \theta'_{k,l} + \phi_{k,l}\right) u(t - D_k) \Bigg], \; 0 \leq t \leq T. \tag{1}$$

where r_k is the distance from interferer $k \in \Phi$ to the origin at the receiver, η is the pathloss coefficient, $h_{k,l}e^{i\phi_{k,l}}$ is the baseband fading coefficient on the l-th subcarrier from interferer k, $e^{i\theta_{k,l}}$ is the baseband data transmitted in the first symbol, $e^{i\theta_{k,l}l}$ is the baseband data transmitter in the second symbol, and $D_k \sim \mathrm{Unif}[0, T]$ is the time that transmission of the second symbol begins, assumed to be uniformly distributed on $[0, T]$.

Using the linear correlation demodulator [4], the desired receiver is subject to interference on subcarrier i corresponding to a frequency $f_c + i\Delta f$. In particular, the in-phase component of the interference contributed by the l-th interfering subcarrier is given by

$$Y_{1,i,l} = \int_0^T Y_l(t) \sqrt{\frac{2}{T}} \cos(2\pi(f_c + i\Delta f)t) \mathrm{d}t, \tag{2}$$

which yields in-phase interference given by

$$Y_{1,i} = \sum_{l=\lfloor -\frac{L}{2} \rfloor}^{\lfloor \frac{L}{2} \rfloor - 1} Y_{1,i,l}. \tag{3}$$

Similarly, the quadrature component of the interference contributed by the l-th interfering subcarrier is given by

$$Y_{2,i,l} = -\int_0^T Y(t) \sqrt{\frac{2}{T}} \sin(2\pi(f_c + i\Delta f)t) \mathrm{d}t, \tag{4}$$

yielding quadrature interference given by

$$Y_{2,i} = \sum_{l=\lfloor -\frac{L}{2} \rfloor}^{\lfloor \frac{L}{2} \rfloor - 1} Y_{2,i,l}. \tag{5}$$

In this paper, we are concerned with characterizing the interference random vector

$$\mathbf{Y} = [Y_{1,1}, Y_{2,1}, \ldots, Y_{1,K}, Y_{2,K}], \tag{6}$$

which consists of the inphase and quadrature interference components on each subcarrier of the desired link. In the following, we derive a tight approximation of the statistics of $\mathbf{Y}$ as $L \to \infty$.

3. Multivariate α-Stable Interference Model

In this section, we derive an approximation for the interference statistics for sufficiently large L. As the receiver exploits the correlation demodulator, the in-phase component of the signal observed on subcarrier i of the receiver due to subcarrier l of the interfering system is given by

$$\begin{aligned}
Y_{1,i,l} &= \int_0^T Y_l(t)\sqrt{\frac{2}{T}}\cos(2\pi(f_c + i\Delta f t)\mathrm{d}t \\
&= \sum_{k\in\Phi} r_k^{-\eta/2} h_{k,l}\frac{P}{T}\int_0^T \big[\cos(2\pi(f_c + l\Delta f')t + \theta_{k,l} + \phi_{k,l})u(D_k - t) \\
&\quad + \cos(2\pi(f_c + l\Delta f')t + \theta'_{k,l} + \phi_{k,l})u(t - D_k)\big]\cos(2\pi(f_c + i\Delta f)t)\mathrm{d}t \\
&\approx \frac{P}{T}\sum_{k\in\Phi} r_k^{-\eta/2} h_{k,l} \\
&\quad \cdot \Bigg[\frac{1}{2\pi(l\Delta f' - i\Delta f)}\big(\sin(2\pi(l\Delta f' - i\Delta f)D_k + \theta_{k,l} + \phi_{k,l}) - \sin(\theta_{k,l} + \phi_{k,l})\big) \\
&\quad + \frac{1}{2\pi(l\Delta f' - i\Delta f)}\sin(2\pi(l\Delta f' - i\Delta f)T + \theta'_{k,l} + \phi_{k,l}) \\
&\quad - \frac{1}{2\pi(l\Delta f' - i\Delta f)}\sin(2\pi(l\Delta f' - i\Delta f)D_k + \theta'_{k,l} + \phi_{k,l})\Bigg] \\
&= \sum_{k\in\Phi}\frac{P r_k^{-\eta/2}}{T} X_{1,k,l},
\end{aligned} \tag{7}$$

