Bargaining Failures and Merger Policy

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Abstract

We study approval rules in a model where horizontal merger proposals arise endogenously as the outcome of negotiations among the firms in the industry. We make two main points. First, relatively inefficient merger proposals succeed with positive probability. That is, the negotiation process may result in a particular merger agreement despite the existence of an alternative one that would generate higher profits and higher consumer surplus. Second, the antitrust authority should optimally commit to an approval rule that is more stringent for all mergers than the optimal ex-post rule.

KEYWORDS: endogenous mergers, merger policy, inefficient bargaining, synergies, antitrust, flexible protocol.

JEL classification numbers: L13, L41

1 Introduction

A horizontal merger may increase market power and hence reduce welfare. But a merger may also create social value if it generates cost savings. One important task for competition authorities is to examine merger proposals and evaluate the balance between these two effects. Approval decisions are typically viewed as discretionary: authorities consider each merger proposal in isolation and approve it only if it improves social welfare. That is, if cost savings outweigh the effects of enhanced market power. However, a growing literature has studied the potential advantages of rules over discretion. This literature has discussed how information asymmetries and dynamic considerations may justify the use of rules.[1] Additionally, Armstrong and Vickers (2010) and Nocke and Whinston (2013) have argued that an ex-ante rule may improve the selection of merger proposals. We take the latter approach. As Armstrong and Vickers (2010) point out, authorities are in the position of a principal dealing with an agent that chooses a project from a feasible set that only the agent can observe. The principal can influence the behavior of the agent, not through monetary rewards, but by "specifying what the agent is and is not allowed to do". Armstrong and Vickers (2010), and subsequently Nocke and Whinston (2013) in more detail, have discussed the optimal merger policy when the authority faces a conflict of interest with one firm (the acquirer) that can choose between different merger partners. The conflict of interest arises from a possible misalignment between the profitability of these alternative mergers and their effect on consumer surplus (or on a weighted average of profits and consumer surplus). They show that a discretionary policy that approves all consumer surplus enhancing mergers is not ex-ante optimal. A more stringent policy for mergers that involve larger firms can improve the selection of merger proposals, by discriminating against mergers that generate higher profits but lower consumer surplus.

In this paper we begin by sidestepping the conflict of interest between authorities and the industry, and focus instead on a problem that arises when the principal deals with multiple agents: bargaining failures. A merger proposal is often the result of negotiations involving several firms with conflicting interests. Even if, as a whole, the interests of

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[1]See Bensako and Spulber (1993) and Nocke and Whinston (2010).

the "industry", aggregate profits, move parallel to those of the authority, negotiations among alternative merging partners do not necessarily result in the maximization of total industry profits. We make two main points. First, relatively inefficient merger proposals succeed with positive probability. If no firm is exogenously designated an essential member of all feasible mergers, and since side payments between merging and not merging firms are not possible, then the negotiation process will sometimes select a merger agreement despite the existence of an alternative one that would generate higher aggregate profits and higher consumer surplus. Second, because of these bargaining failures, the antitrust authority should optimally commit to an approval rule that is more stringent than the optimal discretionary policy for all mergers. The optimal rule balances the benefits (a more stringent rule reduces the probability that the best merger is beaten out by a less desirable option) with the costs (some merger proposals that generate positive, but small, gains in consumer surplus will be blocked.)

We study an abstract model that encompasses several standard oligopoly models, including Cournot competition with homogeneous products and Dixit's (1979) oligopoly model with differentiated products and linear demand. In the benchmark model, any pair among three ex-ante symmetric firms may agree to merge and realize cost efficiencies that vary across pairs. Hence, the resulting surplus will depend on the identity of the merging firms. Ex-ante symmetry guarantees that the most profitable merger is also the one that achieves the highest consumer surplus. Thus, we abstract from any conflict of interest between antitrust authorities and the "industry", so that the effects of the bargaining process are more transparent. We envision the negotiations that lead to the submission of a merger proposal as a flexible process in which each firm may bargain bilaterally and simultaneously with any other firm. In order to formalize these ideas we assume that firms use a specific non-cooperative bargaining protocol that, in contrast to most protocols used in this literature, does not place any artificial restriction on the endogenous likelihood of different mergers. The bargaining protocol always generates a unique prediction, and if the relative synergies of different mergers are not sufficiently different then the outcome of the bargaining process is sometimes inefficient from the industry's point of view.

As discussed in Section 5, bargaining failures have been neglected by the literature on

endogenous mergers and merger policy, but not by the more abstract theory of coalition formation. Indeed, the predictions of our protocol concerning the possibility of inefficient outcomes are perfectly in line with this literature. The merit of our protocol is to generate clean predictions (unique equilibrium) and an intuitive characterization of bargaining failures. In particular, inefficient outcomes are predicted if and only if the core of the underlying cooperative game is empty. Hence, the main qualitative results of the paper are not an artifact of a specific protocol and would also be obtained if we used instead a more standard protocol.[2]

The connection between the emptiness of the core and a positive probability of an inefficient outcome is not accidental. When the core is empty there is no single bilateral agreement that is immune to a counteroffer by the firm left out of the deal. In other words, if the prediction were that one of the mergers occurs with probability one, then the left out firm would gain by offering a better deal to one of the merging partners. Instead, for these cases, in our game all three firms have a positive probability of being part of the successful merger in equilibrium, and each firm is actually indifferent about its merging partner. But then the best merger will not happen with probability one. A more stringent merger policy may restore the non-emptiness of the core by prohibiting mergers that result in modest welfare gains, but that are able to challenge more socially desirable mergers.

Our paper is closely related to three different strands of the literature: endogenous mergers, optimal merger approval rules, and non-cooperative bargaining. Many studies have focused on how mergers are endogenously determined. This literature typically ignores merger control. Some authors (Barros, 1998; and Horn and Persson, 2001) have approached the problem using cooperative solution concepts for games in partition function form, since a merger creates externalities on non-merging firms. Other authors (Kamien and Zang, 1990; Gowrisankaran, 1999; Inderst and Wey, 2004; Fridolffson and Stennek, 2005a; Qiu and Zhou, 2007; and Nocke and Whinston, 2013) have set up non-cooperative games where both the market structure and the division of surplus are determined si-

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[2]Our protocol asymptotically implements a new solution concept for cooperative games that we have developed elsewhere (Burguet and Caminal, 2011).

multaneously. Some of these models set restrictions on the subsets of firms that can participate in a merger. For example, Inderst and Wey (2004) assume that there is an exogenously designated target and in Nocke and Whinston (2013) it is the acquirer who is determined exogenously. In a similar spirit, Gowrisankaran (1999) assumes that the largest firm is the only one that can acquire a smaller firm. Qiu and Zhou (2007) propose a more flexible bargaining protocol but nature still plays a decisive role. In fact, when all potential mergers are profitable and attractive their protocol predicts that the outcome would be determined by nature's exogenous choice. Hence, these studies do not contemplate the possibility that each member of, say, a three-firm group considers merging with each of the other two, which, as we discussed above, is a crucial ingredient in the emergence of bargaining failures.[3] In other studies, these restrictions on the feasible merger combinations are not imposed, but all mergers are assumed symmetric, so that bargaining failures are impossible (Kamien and Zang, 1990; and Fridolfsson and Stennek, 2005a).[4]

The literature on optimal merger approval rules can be traced back to Besanko and Spulber (1993). We have already commented the work by Armstrong and Vickers (2010) on delegated project choice. Nocke and Whinston (2013) apply their ideas to merger control in a Cournot model with endogenous determination of merger proposals. They show that the optimal ex-ante rule is more stringent for mergers that cause a larger change in the (naively computed) Herfindahl index. We borrow from them the specification of the merger review process.[5] As discussed above, we place the focus on the negotiations among firms when none of them is exogenously designated to be an essential member of any feasible merger. In Section 3 we discuss the consequences of ex-ante asymmetries.

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[3]In some real world cases a particular firm (perhaps, in financial distress) may appear as the natural target. This was the case, for instance, in the Nestlé-Perrier case following the 'benzene scandal', where Nestlé and the Agnelli's group competed to acquire the troubled French company. In other situations the industry may require an increase in concentration, but *a priori* all possible subsets could sensibly attempt a cost-reducing merger. A recent example is the US airlines industry. Before announcing their merger with Continental in 2010, United had been reported negotiating with US Airways. Moreover, the press speculated about almost all possible bilateral mergers involving these three firms plus American.

[4]Kamien and Zang (1990) assume that each firm simultaneously sets an asking price and bids for each of the other firms. Much closer to our modeling approach, Fridolfsson and Stennek (2005a) set up a dynamic bargaining game, which will be discussed below.

[5]Our discussion of dynamic merger policy (Section 4) is also related to Motta and Vasconcelos (2005), Qiu and Zhou (2007), and Nocke and Whinston (2010).

Asymmetries exacerbate the expected consumer surplus losses associated to the bargaining process, as compared to a first best where the merger that maximizes consumer surplus was always implemented. Numerical simulations indicate that the effectiveness of the ex-ante optimal rule increases with asymmetries. This optimal ex-ante policy is more stringent for all possible mergers, but much more so for mergers that involve a higher increase in the (naively computed) Herfindhal index. Hence, the insights of Nocke and Whinston (2013) and our benchmark model are complementary. However, under asymmetry the interaction between the conflict of interest and the bargaining failure is characterized by some subtleties. Indeed, as in the symmetric case, when asymmetries are moderate private bargaining failures (that is, firms' failure to maximize industry profits) add to the losses provoked by the conflict of interest discussed in Nocke and Whinston (2013). Consequently, in our framework optimal approval rules are more stringent than in the more standard, pure conflict of interest, setup. However, for large asymmetries, private bargaining failures actually attenuate the effects of the conflict of interest. In other words, for large asymmetries, our bargaining protocol predicts that the probability of success of the efficient merger is higher than in the case merger selection were exclusively determined by the maximization of aggregate profits. As a result, the optimal approval rule for mergers involving larger firms is less stringent than in the alternative scenario.

