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# Polarization imaging of a sinusoidal amplitude grating with partially polarized and partially coherent light

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## ABSTRACT

As an advanced imaging technique, the polarization imaging has attracted more and more interests and many applications have been developed in the fields, such as biomedical diagnostics, target identification and remote sensing due to its unique ability to detect the polarization information of objects. On the other hand, the grating with its periodic spatial structure has been used widely in various imaging systems including but not limited to holographic imaging, Talbot effect of self-imaging, and imaging spectrometer. Furthermore, a sinusoidal amplitude grating is of considerable interest in image analysis and optical system characterization. Although many techniques with applications of the gratings have been developed in the last decades, few investigations have been made to the sinusoidal amplitude grating in a polarization imaging system with arbitrary illumination condition of polarization and coherence. In this paper, the polarization imaging of a sinusoidal amplitude object illuminated with a partially polarized and partially coherent light is investigated. With the help of the unified theory of polarization and coherence, we have extended the use of sinusoidal trace analysis in the evaluation of optical system performance and presented theoretical analysis on the Stokes images of a sinusoidal amplitude grating.

**Keywords:** sinusoidal amplitude grating, polarization imaging, partially polarized and partially coherent light

## 1. INTRODUCTION

Since the introduction of transfer function techniques for image analysis, a variety of methods have been evolved for the measurement of the transfer function. The most popular methods have been scanning techniques of which one whole class involves the Fourier analysis of the image of sinusoidal gratings. An excellent review of this and other techniques for measuring transfer function are to be found in an article by MUKATA [1965]. Because of the importance of the use of sine wave targets in image analysis it is of some considerable interest to evaluate the image of such an object under various coherence conditions of the illumination. Two sets of authors have studied this problem from the point of view of examining the ambiguity of the transfer function of a system where the object is illuminated with partially coherent light (BECHERER and PARRENT [1967]; SWING and CLAY [1967]). Both sets of authors considered only a one-dimensional imaging system for the sake of simplicity. The first named authors discussed the problem for an object with a sinusoidal amplitude transmittance. This, of course, is not the usual "sine wave target" which has been adopted to mean a sine wave in intensity - an outcome of the purely incoherent analysis. Following the work of Becherer and Parrent, Swing and Clay performed the same type of analysis for the more usual sine wave target with a sinusoidal intensity transmittance. Earlier HOPKINS [1953] had shown that the image of periodic line structures in partially coherent light requires for its specifications a set of cross transfer factors for each pair of frequencies.

The literature contains a number of papers dealing with the image of a sinusoidal and the transfer function of a system for sine-wave object[1~6]. The use of sinusoidal trace analysis in the evaluation of system performance is now well established. All these authors showed the basic differences in the image of sine-wave object. But they don't consider the polarimetric properties of light. Theoretical results are presented about polarization image of sine wave with partially polarized and partially coherent light. The theoretical results are presented. The dependence of spatial frequency response upon the Stokes parameter and the obscuration ratio of the aperture have been discussed.

## 2. NONLINEARITY IN IMAGING WITH PARTIALLY COHERENT LIGHT

The basic quantity in the vector theory of partial coherence is the generalized Stokes parameters defined by

$$\begin{cases} S_0^{in}(x_1, x_2) = \left\langle \tilde{E}_x(x_1, t)\tilde{E}_x^*(x_2, t) + \tilde{E}_y(x_1, t)\tilde{E}_y^*(x_2, t) \right\rangle \\[4pt] S_1^{in}(x_1, x_2) = \left\langle \tilde{E}_x(x_1, t)\tilde{E}_x^*(x_2, t) - \tilde{E}_y(x_1, t)\tilde{E}_y^*(x_2, t) \right\rangle \\[4pt] S_2^{in}(x_1, x_2) = \left\langle \tilde{E}_x(x_1, t)\tilde{E}_y^*(x_2, t) + \tilde{E}_x^*(x_2, t)\tilde{E}_y(x_1, t) \right\rangle \\[4pt] S_3^{in}(x_1, x_2) = \left\langle i[\tilde{E}_x^*(x_2, t)\tilde{E}_y(x_1, t) - \tilde{E}_x(x_1, t)\tilde{E}_y^*(x_2, t)] \right\rangle \end{cases} \tag{1}$$

