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Standard Model and New Physics contributions to K_L and K_S into four leptons

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Abstract We study the K_L and K_S decays into four leptons ($e\bar{e}e\bar{e}$, $\mu\bar{\mu}\mu\bar{\mu}$, $e\bar{e}\mu\bar{\mu}$) where we use a form factor motivated by vector meson dominance, and show the dependence of the branching ratios and spectra from the slopes. A precise determination of short distance contribution to $K_L \rightarrow \mu\mu$ is affected by our ignorance on the sign of the amplitude $\mathcal{A}(K_L \rightarrow \gamma\gamma)$ but we show a possibility to measure the sign of this amplitude by studying K_L and K_S decays in four leptons. We also investigate the effect of New Physics contributions for these decays

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1 Introduction

The recent LHCb measurement on $K_S \rightarrow \mu\bar{\mu}$ [1] is getting closer to the Standard Model (SM) prediction

$$\text{Br}(K_S \rightarrow \mu\bar{\mu})|_{\text{LHCb}} < 9 \times 10^{-9} \text{ at } 90\% \text{ CL} \quad (1)$$

$$\text{Br}(K_S \rightarrow \mu\bar{\mu})|_{\text{SM}} = (5.0 \pm 1.5) \times 10^{-12}, \quad (2)$$

and this has motivated our interest in studying other feasible decays at LHC [2] or other facilities: decays of $K_{L,S}$ into two Dalitz pairs ($K_{L,S} \rightarrow \mu\bar{\mu}\mu\bar{\mu}$, $e\bar{e}\mu\bar{\mu}$, $e\bar{e}e\bar{e}$). These decays have received attention before. Compared to the previous literature [3–8], in this paper we have introduced a form factor, motivated by vector meson dominance and a good behaviour at short distance [9], which is particularly important for $K_{L,S} \rightarrow \mu\bar{\mu}\mu\bar{\mu}$, the one more easily detectable at LHCb. We study the dependence of the spectra and the branching ratio from the linear and quadratic slopes of the form factor.

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We show also that the measurement of the time interference of $\mathcal{A}(K_L \rightarrow \ell\bar{\ell}\ell\bar{\ell})$ with $\mathcal{A}(K_S \rightarrow \ell\bar{\ell}\ell\bar{\ell})$ would allow the determination of the sign of $\mathcal{A}(K_L \rightarrow \gamma\gamma)$, this observable indeed depends linearly from $\mathcal{A}(K_L \rightarrow \gamma\gamma)$. This experimental determination is very welcome since would allow CKM stringent tests [10].

We also discuss two possible New Physics (NP) models that can be studied by measuring measurements of $\mathcal{A}(K_{L,S} \rightarrow \ell\bar{\ell}\ell\bar{\ell})$:

- (i) a direct NP coupling for $K_L\gamma\gamma$.
- (ii) a Bremsstrahlung part from $K_{L,S} \rightarrow \mu\bar{\mu}$.

We discuss in order: the chiral perturbation theory (ChPT) and vector meson dominance (VMD) description of $K_{L,S}$ decays in section 2 and, in section 3, the results associated (including the kinematics). The different possibilities of interferences are discussed in section 4 including the Bremsstrahlung contributions and the CP-violation in the K_S decays. The appendix contains some detailed expressions for the amplitudes.

2 Chiral perturbation theory description of $K_{L,S} \rightarrow \ell\bar{\ell}\ell\bar{\ell}$

2.1 $K_L \rightarrow \ell\bar{\ell}\ell\bar{\ell}$

$K_L \rightarrow \mu^+\mu^-$ decay receives large long distance (LD) contributions and small short distance (SD) contributions: to disentangle the small but interesting short distance contribution an accurate description of the long distance contribution $K_L \rightarrow \gamma(q_1)\gamma(q_2) \rightarrow \mu^+\mu^-$ is required. To this purpose the authors of ref. [9] introduce a form factor $F_L(q_1^2, q_2^2)$ motivated by the assumption that VMD plays a crucial role in the matching between

short and long distances

$$F_L(q_1^2, q_2^2) \doteq F_L(0, 0) \left[1 + \alpha_L \left(\frac{q_1^2}{q_1^2 - M_V^2} + \frac{q_2^2}{q_2^2 - M_V^2} \right) + \beta_L \frac{q_1^2 q_2^2}{(q_1^2 - M_V^2)(q_2^2 - M_V^2)} \right], \quad (3)$$

$F_L(0, 0)$ is a constant fixed by the experimental width $\Gamma(K_L \rightarrow \gamma\gamma)$. The duality properties of this form factor are implemented by determining possibly α_L and β_L in the low energy expansion from experiments and imposing a phenomenological matching with the QCD short distance result [9]. We match the SD non-zero value with the form factor at short distance

$$1 + 2\alpha_L + \beta_L = 0.3. \quad (4)$$

As shown in [9], experiments, mainly from $K_L \rightarrow \gamma\gamma^*$ decay, fix the value of $\alpha_L = -1.69 \pm 0.08$ [11], while the experimental determination of β_L from $K_L \rightarrow \bar{\ell}\ell\ell$ would allow a test of saturation with one resonance (ρ) of the sum rule in eq. (4). Since this experimental determination is still missing either we rely on β_L from eq. (4) or, as we will do, we plot $\text{Br}(K_L \rightarrow \bar{\ell}\ell\ell)$ as function of β_L in figure 2.

