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Xueliang Zhu, Jacob Beaumont, Baixin Chen, Mitsuo Takeda, Wei Wang, "Decomposition for coherence current of mutual coherence function for stochastic optical field," Proc. SPIE 11567, AOPC 2020: Optical Sensing and Imaging Technology, 1156705 (5 November 2020); doi: 10.1117/12.2572765

Event: Applied Optics and Photonics China (AOPC 2020), 2020, Beijing, China

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Decomposition for Coherence Current of Mutual Coherence Function for Stochastic Optical Field

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ABSTRACT

In analogy to the velocity decomposition in continuum mechanics, we introduce two new tensors, referred to as the deformation-rate tensor and rotation-rate tensor, to the optical coherence theory for stochastic optical field, and decompose a coherence current vector into its translation, rotation and deformation components to study the optical coherence dynamics. To investigate the optical coherence propagation and evolution, we have conducted an experiment with results given to demonstrate the two newly introduced tensors.

Keywords: Coherence Current, Optical Coherence Function, Stochastic Optical Field

1. INTRODUCTION

Optical fields are inherently of a statistical nature and the cross-correlation between the fluctuating fields known as the coherence function is a quantity of great importance in understanding the statistics of stochastic optical field. [1-2] During the recent years, a new concept of coherence current has been introduced and observed when a generic coherence vortex was experimentally observed. [3-4] Just as the unique role of Poynting vector in Electrodynamics, the coherence current plays a critical role to describe the propagation of optical coherence, where its magnitude is proportional to the fringe contrast and its vectoral direction points along the propagation of optical coherence.

In continuum mechanics, the strain-rate tensor or rate-of-strain tensor is a physical quantity that describes the rate of change of the deformation of a material. It can be defined as the derivative of the strain tensor with respect to time, or as the symmetric component of the gradient (derivative with respect to position) of the flow velocity. In fluid mechanics it also can be described as the velocity gradient, a measure of how the velocity of a fluid changes between different points within the fluid.[5] The concept has implications in a variety of areas of physics and engineering, including magnetohydrodynamics, mining and water treatment. Since the strain rate tensor is a purely kinematic concept that describes the macroscopic motion of the material, therefore, it does not depend on the nature of the material, or on the forces and stresses that may be acting on it; and it applies to any continuous medium, whether solid, liquid or gas.

In this paper, we will follow the approach of velocity decomposition developed for fluid dynamics, and decompose the coherence current vector for optical coherence function of stochastic optical field. After introduction of two new tensors, referred to as the deformation-rate tensor and rotation-rate tensor, we are able to decompose a coherence current vector into its translation, rotation and deformation components to study the optical coherence dynamics. Experiments have been conducted to visualize these two newly introduced tensors to investigate the optical coherence propagation and evolution.

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AOPC 2020: Optical Sensing and Imaging Technology, edited by Xiangang Luo, Yadong Jiang, Jin Lu, Dong Liu, Proc. of SPIE Vol. 11567, 1156705 · © 2020 SPIE
CCC code: 0277-786X/20/$21 · doi: 10.1117/12.2572765

2. DECOMPOSITION OF COHERENCE CURRENT

To understand the kinematics of optical coherence, we can start our analysis from the coherence current given by $\mathbf{T}(\mathbf{r}_1,\mathbf{r}_2) = -j\alpha\bar{k}c[J^*\nabla_1 J - J\nabla_1 J^*]$, where α is a positive constant, $\bar{k} = 2\pi\bar{\nu}/c$ is the mean value of wave number, c is the constant light speed , $J(\mathbf{r}_1,\mathbf{r}_2) = \langle u^*(\mathbf{r}_1)u(\mathbf{r}_2)\rangle$ is the mutual intensity of optical field with $\langle \mathrm{L} \rangle$ indicating an ensemble average [4-5]. Without loss of generality, we will restrict our discussions to the mutual coherence function for variation of location for the $\mathbf{r}_1$ by keeping $\mathbf{r}_2$ fixed. Similar to the well-known Poynting vector in an electromagnetic field, the coherence current vector $\mathbf{T}(\mathbf{r}_1,\mathbf{r}_2)$, proportional to the linear coherence momentum, represents the directional optical coherence flux with its magnitude indicating the fringe contrast and its vectorial direction pointing in the direction of propagation. The coherence current displaced from $\mathbf{r}_1$ by a small vector $d\mathbf{r}_1$, can be approximated after a Taylor series expansion:

$$\mathbf{T}(\mathbf{r}_1 + d\mathbf{r}_1,\mathbf{r}_2) \approx \mathbf{T}(\mathbf{r}_1,\mathbf{r}_2) + \nabla_1\mathbf{T}(\mathbf{r}_1,\mathbf{r}_2)\cdot d\mathbf{r}_1, \tag{1}$$

where $\nabla_1\mathbf{T}(\mathbf{r}_1,\mathbf{r}_2)$, the gradient of the coherence current is a 3×3 tensor which could be understood as a linear map that takes a displacement vector $d\mathbf{r}_1$ to the corresponding change in the coherence current. Note the fact that any matrix can be decomposed into the sum of a symmetric matrix and an anti-symmetric matrix. Applying this to $\nabla_1\mathbf{T}(\mathbf{r}_1,\mathbf{r}_2)$ and following the same operation used in continuum mechanics, we arrive at a sum of symmetric and antisymmetric components. Thus, the coherence current may be rewritten as:

$$\mathbf{T}(\mathbf{r}_1 + d\mathbf{r}_1,\mathbf{r}_2) = \mathbf{T}(\mathbf{r}_1,\mathbf{r}_2) + \boldsymbol{\Omega}(\mathbf{r}_1,\mathbf{r}_2)\times d\mathbf{r}_1 + \boldsymbol{\varepsilon}(\mathbf{r}_1,\mathbf{r}_2)\cdot d\mathbf{r}_1, \tag{2}$$

where '$\times$' and '$\cdot$'are tensor's cross product and dot product, respectively, $\boldsymbol{\Omega}$ and $\boldsymbol{\varepsilon}$ are the *rate of coherence rotation* tensor and the *rate of coherence deformation* tensor given by

$$\boldsymbol{\Omega}(\mathbf{r}_1,\mathbf{r}_2) = \begin{bmatrix} 0 & -\Omega_z(\mathbf{r}_1,\mathbf{r}_2) & \Omega_y(\mathbf{r}_1,\mathbf{r}_2) \\ \Omega_z(\mathbf{r}_1,\mathbf{r}_2) & 0 & \Omega_x(\mathbf{r}_1,\mathbf{r}_2) \\ -\Omega_y(\mathbf{r}_1,\mathbf{r}_2) & -\Omega_x(\mathbf{r}_1,\mathbf{r}_2) & 0 \end{bmatrix} = \begin{bmatrix} 0 & -0.5\left(\frac{\partial T_y}{\partial x_1} - \frac{\partial T_x}{\partial y_1}\right) & 0.5\left(\frac{\partial T_x}{\partial z_1} - \frac{\partial T_z}{\partial x_1}\right) \\ 0.5\left(\frac{\partial T_y}{\partial x_1} - \frac{\partial T_x}{\partial y_1}\right) & 0 & 0.5\left(\frac{\partial T_z}{\partial y_1} - \frac{\partial T_y}{\partial z_1}\right) \\ -0.5\left(\frac{\partial T_x}{\partial z_1} - \frac{\partial T_z}{\partial x_1}\right) & -0.5\left(\frac{\partial T_z}{\partial y_1} - \frac{\partial T_y}{\partial z_1}\right) & 0 \end{bmatrix}, \tag{3}$$