$S_i^{in}(x_1, x_2)$, $i = 0, 1, 2, 3$ is the generalized Stokes parameters in the object plane of an optical imaging system in Eq. (1). The sharp brackets in Eq. (1) indicate a long time average and $E(x, t)$ is the analytic signal associated with the optical disturbance at the point $x$ and time $t$. A more useful form for application to image analysis is obtained by considering the object to be transilluminated. This is certainly the case in all uses of microscopes, enlargers, and micro densitometers in image evaluation. For transilluminated objects, there $p(x)$ is the amplitude transmittance of the object.

Then the expression for the generalized Stokes parameters $S_i^{im}(x_1, x_2)$, $i = 0, 1, 2, 3$ in the image plane of an optical imaging system is

$$S_i^{im}(x_1, x_2) = \iint_{-\infty}^{\infty} S_i^{in}(\xi_1, \xi_2)p(\xi_1)p^*(\xi_2)h(x_1 - \xi_1)h^*(x_2 - \xi_2)d\xi_1 d\xi_2 \quad i = 0, 1, 2, 3 \tag{2}$$

In Eq. (2), $S_i^{im}(x_1, x_2)$ are the generalized Stokes parameters in the image plane, $p(\xi)$ is the amplitude transmittance of the object, and $h(x - \xi)$ is essentially the spatially stationary amplitude impulse response of the imaging system. However, when the detection process is included in the analysis, the generalized Stokes parameters in the image plane are the quantity of interest. For transilluminated objects, this generalized Stokes parameters are found from Eq. (2) and the definition (1) to be

$$S_i^{im}(x) = S_i^{im}(x, x) = \iint_{-\infty}^{\infty} S_i^{in}(\xi_1, \xi_2)p(\xi_1)p^*(\xi_2)h(x - \xi_1)h^*(x - \xi_2)d\xi_1 d\xi_2 \quad i = 0, 1, 2, 3 \tag{3}$$

For the important practical case where $S_i^{in}(\xi_1, \xi_2)$ is spatially stationary, i.e., for

$$S_i^{in}(\xi_1, \xi_2) = S_i^{in}(\xi_1 - \xi_2) \tag{4}$$

Eq. (3) can be written

$$S_i^{im}(x) = \iint_{-\infty}^{\infty} S_i^{in}(\xi_1 - \xi_2)p(\xi_1)p^*(\xi_2)h(x - \xi_1)h^*(x - \xi_2)d\xi_1 d\xi_2 \quad i = 0, 1, 2, 3 \tag{5}$$

From Eq. (5) it is clear that, for transilluminated objects, the transition from object intensity $p(\xi_1)p^*(\xi_2)$ to image intensity $S_i^{im}(x)$ is nonlinear. The significance of this conclusion is that the customary image evaluation techniques and criteria are not, in general, applicable to such systems. Knowing how such a system images sine-wave does not permit us to describe how it images other objects. Furthermore, the same optical system and object could be expected to yield different polarization image $S_i^{im}(x)$ if the coherence of the illumination represented by $S_i^{in}(\xi_1 - \xi_2)$ were varied.

Since systems of this type are inherently nonlinear, it is impossible to characterize them by a transfer function. Because of its usefulness, and because it outlines an analytical technique, it will be worthwhile to go through the transformation in detail. Symmetrical transforms will be employed. The spectra are given by.

$$\mathbf{S}_i^{im}(f_x) = \int S_i^{im}(x)e^{-2\pi i f_x x}dx \tag{6}$$

When Eqs. (5) and (6) are combined, and the integrand regrouped, we find

$$\mathbf{S}_i^{im}(f_x) = \iint_{-\infty}^{\infty} S_i^{in}(\xi_1 - \xi_2)p(\xi_1)p^*(\xi_2)\{\int h(x - \xi_1)h^*(x - \xi_2)e^{-2\pi i f_x x}dx\}d\xi_1 d\xi_2 \tag{7}$$