The value of $F_L(0, 0)$ is fixed through the amplitude of $K_L \rightarrow \gamma^*(q_1, \epsilon_1) \gamma^*(q_2, \epsilon_2)$

$$\mathcal{A}(K_L \rightarrow \gamma^* \gamma^*) = i\varepsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma \epsilon_1^\mu \epsilon_2^\nu F_L(q_1^2, q_2^2), \quad (5)$$

with the effective lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{F_L(0, 0)}{8} \varepsilon_{\mu\nu\rho\sigma} K_2 F^{\mu\nu} F^{\rho\sigma} + \text{h.o.t.}, \quad (6)$$

we can directly connect $F_L(0, 0)$ to the branching ratio

$$\Gamma(K_L \rightarrow \gamma\gamma) = \frac{M_K^3 |F_L(0, 0)|^2}{64\pi} = (7.16 \pm 0.05) \times 10^{-21} \text{ GeV}, \quad (7)$$

and therefore (for the numerical evaluation, we will use the central value only),

$$|F_L(0, 0)| = \left[\frac{64\pi \Gamma(K_L \rightarrow \gamma\gamma)}{M_K^3} \right]^{1/2} = (5.61 \pm 0.06) \times 10^{-9} \text{ GeV}^{-1}. \quad (8)$$

2.2 $K_S \rightarrow \bar{\ell}\ell\ell$

The first non-trivial ChPT contribution to $K_S \rightarrow \gamma^* \gamma^*$ appears at $\mathcal{O}(p^4)$: no counterterms are allowed by chiral symmetry at this order implying that chiral loops are finite [4]. In this paper we want to account for two important $\mathcal{O}(p^6)$ effects:

- (i) We need to add local $\mathcal{O}(p^6)$ contributions to the $\mathcal{O}(p^4)$ $K_S \rightarrow \gamma\gamma$ to match exactly the experimental value.
- (ii) Potentially important vector meson dominance contribution $\mathcal{O}(p^6)$ to $K_S \rightarrow \gamma^* \gamma^*$ generated by the $\mathcal{O}(p^4)$ electromagnetic form factor of the pion.

We discuss the strategy to account for these effects. Writing the amplitude of $K_S \rightarrow \gamma^*(q_1, \epsilon_1) \gamma^*(q_2, \epsilon_2)$

$$\mathcal{A}(K_S \rightarrow \gamma^* \gamma^*) = i[(q_1 \cdot q_2)g^{\mu\nu} - q_2^\mu q_1^\nu] \epsilon_{1\mu} \epsilon_{2\nu} F_S(q_1^2, q_2^2), \quad (9)$$

we can obtain the PDG experimental value $F_S(0, 0)$ [11],

$$\Gamma(K_S \rightarrow \gamma\gamma) = \frac{M_K^3 |F_S(0, 0)|^2}{64\pi} = (1.93 \pm 0.12) \times 10^{-20} \quad (10)$$

adding an $\mathcal{O}(p^6)$ local term to the $\mathcal{O}(p^4)$ $K_S \rightarrow \gamma\gamma$ chiral loop [12] as done in ref. [13], then (here too, we will use only the central value for the numerical evaluation)

$$|F_S(0, 0)| = \left[\frac{64\pi \Gamma(K_S \rightarrow \gamma\gamma)}{M_K^3} \right]^{1/2} = (3.38 \pm 0.03) \times 10^{-9} \text{ GeV}^{-1}. \quad (11)$$

We want also to add the potentially important vector meson dominance contribution $\mathcal{O}(p^6)$ to $K_S \rightarrow \gamma^* \gamma^*$: this is generated by the $\mathcal{O}(p^4)$ electromagnetic form factor of the pion; this problem was already studied to evaluate the potentially important $\mathcal{O}(p^6)$ VMD contribution to $K_L \rightarrow \pi^0 \gamma^* \gamma^*$ ([13] and references therein). The leading chiral contribution to $K_L \rightarrow \pi^0 \gamma\gamma$ appears at $\mathcal{O}(p^4)$: no counterterms are allowed by chiral symmetry at this order implying that chiral loops are finite. Large VMD and unitarity $\mathcal{O}(p^6)$ corrections to $K_L \rightarrow \pi^0 \gamma\gamma$, as required by phenomenology have been investigated [5]. In ref. [13] the effects of the pion electromagnetic form factor to the pion loop amplitude $\mathcal{A}(K_L \rightarrow \pi^0 \gamma^*(q_1) \gamma^*(q_2))$ have been studied: they suggest to approximate this amplitude as the product of the amplitude with the photons on shell multiplied a form factor like the one in eq. (3). Very similarly to ref. [13] (and references therein) to include this VMD contribution we suggest to approximate the full amplitude as

$$F_S(q_1^2, q_2^2) \doteq F_S(0, 0) \left[1 + \alpha_S \left(\frac{q_1^2}{q_1^2 - M_V^2} + \frac{q_2^2}{q_2^2 - M_V^2} \right) + \beta_S \frac{q_1^2 q_2^2}{(q_1^2 - M_V^2)(q_2^2 - M_V^2)} \right]. \quad (12)$$

where $F_S(0, 0)$ is the $\mathcal{O}(p^4)$ $K_S \rightarrow \gamma\gamma$ chiral loop loop amplitude, with on-shell photons, plus a local $\mathcal{O}(p^6)$ as

discussed in connection with eq. (11). Differently from $K_L \rightarrow \gamma^* \gamma^*$, sum rule in eq. (4) due to the vanishing SD contribution the limit $q_{1,2}^2 \gg M_V^2$ imposes the constraint [9]:

$$1 + 2\alpha_S + \beta_S = 0, \quad (13)$$

reducing the number of unknown parameters to one.

In principle the off-shell photon behavior of $\mathcal{O}(p^4)$ $\mathcal{A}(K_S \rightarrow \gamma^*(q_1)\gamma^*(q_2))$ from ref. [4] could affect α_S , β_S or add other gauge invariant structures but we have checked that these effects are negligible¹ to potentially large effects from VMD.