and

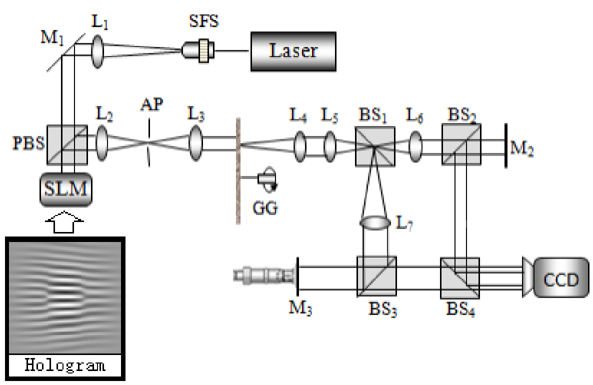
$$\underline{\boldsymbol{\varepsilon}}(\mathbf{r}_1,\mathbf{r}_2) = \begin{bmatrix} \varepsilon_{xx}(\mathbf{r}_1,\mathbf{r}_2) & \varepsilon_{xy}(\mathbf{r}_1,\mathbf{r}_2) & \varepsilon_{xz}(\mathbf{r}_1,\mathbf{r}_2) \\ \varepsilon_{yx}(\mathbf{r}_1,\mathbf{r}_2) & \varepsilon_{yy}(\mathbf{r}_1,\mathbf{r}_2) & \varepsilon_{yz}(\mathbf{r}_1,\mathbf{r}_2) \\ \varepsilon_{zx}(\mathbf{r}_1,\mathbf{r}_2) & \varepsilon_{zy}(\mathbf{r}_1,\mathbf{r}_2) & \varepsilon_{zz}(\mathbf{r}_1,\mathbf{r}_2) \end{bmatrix} = \begin{bmatrix} \frac{\partial T_x}{\partial x_1} & 0.5\left(\frac{\partial T_y}{\partial x_1} + \frac{\partial T_x}{\partial y_1}\right) & 0.5\left(\frac{\partial T_x}{\partial z_1} + \frac{\partial T_z}{\partial x_1}\right) \\ 0.5\left(\frac{\partial T_y}{\partial x_1} + \frac{\partial T_x}{\partial y_1}\right) & \frac{\partial T_y}{\partial y_1} & 0.5\left(\frac{\partial T_z}{\partial y_1} + \frac{\partial T_y}{\partial z_1}\right) \\ 0.5\left(\frac{\partial T_x}{\partial z_1} + \frac{\partial T_z}{\partial x_1}\right) & 0.5\left(\frac{\partial T_z}{\partial y_1} + \frac{\partial T_y}{\partial z_1}\right) & \frac{\partial T_z}{\partial z_1} \end{bmatrix}. \tag{4}$$

The antisymmetric tensor $\boldsymbol{\Omega}(\mathbf{r}_1,\mathbf{r}_2)$ represents a rigid body rotation of the coherence current along the running point $\mathbf{r}_1$ where the corresponding coherence vorticity vector is given by $(\Omega_x,\Omega_y,\Omega_z)^t = \nabla_1\times\mathbf{T}(\mathbf{r}_1,\mathbf{r}_2)/2$ with a superscript 't' indicating a matrix transpose. The coherence vorticity is a key characteristic of coherence current and have been used to explain the coherence vortex [4-5]. The symmetric coherence deformation rate tensor $\boldsymbol{\varepsilon}(\mathbf{r}_1,\mathbf{r}_2)$ describes optical coherence expansion or contraction $\varepsilon_{mn},(m=n)$ and optical coherence shear $\varepsilon_{mn},(m\neq n)$ resulting from optical coherence gradients in the inner of the volume. Therefore, we can refer to the diagonal components in this tensor as the normal strain rates of optical coherence and the off-diagonal components as shear strain rates of optical coherence. From

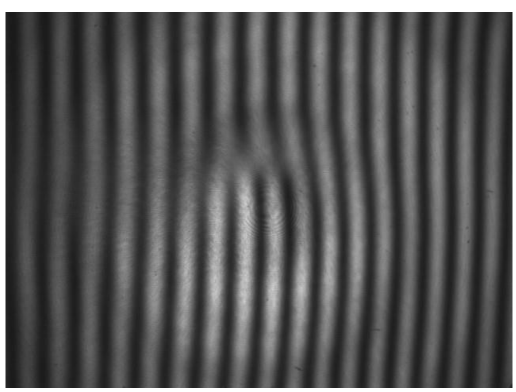
Eq. (2)-(4), we can see that, to the first order in the linear dimensions of a small region surrounding the position $\mathbf{r}_1$, the coherence current vector $\mathbf{T}$ stemming from the correlation of optical fields at the point $\mathbf{r}_1 + d\mathbf{r}_1$ and $\mathbf{r}_2$ consists, in effect, of the superposition of a uniform translation of coherence current $\mathbf{T}(\mathbf{r}_1, \mathbf{r}_2)$, a pure deforming motion characterized by the rate of optical coherence deformation tensor $\boldsymbol{\varepsilon}(\mathbf{r}_1, \mathbf{r}_2)$ which itself may be decomposed into an isotropic expansion or contraction and a shearing motion without change of its volume, and a rigid-body rotation with angular velocity $(\Omega_x, \Omega_y, \Omega_z)^t$.