Take $\xi$ and $\sigma$ to be conjugate variables in $\bar{\xi}$ and $\bar{\sigma}$, respectively. We can then evaluate the inner integral. Thus,

$$\int h(x - \xi_1)h^*(x - \xi_2)e^{-2\pi i f_x x}dx = e^{-2\pi i \xi_1 f_x}\int \mathbf{H}(f_x - \sigma)\mathbf{H}^*(\sigma)e^{2\pi i(\xi_1 - \xi_2)\sigma}d\sigma \tag{8}$$

When we insert this in Eq. (7) and regroup, we find

$$\mathbf{S}_i^{im}(f_x) = \iint_{-\infty}^{\infty} \mathbf{H}(f_x - \sigma)\mathbf{H}^*(\sigma)p^*(\xi_2)\{\int S_i^{in}(\xi_1 - \xi_2)p(\xi_1)e^{-2\pi i(f_x - \sigma)\xi_1}d\xi_1\}e^{-2\pi i \sigma \xi_2}d\sigma d\xi_2 \tag{9}$$

Take $\zeta$ and $\gamma$ to be conjugate variables in $\bar{\xi}$. We can then evaluate the inner integral. Thus,

$$\int S_i^{in}(\xi_1-\xi_2)p(\xi_1)e^{-2\pi i(f_x-\sigma)\xi_1}d\xi_1 = e^{-2\pi i\xi_2(f_x-\sigma)}\int \mathbf{S}_i^{in}(f_x-\sigma-\gamma)\mathbf{P}(\gamma)e^{2\pi i\xi_2\gamma}d\gamma \tag{10}$$

When we insert this in Eq. (9) and make a final regrouping, we have

$$\mathbf{S}_i^{im}(f_x)=\int\int_{-\infty}^{\infty}\mathbf{S}_i^{in}[f_x-(\sigma+\gamma)]\mathbf{H}(f_x-\sigma)\mathbf{H}^*(\sigma)\mathbf{P}(\gamma)\{\int p^*(\xi_2)e^{-2\pi i(f_x-\gamma)\xi_2}d\xi_2\}d\sigma d\gamma \tag{11}$$

The inner integral can immediately be recognized as a Fourier transform with $\xi_2$ and $(f_x\text{-}\gamma)$ as conjugate variables. Thus, the polarization image spectra for the one-dimensional case of image formation are given by

$$\mathbf{S}_i^{im}(f_x)=\int \mathbf{P}(\gamma)\mathbf{P}^*(f_x-\gamma)\{\int \mathbf{S}_i^{in}[f_x-(\sigma+\gamma)]\mathbf{H}(f_x-\sigma)\mathbf{H}^*(\sigma)d\sigma\}d\gamma \tag{12}$$

Where the grouping of terms has been deliberate, to point out the source of the nonlinearity. The inner integral contains the optical-system and illumination characteristics. It is a function of two spatial frequencies and has been referred to as the "transmission cross coefficient". In only a vague sense does it constitute a transfer function, and is not used as one in Eq. (12). Therefore, for an arbitrary mutual polarization image, the behavior of an optical system is inherently nonlinear. The normal linear multiplicative relation between object spectrum and system transfer function is no longer applicable, and the current technique of optical system analysis through the cascading of component transfer functions is clearly subject to error.

In Eq. (12) the inner integral is characteristic of the instrument and the illumination while the factors $\mathbf{P}(\gamma)$ and $\mathbf{P}^*(f_x-\gamma)$ are determined solely by the object. However, the right side of Eq. (12) is not in the form of a product of object spectrum and transfer function as it would be if the system were linear. The inner integral in Eq. (12) has been referred to as a generalized transfer function, but that nomenclature is rather misleading since the function is not used as a transfer function. A better terminology may be the more cumbersome one introduced by Wolf, i.e., the "transmission cross coefficient."

