Hybrid modified gravity unifying local tests, galactic dynamics and late-time cosmic acceleration[*]

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Abstract

The non-equivalence between the metric and Palatini formalisms of $f(R)$ gravity is an intriguing feature of these theories. However, in the recently proposed hybrid metric-Palatini gravity, consisting of the superposition of the metric Einstein-Hilbert Lagrangian with an $f(\mathcal{R})$ term constructed à la Palatini, the "true" gravitational field is described by the interpolation of these two non-equivalent approaches. The theory predicts the existence of a light long-range scalar field, which passes the local constraints and affects the galactic and cosmological dynamics. Thus, the theory opens new possibilities for a unified approach, in the same theoretical framework, to the problems of dark energy and dark matter, without distinguishing a priori matter and geometric sources, but taking their dynamics into account under the same standard.

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There has been considerable interest in modifications of the geometric part of Einstein's field equations, mainly motivated by the late-time cosmic acceleration and dark matter issues [1–7]. In particular, gravitational actions consisting of more general combinations of curvature invariants than the pure Einstein-Hilbert term have been investigated extensively [8–16]. Einstein though was more satisfied with the geometric part of the equations, and has been quoted to say that while the "left-hand side is carved of marble, the right-hand side is made of straw." However, in generalized gravity theories, the problem of coupling matter to gravity is often reduced to the question of which frame matter resides in with respect to gravity. The matter Lagrangian and the corresponding stress-energy tensor are defined in the usual way, but the metric that matter couples to may be related by a conformal, or more generally a disformal, transformation to the gravitational metric. Apart from specific non-conservation terms in the continuity equations, the structure of the theory is retained.

In this context, a new class of $f(\mathcal{R})$ gravity theories, denoted C-theories [17], were considered. In the latter, the connection was related to the conformally scaled metric $\hat{g}_{\mu\nu} = C(\mathcal{R})g_{\mu\nu}$ with a scaling dependence on the scalar curvature $\mathcal{R}$. It was shown that the Einstein and Palatini gravities were obtained as special limits, and in addition to this, C-theories include completely new physically distinct gravity theories even when $f(\mathcal{R}) = \mathcal{R}$. With nonlinear $f(\mathcal{R})$, C-theories interpolate and extrapolate the Einstein and Palatini cases and avoid some of their conceptual and observational problems. In an earlier work [18], the known equivalence between higher order gravity theories and scalar tensor theories was generalized to a new class of theories. More specifically, in the context of the Palatini formalism, where the metric and connection are treated as independent variables (see [19] for a recent review), the Lagrangian density was generalized to a function of the Ricci scalar computed from the metric, and a second Ricci scalar computed from the connection. It was shown that these theories can be written as tensor-multi-scalar theories with two scalar fields.

More radically, one may modify the response of matter to gravity by defining an action, which depends nonlinearly upon the matter Lagrangian [20–25], or its trace [26–28]. Generally, the motion is non-geodesic, and takes place in the presence of an extra force orthogonal to the four-velocity [29]. In fact, in these cases the motion of matter is typically altered already in flat Minkowski space, and one may expect instabilities due to new nonlinear interactions within the matter sector.

A natural way to obtain solely gravitational modifications of the behaviour of matter

emerges in the Palatini formulation of extended gravity actions. There the relation between the independent connection and the metric turns out to depend upon the trace of the matter stress-energy tensor in such a way that the field equations effectively feature extra terms given by the matter content. However, since the extra terms are fourth order in derivatives, the theory is problematic both at the theoretical and phenomenological levels [30].

In the present paper, we propose a simple generalization of the models resulting from modified gravity actions within the Palatini formalism, where such problems are absent. An equivalent formulation is presented as a particular one-parameter class of scalar-tensor theories. In these theories the scalar field mediates the new type of coupling to the matter. By a constraint that is an identity within this class of theories, the scalar field can be algebraically eliminated in terms of the matter content and the Ricci curvature in the field equations. Generally the field equations then feature fourth order derivatives of both the metric and the matter fields. This provides a consistent and covariant means to modify the appearance of matter fields in the field equations, without affecting the conservation laws. This also opens a new perspective on the $f(R)$ theories, which are the special limits of the one-parameter class of theories where the scalar field depends solely on the stress energy trace (Palatini version) or solely on the Ricci curvature (metric version).