3 Kinematics and results

3.1 $K_L \rightarrow \ell_1 \bar{\ell}_1 \ell_2 \bar{\ell}_2$

The cases that we calculated are $\ell_1 = \ell_2 = e$, $\ell_1 = \ell_2 = \mu$ and the composite case $\ell_1 = \mu$ and $\ell_2 = e$ (see appendices for more detailed expressions). For each branching ratio, we have to use the phase space measure based on 5 different variables completely determining the system. We choose two momenta and three angles. Thus we have to make a geometric treatment cf. fig. 1 and we will use the Cabibbo-Maksymovych approach [14]

$$d\Phi_4 = \frac{\pi^2}{2^5 M_K^2} \sigma_1 \sigma_2 X dq_1^2 dq_2^2 d(\cos \theta_1) d\phi d(\cos \theta_2), \quad (14)$$

where

$$X = \frac{1}{2} \lambda^{1/2}(M_K^2, q_1^2, q_2^2), \quad (15)$$

$$\sigma_i = \left(1 - 4 \frac{m_i^2}{q_i^2}\right)^{1/2}, \quad i = 1, 2 \quad (16)$$

and λ is the well-known Källén function,

$$\lambda(x, y, z) = (x - y - z)^2 - 4yz. \quad (17)$$

Here the integrations bounds are (m here stands for the smallest mass between the leptons ℓ_i):

$$\begin{cases} 4m_\mu^2 \leq q_1^2 \leq (M_K - 2m)^2, \\ 4m^2 \leq q_2^2 \leq (M_K - \sqrt{q_1^2})^2, \\ 0 \leq \theta_1, \theta_2 \leq \pi, \\ 0 \leq \phi \leq 2\pi. \end{cases} \quad (18)$$

¹Numerically we have found that these effects generate α_S and β_S at $\mathcal{O}(0.1)$, other effects are substantially smaller. Also we have checked that our parametrization of the off-shell photon behavior of $\mathcal{O}(p^4)$ of $\mathcal{A}(K_S \rightarrow \gamma^* \gamma^*)$ from ref. [4] in terms of α_S and β_S reproduce well $\text{Br}(K_S \rightarrow 4\ell)$ as described in ref. [7].

Then, any differential decay width is given from the corresponding amplitude \mathcal{M} by

$$d\Gamma(K_L \rightarrow \ell_1 \bar{\ell}_1 \ell_2 \bar{\ell}_2) = \frac{(2\pi)^{-8}}{2M_K} |\mathcal{M}|^2 d\Phi_4. \quad (19)$$

We give the results in table 1 and the evolution of the various branching ratios according to β_L is illustrated on fig. 2.

3.2 $K_S \rightarrow \ell_1 \bar{\ell}_1 \ell_2 \bar{\ell}_2$

In the same manner, we consider like for K_L the cases $\ell_1 = \ell_2 = e$, $\ell_1 = \ell_2 = \mu$ and the composite case $\ell_1 = e$ and $\ell_2 = \mu$ (see appendices). We present here our values for these decays in table 2 and their values according to the variations of α_S/α_L (cf. fig. 3).

4 Interferences

4.1 SM CP conserving interferences

Both determinations of the sign and of the value of $K_L \rightarrow \gamma\gamma$ are very challenging, as it has been shown in [10]. Indeed, the sign of $\mathcal{A}(K_L \rightarrow \gamma\gamma)$ is responsible for the increase or decrease of the interference contribution between short and long distance contributions in the decay $K_L \rightarrow \mu\mu$. From the CKM matrix point of view, it means that one can constrain more the $\bar{\rho}$ parameter. We propose an experimental analysis through the interferences of K_L and K_S into four leptons to fix the sign.

Since the K_L and K_S are composite systems in the point of view of CP violation, we have to take into account this fact. It means that from now we cannot longer identify K_1 and K_2 to K_S and K_L , but

$$\begin{pmatrix} K_S \\ K_L \end{pmatrix} = \frac{1}{\sqrt{1+|\varepsilon|^2}} \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{pmatrix} \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \quad (20)$$

$$= \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} K_0 \\ \bar{K}^0 \end{pmatrix}, \quad (21)$$

with $\text{Re } \varepsilon = 1.66 \times 10^{-3}$ and $\text{Im } \varepsilon = 1.57 \times 10^{-3}$.

First, to take into account the CP asymmetry, but in the CP conserving limit ($\varepsilon = 0$), a pertinent observable to measure the oscillations between K_S and K_L is according to [15, 16],

$$\begin{aligned} A^{LS}(t) &= \frac{2e^{-\Gamma t} \int d\Phi_4 f(X, Y) \text{Re} A_L \text{Re} A_S}{\int d\Phi_4 [e^{-\Gamma_S t} |A_S|^2 + e^{-\Gamma_L t} |A_L|^2]} \cos(\Delta M t), \end{aligned} \quad (22)$$

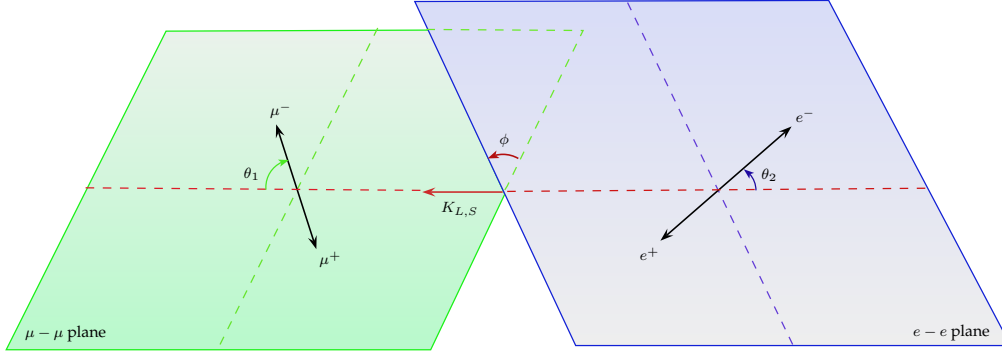


Fig. 1 Kinematics variables for the decays of the $K_{L,S}$ into 2 Dalitz pairs.