3. EXPERIMENTAL DEMONSTRATION

To study the evolution of the coherence currents associated with a coherence vortex, an experiment has been conducted to synthesize a phase singularity in the optical coherence function. Fig.2(a) shows the experimental geometry for coherence synthesis and visualization based on the principle of coherence holography [7]. A computer-generated hologram encoded with vortex information is projected onto a rotating ground glass to serve as a spatially incoherent light source. For coherence visualization, two telescope systems with different magnification have been used for two arms of the Michelson interferometer to introduce the radial shearing. From the recorded interferogram as shown in Fig.1(b), we have reconstructed the complex-valued mutual intensity and measured the visibility of the fringes by using the Fourier transform method [8]. By calculation of coherence current from its definition, we obtained the distribution of the coherence current vector and conducted its decomposition from the two newly introduced tensors based on their definition to study the evolution of the optical coherence for stochastic optical field.



(a)



(b)

Fig.1. Experimental setup of coherence holography (a); and the recorded interferogram (b).

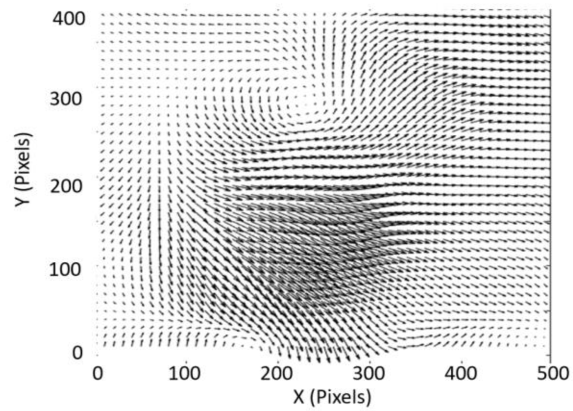


Fig.2 Quiver plot of the coherence currents reconstructed from the recorded interferogram

Figure 2 shows the quiver plot of the coherence current reconstructed from the recorded interferogram as shown in Fig. 1(b), where the length of each arrow indicates the magnitude of local coherence current with its arrow direction pointing out the corresponding orientation of coherence current vector. As expected, the coherence current circulates around the coherence vortex located at the centers of fork-like fringe pattern.

Since we are only carrying out a 2-dimensional analysis of the coherence current from the recorded interferogram, the rate of rotation tensor $\mathbf{\Omega}$ and rate of deformation tensor $\boldsymbol{\varepsilon}$ has been simplified by eliminating some tensor components associated with the $\hat{z}$ coordinate. Following such a simplification, the tensor components $\Omega_z, \varepsilon_{xx}, \varepsilon_{yy}$ and $\varepsilon_{xy} (= \varepsilon_{yx})$ remain and can be reconstructed from the numerical calculations based on their definitions in Eqs. (3) and (4).

Figure 3 shows the distribution of Ω_z indicating the distribution of the vorticity as the rate of rotation around the $\hat{z}$ axis. As expected, a large value of Ω_z can be observed at the location where a coherence vortex exits. When the value of Ω_z becomes negative, the local direction for the rate of rotation changes its direction.