The merger problem we take up in this paper is similar (and equivalent, for some parameter values) to what has been termed the three-person/three-cake problem (see, for instance, Binmore, 1985), or in general a (restricted) game of coalition formation. Non cooperative analyses of this sort of problems abound, and most use one version or another of a dynamic proponent-respondent game in the Rubinstein-Stahl tradition. (See Ray, 2007, for a general discussion including games with externalities, and Compte and Jehiel, 2010, for a recent example.) As we have already mentioned, we use a less standard game where the ordering of proposers is endogenous. That is, the agreed outcome is not a consequence of any arbitrary order of proposals: on the contrary, both order and outcome, are jointly determined by the primitives of the bargaining problem.[6]

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[6]We must agree with Ray in that "a theory that purports to yield solutions that are independent of proposer ordering is suspect." The key for our unique prediction is that our theory includes an endogenous determination of this "order of proponents".

The benchmark model describes the implications of alternative market structures using reduced-forms. Both the model and the main results are presented in Section 2. In Section 3 we maintain the assumption of three initial firms but allow for ex-ante asymmetries and in Section 4 we discuss sequential mergers by considering an industry with four initial firms. In both cases we show that bargaining failures are compounded with the conflict of interest between authorities and the industry. In Section 5 we discuss the robustness of the results to changes in the bargaining protocol. Finally, Section 6 contains a brief summary and comments on several additional issues.

2   The benchmark model

We consider an industry where firms compete in the market but also bargain about the possibility of submitting a merger proposal. We embed bargaining and competition in a dynamic but stationary setting. First, merger opportunities, i.e., potential synergies, do not evolve with time. Second, we restrict ourselves to "stationary" equilibria, so that firms' strategies do not depend on history. That is, firms play the one-shot equilibrium strategies in the market stage, and ignore past moves in the bargaining stage.

Time is a discrete variable indexed by $t = 0, 1, 2, ...$ (infinite horizon). At the beginning of the game there are three identical firms: $1, 2, 3$. The following sequence of moves takes place in $t = 0$:

a) Competition authorities announce a rule for approving mergers that will remain fixed forever (full commitment).

b) All firms, but not authorities, learn the synergies that would result from each merger. In other words, the marginal cost for the firm resulting from a merger between firms i and j, denoted by c_{ij}, for all (i, j), $i, j = 1, 2, 3, i \neq j$, becomes firms' common knowledge.

c) The three firms bargain about the possible submission of a merger proposal. They negotiate bilaterally within a protocol specified below. If two firms agree on submitting a merger proposal, authorities will learn the cost of the merged firm, and approve the merger if and only if it complies with the announced rule.

d) If no merger has been authorized, then the triopoly game is played in the market; otherwise, the duopoly game is played where the merged firm competes with the stand alone firm.

In each later period t, $t > 0$, if a merger was successful in the past then the existing firms keep playing the duopoly game. If no merger proposal was ever submitted, the game follows c) and d) above. That is, again firms engage in bargaining followed by competition.

Authorities are assumed to maximize the expected present value of consumer surplus, as it appears to be the case in the real world. Both firms and competition authorities discount the future at the rate r. We will focus on the case that r is arbitrarily small, so that the friction built in the bargaining protocol is also arbitrarily small.

2.1 Static competition in a nutshell

The effect of synergies on the distribution of profits and consumer surplus will be represented by exogenous functions. The three initial firms have access to the same constant returns to scale technology and hence face the same marginal cost c_0. The equilibrium level of profits of a single firm under triopoly is denoted by π_0, and the level of consumer surplus by CS_0.

Only two-firm mergers generate synergies, which can be different for different pairs, and as a result authorities will never allow a merger to monopoly.[7] As a notational convention, firms 1 and 2 are the partners to the most efficient and profitable merger. Of course, in making this convention, we must assume that the identity of the firms is common knowledge for firms, but unknown to authorities. For simplicity, we also assume that the other two mergers are symmetric; i.e., $c_{13} = c_{23} \geq c_{12}$. We thus assume that from the authorities' point of view (c_{12}, c_{13}) are random variables distributed according to some density function $h(c_{12}, c_{13})$ that has no mass points and takes strictly positive values on $C \equiv \{(c_{12}, c_{13}) \mid 0 \leq c_{12} \leq c_{13} \leq c_0\}$, but $h(c_{12}, c_{13}) = 0$, if $(c_{12}, c_{13}) \notin C$. Also for simplicity, we restrict attention to the case that approval rules take the form of a

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[7]Unless otherwise specified, we assume that implementing a merger involves no cost.

threshold value, $\bar{c}$. Thus, at the beginning of the game authorities announce a cut-off, $\bar{c}$, and commit to approve a merger proposal if and only if the marginal cost of the merged firm is lower than $\bar{c}$.[8]

Let us now consider the duopoly game. Suppose a merger between firms i and j, with marginal cost c_{ij}, has been approved. Then the merged firm faces a stand alone firm, k, $k \neq i, j$, with marginal cost c_0. Per period profits of the merged and stand alone firms are denoted by $\pi_{ij}(c_{ij})$ and $\pi_k(c_{ij})$, respectively. Consumer surplus is denoted by $CS(c_{ij})$. In the appendix, the following assumptions are derived from first principles for some models of market competition:

(A.1) There exists a value of the marginal cost of the merged firm, c_n, $0 < c_n < c_0$, such that $CS(c_n) = CS_0$. Moreover, $CS(c_{ij})$ is a continuously differentiable function with $\frac{dCS}{dc_{ij}} < 0$. Hence, a merger increases consumer surplus if and only if $c_{ij} \leq c_n$.

(A.2) $\pi_{ij}(c_{ij})$ is continuously differentiable with $\frac{d\pi_{ij}}{dc_{ij}} < 0$. Moreover, $\pi_{ij}(c_n) > 2\pi_0$. Hence, any merger that is socially desirable is also profitable for the merging firms.

(A.3) $\pi_k(c_{ij})$ is continuously differentiable with $\frac{d\pi_k}{dc_{ij}} > 0$. Moreover, $\pi_k(c_n) = \pi_0$. Hence, any merger that is socially desirable is detrimental to the stand alone firm.

In imposing (A.3) we implicitly assume that for all $c_{ij} > 0$ the stand alone firm, k, remains active. In the models discussed at the end of this subsection, there may exist realizations of the marginal cost for which the stand alone firm leaves the market and then the merged firm becomes a monopolist. To simplify the presentation we will ignore this possibility.

Since we focus for the moment on the case $\bar{c} \leq c_n$, assumptions (A.1) through (A.3) imply that a feasible merger is not only profitable but also attractive: all mergers that would be authorized result in profits for the merged firm that exceed the joint profits in the status quo. Moreover, all firms prefer to be part of a merger rather than be left out.

For a given realization of (c_{12}, c_{13}), π_{12}, π_3 denote the distribution of profits when the efficient merger takes place, and π_{13} $(= \pi_{23})$, π_2 $(= \pi_1)$ when one of the less efficient merger materializes. Thus, these four numbers, $\pi_{12}, \pi_{13}, \pi_2, \pi_3$, are the crucial parameters of the bargaining game. Assumptions (A.2) and (A.3) imply that if $c_{ij} \leq c_n$ then $\frac{1}{2}\pi_{12} \geq$

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[8]In a related setup Nocke and Whinston (2013) discuss in detail the optimality of cut-off policies.

$\frac{1}{2}\pi_{13} > \pi_0 \geq \pi_2 \geq \pi_3.$

Finally we assume:

(A.4) For all $c_{ij} \leq c_n, -\frac{d\pi_{ij}}{dc_{ij}}(c_{ij}) > 2\frac{d\pi_k}{dc_{ij}}(c_{ij})$

This assumption will contribute to pin down the comparative statics of the bargaining outcome, but is not crucial for the main result on the predicted merger and on merger rules. It is important to notice that (A.4) implies that, if all mergers enhance consumer surplus, $c_{12} \leq c_{13} \leq c_n$, then aggregate profits under the efficient merger are higher than under a less efficient one: $\pi_{12} + \pi_3 \geq \pi_{13} + \pi_2$. The reason is that starting at $c_{12} = c_{13}$, any reduction in c_{12} increases π_{12} more than what it reduces π_3. Hence, under (A.4) private and social goals are aligned as far as the ranking of mergers is concerned. This simply clarifies the nature of the bargaining failures.

In some of the most standard models of oligopoly, the equilibrium profits and consumer surplus satisfy these assumptions. That is, our model captures in reduced form those models. As we show in the appendix, these include the Cournot model when the inverse demand function $p(Q)$ is strictly decreasing and satisfies $p'(Q)Q + p"(Q) < 0$.[9] Differentiated-goods models in the spirit of Dixit (1979) also result in profits and consumer surplus that satisfy (A.1) through (A.4). For instance, assume that the representative consumer's utility function is quadratic in the varieties produced by three firms, $i = 1, 2, 3$, and additively separable in the numeraire good, x,

$$(1) \qquad U(\mathbf{q}, x) = x + \alpha \sum_{i=1}^{3} q_i - \frac{\beta}{2} \sum_{i=1}^{3} q_i^2 - \frac{\gamma}{2} \sum_{i=1}^{3} \prod_{j \neq i} q_i q_j,$$

where $\alpha > 0$, $\beta > \gamma > 0$, q_i represents the quantity of variety i consumed, and $\mathbf{q} = (q_1, q_2, q_3)$. We show in the appendix that profits and consumer surplus satisfy (A.1) through (A.4), independently of whether firms compete in prices or quantities, as long as the products are not too close substitutes.

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[9]Some additional conditions are necessary for $c_n > 0$, which is part of assumption (A.1), both in the Cournot model and in the differentiated product model. If these restrictions are violated then the probability that a merger is able to increase consumer surplus is zero, and so the problem is not interesting.

2.2   The bargaining game

We model the negotiation between firms leading to a merger proposal as a bargaining protocol that is more flexible, in a sense that will be specified below, than most protocols used in the literature. In Section 5 we will argue that the main qualitative results about merger policy would also hold if inestead we used a more standard protocol. However, these other protocols fail to offer clear predictions (multiplicity of equilibria) and tend to distort the distribution of surplus in favor of the weaker player, causing non intuitive comparative statics and discontinuities of the outcomes.

We model firms' negotiations as a game that is repeated in each period until an agreement is reached. There are two elements to any agreement: the identity of the merging firms, and the division of surplus resulting from the merger. Thus, in each period the protocol first allows firms to endogenously select the negotiating partners, and then allows those partners to discuss the realization of the merger and the division of the corresponding surplus. The first part of the protocol is as follows:

Selection of negotiating partners

(1) Nature selects one of the three firms, each with probability $\frac{1}{3}$. Let that firm be A.

(2) Firm A invites one of the other two firms to become its negotiation partner. Let that firm be B.