Table 1 Results for the branching ratios of K_L decays

	$\alpha_L = \beta_L = 0$		$\alpha_L = -1.63$ and $\beta_L = 0.3 - 1 - 2\alpha_L$	Experiment
	This work	Miyazaki <i>et al.</i> [3]	This Work	PDG [11]
$K_L \rightarrow \mu\bar{\mu}\mu\bar{\mu}$	4.82×10^{-13}	5.17×10^{-13}	8.78×10^{-13}	—
$K_L \rightarrow e\bar{e}e\bar{e}$	3.40×10^{-8}	3.22×10^{-8}	3.65×10^{-8}	$(3.56 \pm 0.21) \times 10^{-8}$
$K_L \rightarrow \mu\bar{\mu}e\bar{e}$	1.55×10^{-9}	7.77×10^{-10}	2.51×10^{-9}	$(2.69 \pm 0.27) \times 10^{-9}$

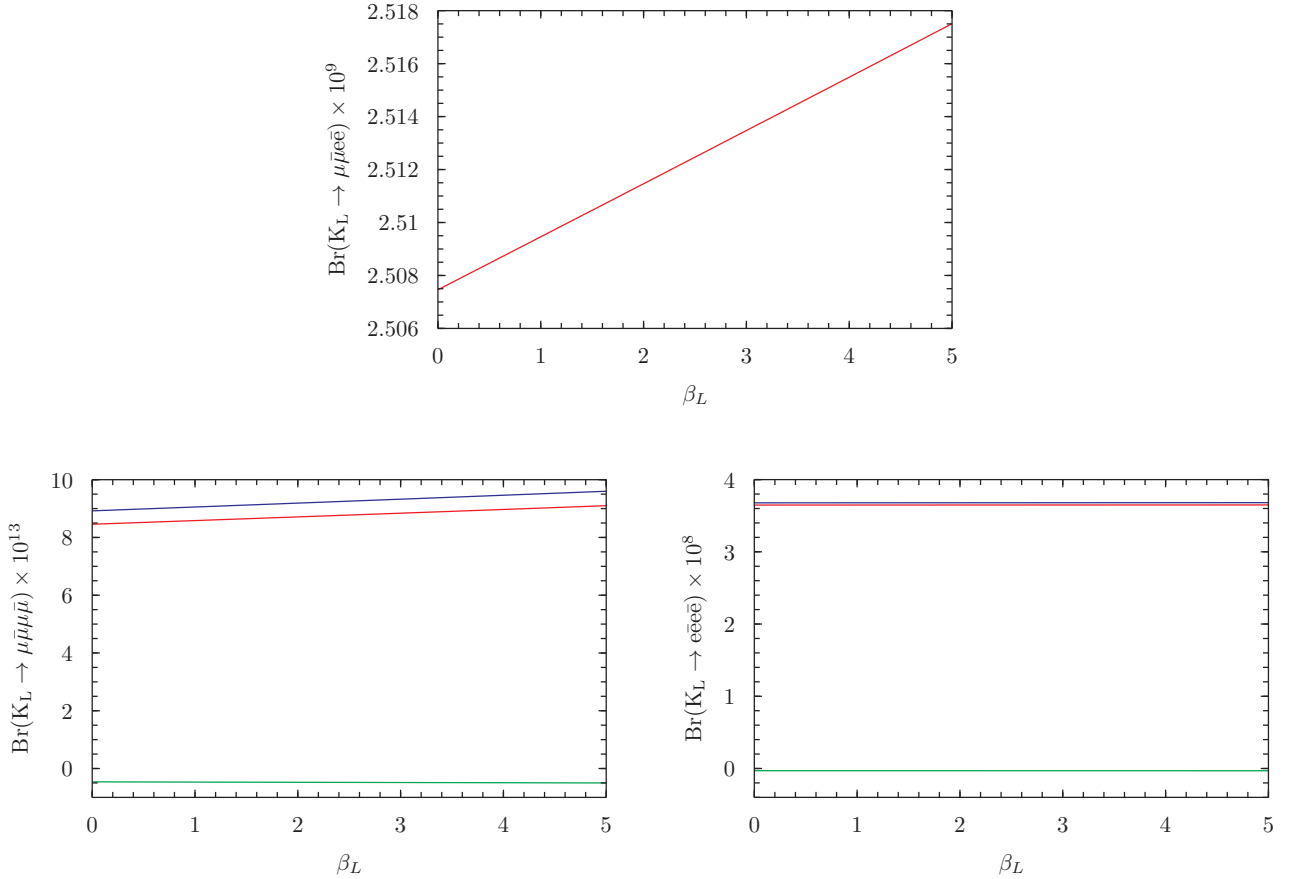
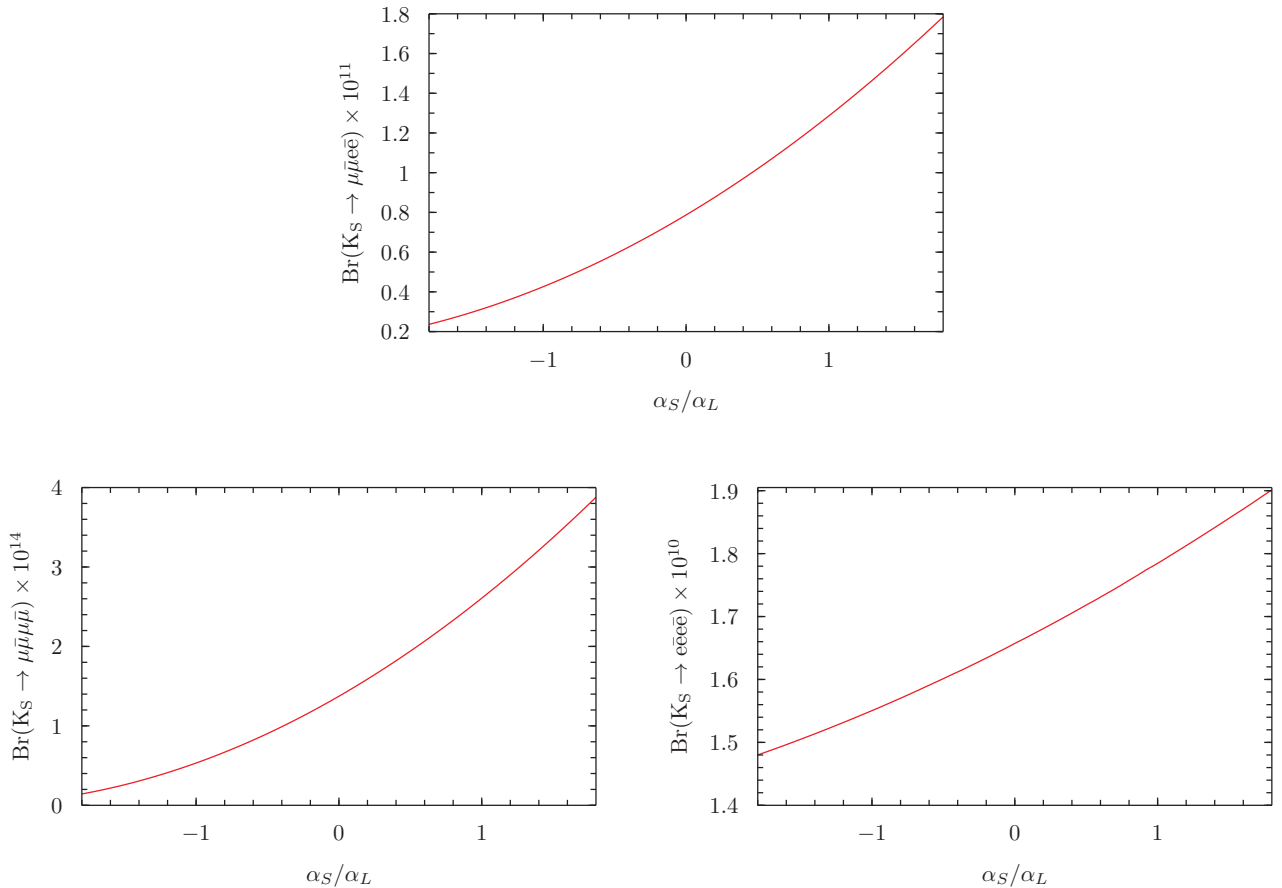