More specifically, consider a one-parameter class of scalar-tensor theories in which the scalar field is given as an algebraic function of the trace of the matter fields and the scalar curvature [31]:

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2} \phi R - \frac{D-1}{2(D-2)\left(\Omega_A - \phi\right)} (\partial \phi)^2 - V(\phi) \right], \tag{1}$$

The theories can be parameterized by the constant Ω_A. The limiting values $\Omega_A = 0$ and $\Omega_A \to \infty$ correspond to scalar-tensor theories with the Brans-Dicke parameter $\omega = -(D-1)/(D-2)$ and $\omega = 0$. These limits reduce to $f(R)$ gravity in the Palatini and the metric formalism, respectively.

This sheds further light on the nature of $f(R)$ gravity theories. They can be understood as the – quite special – limits of the action (1). For any value of Ω_A, the field can be algebraically eliminated in terms of R and T from the equations of motion. For any finite value of Ω_A, its value depends both on the matter and curvature. In particular, $\Omega_A = 1$ corresponds to the case where the deviation from the Einstein theory is given directly by the deviation from the trace equation $X = -\kappa^2 T + (D/2 - 1)R$. In the limit $\Omega_A \to \infty$

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the propagating mode is given solely by the curvature, $\phi(R,T) \to \phi(R)$, and in the limit $\Omega_A \to 0$ solely the matter fields $\phi(R,T) \to \phi(T)$. In the general case the field equations are fourth order both in the matter and in the metric derivatives.

More specifically, the intermediate theory with $\Omega_A = 1$, corresponds to the hybrid metric-Palatini gravity theory proposed in [32], where the action is given by

$$S = \int d^D x \sqrt{-g} \left[R + f(\mathcal{R}) + 2\kappa^2 \mathcal{L}_m \right] . \tag{2}$$

which, in $D = 4$, can be recast into a scalar-tensor representation [32–34] given by the action

$$S = \frac{1}{2\kappa^2} \int d^4 x \sqrt{-g} \left[(1+\phi)R + \frac{3}{2\phi} \nabla_\mu \phi \nabla^\mu \phi - V(\phi) \right] + S_m , \tag{3}$$

where S_m is the matter action, $\kappa^2 = 8\pi G/c^3$, and $V(\phi)$ is the scalar field potential. Note that the gravitational theory given by Eq. (3) is similar to a Brans-Dicke scalar-tensor action with parameter $w = -3/2$, but differs in the coupling to curvature.

The variation of this action with respect to the metric tensor provides the field equations

$$(1+\phi)G_{\mu\nu} = \kappa^2 T_{\mu\nu} + \nabla_\mu \nabla_\nu \phi - \Box \phi g_{\mu\nu} - \frac{3}{2\phi} \nabla_\mu \phi \nabla_\nu \phi + \frac{3}{4\phi} \nabla_\lambda \phi \nabla^\lambda \phi g_{\mu\nu} - \frac{1}{2} V g_{\mu\nu}, \tag{4}$$

where $T_{\mu\nu}$ is the matter stress-energy tensor. The scalar field ϕ is governed by the second-order evolution equation

$$-\Box \phi + \frac{1}{2\phi} \nabla_\mu \phi \nabla^\mu \phi + \frac{1}{3} \phi \left[2V - (1+\phi)\frac{dV}{d\phi} \right] = \frac{\phi \kappa^2}{3} T , \tag{5}$$

which is an effective Klein-Gordon equation [32, 33].

In the weak field limit and far from the sources, the scalar field behaves as $\phi(r) \approx \phi_0 + (2G\phi_0 M/3r)e^{-m_\varphi r}$; the effective mass is defined as $m_\varphi^2 \equiv (2V - V_\phi - \phi(1+\phi)V_{\phi\phi})/3|_{\phi=\phi_0}$, where ϕ_0 is the amplitude of the background value. The metric perturbations yield

$$h_{00}^{(2)}(r) = \frac{2G_{\text{eff}} M}{r} + \frac{V_0}{1+\phi_0} \frac{r^2}{6} , \qquad h_{ij}^{(2)}(r) = \left(\frac{2\gamma G_{\text{eff}} M}{r} - \frac{V_0}{1+\phi_0} \frac{r^2}{6} \right) \delta_{ij} , \tag{6}$$

where the effective Newton constant G_{eff} and the post-Newtonian parameter γ are defined as

$$G_{\text{eff}} \equiv \frac{G}{1+\phi_0} \left[1 - (\phi_0/3) e^{-m_\varphi r} \right] , \qquad \gamma \equiv \frac{1 + (\phi_0/3) e^{-m_\varphi r}}{1 - (\phi_0/3) e^{-m_\varphi r}} . \tag{7}$$