where the approximation is tight when $f_c T \gg 1$. Using a similar argument, the quadrature component of the signal observed on subcarrier i of the receiver due to subcarrier l of the interfering system is given by

$$\begin{aligned}
Y_{2,i,l} \approx &\frac{P}{T}\sum_{k\in\Phi} r_k^{-\eta/2} h_{k,l} \\
&\cdot\left[\frac{1}{2\pi(l\Delta f' - i\Delta f)}\left(-\cos(2\pi(l\Delta f')D_k + \theta_{k,l} + \phi_{k,l}) + \cos(\theta_{k,l} + \phi_{k,l})\right)\right. \\
&- \frac{1}{2\pi(l\Delta f' - i\Delta f)}\sin(2\pi(l\Delta f' - i\Delta f)T + \theta'_{k,l} + \phi_{k,l}) \\
&\left. + \frac{1}{2\pi(l\Delta f' - i\Delta f)}\sin(2\pi(l\Delta f' - i\Delta f)D_k + \theta'_{k,l} + \phi_{k,l})\right] \\
= &\sum_{k\in\Phi}\frac{P r_k^{-\eta/2}}{T} X_{2,k,l}.
\end{aligned} \tag{8}$$

We now consider

$$\begin{aligned}
Y_{1,i} &= \sum_{l=\lfloor -\frac{L}{2}\rfloor}^{\lfloor \frac{L}{2}\rfloor - 1} Y_{1,i,l} \\
&= \sum_{l=\lfloor -\frac{L}{2}\rfloor}^{\lfloor \frac{L}{2}\rfloor - 1} \sum_{k\in\Phi} \frac{r_k^{-\eta/2}}{T} X_{1,k,l} \\
&= \sum_{k\in\Phi} \frac{\sqrt{L} r_k^{-\eta/2}}{T} \frac{1}{\sqrt{L}} \sum_{l=\lfloor -\frac{L}{2}\rfloor}^{\lfloor \frac{L}{2}\rfloor - 1} X_{1,k,l}.
\end{aligned} \tag{9}$$

Conditioned on D_k, the terms $\{X_{1,k,l}\}_l$ are independent and identically distributed zero mean Gaussian random variables, with variance independent of D_k. This is due to the fact that $\Theta_{k,l} = \theta_{k,l} + \phi_{k,l}$ has distribution

$$\Pr(\theta_{k,l} + \phi_{k,l} \leq x) = \sum_{j=1}^{M} \frac{1}{M}\left(\frac{(x-\theta_j)}{2\pi}\mathbf{1}_{\{0\leq x-\theta_j<2\pi\}} + \mathbf{1}_{\{x-\theta_j>2\pi\}}\right), \tag{10}$$

where θ_j is the j-th symbol in the M-PSK constellation. We then have, for any $z \in \mathbb{R}$,

$$\mathbb{E}[\sin(z+\Theta)] = \sum_{j=1}^{M}\frac{1}{M}\mathbb{E}[\sin(z+\theta_j+\phi)] = 0. \tag{11}$$

Similarly,

$$\mathbb{E}[\sin^2(z+\Theta)] = \sum_{j=1}^{M} \frac{1}{M}\mathbb{E}[\sin^2(z+\theta_j+\phi)] = \frac{1}{2}. \tag{12}$$

Let $\sigma_Y^2 = \mathrm{Var}(X_{1,k,l}) = \mathrm{Var}(X_{2,k,l})$. As $L \to \infty$, by the central limit theorem, we then obtain

$$Y_{1,i} \approx \tilde{Y}_{1,i} = \sum_{k\in\Phi} \frac{P\sqrt{L}r_k^{-\eta/2}}{T} G_{1,k}, \tag{13}$$

where $G_{1,k} \sim \mathcal{N}(0,\sigma_Y)$. Similarly,

$$Y_{2,i} \approx \tilde{Y}_{2,i} \sum_{k\in\Phi} \frac{P\sqrt{L}r_k^{-\eta/2}}{T} G_{2,k}, \tag{14}$$

where $G_{2,k} \sim \mathcal{N}(0,\sigma_Y)$.