Fig. 2 Branching ratio of $K_L \rightarrow \ell_1 \bar{\ell}_1 \ell_2 \bar{\ell}_2$ vs. β_L . We fixed $\alpha_L = -1.63$ [9]. For the cases $\ell_1 = \ell_2$, the red line is the total branching ratio, the blue one is the contribution of $|\mathcal{M}_A|^2$ and the green one is the contribution of the interference term $\mathcal{M}_A \mathcal{M}_B^*$.

Table 2 Results for the branching ratios of K_S decays. Notice that there are no experimental results.

	This work			Birkfellner [7]
	$\alpha_S = \beta_S = 0$	$\alpha_S = 0$ and $\beta_S = -1 - 2\alpha_S$	$\alpha_S = \alpha_L$ and $\beta_S = -1 - 2\alpha_S$	
$K_S \rightarrow \mu\bar{\mu}\mu\bar{\mu}$	1.40×10^{-14}	1.37×10^{-14}	2.61×10^{-14}	1×10^{-14}
$K_S \rightarrow e\bar{e}e\bar{e}$	1.66×10^{-10}	1.66×10^{-10}	1.78×10^{-10}	7×10^{-11}
$K_S \rightarrow \mu\bar{\mu}e\bar{e}$	7.88×10^{-12}	7.87×10^{-12}	1.29×10^{-11}	8×10^{-12}

**Fig. 3** Branching ratio of $K_S \rightarrow \ell_1 \bar{\ell}_1 \ell_2 \bar{\ell}_2$ vs. α_S/α_L . We fixed $\alpha_L = -1.63$ [9] and $\beta_S = -1 - 2\alpha_S$ using (13).

for some weight function $f(X, Y)$ and we will choose here $f(X, Y) \doteq \text{sgn}(\cos \phi \sin \phi)$,

$$\Delta M = M_L - M_S \quad \text{and} \quad \Gamma = \frac{1}{2} (\Gamma_L + \Gamma_S). \quad (23)$$

We can easily obtain this function of time in our calculations since we can evaluate each part and we present our results for the three different channels on the fig. 4. Since F_S and F_L depend respectively on α_S and β_L , we take the arbitrariness to give the plots for three different values of $\alpha_S = \{-3, 0, 3\}$ and using the short-distance constraint $1 + 2\alpha_S + \beta_S = 0$ whereas the value of α_L is fixed to -1.63 and we use the sum rule $1 + 2\alpha_L + \beta_L = 0.3$ [9].

We want to stress here that the α_L as the value of the slope of the form factor does not fix the sign of $K_L \rightarrow \gamma\gamma$, as explained in the second DAΦNE book [17], it keeps an ambiguity. If we assume the VMD model for the weak form factor, this ambiguity can be removed as it has been shown in [18] and confirmed by other theoretical considerations in [19]. In our approach, we take that the dominant low energy contribution is coming from the pion pole, implying then

$$\text{sgn}[\mathcal{A}(K_L \rightarrow \gamma\gamma)] = \text{sgn}[\mathcal{A}(K_L \rightarrow \pi^0 \rightarrow \gamma\gamma)], \quad (24)$$

thus we will do all the following analysis under this statement. But of course, the experimental interferences analysis that we propose allows us to remove the ambiguity since the shape is fixed by the sign of $K_L \rightarrow \gamma\gamma$.