(3) Firm B accepts or rejects the invitation. If it accepts, then firms (A, B) enter into the negotiation stage. Otherwise, firms (B, C) enter into the negotiation stage.

The second part, i.e., the negotiations between either (A, B) or (B, C) (let (F, E) represent in general that pair of firms), could be modeled in a variety of equivalent ways and we choose the simplest.

Actual negotiation between F and E.

(4) Nature selects one of the two firms, each with probability $\frac{1}{2}$. Let that firm be F.

(5) Firm F makes an offer to firm E: θ_F^E, understood as the per-period profits that E retains if merged with F.

(6) Firm E accepts or rejects F's offer. If E accepts then it gets θ_F^E per period, that is, $\frac{1+r}{r}\theta_F^E$ discounted total payoff; firm F gets $\pi_{FE} - \theta_F^E$, that is, $\left(\pi_{FE} - \theta_F^E\right)\frac{1+r}{r}$ discounted total payoff; and the bargaining ends. If E rejects the offer then all firms obtain in that

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period the equilibrium profits of the triopoly game, π_0, and bargaining is resumed in the next period.

Steps (1) to (6) describe the timing of the perfect-information, stage game played in each period until an agreement is reached. The protocol's most novel feature is step (3) and it is this feature what makes our protocol sufficiently flexible.[10] By introducing it, nature's choice in step (1) does not impose upper or lower bounds on the probability that any given firm is part of a successful merger in any given period. We discuss the consequences of alternative specifications of the bargaining protocol in Section 5.

We focus on Markov Perfect equilibrium (MPE) outcomes when bargaining frictions are negligible.[11] Thus, we will state results for the limit of equilibria as $r \to 0$. A stationary strategy for a firm includes a probability distribution over the two potential invitees when the firm becomes firm A in step (2), and a probability distribution over the two potential answers in step (3), if the firm is invited in step (2). Also, a strategy includes an offer to be made to each potential partner in step (5) if the firm becomes firm F in step (4), and an answer to every possible offer received from either of the two possible partners if it becomes firm E in step (5). For simplicity and keeping with the stationarity assumption, we will only consider equilibria where B's choice of partner for steps (4) to (6) does not depend on who played the role of A, and offers and answers in (5) and (6) depend on the identity of E and F, but not on who played the role of A and B.

In the trivial case that $\overline{c} < c_{12} \leq c_{13}$ no merger would pass the requirements of the authorities, and hence there is no room for negotiations. Almost as trivial is the case

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[10]Note that we could have added a trivial possibility in (3): that firm C rejects being part of the negotiations with B, and then the game moves to the next period without agreement. Without adding this possibility, firm C can always reject any offer in (6) if it becomes firm E, and make an offer that will be rejected for sure, if it is firm E. Thus any equilibrium outcome in the extended game where that option is played with positive probability would be also an equilibrium outcome of our game. We can also argue that nothing would change by adding yet another possibility, that firm C rejects firm B's invitation and instead invites firm A. The key is that: 1) any (equilibrium) probability distribution over the three possible pairs in the extended protocol can be obtained with (mixed) strategies in the protocol proposed here; and, 2) with the same continuation strategies for the negotiations stage, those strategies would also be equilibria in our protocol.

[11]As we let $r \to 0$ both bargaining rounds and market decisions become more frequent. Like most of the bargaining literature, we wish to focus on the limiting case where bargaining rounds are arbitrarily close in order to avoid introducing artificial rigidities. Linking the timing of bargaining moves and market decisions facilitates the analysis considerably by preserving the stationarity of the game.

$c_{12} \leq \bar{c} < c_{13}$, where only one merger is feasible. In the unique equilibrium for that case, the efficient merger is agreed in period 0 and firms 1 and 2 obtain an expected, per period payoff $\frac{\pi_{12}(c_{12})}{2}$, whereas firm 3 obtains $\pi_3(c_{12})$.

The most interesting case is $c_{12} \leq c_{13} \leq \bar{c}$, where three mergers are feasible. For the moment, let us focus on the case $\bar{c} \leq c_n$. Since all acceptable mergers are *profitable* and *attractive*, a merger is bound to occur immediately with probability one. However, there is still a question as to the identity of the firms involved. The following proposition characterizes the unique equilibrium outcome.

Proposition 1 *Suppose that all three mergers are feasible. Then, for r sufficiently small, there exists a unique MPE outcome, both in payoffs and probability distribution over mergers. A merger always occurs with probability 1 in the first period. (i) If $\left(\frac{1}{2} + r\right) \pi_{12} - (1 + r) \pi_{13} + \pi_3 \geq 0$ then the efficient merger between firms 1 and 2 occurs with probability one, (ii) if $\left(\frac{1}{2} + r\right) \pi_{12} - (1 + r) \pi_{13} + \pi_3 < 0$ then all three potential mergers take place with positive probability. In particular, the probability of the efficient merger is:*

$$(2) \qquad d = \frac{\pi_{12} - 2\pi_2 + 4r(\pi_{12} - \pi_{13})}{-\pi_{12} + 4\pi_{13} - 2\pi_2 - 4\pi_3}.$$

Proof. See Appendix. ∎

The uniqueness result indicates that our bargaining protocol offers sharp predictions. Even more important, these predictions include the possibility of inefficient outcomes. Although the formal proof of the proposition is contained in the appendix, it is helpful to present its heuristics here. Consider a very low value of r, so that we can virtually set $r = 0$. Notice that, in this case, the unique equilibrium identified in the proposition is efficient if and only if the core of the cooperative game is not empty. Indeed, the condition

$$(3) \qquad \frac{1}{2}\pi_{12} - \pi_{13} + \pi_3 \geq 0$$

is necessary and sufficient for the core not to be empty and is also necessary and sufficient for the merger between firms 1 and 2 to occur with probability one. Let u_i be the per

period, ex-ante (before Nature moves), equilibrium utility for firm i. When (3) is satisfied, $u_1 = u_2 = \frac{1}{2}\pi_{12}$ and $u_3 = \pi_3$. Firms 1 and 2 always invite each other to negotiate and the existence of alternative feasible mergers is irrelevant (outside option principle): any offer that firm 3 may accept or any request that it would make, i.e., above π_3, would leave at most $\pi_{13} - \pi_3 < \frac{1}{2}\pi_{12}$ for firm 1 or 2, and hence these firms have no incentives to deviate. However, when mergers are not too heterogeneous, and condition (3) fails, a pure strategy equilibrium where firms 1 and 2 always merge is impossible. Indeed, if $d = 1$ the alternative to any deal today would be again a merger between firms 1 and 2 tomorrow, and so we would still have $u_1 = u_2 = \frac{1}{2}\pi_{12}$ and $u_3 = \pi_3$. But then, when invited to negotiate, firm 1 (or, equivalently, firm 2) would prefer to negotiate with firm 3, which firm 3 would accept. Indeed, in the actual negotiation, firm 1 could offer $\theta_1^3 = \pi_3$ if it is the proposer, an offer that would be accepted. (Of course, firm 3 would offer an acceptable offer $\theta_3^1 = \frac{1}{2}\pi_{12}$, which leaves a payoff of $\pi_{13} - \frac{1}{2}\pi_{12} > \pi_3$). This renders the deviation of firm 1 (or firm 2) profitable. Hence, in this region an equilibrium with $d = 1$ does not exist.

The question is then, what could be an equilibrium when the core is empty? This is what part (ii) of the proposition answers. From our discussion above, an equilibrium would have to put positive probability on all three potential mergers. Also, firms 1 and 2 are symmetric, and so if firm 1 negotiates with firm 2, it obtains $\frac{1}{2}\pi_{12}$ per period in expected value. If instead firm 1 negotiates with firm 3, then its expected payoff per period must be $u_1 + \frac{1}{2}(\pi_{13} - u_1 - u_3)$ (the usual Nash split). As usual, mixed strategy equilibrium requires indifference, and for firm 1 to be indifferent,

$$(4) \qquad\qquad u_1 - u_3 = \pi_{12} - \pi_{13}.$$

This equation plus the two Bellman equations:

$$(5) \qquad\qquad u_1 = \frac{1+d}{2}\frac{1}{2}\pi_{12} + \frac{1-d}{2}\pi_2,$$

$$(6) \qquad u_3 = d\pi_3 + (1-d)\left(\pi_{13} - \frac{1}{2}\pi_{12}\right),$$

must be satisfied in equilibrium, and so define u_1, u_3, and d. Indeed, in equilibrium firm 1 is part of the merger with probability $d + \frac{1-d}{2} = \frac{1+d}{2}$, and gets the same payoff, $\frac{1}{2}\pi_{12}$, whether it merges with one firm or the other, as we have just argued. With probability $\frac{1-d}{2}$, firms 2 and 3 merge, and firm 1's payoff is then π_2. This is equation (5). Likewise, with probability d firm 3 is left out of the deal, and so its payoff is π_3. Otherwise, it is part of the deal and, in expected terms, receives the profits of the merged firm minus the payoff of the partner in case of merger, $\frac{1}{2}\pi_{12}$, as we have just discussed. This is equation (6). This completes the heuristic argument behind Proposition 1.

We now discuss the behavior of d, the probability of the efficient merger, when (3) fails. Note that d is always between $\frac{1}{3}$ and 1. If all mergers are equivalent, $c_{12} = c_{13}$, then $d = \frac{1}{3}$. In the other extreme, if $c_{13} - c_{12}$ is sufficiently large, so that $\frac{1}{2}\pi_{12} - \pi_{13} + \pi_3 = 0$, then $d = 1$. In the interior of this region we obtain the following comparative statics:

Remark If $\frac{1}{2}\pi_{12} - \pi_{13} + \pi_3 < 0$ (empty core) then d strictly increases with c_{13} and strictly decreases with c_{12}.

This result is a direct implication of assumption (A4). Hence, it is important to emphasize that, even though players use mixed strategies, the comparative statics are very intuitive. In particular, as c_{12} falls or c_{13} increases, firm $3's$ bargaining position weakens and the likelihood of an inefficient merger strictly decreases.