## 3. DESCRIPTION OF IMAGING SYSTEM

This section contains a description of an optical imaging system for which the transfer function is to be measured. The mutual intensity of the light incident on the object and the amplitude impulse response of the imaging system, and their Fourier transforms, are found in this section. These are used in the following sections to solve the imaging problem and to determine the apparent transfer functions for sinusoidal objects.

Figure 1 indicates a schematic diagram of polarization imaging of sine-wave object under illumination of a partially coherent and partially polarized light source.
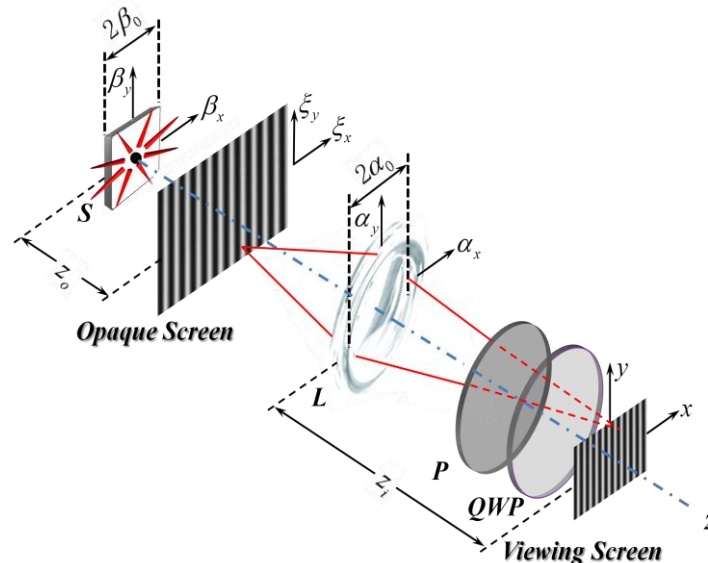


Fig.1. Schematic diagram for polarization imaging of sine-wave object illuminated by partially coherent and partially polarized light source.

For mathematical simplicity, all the treatments here are restricted to one-dimensional variations for light source, object, apertures and images. An incoherent source in the $\beta$ plane illuminates an object in the $\xi$ plane. The $\xi$ plane is imaged

onto the x, y plane by an imaging system with exit pupil in the $\alpha$ plane. The distance in image space is normalized by the lateral magnification of the imaging system and the positive direction in image space is opposite to that in object space. This is done to make the ordinate of a given object point equal to that of its corresponding image point.

An incoherent source (S) with side length $2\beta_0$ illuminates sine-wave object located far away with a distance of $z_o$. For such a light source with uniform irradiance, its Stokes parameters can be written as Eq. (13) with $\hat{s}_i = S_i / \sqrt{\sum_i S_i^2}$ being the normalized Stokes parameters for the tensor wave of light source.

$$S_i(\beta) = \begin{cases} A\hat{s}_i, & |\beta| \leq \beta_0 \\ 0, & |\beta| > \beta_0 \end{cases} \quad (i = 0 \sim 3) \tag{13}$$

A sine-wave object is imaged onto the viewing screen located at a distance of $z_i$ by an imaging system of the lens (L) with its exit pupil of $2\alpha_0$. The generalized Stokes parameters of the incident beam at the object plane is found from the well-known van Cittert-Zernike theorem to be given to a good approximation by

$$S_i^{in}(\xi_1, \xi_2) = \int S_i(\beta) e^{2\pi i \beta(\xi_1 - \xi_2)/\lambda z_o} \, d\beta \quad (i = 0 \sim 3) \tag{14}$$

For points $\xi_1$ and $\xi_2$ close to the optical axis, here $z_o$ is the distance between the incoherent-source plane $\beta$ and the object plane $\xi$. When Eq. (13) is used to evaluate Eq. (14), the generalized Stokes parameters of the incident beam at the object plane is found to be

$$S_i^{in}(\xi_1, \xi_2) = \text{const sinc}\left[2\pi i \beta_0 (\xi_1 - \xi_2)/\lambda z_o\right] \quad (i = 0 \sim 3) \tag{15}$$

The distance from the center to the first zero of the generalized Stokes parameters of the incident beam at the object plane in Eq. (15) will be referred to as the coherence interval of the object illumination.