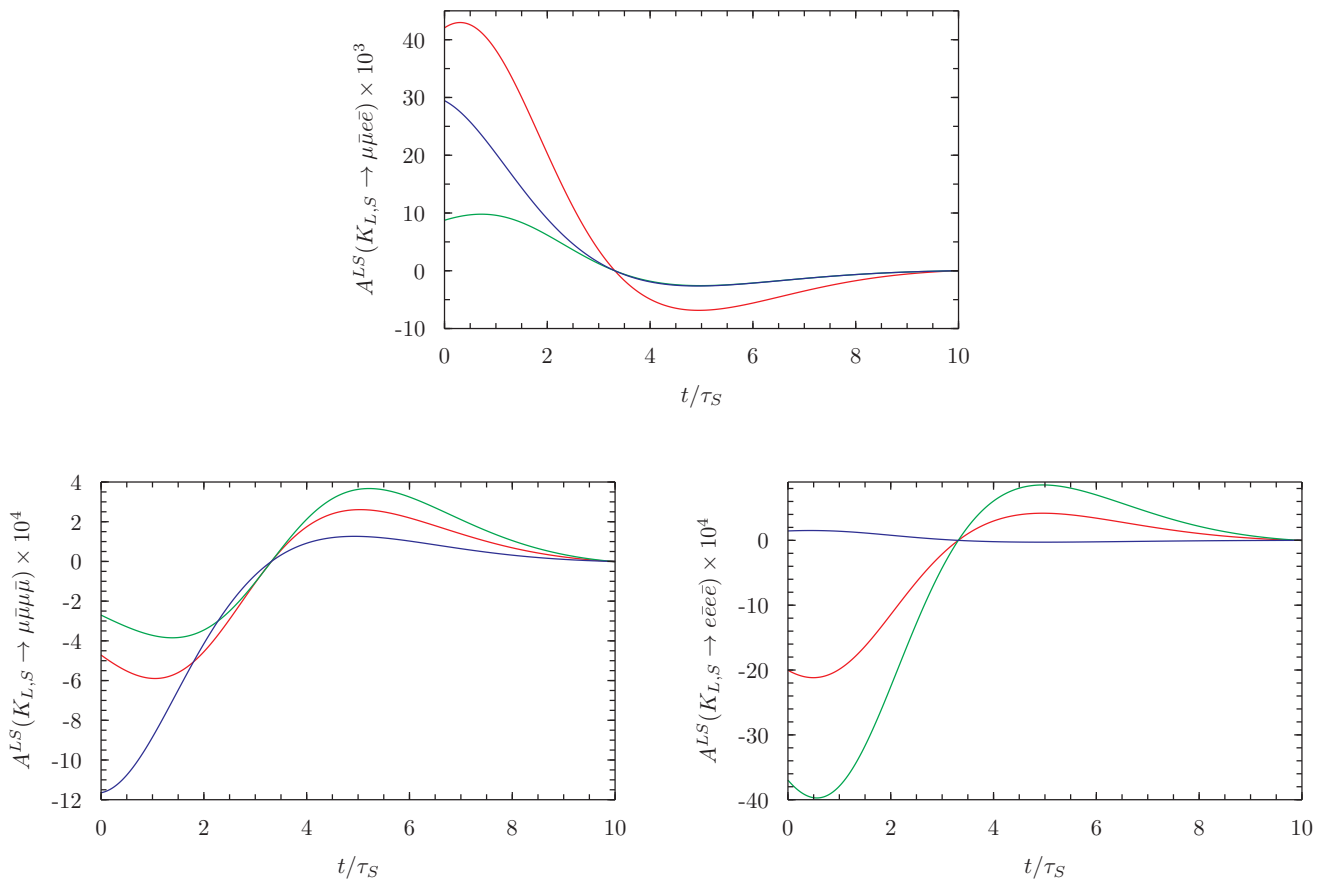


Fig. 4 Interferences between K_L and $K_S \rightarrow \ell_1 \bar{\ell}_1 \ell_2 \bar{\ell}_2$. The red line corresponds to the case $\alpha_S = 0$, the green line is $\alpha_S = -3$ while the blue line is $\alpha_S = 3$. As explained in the text we assume the sign $K_L \rightarrow \gamma\gamma$. The interferences being directly related to this sign, their experimental observations (in shape and amplitude) could confirmed this hypothesis.

4.2 NP contributions to CP violation interferences

As matter of principles, one can question an eventual apparition of New Physics contributions to these decays. Of course, to be fully descriptive we have first taken into account all already permitting contributions and evaluated their size to pretend to see new signatures in experimental results. This is the reason why we decompose all the possible contributions to the amplitude of the decay of K_S as²

$$A_S = |A_1|e^{i\delta} + \varepsilon|A_2|e^{i\delta'} + i|A_1^B| + i|A_1'|e^{i\delta'}, \quad (25)$$

viz.

- $|A_1|e^{i\delta}$ is the K_S CP conserving part. A_1 is just the SM amplitude computed in the section 3. δ is related through the optical theorem to the absorptive part

²We are aware that we do a misuse of writing by exponentiating the amplitude since we do not prove any unitarization of the amplitudes as long as we consider only the first term. But it is quite obvious that faced with the smallness of the numbers, this cannot change a lot the conclusions.

of $\mathcal{A}(K_S \rightarrow \pi\pi \rightarrow \gamma^*\gamma^*)$. It is given by [4]:

$$\arctan \delta = \frac{\text{Im} \mathcal{A}(K_S \rightarrow \pi\pi \rightarrow \gamma^*\gamma^*)}{\text{Re} \mathcal{A}(K_S \rightarrow \pi\pi \rightarrow \gamma^*\gamma^*)}, \quad (26)$$

with:

$$\begin{aligned} \text{Re} \mathcal{A}(K_S \rightarrow \pi\pi \rightarrow \gamma^*\gamma^*) \\ = -\frac{1}{2} \left\{ 1 + r_\pi^2 \left[\ln^2 \left(\frac{1 - \sigma_\pi}{1 + \sigma_\pi} \right) + \pi^2 \right] \right\}, \end{aligned} \quad (27)$$

$$\text{Im} \mathcal{A}(K_S \rightarrow \pi\pi \rightarrow \gamma^*\gamma^*)$$

$$= -\pi r_\pi^2 \ln \left(\frac{1 - \sigma_\pi}{1 + \sigma_\pi} \right), \quad (28)$$

where $r_\pi = M_\pi/M_K$ and $\sigma_\pi = \sqrt{1 - 4r_\pi^2}$. This yields to $\delta \approx -27.54^\circ \approx -0.48065$.