2.3 The ex-ante optimal merger policy

Consider first the case that the policy rule, $\bar{c}$, is lower than c_n. If we denote by $\Delta(c_{ij})$ the change in consumer surplus that results from a merger with efficiency level c_{ij}, $\Delta(c_{ij}) = CS(c_{ij}) - CS_0$, then the expected change in consumer surplus when the approval rule is

$\bar{c}$, can be written as:

$$W\left(\bar{c}\right) = \int_0^{\bar{c}} \int_{c_{12}}^{\bar{c}} \left\{ d\left(c_{12}, c_{13}\right) \Delta\left(c_{12}\right) + \left[1 - d\left(c_{12}, c_{13}\right)\right] \Delta\left(c_{13}\right) \right\} h\left(c_{12}, c_{13}\right) dc_{13} dc_{12} + $$

$$+ \int_0^{\bar{c}} \int_{\bar{c}}^{c_0} \Delta\left(c_{12}\right) h\left(c_{12}, c_{13}\right) dc_{13} dc_{12}.$$

The first term captures the expected change in consumer surplus when all mergers are acceptable, $c_{12} \leq c_{13} \leq \bar{c}$, while the second term captures this effect when the efficient merger is the only acceptable proposal, $c_{12} \leq \bar{c} < c_{13}$.

The effect of a change in $\bar{c}$ on W can be written as:

(7)
$$\frac{dW\left(\bar{c}\right)}{d\bar{c}} = -\int_0^{\bar{c}} \left[1 - d\left(c_{12}, \bar{c}\right)\right] \left[\Delta\left(c_{12}\right) - \Delta\left(\bar{c}\right)\right] h\left(c_{12}, \bar{c}\right) dc_{12} + \int_{\bar{c}}^{c_0} \Delta\left(\bar{c}\right) h\left(\bar{c}, c_{13}\right) dc_{13}.$$

An increase in $\bar{c}$ has two effects on the expected consumer surplus. On the one hand, it intensifies the competition between efficient and less efficient mergers; that is, it expands the cost range where less efficient mergers are also acceptable, causing a discrete fall of the probability of success of the efficient merger, from 1 to $d < 1$. This effect is the first term of (7). Note that for any value of $\bar{c} > 0$ the term is strictly negative: $\Delta\left(c_{12}\right) - \Delta\left(\bar{c}\right)$ is positive for all $c_{12} < \bar{c}$, and for values of c_{12} sufficiently close to $\bar{c}$, $d\left(c_{12}, \bar{c}\right) < 1$. On the other hand, an increase in $\bar{c}$ reduces the possibility that socially desirable mergers are blocked; that is, it expands the area where the efficient merger is the only acceptable proposal. This is the second term of (7). Note that as $\bar{c}$ approaches c_n this second term vanishes. Summing up, starting at $\bar{c} = c_n$ a decrease in $\bar{c}$ raises expected consumer surplus, since the reduction in competition between efficient and less efficient mergers generates a first order gain, while the exclusion of the efficient merger with an efficiency level close to c_n only causes a second order loss. Therefore, $\frac{dW}{d\bar{c}}\left(\bar{c} = c_n\right) < 0$.

Clearly, a value of $\bar{c}$ above c_n cannot be optimal, since both effects have the same negative sign in that range of values: an increase in $\bar{c}$ intensifies competition between the efficient and the inefficient merger and also expands the acceptance of socially undesirable mergers. Summarizing,

Proposition 2 *The optimal ex-ante merger policy is more stringent than the ex-post optimal one; i.e., $\bar{c} < c_n$.*

Thus, an approval rule more stringent than the ex-post optimal rule increases expected consumer surplus by reducing the probability of a bargaining failure.

3 Ex-ante asymmetric firms

In the baseline model firms are assumed to be ex-ante identical, and hence the interests of authorities and the industry are perfectly aligned: the rankings of alternative mergers according to consumer surplus and profits coincided. Nevertheless, the possibility of bargaining failures calls for approval rules that are more stringent for all mergers than the ex-post optimal policy. In a model with ex-ante asymmetric firms Nocke and Whinston (2013) have recently shown that the possibility of a conflict of interest between the authorities and the industry also calls for more stringent approval rules for mergers that include larger firms. In this section we argue that the two insights are complements, in the sense that more stringent approval rules are socially beneficial in more general setups. However, bargaining failures and the conflict of interest interact in non-trivial ways.

In both this and Nocke and Whinston's papers, the inability of the regulator to select the efficient merger results in what we can term a *selection failure* which translates into consumer surplus losses. When firms are ex-ante symmetric, the only source of selection failure is the inability of the bargaining parties to maximize industry profits, which can be labeled *private bargaining failures*. On the other hand, if firms were ex-ante asymmetric and always selected the merger that maximizes industry profits, then no private bargaining failure would exist, but a selection failure would still result from a *conflict of interest*.

To illustrate the interaction of these two sources of selection failures, consider a Cournot model where firms are ex-ante asymmetric and bargain according to our protocol. We show that: (1) Asymmetries increase potential losses linked to the selection failure and changes their nature; in particular, as firms become ex-ante more asymmetric, the conflict of interest between the industry and authorities becomes more salient, whereas private bargaining failures become less of a problem. (2) For large asymmetries,

private bargaining failures may even attenuate the negative consequences of the conflict of interest. (3) The optimal ex-ante rule is only able to prevent a fraction of the losses due to the selection failure, but such a fraction increases with the size of ex-ante asymmetries.

Suppose that demand is linear, $p = 1 - Q$, and that two firms, A and B, have the same cost advantage prior to any merger opportunity. That is, $c_A = c_B \leq c_C = 0.5$, where c_i is the marginal cost of firm i, $i = A, B, C$. Also, in line with the benchmark model and for simplicity, assume that mergers between firm C and either firm A or B result in the same level of synergies, $c_{AC} = c_{BC} = c_{iC}$, and this can be either higher or lower than the marginal cost of the firm resulting from a merger between the two large firms, c_{AB}. In the presence of asymmetries the optimal ex-ante rule can be conditional on the (ex-ante) size of the merging firms. In particular, authorities announce two thresholds: $\overline{c}_{AB}, \overline{c}_{iC}$. Finally, we need to specify the ex-ante distribution of (c_{AB}, c_{iC}). In particular, we assume that with probability λ, (c_{AB}, c_{iC}) is a random vector uniformly distributed on the triangle $\{c_{iC} \in [0, c_A], c_{AB} \leq c_{iC}\}$ and with probability $1 - \lambda$ it is uniformly distributed in $\{c_{iC} \in [0, c_A], c_{AB} \geq c_{iC}\}$. We choose the value of λ so that the ex-ante optimal approval rule is the same for all mergers in the ex-ante symmetric case, i.e., when $c_A = c_B = c_C$.[12]

We have solved this model and obtained that, for all possible mergers, the optimal ex-ante approval rule is more stringent that the ex-post optimal rule, but much more so for mergers that involve a higher increase in the (naively computed) Herfindhal index. This is what we would expect from adding to our benchmark model the conflict of interest. In Table 1 we report, for different values of c_A, the losses in consumer surplus associated with selection failures under alternative selection procedures.[13]

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[12]Note that, if we assumed, for instance, $\lambda = .5$ then, even in the case $c_A = c_B = c_C$, the optimal rule would be characterized by $\overline{c}_{AB} < \overline{c}_{iC}$. The reason is that c_{AC} and c_{BC} are perfectly correlated. As a result, merger AB challenges the efficient merger more often than AC and BC.

[13]We stop at $c_A = 0.35$ because for $c_A \leq 0.325$ the optimal approval rule becomes degenerate (always forbid the merger between the largest firms).

Table 1: Losses in consumer surplus (%)

c_A	Our model	Conflict of int.	Ex-ante vs Ex-post
0.500	6.67	0.00	11.39
0.475	6.24	0.47	17.15
0.450	6.49	2.05	23.88
0.425	7.62	5.05	30.18
0.400	9.75	9.77	35.17
0.375	12.79	16.27	40.27
0.350	16.15	23.84	48.11

In particular, column 1 reports the increase in consumer surplus that would be realized if the most efficient merger was always implemented (first best), measured as a percentage of the increase in consumer surplus when authorities use the ex-post efficient approval rule.[14] Notice that these losses first decrease and then increase with the degree of asymmetry (as c_A decreases). Column 2 reports the losses if instead the merger proposal was always the one that maximizes industry profits, again measured as a percentage of the increase in consumer surplus realized when authorities use ex-post efficiency.[15] That is, column 2 measures the losses caused exclusively by the conflict of interest, assuming away any private bargaining failures. The losses attributed to private bargaining failures can be computed by substracting column 2 from column 1. Under symmetry, there is no loss associated to the conflict of interest, since private and social interests are perfectly aligned. However, as c_A falls these losses increase at an increasing rate. Simultaneously, private bargaining failures become less important and a larger share of selection failures is due to the conflict of interest. For values of c_A equal or below 0.4, the losses generated by the conflict of interest are higher than total selection losses. That is, in expected terms,

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[14]Let CS_1 be the expected consumer surplus (ECS) when the efficient merger is always implemented, CS_2 be the ECS when the merger proposal is the outcome of our bargaining game, provided it is CS enhancing (i.e., when the ex-post efficient rule is used), CS_3 be the ECS if no merger is allowed. Then, column 1 of Table 1 reports $\frac{CS_1 - CS_2}{CS_2 - CS_3} \times 100$.

[15]Let CS_4 be ECS if the merger proposal maximizes industry profits and is CS enhancing. Then, column 2 of Table 1 reports $\frac{CS_1 - CS_4}{CS_4 - CS_3} \times 100$.

private bargaining failures alleviate the negative effects of the conflict of interest.[16] Indeed, the private bargaining failure prevents firms from striking the most profitable deal with probability one, and when the conflict of interest between consumers and firms is large, this may be in the interest of consumers.

In column 3 we measure the efficacy of ex-ante optimal rules in reducing selection losses. We report the percentage of the consumer surplus loss reported in column 1 that is avoided when the authority uses the ex-ante optimal approval rule.[17] These gains in surplus when the approval rule takes into account selection failures are modest in the case of symmetry, but increase rapidly as c_A falls.

4   More than three firms

The main predictions of the benchmark model do not hinge on the assumption that there are initially three firms in the market. However, the presence of more than three firms opens the door to multiple mergers and so adds a dynamic dimension that is absent in the three-firm case. As an illustration, we now discuss an industry with four ex-ante identical firms and draw two main lessons that are likely to hold for any number of firms: (i) when only one merger is feasible then Proposition 1 can be readily extended; (ii) when more than one merger may end up materizalizing, then merger negotiations are typically characterized by both a conflict of interest, as in Nocke and Whinston (2013), and also by private bargaining failures, as in our benchmark model.