The Fourier transform of the generalized Stokes parameters of the incident beam at the object plane Eq. (15) is

$$\mathbf{S}_i^{in}(f_x) = \int S_i^{in}(\xi_1 - \xi_2) e^{2\pi i f_x(\xi_1 - \xi_2)} d(\xi_1 - \xi_2) = A \begin{cases} \hat{s}_i, & |f_x| \leq \beta_0/\lambda z_0 = f_1 \\ 0, & |f_x| > \beta_0/\lambda z_0 = f_1 \end{cases} \tag{16}$$

Equation (16) serves to define the parameter $f_1$.

To emphasize the coherence effects and to minimize the complications arising from aberrations, the restriction to diffraction-limited optics is imposed. The amplitude in the exit pupil of the imaging system due to a point object is taken to be

$$A(\alpha) = const \begin{cases} 1, & |\alpha| \leq \alpha_0 \\ 0, & |\alpha| > \alpha_0 \end{cases} \tag{17}$$

Where $\alpha_0$ is constant. Under the usual approximations which characterize Fraunhofer diffraction, the amplitude impulse response corresponding to Eq. (17) is

$$h(x) = \int A(\alpha) e^{2\pi i \alpha x/\lambda z_i} d\alpha = const \sin c\left[2\pi i \alpha_0 x/\lambda z_i\right] \tag{18}$$

Where $z_i$ is the distance from the plane $\alpha$ of the exit pupil to the image plane $x$. The distance from the center to the first zero of the impulse response function in Eq. (18) will be referred to as the size of the imaging system's diffraction pattern. The Fourier transform of Eq. (18) is

$$\mathbf{H}(f_x) = \int h(x) e^{2\pi i f_x x} dx = const \begin{cases} 1, & |f_x| \leq \alpha_0/\lambda z_i = f_0 \\ 0, & |f_x| > \alpha_0/\lambda z_i = f_0 \end{cases} \tag{19}$$

Equation (19) serves to define the parameter $f_0$.

## 4. POLARIZATION IMAGE FOURIER TRANSFORM FOR SINE WAVE OBJECT

The object to be considered is a sinusoidal object with amplitude transmittance

$$p(\xi) = 1 + \cos 2\pi f_{x_0} \xi \tag{20}$$

where $f_{x_0}$ is the spatial frequency. The Fourier transform of Eq. (20) is

$$\mathbf{P}(r) = \delta(r) + \frac{1}{2}\delta(r - f_{x_0}) + \frac{1}{2}\delta(r + f_{x_0}) \tag{21}$$

The corresponding intensity transmittance of this object is

$$|p(\xi)|^2 = 3/2 + 2\cos 2\pi f_{x_0}\xi + 1/2(\cos 2\pi 2 f_{x_0}\xi) \tag{22}$$

The intensity transmittance of the sine-wave object often used in laboratory measurements of the optical transfer function has a single spatial-frequency component rather than the two-spatial-frequency components which appear in Eq. (22). When the imaging system is linear, there is no fundamental difference between these two objects. When the system becomes nonlinear, the sine-wave loses its value as a test object.

The necessity of specifying the amplitude transmittance of the object for use in Eqs.(10) or (11) when the illumination is partially coherent makes a simple object of the form shown by Eq. (20) particularly suitable for the present analysis. The object represented by Eqs. (20) and (22) is, of course, physically realizable.

Since the object illumination is partially coherent, it is necessary to describe the experiment very carefully. The resulting image is shown to have an intensity function of the form

$$S_i^{im}(x) = A_i + B_i \cos 2\pi f_{x_0} x + C_i \cos 2\pi 2 f_{x_0} x \tag{23}$$

Where, of course, $A_i$, $B_i$, and $C_i$ are yet to be determined. The ratio of the image modulation in terms of intensity to the object modulation, in terms of intensity, will be calculated. This ratio will be calculated for both spatial frequency components separately, following the practice which would be used under the condition of incoherent object illumination for an object whose intensity contained more than one spatial-frequency component. If the system was linear in intensity, these modulation ratios for the components of frequency $f_{x_0}$ and $2f_{x_0}$ should be the same except for the scale factor of 2 in spatial frequency.