- $\varepsilon|A_2|e^{i\delta'}$ is the K_L CP-violating part. A_2 is the SM amplitude and ε parametrizes the indirect CP violation. δ' is related through the optical theorem to

the absorptive part of $\mathcal{A}(K_L \rightarrow \mu\bar{\mu})$. It is given by:

$$\arctan \delta' = \frac{\text{Im } \mathcal{A}(K_L \rightarrow \mu\bar{\mu})}{\text{Re } \mathcal{A}(K_L \rightarrow \mu\bar{\mu})}, \quad (29)$$

with [20, 21]:

$$\begin{aligned} \text{Re } \mathcal{A}(K_L \rightarrow \mu\bar{\mu}) &= \frac{1}{4\sigma_\mu} \ln^2 \left(\frac{1 - \sigma_\mu}{1 + \sigma_\mu} \right) + \frac{1}{\sigma_\mu} \text{Li}_2 \left(\frac{\sigma_\mu - 1}{\sigma_\mu + 1} \right) \\ &+ \frac{\pi^2}{12\sigma_\mu} + 3 \ln \frac{m_\mu}{\mu} + \chi(\mu), \end{aligned} \quad (30)$$

$$\text{Im } \mathcal{A}(K_L \rightarrow \mu\bar{\mu}) = \frac{\pi}{2\sigma_\mu} \ln \left(\frac{1 - \sigma_\mu}{1 + \sigma_\mu} \right), \quad (31)$$

where $r_\mu = m_\mu/M_K$, $\sigma_\mu = \sqrt{1 - 4r_\mu^2}$ and $\chi(M_\rho) = 3.3$. We get $\delta' \approx -82.48^\circ \approx -1.43952$.

- $i|A_1^B|$ is the K_S Bremsstrahlung CP-violating part. For more details see the appendices.
- $|A_1'|e^{i\delta'}$ is the K_S CP-violating part. A_1' is a CP-violation part of the K_S coming from a coupling to

the photons similar to the one of the K_L : $K_S \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$. Consequently A_1' would be proportional to A_2 . An estimation of the relation between A_1' and A_2 gives: $A_1' \doteq \xi A_2$ with $\xi \sim 10^{-1}$ [10]. The strong phase δ' is the same as for the second term due to universality, the coupling of the $K_{S,L}$ to the two photons in this case being similar. This constitutes our NP implementation.

Contrary to usual asymmetries prescriptions to have a relevant observable to distinguish the most important contribution, in the case of identical leptons pairs, we have to consider the following phase space integration

$$\int_0^\pi d\phi \int \frac{d\Phi_4}{d\phi} \text{sgn}(\cos \phi \sin \phi) |A_S|^2. \quad (32)$$

A straightforward computation of all parts, illustrated on the fig. 5 only for the channel into four muons, show the A_1' NP part is dominant as expected, assuming that it is universal and just an approximation to the dominant behaviour.

Since the hypothesis of the dominant part is coming from the NP contribution one can suppose now to generate as an observable $A^{LS}(t)$ such as

$$A^{LS}(t) = \frac{e^{-\Gamma t} \int_0^{\phi_0} d\phi \int \frac{d\Phi_4}{d\phi} \text{sgn}(\cos \phi \sin \phi) \left[\text{Re}(A_L A_S^*) \cos(\Delta M t) + \text{Im}(A_L A_S^*) \sin(\Delta M t) \right]}{\int_0^{\phi_0} d\phi \int \frac{d\Phi_4}{d\phi} [e^{-\Gamma_S t} |A_S|^2 + e^{-\Gamma_L t} |A_L|^2]}, \quad (33)$$

where

$$A_L = |A_2|,$$

$$A_S = |A_1|e^{i\delta} + i|A_1'|e^{i\delta'} = |A_1|e^{i\delta} + i\xi|A_2|e^{i\delta'}$$

and ϕ_0 is the angle that maximizes $A^{LS}(t)$, $\phi_0 = \pi$ for the case where the four leptons are identical and $\phi_0 = \pi/2$ for $K_S \rightarrow \bar{\mu}\mu e\bar{e}$ [22, 23].

Therefore one obtains the results of fig. 6. It is obvious that they results expected in our calculation are made under the assumptions of the sign of the amplitude $\mathcal{A}(K_L \rightarrow \gamma\gamma)$, if experimentally one observes the same kind of curves (after fixing α_L from the decays) it means that our hypothesis for the sign is correct, if the curves are symmetric about the horizontal axis it implies the opposite sign naturally.

5 Conclusions

We have shown that it is possible to obtain good predictions for the branching ratios for the decays of the

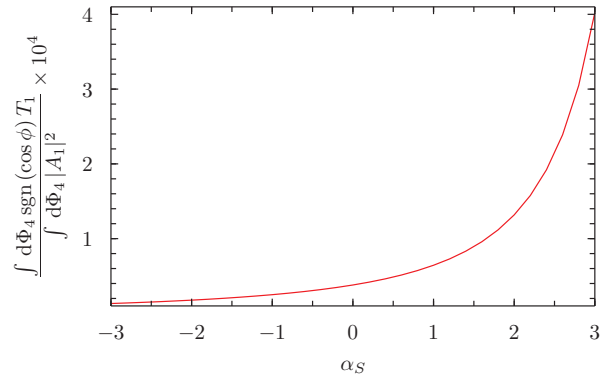


Fig. 5 Dominant NP contribution to $K_S \rightarrow 4\mu$ for the direct CP violation contribution and where we use the short notation $T_1 = (|\varepsilon|^2 + \xi^2 + 2\xi \text{Im } \varepsilon) |A_2|^2$.