To facilitate the presentation, consider the Cournot model with linear demand, $p = 1 - Q$. Assume that only bilateral mergers can generate synergies. A merger between firms i and j result in costs c_{ij}, $i, j = 1, ..., 4$. However, authorities may still accept two (sequential) merger proposals if they both generate sufficient synergies.

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[16]The absolute value of these figures is very sensitive to changes in the density function. In this example, we are assuming that the two possible realizations of synergies are only slightly correlated, and hence with a relatively high probability the realizations are sufficiently different so that the efficient merger occurs with probability one. Therefore, these figures would be larger if these two realizations were more positively correlated.

[17]Let CS_5 be the ECS when the merger proposal is the outcome of our bargaining game given the optimal ex-ante rule. Then, column 3 of Table 1 reports $\frac{CS_5 - CS_2}{CS_1 - CS_2} \times 100$.

We need modifying the bargaining protocol only in stage 1, to let Nature choose each firm with probability $\frac{1}{4}$, and stage 3, to allow firm B to choose either A or any of the other two firms as negotiation partner. If there is room for two mergers, we may also assume that after the first successful merger, the remaining two firms keep bargaining according to the protocol described in stages (4) to (6) and continue to do so in every period until an agreement is reached.

Suppose that authorities behave myopically and accept any merger proposal that reduces the price. Let us denote by c_{nl} the marginal cost of the lth merger, $l = 1, 2$, that leaves the market price unchanged. In our example $c_{n1} = \frac{-1+6c_0}{5}$ and $c_{n2}(c^F) = \frac{-1+6c_0-c^F}{4}$, where c^F is the marginal cost of the first approved merger. Note that any merger proposal with a marginal cost lower than c_{n1} will also be approved in the second round.

For some realizations of synergies only one merger is feasible.[18] Then, (a slightly modified version of) Proposition 1 readily applies.

The case where multiple mergers are feasible requires more elaboration. Suppose first that $c_{12} \leq c_{34} \leq c_{n1}$, and $c_{ij} \in [c_{12}, c_{34}]$ for any other merger ij. Thus, there are two types of firms. Firms 1 and 2 are of a good type and they together generate the highest level of synergies. Firms 3 and 4 are of a bad type, and their merger generates the lowest level of synergies. A mixed merger, involving one good and one bad firm, generates an intermediate level of synergies. Note that, in this example, instead of the three structures in the benchmark model, only two distinct market structures can emerge: (i) an asymmetric duopoly, where $(1,2)$ and $(3,4)$ compete and obtain π_{12} and π_{13} respectively, or (ii) a symmetric duopoly, where the result of two mixed mergers compete, each obtaining π_{13}, with $\pi_{13} \in (\pi_{34}, \pi_{12})$. Whether firm 1 merges with firm 3 or 4, and so firm 2 merges with 4 or 3, cannot make a difference in payoffs for any player. The consequence is that the core of the game is never empty. As in our benchmark model, this means that no private bargaining failure will occur: the equilibrium of the bargaining game results in an asymmetric duopoly if $\pi_{12} + \pi_{34} \geq 2\pi_{13}$, and in a symmetric one otherwise. However, a conflict of interest between social and private objectives emerges, since the asymmetric

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[18]For instance, suppose that $c_{12} \leq c_{13} = c_{23} \leq c_{n1}$ and $c_{14} > c_{n2}(c_{23})$, $c_{24} > c_{n2}(c_{13})$, $c_{34} > c_{n2}(c_{12})$. Alternatively, consider the case $c_{12} \leq c_{ij} \leq c_{n1}$, for $i, j \neq 1, 2$, but suppose now that firms incur in a fixed cost of merging. For some range of the fixed cost, the market can only support one merger.

duopoly maximizes consumer surplus if and only if $c_{12} + c_{34} \leq 2c_{13}$. If c_{13} is smaller but close to $\frac{c_{12}+c_{34}}{2}$, then profits are higher under an asymmetric configuration, since asymmetries reduce the intensity of competition, but consumer surplus is larger with the symmetric configuration. Therefore, from a consumer viewpoint, merger proposals are biased towards market configurations that are too asymmetric.

But the conflict of interest is not the only selection problem in the four-firm case. Private bargaining failures may also be an outcome of negotiations. Indeed, assume now that $c_{12} \leq c_{13} = c_{23} \leq c_{14} = c_{24} \leq c_{34} \leq c_{n1}$. Three possible market configurations are possible again: (i) a duopoly with a merger of firms 1 and 2 competing with a merger between firms 3 and 4, (ii) a duopoly with a merger between firms 1 and 3 competing with a merger between firms 2 and 4, and (iii) a duopoly where firms 1 and 2 swap positions with respect to (ii). Now, the core may be empty again. In particular, this is the case if $\pi_{12} + \pi_{34} \in (\pi_{13} + \pi_{24}, 2\pi_{13})$, and then the profit maximizing market configuration will not be the equilibrium outcome with probability one.

5   The bargaining protocol

In this section we argue that the main results of the paper are not an artifact of the specific bargaining protocol we have used, but rather the consequence of more fundamental causes. However, our modeling choice is not irrelevant, since our protocol has important advantages over more standard protocols.

Bargaining failures have been overlooked in the literature on endogenous mergers, which has either assumed that one of the firms is exogenously selected as target or acquirer (Inderst and Wey (2004), and Nocke and Whinston (2013)) or, alternatively, ex-post symmetric setups (all mergers generate the same synergies, like in Fridolfsson and Stenek, 2005a). In both cases, bargaining failures are absent by construction.

A merger is the result of a process of coalition formation, and the literature on this topic (see, for instance, Chaterjee et al. (1993), Okada (1996), and Compte and Jehiel (2010)) has long established that inefficiencies are likely equilibrium results.[19] Moreover,

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[19]See also Ray (2007) for an excellent overview.

the existence of these inefficiencies is closely related to the emptiness of the core of the underlying cooperative game. For instance, Chaterjee et al. (1993) prove (for strictly superadditive games) that any efficient, stationary equilibrium (selection) of any *rejector proposes* protocol must converge to a point in the core.[20] This result immediately implies that inefficiencies have to be part of the equilibrium for these protocols when the core is empty. Our underlying cooperative game is not strictly superadditive: the grand coalition cannot form, but it is intuitive that this can only turn efficiency even more problematic. The main alternative to this type of protocol is a *random proponent* protocol.[21] It is a simple exercise to show (details upon request) that in our setting all *rejector proposes* protocols as well as all *random proponent* protocols result in inefficiencies if the core is empty.[22]

Our simple protocol shares with those classes of protocols the property that inefficient mergers are part of the equilibrium when the core is empty. However, it has some important advantages over them: (i) it offers sharp predictions (uniqueness of equilibria), (ii) provides an intuitive characterization of bargaining failures, and (iii) delivers intuitive comparative statics. For example, in our setting, a random proponent protocol with equal probabilities (all players have the same chances of being chosen as proponent) as in Fridolfsson and Stennek (2005a) would offer the same qualitative predictions (details are available upon request).[23] However, such protocol generates multiple equilibria for some range of parameters, and hence an equilibrium selection criterion is needed. Also, the non-emptiness of the core is a necessary, but not sufficient, condition for the efficiency of an equilibrium. Finally, the comparative statics are neither smooth nor intuitive.[24] All these drawbacks can be attributed to the rigidities imbedded in the protocol. In partic-

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[20]A proponent makes a take-it-or-leave-it offer to a subset of active players. If they accept, then the coalition is irreversibly formed. Otherwise the game moves to the next period, where the first player that rejected the last offer is the new proponent.

[21]After each rejection, the new proposer is randomly selected using the same random device.

[22]The term *efficiency* here refers to whether the merger that maximizes industry profits is formed.

[23]Fridolfsson and Stennek assume that all firms are ex-ante identical and all megers result in the same level of synergies. In addition, they frame their game in continuous time and bidding rounds occur at random points in time. However, they also focus on the limit case when the expected difference between two bidding rounds goes to zero. This is equivalent to the deterministic version we discuss here.

[24]In particular, in our protocol the degree of efficiency, d, increases as c_{12} falls, when the core is empty, This is a natural result, since a lower c_{12} should enhance the bargaining power of firms 1 and 2. In contrast, in Fridolfsson and Stennek d is constant and equal to $\frac{2}{3}$ for a full range of values of c_{12}.

ular, each player has the same chance of becoming the next proponent, which in a game with discounting grants the weakest player, player 3, a significant amount of bargaining power.[25]

Our protocol is more flexible in the sense that the probability that a particular firm becomes the proponent is endogenous. As a result, the relative bargaining power of the different firms is exclusively given by the fundamentals of the model. As a result equilibrium is uniquely determined. Moreover, if the core is not empty, firms 1 and 2 are able to avoid firm 3 and the equilibrium is efficient. If the core is empty firms 1 and 2 cannot ignore firm 3, who is always able to make a destabilizing offer. However, the probability that firm 3 is the proponent is not exogenous but depends on parameter values. In particular, as the game approaches the region where the core is not empty, such probability goes to zero.

6 Concluding remarks

In this paper we have made two main points: (a) passive merger policy opens the door to relatively inefficient mergers because of potential bargaining failures, and (b) a commitment to a more stringent policy rule may alleviate this inefficiency. We have first illustrated these ideas in a stylized model with three ex-ante identical firms. The model encompasses several standard oligopoly models. We have also showed that ex-ante asymmetries exacerbate the losses from the bargaining process and so the optimal ex-ante rule is more stringent for all mergers, but particularly so for those mergers that include larger firms. Finally, we have argued that the results are likely to hold in the general case where an arbitrary number of firms are involved in a dynamic merging process.

The working paper version (Burguet and Caminal, 2012) also studies the optimal ex-ante rule when authorities can use more than one instrument. In particular, relaxing the assumption that authorities perfectly observe the synergies generated by a merger proposal, we assume that they have access only to a noisy signal, whose quality depends

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[25]Notice that in any equilibrium without delay the probability of the efficient merger, d, is bounded above by $\frac{2}{3}$.

on the effort exerted by the partners in the potential merger. That is, authorities allow for an *efficiency defense* but they set the information standards that are required to substantiate such efficiency claims. Merger policy will then consist of two instruments: the minimum quality of the noisy signal and the threshold of its realization. There we show that bargaining failures still induce authorities to commit to a threshold level of the noisy signal that is lower than what is ex-post optimal, but they may or may not require the signal to be of higher quality.