To begin, the integral in Eq. (11) is again written in the form

$$\mathbf{S}_i^{im}(f_x) = \int[\int \mathbf{P}(r)\mathbf{P}(f_x - r)\mathbf{S}_i^{in}(f_x - \sigma - r)dr]\mathbf{H}(f_x - \sigma)\mathbf{H}^*(\sigma)d\sigma \tag{24}$$

Where Eq. (18) and Eq. (20) have been used to write $h = h^*$ and $p = p^*$. By substituting Eq. (21) into Eq. (24), using Eq. (19) to write $\mathbf{H}(-\sigma) = \mathbf{H}(\sigma)$, and taking the inverse Fourier transform of both sides of Eq. (24), we find that

$$
\begin{aligned}
S_i^{im}(x) =& \int \mathbf{S}_i^{in}(-\sigma)\mathbf{H}(-\sigma)\mathbf{H}(\sigma)d\sigma \\
&+ \frac{1}{4}\int \mathbf{H}(\sigma)\left[\mathbf{S}_i^{in}(-\sigma - f_{x_0})\mathbf{H}(-\sigma) + \mathbf{S}_i^{in}(-\sigma + f_{x_0})\mathbf{H}(-\sigma)\right]d\sigma \\
&+ \int\left[\mathbf{S}_i^{in}(f_{x_0} - \sigma)\mathbf{H}(f_{x_0} - \sigma) + \mathbf{S}_i^{in}(-\sigma)\mathbf{H}(f_{x_0} - \sigma)\right]\mathbf{H}(\sigma)d\sigma\cos 2\pi f_{x_0} x \\
&+ \frac{1}{2}\int\left[\mathbf{S}_i^{in}(f_{x_0} - \sigma)\mathbf{H}(2f_{x_0} - \sigma)\mathbf{H}(\sigma)\right]d\sigma\cos 2\pi 2 f_{x_0} x \\
=& A_i + B_i\cos 2\pi f_{x_0} x + C_i\cos 2\pi 2 f_{x_0} x
\end{aligned}
\tag{25}
$$

where the coefficients $A_i$, $B_i$, and $C_i$ are determined by using Eq. (16) and (19) to evaluate the integrals in Eq. (25). The result is

$$
A_i = 2\hat{s}_i\begin{bmatrix} f_0 \\ f_1 \end{bmatrix} + \frac{1}{2}\hat{s}_i\begin{cases} 2f_0 & f_0 < f_1 \quad and \quad 0 \le f_{x_0} \le f_1 - f_0 \\ f_0 + f_1 - f_{x_0} & f_0 < f_1 \quad and \quad f_1 - f_0 \le f_{x_0} \le 2f_0 \\ 2f_1 & f_0 > f_1 \quad and \quad 0 \le f_{x_0} \le f_0 - f_1 \\ f_0 + f_1 - f_{x_0} & f_0 > f_1 \quad and \quad f_0 - f_1 < f_{x_0} \le f_0 + f_1 \\ 0 & f_0 > f_1 \quad and \quad f_0 + f_1 < f_{x_0} \le 2f_0 \end{cases} \tag{26}
$$

$$B_i = 2\hat{s}_i \begin{cases} (2f_0 - f_{x_0}) & f_0 < f_1 \quad and \quad 0 \le f_{x_0} \le 2f_0 \\ 0 & f_0 < f_1 \quad and \quad 2f_0 < f_{x_0} \\ 2f_1 & f_0 > f_1 \quad and \quad 0 \le f_{x_0} \le f_0 - f_1 \\ (f_0 + f_1 - f_{x_0}) & f_0 > f_1 \quad and \quad f_0 - f_1 < f_{x_0} \le f_0 + f_1 \\ 0 & f_0 > f_1 \quad and \quad f_0 + f_1 < f_{x_0} \le 2f_0 \end{cases} \tag{27}$$