Writing

$$\tilde{Y}_{1,i} = \frac{P\sqrt{L}}{T} \sum_{k\in\Phi} (r_k^{-\eta/4})^2 G_{1,k} \tag{15}$$

and applying the mapping theorem for Poisson point processes [3], it follows that $\tilde{Y}_{1,i}$ is a symmetric $4/\eta$-stable random variable by the LePage series expansion. For more details, see [3, 4]. By the same argument, $\tilde{Y}_{2,i}$ is also a symmetric $4/\eta$-stable random variable.

We now turn to the random vector $\mathbf{Y}$. So far, we have shown that each marginal of $\mathbf{Y}$, i.e., $Y_{1,1}, Y_{2,1}, \ldots$, is well approximated by an identically distributed symmetric α-stable random variable. If each term $G_{1,1}, G_{2,1}, \ldots$ is independent, then by [8, 2], it follows that $\mathbf{Y}$ is a sub-Gaussian α-stable random vector with underlying Gaussian random vector $\mathbf{G} \sim \mathcal{N}(0, \sigma\mathbf{I})$.

However, in the present case, the terms $G_{1,1}, G_{2,1}, \ldots$ are not in general independent due to the presence of $\{\theta_{k,l}, \phi_{k,l}, h_{k,l}\}$ in the expressions for $Y_{1,i}$ and $Y_{2,i}$. As such, there is a non-zero correlation between each pair $Y_{j,i}, Y_{j',i'}$ for $j \neq j'$ and $i, i' \in \{1, \ldots, \mathcal{I}\}$. An approximation for the statistics of $\mathbf{Y}$ is given in Theorem 1.

Theorem 1. $\tilde{\mathbf{Y}}$ is a sub-Gaussian $4/\eta$-stable random vector with a non-isotropic underlying Gaussian random vector.

Proof. We first write

$$\begin{aligned}\tilde{\mathbf{Y}} &= \frac{\sqrt{L}}{T}\sum_{k\in\Phi} r_k^{-\eta/2}\mathbf{G}_k \\ &= \frac{\sqrt{L}}{T}\mathbf{C}\sum_{k\in\Phi} r_k^{-\eta/2}\tilde{\mathbf{G}}_k, \end{aligned} \tag{16}$$

where $\tilde{\mathbf{G}}_k \sim \mathcal{N}(0, \sigma_Y \mathbf{I})$ and $\mathbf{C}$ is the Cholesky decomposition of the covariance matrix of $\mathbf{G}_k$. It then follows that $\sum_{k\in\Phi} r_k^{-\eta/2}\tilde{\mathbf{G}}_k$ is a sub-Gaussian α-stable random vector with isotropic underlying Gaussian random vector $\mathbf{Z}$ by [8, 2]. Using the fact that $\sum_{k\in\Phi} r_k^{-\eta/2}\tilde{\mathbf{G}}_k$ is sub-Gaussian α-stable,

$$\tilde{\mathbf{Y}} = \frac{\sqrt{L}}{T}\mathbf{C}\sqrt{A}\mathbf{Z}, \tag{17}$$

where A is a skewed stable random variable (see Definition 1), it then follows that $\tilde{\mathbf{Y}}$ is a sub-Gaussian α-stable random vector with non-isotropic underlying Gaussian random vector. ∎

4. Conclusion

Multivariate interference models naturally arise in OFDM-based communication systems, such as NB-IoT. In this paper, we considered the impact of a mismatch between the subcarrier spacing of the desired link and the interfering signals. In the case of interferer locations forming a Poisson spatial field, we have shown that the interference is well approximated by a sub-Gaussian α-stable model with non-isotropic underlying Gaussian random vector when the number of interferer subcarriers L is large, bearing similarities with the analysis of passband additive symmetric α-stable noise in [10]. The result forms a basis for improved receiver design in IoT systems.