Two important issues remain to be briefly discussed. First, private bargaining failures could be eliminated (industry profits could be maximized) if firms were allowed to reach agreements that include monetary transfers between merging and non merging firms. Second, competition authorities may care not only about the effect of a merger on consumer surplus, but may also put some weight on industry profits.

One of the crucial characteristics of our bargaining protocol is that only bilateral agreements are feasible. In our benchmark model, a merger that results in higher consumer surplus also results in higher industry profits. Thus, bargaining failures are mainly associated with the inability of the merging partners to compensate the outsider. Alternatively, if firms could agree on transfers from merging firms to third parties, then we would expect that the probability of a relatively inefficient merger would be substantially lower. Obviously, allowing for transfer payments among firms is a highly controversial policy prescription, as it could be used to implement collusive arrangements. Moreover, when ex-ante asymmetries are sufficiently large, then we showed that private bargaining failures alleviate the inefficiencies associated with the conflict of interest. Hence, allowing side payments between merging and non-merging firms would reduce consumer surplus. Finally, the use of transfer payments may also be counterproductive if there exists the possibility of preemptive mergers; that is, when firms are willing to participate in a profit-reducing merger in order to avoid a worst situation: being left out of the deal. In this case, since a merger lowers aggregate profits but raises consumer surplus, if transfer payments are allowed then firms may be able to reach an overall agreement that eliminates the merger equilibrium.[26]

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[26]Fridolfsson and Steneck (2005b) have noticed that divestiture clauses in merger proposals can actually

Extending the analysis to the case when competition authorities maximize a weighted average of consumer surplus and profits involves considering mergers that are profitable but unattractive, in the sense that firms prefer not to participate and be left out of the merger. Fridolfsson and Stennek (2005a and 2005b) show that the bargaining game is a war of attrition in such case, and when all mergers are symmetric there are multiple equilibria. In particular, there is an equilibrium where a merger takes place later in the game (delay). In our model with asymmetric mergers it can be shown that for some parameter values there is still a probability that the relatively inefficient merger succeeds. Hence the identity of merging firms is an issue also in this scenario. Consequently, the main insight of our paper seems to extend quite directly to the case where authorities have a more general objective function.

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7 Appendix

7.1 Market models

We show that both the Cournot model and a differentiated product model in the spirit of Dixit (1979), fit the reduced form model of Section 2 under appropriate parameter restrictions.

First, we analyze the differentiated product model with utility function (1). We restrict attention to parameter values that imply $c_n > 0$. Thus, we let $(3\beta + 2\gamma)c_0 > \gamma\alpha$ if competition is in quantities, and $\gamma(\beta - \gamma)\alpha < (\beta + \gamma)(2\beta - \gamma)c_0$ if competition is in prices. Also, in order to simplify the discussion we will only consider cases where $q_k > 0$. This requires that $\beta\alpha > (\beta + \gamma)c_0$ if competition is in quantities and $(\beta - \gamma)\alpha > (\beta - \frac{\gamma^2}{\beta+\gamma})c_0$,[27] if competition is in prices.

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be equivalent to transfer payments between merged and non-merged firms. They have shown that when mergers are profitable but unattractive firms play a war of attrition. Whether or not divestiture clauses may help in implementing the aggregate profit maximizing outcome in alternative scenarios remains an open question.

[27] In price competition, if the intercept of the demand for firm k is below c_0, then the equilibrium price for firm k is c_0.

The inverse demand function for each good is

$$(8) \qquad p_i = \frac{\partial U(\mathbf{q}, x)}{\partial q_i} = \alpha - \beta q_i - \gamma \sum_{j \neq i} q_j,$$

where p_i is the price of variety i in terms of the numeraire. Thus, the consumer surplus as a function of $(\mathbf{q}, x)$ can be written as

$$\widehat{CS}(\mathbf{q}, x) = U(\mathbf{q}, x) - \sum_{i=1}^{3} p_i q_i = x + \frac{\beta - \gamma}{2} \sum_{i=1}^{3} q_i^2 + \frac{\gamma}{2} (\sum_{i=1}^{3} q_i)^2.$$

Quantity competition: In the triopoly game each firm solves

$$\max_{q_i} \left(\alpha - c_0 - \beta q_i - \gamma \sum_{j \neq i} q_j \right) q_i,$$

and so firm i's reaction function is

$$(9) \qquad q_i = R_0(q_{-i}) = \frac{\alpha - c_0 - \gamma \sum_{j \neq i} Q_j}{2\beta}.$$

Then the symmetric equilibrium output of each firm, q_0, is

$$(10) \qquad q_0 = \frac{\alpha - c_0}{2(\beta + \gamma)}.$$

Assume firms i and j merge and the cost of each variety is c_{ij}. Firm k's reaction function is still (9). For the merged firm the first order condition that determines q_i is now

$$(11) \qquad \alpha - c_{ij} - 2\beta q_i - 2\gamma q_j - \gamma q_k = 0,$$

and similarly for q_j. In equilibrium, $q_i = q_j$. Thus, let $q_{ij}(c_{ij})$ denote this common quantity of varieties i and j, and $q_k(c_{ij})$ denote the equilibrium quantity for firm k. These

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values are

$$
(12) \qquad
\begin{aligned}
q_{ij}(c_{ij}) &= \frac{2\beta\,(\alpha - c_{ij}) - \gamma(\alpha - c_0)}{4\beta(\beta + \gamma) - 2\gamma^2}, \\
q_k(c_{ij}) &= \frac{2(\gamma + \beta)(\alpha - c_0) - 2\gamma\,(\alpha - c_{ij})}{4\beta(\beta + \gamma) - 2\gamma^2}.
\end{aligned}
$$

We let c_n be the value of c_{ij} that satisfies $q_{ij}(c_{ij}) = q_0$, where q_0 is defined in (10). We will see below that this value is well defined and smaller than c_0. Since the reaction function of firm k has not changed, then also $q_k(c_n) = q_0$. Finally, $CS(c_n) = CS_0$.

Price competition: Assume now that firms set prices. We still have that the inverse demand system is given by (8). Inverting that system, we obtain the demand system,

$$
q_i = \frac{1}{D}\left[\alpha(\beta - \gamma) - (\beta + \gamma)p_i + \gamma(p_j + p_k)\right],
$$

where $D = (\beta - \gamma)(\beta + 2\gamma)$. We write this as

$$
q_i = \frac{1}{D}\left[A - Bp_i + G(p_j + p_k)\right],
$$

where $A = \alpha(\beta - \gamma)$, $B = (\beta + \gamma)$ and $G = \gamma$. The first order condition for profit maximization by firm i is

$$
(13) \qquad A + Bc_0 - 2Bp_i + G(p_j + p_k) = 0.
$$

Solving this system we obtain the pre-merger equilibrium prices as

$$
p_0 = \frac{A + Bc_0}{2(B - G)}.
$$

Assume firms i and j merge. Firm k's first order condition is still (13), whereas for the merged firm in its variety i the first order condition is

$$
A + (B - G)c_{ij} - (2B - G)p_i + G(p_j + p_k) = 0.
$$

Solving this system for $p_i = p_j$, we obtain

$$p_{ij} = \frac{A(G + 2B) + 2B(B - G)c_{ij} + BGc_0}{4B(B - G) - 2G^2},$$

$$p_k = \frac{AB + B(B - G)c_0 + G(B - G)c_{ij}}{2B(B - G) - G^2}.$$

The quantities are

$$q_{ij} = \frac{1}{D}\left[A - (B - G)p_{ij} + Gp_k\right],$$

$$q_k = \frac{1}{D}\left[A - 2Bp_k + 2Gp_{ij}\right].$$

Let c_n be such that $p_k(c_n) = p_0$. Note again that $p_{ij}(c_n) = p_0$. Indeed, the reaction function of firm k has not changed, so that any other prices by the merged firms would imply a different price by firm k.

Lemma 1. *Whether firms compete in quantities or prices, assumptions (A.1) through (A.4) are satisfied.*

The proof of Lemma 1 can be found in the online appendix.

We now analyze the Cournot model under. In particular, assume $p(Q)$ is twice continuously differentiable, and for all Q such that $p(Q) > 0$, satisfy (i) $p'(Q) < 0$, (ii) $p'(Q) + p''(Q)Q < 0$, and (iii) $\lim_{Q \to \infty} p(Q) = 0$. Condition (ii) implies that individual profit functions are strictly concave. The first order condition for firm i's profit maximization problem, whether in duopoly or triopoly, is

(14) $$p'(Q)Q_i + p(Q) - c_i = 0.$$

Also, condition (ii) plus constant marginal costs imply that the reaction function of a firm is decreasing in the output of rival firms with slope greater than -1. Finally, in the duopoly game, a higher value of c_{ij} results in lower q_{ij}, and hence higher q_k and lower aggregate output Q. (See, for instance, Proposition 2.4 in Corchón, 1996.) Thus, CS is decreasing in c_{ij}.

Lemma 2. *Under conditions (i), (ii), and (iii), the Cournot model satisfies (A.1) through (A.4) if $c_n > 0$.*

The proof of Lemma 2 is also in the online appendix.

If $c_n \not> 0$, then there is no merger that can result in an increase in consumer surplus,

so that the problem is not interesting. If the condition

$$(15) \qquad\qquad p'(3q_0)\,2q_0 + p(3q_0) > 0$$

holds then in the duopoly game a merged firm with marginal costs $c_{ij} = 0$ will choose a level of output higher than $2q_0$, which implies that aggregate output will be higher than premerger output $3q_0$. Therefore, by the monotonicity of $CS(c_{ij})$, $c_n > 0$. It can be shown that condition (15) is equivalent to the elasticity of demand at $3q_0$ being higher than $\frac{2}{3}$. A sufficient assumption involving only the primitives that implies (15) is that $p(0) - c_0$ is positive but sufficiently small. Under these assumptions, the Cournot model results in profits and consumer surplus that fit our reduced form model.