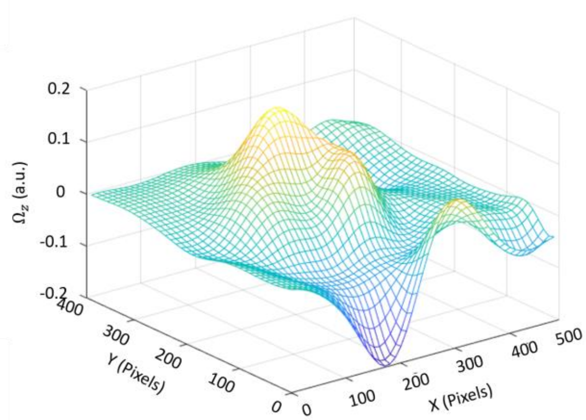
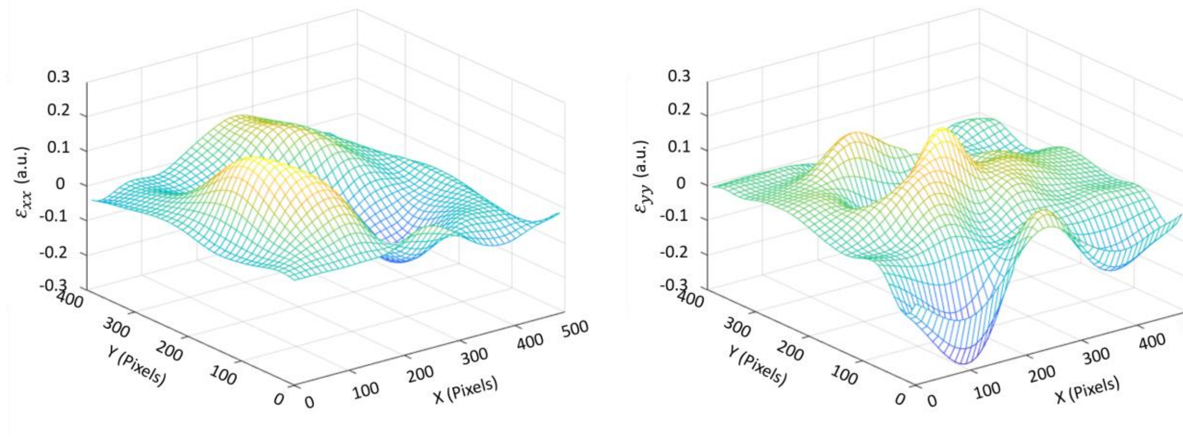


Fig. 3 Distribution of the tensor component Ω_z for the rate of coherence rotation tensor



(a) (b)

Fig. 4 Distribution of the tensor components ε_{xx} and ε_{yy} for the rate of coherence deformation tensor

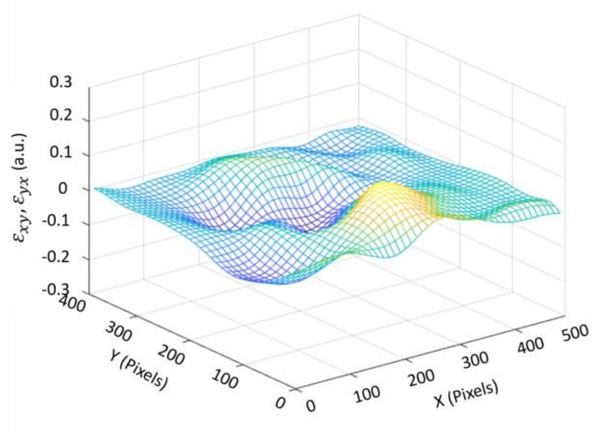


Fig. 5 Distribution of the tensor components ε_{xy} and ε_{yx} for the rate of coherence deformation tensor

Figure 4(a) and (b) give the distributions of the normal rates of deformation for the coherence current $\mathbf{T}(\mathbf{r}_1, \mathbf{r}_2)$ along the $\hat{x}$ and $\hat{y}$ directions, respectively, where the normal deformation rates for this example can be observed and analyzed from these diagonal tensor components ε_{xx} and ε_{yy}. The corresponding shearing deformation rates of $\mathbf{T}(\mathbf{r}_1, \mathbf{r}_2)$, i.e. $\varepsilon_{xy} (= \varepsilon_{yx})$ have been given in Fig. 5 indicating a distribution of change rate in shape for the coherence current.

4. CONCLUSIONS

In summary, we have adopted the theorem developed for fluid dynamics to study the dynamics of the optical coherence for stochastic optical field. By introducing the deformation-rate tensor and rotation-rate tensor to statistical optics, we have investigated the evolution of the coherence current by its decomposition into a translation, rotation and deformation contributions. Experiments have been conducted to demonstrate these two newly introduced tensors.

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