As is clear from the above expressions, the coupling of the scalar field to the local system depends on ϕ_0. If $\phi_0 \ll 1$, then $G_{\text{eff}} \approx G$ and $\gamma \approx 1$ regardless of the value of m_φ^2. This

contrasts with the result obtained in the metric version of $f(R)$ theories, and, as long as ϕ_0 is sufficiently small, allows to pass the Solar System tests, even if the scalar field is very light.

In the modified cosmological dynamics, consider the spatially flat Friedman-Robertson-Walker (FRW) metric $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$, where $a(t)$ is the scale factor. Thus, the modified Friedmann equations take the form

$$3H^2 = \frac{1}{1+\phi}\left[\kappa^2\rho + \frac{V}{2} - 3\dot\phi\left(H + \frac{\dot\phi}{4\phi}\right)\right],\tag{8}$$

$$2\dot H = \frac{1}{1+\phi}\left[-\kappa^2(\rho+P) + H\dot\phi + \frac{3}{2}\frac{\dot\phi^2}{\phi} - \ddot\phi\right]\tag{9}$$

respectively.

The scalar field equation (5) becomes

$$\ddot\phi + 3H\dot\phi - \frac{\dot\phi^2}{2\phi} + \frac{\phi}{3}[2V - (1+\phi)V_\phi] = -\frac{\phi\kappa^2}{3}(\rho - 3P).\tag{10}$$

As a first approach, consider a model that arises by demanding that matter and curvature satisfy the same relation as in GR. Taking

$$V(\phi) = V_0 + V_1\phi^2,\tag{11}$$

the trace equation automatically implies $R = -\kappa^2 T + 2V_0$ [32, 33]. As $T \to 0$ with the cosmic expansion, this model naturally evolves into a de Sitter phase, which requires $V_0 \sim \Lambda$ for consistency with observations. If V_1 is positive, the de Sitter regime represents the minimum of the potential. The effective mass for local experiments, $m_\varphi^2 = 2(V_0 - 2V_1\phi)/3$, is then positive and small as long as $\phi < V_0/V_1$. For sufficiently large V_1 one can make the field amplitude small enough to be in agreement with Solar System tests. It is interesting that the exact de Sitter solution is compatible with dynamics of the scalar field in this model.

Relative to the galactic dynamics, a generalized virial theorem, in the hybrid metric-Palatini gravity, was extensively analyzed [35]. More specifically, taking into account the relativistic collisionless Boltzmann equation, it was shown that the supplementary geometric terms in the gravitational field equations provide an effective contribution to the gravitational potential energy. The total virial mass is proportional to the effective mass associated with the new terms generated by the effective scalar field, and the baryonic mass. This shows that the geometric origin in the generalized virial theorem may account for the well-known

virial mass discrepancy in clusters of galaxies. In addition to this, astrophysical applications of the model where explored, and it was shown that the model predicts that the effective mass associated to the scalar field, and its effects, extend beyond the virial radius of the clusters of galaxies. In the context of the galaxy cluster velocity dispersion profiles predicted by the hybrid metric-Palatini model, the generalized virial theorem can be an efficient tool in observationally testing the viability of this class of generalized gravity models. Thus, hybrid metric-Palatini gravity provides an effective alternative to the dark matter paradigm of present day cosmology and astrophysics.

In a monistic view of Physics, one would expect Nature to make somehow a choice between the two distinct possibilities offered by metric and Palatini formalisms. We have shown, however, that a theory consistent with observations and combining elements of these two standards is possible. Hence gravity admits a diffuse formulation where mixed features of both formalisms allow to successfully address large classes of phenomena.

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