7.2   Proof of Proposition 1

We provide a proof of Proposition 1 by investigating a set of properties that any equilibrium would have to satisfy. These properties impose restrictions on the equilibrium outcomes which, as r approaches 0, identifies a unique outcome. Finally, we find equilibrium strategies that result in such outcome. Before turning to the formal details, we offer an intuitive description of these properties. First, some merger must be strictly better for the firms involved than passing on the opportunity to merge. Moreover, it cannot be the case that this is so for exactly two of the three mergers. That would leave the firm not involved in the third merger with too much leverage, so that it should expect a payoff as large as the total surplus of at least one of its two possible mergers, which is a contradiction. Now, in terms of firms' preferences for merging partner, there could be no cycles, where every firm prefers (some strictly so) to merge with another firm that itself prefers to merge with the third one. This would basically preclude agreements. So either all firms are indifferent as to the choice of merger partner, or there is a pair of firms who strictly prefer merging with each other. In the latter case, these firms must be firms 1 and 2 and, since nothing prevents these firms to negotiate (in our protocol), they would merge with probability 1. But then firms 1 and 2 would be in a symmetric position vis a vis each other, and so they must expect the same payoff. These properties define the two possible types of equilibria: one where firms 1 and 2 merge with probability 1 and share equally the surplus of their merger, and one where everyone is indifferent as to the choice of merger partner. The latter can only happen when asymmetries (expost) are not large, and the opposite holds for the former.

We now turn to the formal proof. A strategy for firm i consists of $(\mu_i^j, \lambda_i^j, \lambda_i^k)$ for the selection of negotiating partners and $(\theta_i^j, \rho_i^j, \theta_i^k, \rho_i^k)$ for the actual negotiation phase. μ_i^j is the probability that firm i invites firm j to be its negotiation partner in node (2), if i is chosen by nature in node (1). Given the definition of the game, the probability that i proposes k is $\mu_i^k = 1 - \mu_i^j$. λ_i^j is the probability that firm i accepts firm j's invitation to

become a negotiation partner in node (3), and λ_i^k is the probability that i accepts firm k's invitation. In line with the restriction to stationary strategies, we will assume that $\lambda_i^j = 1 - \lambda_i^k$. That is, if invited in step (2), firm i chooses its partner for the negotiation phase independently of who gave it that possibility, firm k or firm j. Therefore, in case nature chooses firm i, the probability that firms (i,j) negotiate in nodes (5) and (6) is $\mu_i^j \lambda_j^i$, the probability that (i,k) negotiate is $\mu_i^k \lambda_k^i = \left(1 - \mu_i^j\right) \lambda_k^i$, and the probability that (j,k) negotiate is $\mu_i^j \lambda_j^k + \mu_i^k \lambda_k^j = \mu_i^j \left(1 - \lambda_j^i\right) + \left(1 - \mu_i^j\right) \left(1 - \lambda_k^i\right)$. Also, θ_i^j is the (per period) offer that firm i makes to firm j with probability ρ_i^j in node (5) if the former is chosen by nature in node (4) as the proponent. θ_i^k and ρ_i^k are the corresponding values in a negotiation with k. In order to avoid open-set technical problems, and also to save in notation, we assume that in node (6) the respondent accepts with probability one any offer above or equal to the value of continuation. That is why we do not include these decisions in the definition of a strategy. As we will see in the analysis below, this is innocuous and in particular does not rule out the possibility of delay in case of indifference.[28] Then, in any equilibrium $\theta_i^j = \frac{r\pi_0 + u_j}{1+r}$ whenever $\pi_{ij} > \frac{u_i + u_j + 2r\pi_0}{1+r}$ (and we can also restrict to such offer when $\pi_{ij} = \frac{u_i + u_j + 2r\pi_0}{1+r}$ and $\rho_i^j > 0$).

Again, note that in line with the restriction to stationary strategies, we are implicitly assuming that the answer to invitations to negotiate in node (3) and the offer in node (5) do not depend on who made the invitation to negotiate or who answered to that invitation, but only on the identity of the partner.

Let us denote u_i^{ij} the equilibrium per-period payoff that firm i expects in step (4) before nature chooses who will make an offer, and given that firms $(i.j)$ will be negotiating. That is, in any node of the extensive form game reached immediately after i has chosen j as the negotiation partner, or j has chosen i. Finally, let us d_{ij} denote the probability that merger (i,j) succeeds. We derive several properties of any equilibrium outcome.

7.2.1   Property 1: There is a positive surplus in at least one negotiation (i,j):

$$\pi_{ij} > \frac{u_i + u_j + 2r\pi_0}{1+r}.$$

Suppose not; i.e., for all (i,j)

$$(16) \qquad\qquad \pi_{ij} \leq \frac{u_i + u_j + 2r\pi_0}{1+r},$$

which implies that whenever firm i is one of the negotiation partners it gets $\frac{u_i + r\pi_0}{1+r}$, whether the merger materializes or not. Then, $\forall i$, $u_i \leq d_{jk}\pi_i + (1 - d_{jk}) \max\left\{\pi_0, \frac{u_i + r\pi_0}{1+r}\right\}$. Hence,

---

[28]Indeed, apart from open-set issues, in a SPE there could be indifference between accepting and rejecting a partner's offer only if the sum of the continuation values for both partners is equal to what they have to share. In ths case, the fact that the proponent can choose any value ρ in $[0, 1]$ already allows for any probability of delay.

$u_i \leq \pi_0$. Therefore, $\frac{u_i + u_j + 2r\pi_0}{1+r} \leq 2\pi_0 < \pi_{ij}$. We have reached a contradiction.

7.2.2 Property 2: It cannot be the case that there is a strictly positive surplus in exactly two negotiations .

Suppose that in two and only two negotiations there is a strictly positive surplus, i.e.,

$$
\begin{aligned}
\frac{u_i + u_j + 2r\pi_0}{1+r} &\geq \pi_{ij}, \\
\frac{u_i + u_k + 2r\pi_0}{1+r} &< \pi_{ik}, \\
\frac{u_j + u_k + 2r\pi_0}{1+r} &< \pi_{jk}.
\end{aligned}
$$

These inequalities imply that:

(17)
$$
u_k + r\pi_0 < \frac{1+r}{2} \left(\pi_{ik} + \pi_{jk} - \pi_{ij} \right).
$$

Since $u_i^{ik} > \frac{u_i + r\pi_0}{1+r} = u_i^{ij}$ then $\lambda_i^k = (\rho_i^k =) 1$. Similarly, $\lambda_j^k = 1$. As a result, $d_{ij} = 0$ and $d_{ik} + d_{jk} = 1$. Hence, we can write:

$$
\begin{aligned}
u_i &= d_{ik}\frac{1}{2}\left(\pi_{ik} + \frac{u_i - u_k}{1+r} \right) + (1 - d_{ik})\pi_i, \\
u_j &= d_{ik}\pi_j + (1 - d_{ik})\frac{1}{2}\left(\pi_{jk} + \frac{u_j - u_k}{1+r} \right), \\
u_k &= d_{ik}\frac{1}{2}\left(\pi_{ik} + \frac{u_k - u_i}{1+r} \right) + (1 - d_{ik})\frac{1}{2}\left(\pi_{jk} + \frac{u_k - u_j}{1+r} \right).
\end{aligned}
$$

For any $d_{ik} \in [0, 1]$, the solution of this system for u_k plus $r\pi_0$ is larger than the right hand side of (17). We have reached a contradiction.

7.2.3 Property 3: If firm i strictly prefers to negotiate with firm j and vicev-ersa, then $i = 1$ and $j = 2$.

Consider first the case where there is only one negotiation with a strictly positive surplus. Then we show that it has to be the negotiation between firms i and j. Indeed, if $\pi_{ij} >$

$\frac{u_i+u_j+2r\pi_0}{1+r}$, then

$$u_i^{ij} = \frac{1}{2}\left(\pi_{ij} + \frac{u_i+r\pi_0}{1+r} - \frac{u_j+r\pi_0}{1+r}\right) > \frac{u_i+r\pi_0}{1+r},$$

whereas $u_i^{ik} = \frac{u_i+r\pi_0}{1+r}$. The same applies to j, so that in equilibrium $\lambda_i^j = \lambda_j^i = 1$. Also, this implies that $\mu_i^j = \mu_j^i = 1$, and then $u_i = u_j = \frac{1}{2}\pi_{ij}$, and $u_k = \pi_k$ Thus, if $\pi_{ij} \le \pi_{ik}$, we have that $\pi_{ij} > \frac{u_i+u_j+2r\pi_0}{1+r}$ and $\pi_{ik} \le \frac{u_i+u_k+2r\pi_0}{1+r}$ implies $u_j = \frac{1}{2}\pi_{ij} < u_k = \pi_k$, which is a contradiction.

Alternatively, if all three negotiations involve a strictly positive surplus, then suppose that $(i,j) = (1,3)$. That is,

$$u_1^{13} > u_1^{12},$$
$$u_3^{13} > u_3^{23}.$$

These inequalities imply that $\lambda_3^1 = \lambda_1^3 = (\rho_1^3 = \rho_3^1 =)1$, and then also $\mu_3^1 = \mu_1^3 = 1$. Thus, $u_1 = u_3 = \frac{1}{2}\pi_{13}$, and $u_2 = \pi_2$. Then

$$
\begin{aligned}
u_1^{12} &= \frac{1}{2}\left(\pi_{12} + \frac{\frac{1}{2}\pi_{13}+r\pi_0}{1+r} - \frac{\pi_2+r\pi_0}{1+r}\right) \\
&> \frac{1}{2}\left(\pi_{13} + \frac{\frac{1}{2}\pi_{13}+r\pi_0}{1+r} - \frac{\frac{1}{2}\pi_{13}+r\pi_0}{1+r}\right) = u_1^{13},
\end{aligned}
$$

which is a contradiction. A similar contradiction would obtain if we assumed that $(i,j) = (2,3)$.

7.2.4 Property 4: Preference cycles cannot occur: If i weakly prefers to negotiate with j, j weakly prefers to negotiate with k, and k weakly prefers to negotiate with i, then they all must be indifferent.