$$C_i = \frac{1}{2}\hat{s}_i \begin{cases} 2f_0 - 2f_{x_0} & f_0 < f_1 \quad and \quad 0 \le f_{x_0} \le f_0 \\ 0 & f_0 < f_1 \quad and \quad f_0 < f_{x_0} \\ 2f_1 & f_0 > f_1 \quad and \quad 0 \le f_{x_0} \le f_0 - f_1 \\ 2f_0 - 2f_{x_0} & f_0 > f_1 \quad and \quad f_0 - f_1 < f_{x_0} \le f_0 \\ 0 & f_0 > f_1 \quad and \quad f_0 < f_{x_0} \end{cases} \tag{28}$$

Thus, the polarization image spectra for the one-dimensional case of image formation are given by

$$\mathbf{S}_i^{im}(f_x) = \int \left[ A_i + B_i \cos 2\pi f_{x_0}\xi + C_i \cos 2\pi 2 f_{x_0}\xi \right] e^{-2\pi i f_x \xi} d\xi$$

$$= A_i \delta(f_x) + \frac{1}{2}B_i \left[ \delta(f_x - f_{x_0}) + \delta(f_x + f_{x_0}) \right] + \frac{1}{2}C_i \left[ \delta(f_x - 2f_{x_0}) + \delta(f_x + 2f_{x_0}) \right] \tag{29}$$

Here the symbol $\begin{bmatrix} f_0 \\ f_1 \end{bmatrix}$ is to be read "$f_0$ or $f_1$, whichever is smaller."

## 5. POLARIZATION IMAGES OF SINE WAVE OBJECT WITH DIFFERENT LIGHT ILLUMINATION

Figure 2 shows plots of the normalized magnitudes $\mathbf{S}_i^{im}(x)/\|\mathbf{S}^{im}(0)\|$ with $\|\mathbf{S}^{im}(0)\| = \sqrt{\sum_i |\mathbf{S}_i^{im}(0)|^2}$ being the normalized Stokes parameters at the viewing plane of this apparent Fourier Spectra for various values of $R = f_0/f_1$. The $x$-axis represents the normalized natural frequency of the grating, and $2f_0$ is taken as the normalized value, and $2f_0$ is also the cutoff frequency of the grating signal.

From examination of the description of the imaging system in the previous section, it is seen that the parameter $R$ is the ratio of the coherence interval of the object illumination to the size of the imaging system's diffraction pattern.

When the normalized Stokes parameters for the tensor wave of light source are as follows: $\hat{s}_0 = 0.89$, $\hat{s}_1 = 0.25$, $\hat{s}_2 = -0.32$, and $\hat{s}_3 = -0.2$, which means Fourier spectra of polarization images of a sinusoidal object with partially polarized light illumination.

Figure 2(a)-(d) show plots of the Fourier spectra of the Stokes images of sine-wave object for various values of $R$ indicating the ratio of coherence degree at the object illumination to the size of the imaging system's diffraction pattern.