A. α-Stable Preliminaries

In this appendix, we overview key properties of α-stable random vectors. The probability density function of an α-stable random variable is parameterized by four parameters: the exponent $0 \leq \alpha \leq 2$; the scale parameter $\gamma \in \mathbb{R}_+$; the skew parameter $\beta \in [-1, 1]$; and the shift parameter $\delta \in \mathbb{R}$. As such, a common notation for an α-stable random variable X is $X \sim S_\alpha(\gamma, \beta, \delta)$. In the case $\beta = \delta = 0$, X is said to be a symmetric α-stable random variable.

In general, α-stable random variables do not have closed-form probability density functions. Instead, they are more compactly represented by their characteristic function, given by [11, Eq. 1.1.6]

$$\mathbb{E}[e^{i\theta X}] = \begin{cases} \exp\left\{-\gamma^\alpha|\theta|^\alpha(1 - i\beta(\mathrm{sign}\theta)\tan\frac{\pi\alpha}{2}) + i\delta\theta\right\}, \\ \qquad \alpha \neq 1 \\ \exp\left\{-\gamma|\theta|(1 + i\beta\frac{2}{\pi}(\mathrm{sign}\theta)\log|\theta|) + i\delta\theta\right\}, \\ \qquad \alpha = 1 \end{cases} \tag{18}$$

It is possible to extend the notion of an α-stable random variable to the multivariate setting. In particular, a random vector $\mathbf{X}$ is a symmetric α-stable random vector if for all $a, b > 0$ there exists $c > 0$ such that

$$a\mathbf{X}^{(1)} + b\mathbf{X}^{(2)} \stackrel{d}{=} c\mathbf{X}, \tag{19}$$

where $\mathbf{X}^{(1)}, \mathbf{X}^{(2)}$ are independent copies of $\mathbf{X}$.

A sufficient condition for a random vector $\mathbf{X}$ to be a symmetric α-stable random vector is that all linear combinations of the marginal distributions for the elements of $\mathbf{X}$ are symmetric α-stable [11]. In general, d-dimensional symmetric α-stable random vectors are also represented via their characteristic function, given by [11]

$$\mathbb{E}[e^{i\boldsymbol{\theta}\cdot\mathbf{X}}] = \exp\left(-\int_{\mathbb{S}^{d-1}} \left|\sum_{k=1}^{d} \theta_k s_k\right|^\alpha \Gamma(d\mathbf{s})\right), \tag{20}$$

where Γ is the unique symmetric measure on the surface of the d-dimensional unit sphere.

In the case that a d-dimensional symmetric α-stable random vector $\mathbf{X}$ is *truly* d-dimensional, there exists a joint probability density function $p_{\mathbf{X}}(\mathbf{x})$ on $\mathbb{R}^d$. Note that a simple necessary and sufficient condition for $\mathbf{X}$ to be truly d-dimensional is for the support of the spectral measure Γ to span $\mathbb{R}^d$ [**?**]. This condition means that degenerate symmetric α-stable random vectors (e.g., when $X_i = X_j$ for some $i \neq j,\ i, j \in \{1, \ldots, d\}$) are not considered.

An important class of α-stable random vectors is the sub-Gaussian α-stable family.

Definition 1. Any vector $\mathbf{X}$ distributed as $\mathbf{X} = (A^{1/2}G_1, \ldots, A^{1/2}G_d)$, where

$$A \sim S_{\alpha/2}((\cos \pi\alpha/4)^{2/\alpha}, 1, 0), \tag{21}$$

and $\mathbf{G} = [G_1, \ldots, G_d]^T \sim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ is called a sub-Gaussian α-stable random vector in $\mathbb{R}^d$ with underlying Gaussian vector $\mathbf{G}$.

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