Suppose not. If there is a strictly positive surplus in all three negotiations, so that all end up in agreement, then $u_i + u_j + 2r\pi_0 \le (1+r)\pi_{ij}$, for all i,j, and then

$$(18) \qquad \pi_{ij} - \frac{u_j+r\pi_0}{1+r} \ge \pi_{ik} - \frac{u_k+r\pi_0}{1+r},$$

$$(19) \qquad \pi_{jk} - \frac{u_k + r\pi_0}{1+r} \geq \pi_{ij} - \frac{u_i + r\pi_0}{1+r},$$

$$(20) \qquad \pi_{ik} - \frac{u_i + r\pi_0}{1+r} \geq \pi_{jk} - \frac{u_j + r\pi_0}{1+r}.$$

If we add up these three inequalities then this can only be satisfied if the three hold with equality. Alternatively, if there is strictly positive surplus only in the negotiation between firms 1 and 2, i.e., $u_1 + u_2 + 2r\pi_0 < (1+r)\pi_{12}$, and for the rest of the pairs the inequality is (weakly) reversed, then $u_2^{23} = \frac{u_2 + r\pi_0}{1+r}$. Assume that 1 prefers to negotiate with 2. For 2 to prefer negotiations with 3, it should hold that $u_2^{23} = \frac{u_2 + r\pi_0}{1+r} \geq \pi_{12} - \frac{u_1 + r\pi_0}{1+r}$, which contradicts $u_1 + u_2 + 2r\pi_0 < (1+r)\pi_{12}$. Similarly if 2 prefers negotiating with 1, and 1 prefers negotiating with 3.

7.2.5 Property 5: If firm 1 strictly prefers to negotiate with firm 2, and viceversa, then $d_{12} = 1$.

If $u_1^{12} > u_1^{13}$ then $\lambda_1^2 = 1$. Similarly, if $u_2^{12} > u_2^{23}$ then $\lambda_2^1 = 1$. Since $u_1^{13} \geq \frac{u_1 + r\pi_0}{1+r}$ and $u_2^{23} \geq \frac{u_2 + r\pi_0}{1+r}$, then $\pi_{12} = u_1^{12} + u_2^{12} > \frac{u_1 + u_2 + 2r\pi_0}{1+r}$. That is, $\rho_1^2 = \rho_2^1 = 1$. Then, $\mu_1^2 = \mu_2^1 = 1$. Indeed, if $\mu_i^3 > 0$, $i = 1, 2$, then i's payoff will be either u_i^{i3} (if 3 accepts the invitation), $\frac{u_i + r\pi_0}{1+r}$ (if 3 refuses but no agreement ensues) or π_2 (if 3 refuses and then agrees with j). Since $u_i^{12} > u_i^{i3} \geq \frac{u_i + r\pi_0}{1+r}$, a deviation to $\mu_i^3 = 0$ is profitable unless $u_i^{12} = \pi_2 \ (< \pi_0)$. But $\frac{u_i + r\pi_0}{1+r} < u_i^{12} = \pi_2$ implies $u_i < \pi_2$, and a strategy of refusing any deal would be a profitable deviation. This contradition shows that $d_{12} = 1$.

7.2.6 Property 6: Firms 1 and 2 obtain the same expected payoff

If the negotiation between 1 and 2 is the only one that generates a strictly positive surplus, then from Property 5, $d_{12} = 1$, which indeed implies $u_1 = u_2 = \frac{1}{2}\pi_{12}$. Suppose now that all three negotiations generate a strictly positive surplus, and $u_1 > u_2$. In this case firm 3 strictly prefers to negotiate with firm 2 rather than firm 1, since

$$u_3^{i3} = \frac{1}{2}\left(\pi_{i3} + \frac{u_3 + r\pi_0}{1+r} - \frac{u_i + r\pi_0}{1+r}\right),$$

and so $u_3^{23} > u_3^{13}$. Then from Property 3, $u_2^{23} \leq u_2^{12}$, and so from Property 4 $u_1^{12} > u_1^{13}$. But $u_1^{12} > u_1^{13}$ implies that $\lambda_1^2 = 1$, and $u_3^{23} > u_3^{13}$ implies that $\lambda_3^2 = 1$. Hence, $d_{13} = 0$ and

so $d_{12} + d_{23} = 1$. Thus,

$$(21) \qquad\qquad u_1 = d_{12}u_1^{12} + d_{23}\pi_2 \le u_1^{12},$$

$$(22) \qquad\qquad u_2 = d_{12}u_2^{12} + d_{23}u_3^{23}.$$

If $u_2^{23} < u_2^{12}$ then from Property 5, $d_{12} = 1$, and equations (21) and (22) imply that $u_1 = u_2$. If $u_2^{23} = u_2^{12}$ then $u_2 = u_2^{12}$, which together with inequality 21 contradicts that $u_1 > u_2$.

7.2.7 Property 7: There are two possible types of equilibria: (I) $u_1^{12} > u_1^{13}$ and

$u_2^{12} > u_2^{23}$, (II) $u_i^{ij} = u_i^{ik}$ for all i, j, k.

Since $u_1 = u_2$ (Property 6), and since either none or both negotiations of 3 with 1 and 2 have strictly positive surplus, then $u_3^{23} = u_3^{13}$. Thus, both the case where $u_2^{12} \le u_2^{23}$ and $u_1^{12} \ge u_1^{13}$, and the case where $u_2^{12} \ge u_2^{23}$ and $u_1^{12} \le u_1^{13}$, would violate Property 4 unless all inequalities hold with equality. Thus, besides the case where all firms are indifferent, there are two other cases to consider: (a) $u_2^{12} < u_2^{23}$, $u_1^{12} < u_1^{13}$ and (b) $u_2^{12} > u_2^{23}$, $u_1^{12} > u_1^{13}$. Case (a) cannot be part of an equilibrium. Indeed, in case (a) $\lambda_1^2 = \lambda_2^1 = 0$, which implies $d_{12} = 0$, and $\pi_{13} > \frac{u_1 + u_3 + 2r\pi_0}{1+r}$ and $\pi_{23} > \frac{u_2 + u_3 + 2r\pi_0}{1+r}$. Therefore, $d_{13} + d_{23} = 1$, and so:

$$
\begin{aligned}
u_1 &= d_{13}\frac{1}{2}\left(\pi_{13} - \frac{u_3 - u_1}{1+r}\right) + d_{23}\pi_2, \\
u_2 &= d_{13}\pi_2 + d_{23}\frac{1}{2}\left(\pi_{13} - \frac{u_3 - u_2}{1+r}\right), \\
u_3 &= \frac{1}{2}\left(\pi_{13} - \frac{u_1 - u_3}{1+r}\right).
\end{aligned}
$$

Since $u_1 = u_2$ then $d_{13} = d_{23}$, and solving the above system we obtain $u_3 = \frac{(1+2r)\pi_{13} - \pi_2}{1+4r} > \frac{\pi_{13}}{2}$. As a result $u_1^{13} < \frac{\pi_{13}}{2} < \frac{\pi_{12}}{2} = u_1^{12}$, and we reach a contradiction in case (a). $\square$

We can now proceed to characterize the two types of equilibria.

7.2.8 Equilibrium type I

Consider an equilibrium with $u_1^{12} > u_1^{13}$ and $u_2^{12} > u_2^{23}$. From Property 4, $d_{12} = 1$. For instance, $\lambda_1^2 = \lambda_2^1 = \mu_1^2 = \mu_2^1 = \rho_1^2 = \rho_2^1 = 1$. Hence

$$u_1 = u_2 = \frac{1}{2}\pi_{12},$$

$$u_3 = \pi_3.$$

Thus, a profitable deviation for either firm 1 or firm 2 exists if and only if $\frac{1}{2}\pi_{12} < \frac{1}{2}\left(\pi_{13} - \frac{\pi_3}{1+r} + \frac{1}{2(1+r)}\pi_{12}\right)$. Indeed, the right hand side of the inequality is player i's expected payoff, $i = 1, 2$, when invited in step (2) if $\lambda_i^3 = 1$, whereas the left hand side is what she expects in that situation when $\lambda_i^3 = 0$. Therefore, $d_{12} = 1$ is an equilibrium if and only if:

$$\left(\frac{1}{2} + r\right)\pi_{12} - (1 + r)\pi_{13} + \pi_3 \geq 0.$$

7.2.9 Equilibrium type II

Consider an equilibrium with $u_i^{ij} = u_i^{ik}$ for all i, j, k. If $u_i^{ij} = u_i^{ik} = \frac{u_i + r\pi_0}{1+r}$, that implies $u_j^{ij} = \frac{u_j + r\pi_0}{1+r}$ and $u_k^{ik} = \frac{u_k + r\pi_0}{1+r}$. Then, since $u_k^{ik} = u_k^{jk}$ and $u_j^{ij} = u_j^{jk}$, we reach a contradiction with Property 1. Thus, $u_i^{ij} = u_i^{ik} > \frac{u_i + r\pi_0}{1+r}$, so that by Property 2 there is positive surplus in all negotiations. Thus, all three negotiations would end in agreement. Thus, $d_{12} + d_{13} + d_{23} = 1$ and

$$
\begin{aligned}
u_1 &= (d_{12} + d_{13})\frac{1}{2}\pi_{12} + d_{23}\pi_2, \\
u_2 &= (d_{12} + d_{23})\frac{1}{2}\pi_{12} + d_{13}\pi_2, \\
u_3 &= d_{12}\pi_3 + (d_{13} + d_{23})\left(\pi_{13} - \frac{1}{2}\pi_{12}\right).
\end{aligned}
$$

Since $u_1 = u_2$ then $d_{13} = d_{23}$. If we let $d_{12} = d$, and hence $d_{13} = d_{23} = \frac{1-d}{2}$, and since

(23)
$$\pi_{12} - \frac{u_3 + r\pi_0}{1+r} = \pi_{13} - \frac{u_1 + r\pi_0}{1+r},$$

we can solve this system to obtain

$$d = \frac{\pi_{12} - 2\pi_2 + 4r\left(\pi_{12} - \pi_{13}\right)}{-\pi_{12} + 4\pi_{13} - 2\pi_2 - 4\pi_3}.$$

The numerator is positive, and so $\left(\frac{1}{3} \leq\right) d < 1$ if and only if the denominator is larger than the numerator. That is, if $\left(\frac{1}{2} + r\right)\pi_{12} - \left(1 + r\right)\pi_{13} + \pi_3 < 0$. One such equilibrium would be $\lambda_i^j = \frac{1}{2}$, for all i, j, and $\mu_1^2 = \mu_2^1 = \frac{3d-1}{2}$ and $\mu_3^1 = \frac{1}{2}$. Finally, if $\left(\frac{1}{2} + r\right)\pi_{12} - \left(1 + r\right)\pi_{13} + \pi_3 \geq 0$ then for r sufficiently small $\frac{u_1 + u_3 + 2\pi_0}{1+r} > \frac{1}{2}\pi_{13}$, and firms 1 and 2 cannot be indifferent between negotiating with each other or with firm 3.

Summarizing, for any r sufficiently close to 0 the equilibrium exists and the equilibrium outcome is unique. Q.E.D.

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