When a sine-wave object is illuminated by partially polarized light, their normalized magnitudes Fourier spectra of polarization images change with various values of $R$ and the cutoff frequency of the grating signal, which means more information and details about the object can be recognized and detected.
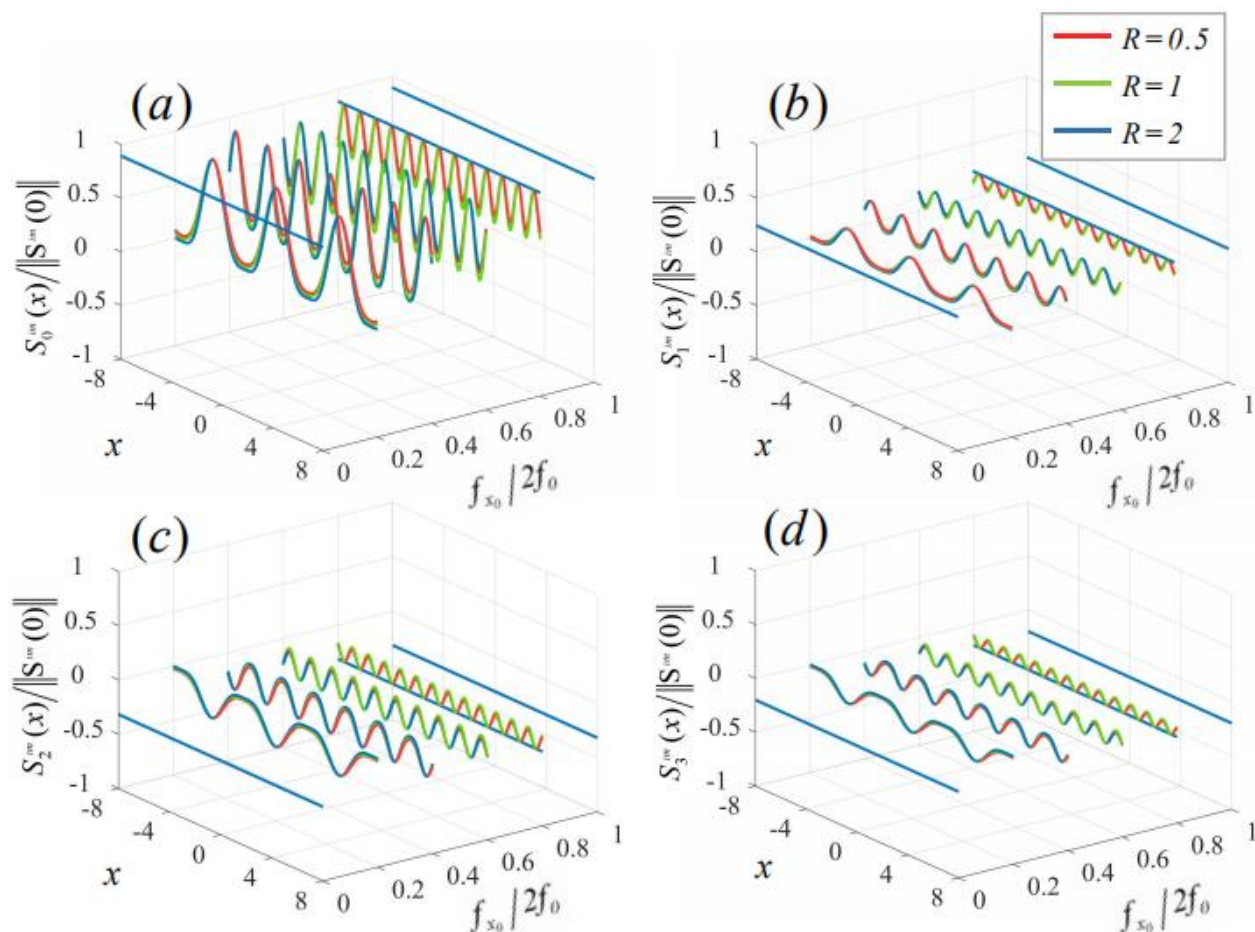
Fig.2. The normalized Fourier spectra for Stokes images of a sinusoidal object illuminated by partially polarized light for various values of $R = f_0/f_1$ and the normalized frequency $f_{x_0}/2f_0$.

# 6. CONCLUSIONS AND RESULTS

In summary, the polarization imaging as an advanced sensing technique has attracted more and more interests due to its unique ability to detect the polarization information of objects, which will be beneficial to many applications, such as biomedical diagnostics and target detection, recognition and identification. In all the above figures, when the normalized frequency $f_{x_0}/2f_0$ is greater than 0.5, the double frequency disappears. When $R=2$ and the normalized frequency $f_{x_0}/2f_0$ is greater than about 0.75, the frequency once and frequency twice disappear altogether, leaving only the DC component. When the normalized frequency is the same, Stokes intensity of $R=0.5$ almost coincides with that of $R=1$ while Stokes intensity decreases obviously when $R=2$.

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