K_L into four leptons comparing to the experimental data through a vector meson dominance inspired form factor. It is natural then to consider that the same approach is pertinent for the case of the K_S into four

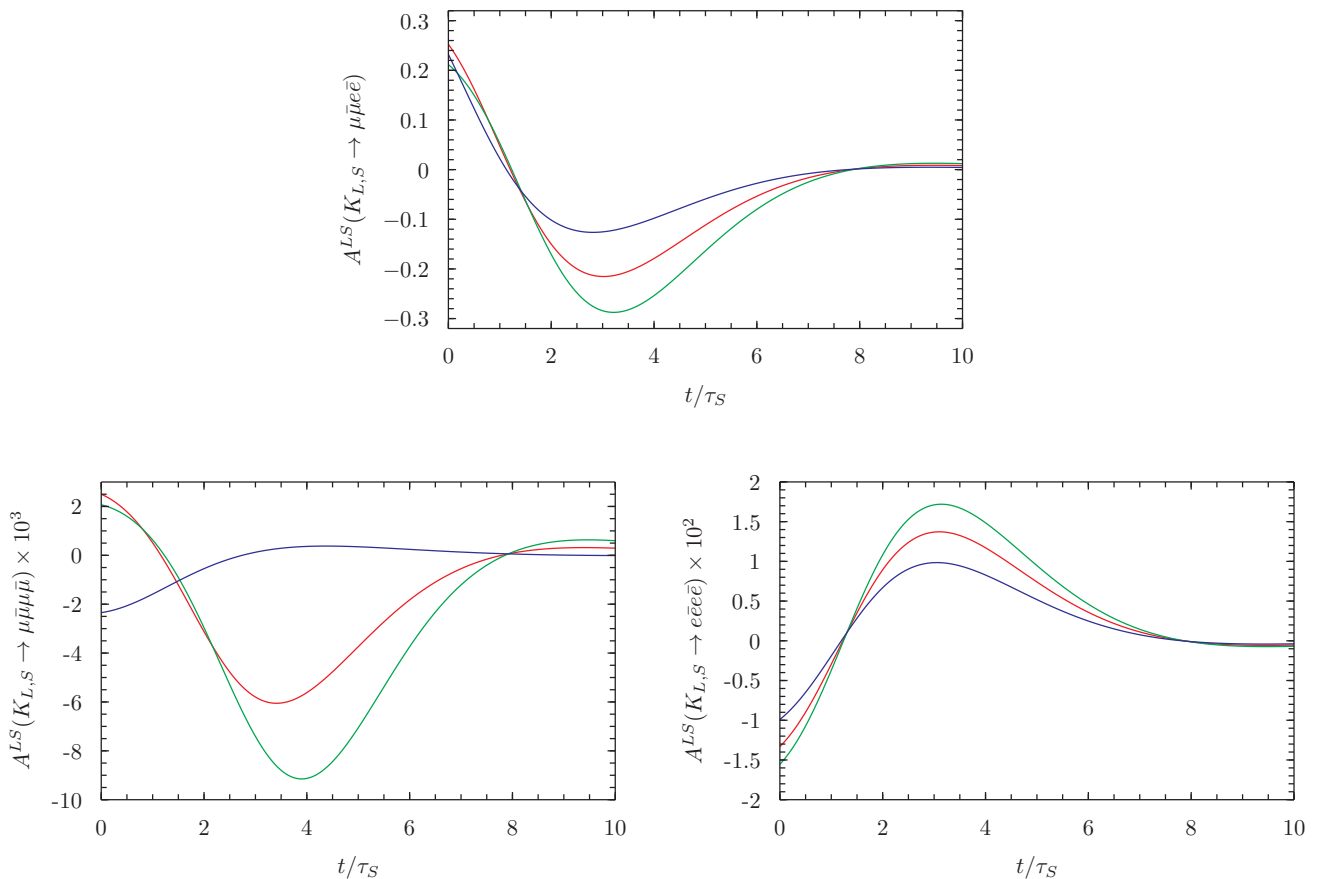


Fig. 6 New Physics contributions to the CP violation interferences. The red line corresponds to the case $\alpha_S = 0$, the green line is $\alpha_S = -3$ while the blue line is $\alpha_S = 3$. Here too we make the same assumption for the sign of $K_L \rightarrow \gamma\gamma$.

leptons, since the model is more constrained from short distance behaviour. Since this short distance behaviour is model dependant in our approach, one can emphasize that even if our assumptions of a VMD form factor type, one could ever consider the slope (α) by itself and see it as the first derivative of the form factor experimentally observe in a model independent way.

A direct consequence of the experimental data in our approach would be to fix the α and β parameters for K_L and K_S and then give the sign of $\mathcal{A}(K_L \rightarrow \gamma\gamma)$ (for a sufficient accuracy of course). Moreover, we have shown that a simple assumption on the existence of a NP operator in the lagrangian could be verified with interferences in those decays.

It appears that now it is important to obtain some experimental data in these channels involving the K_S decays (particularly the muons ones) and considering our predictions, we hope that the LHCb processes for tagging the muons allow us to reach a sufficient level of accuracy. We think also, that it could be easier to identify the decays rates containing electrons through the ones involving pions decays.

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Appendix A: Detailed expressions for amplitudes

Appendix A.1: The K_S decays amplitudes

For the cases where $\ell_1 = \ell_2$, there exist 4 diagrams that can be reduced to two different amplitudes \mathcal{M}_A and \mathcal{M}_B ,

$$\begin{aligned} \mathcal{M}_A = e^2 \frac{F_S((p_1 + p_2)^2, (p_3 + p_4)^2)}{(p_1 + p_2)^2 (p_3 + p_4)^2} \\ \times [(p_1 + p_2) \cdot (p_3 + p_4) g^{\mu\nu} - (p_1 + p_2)^\nu (p_3 + p_4)^\mu] \\ \times [\bar{u}(p_1) \gamma_\mu v(p_2)] [\bar{u}(p_3) \gamma_\nu v(p_4)] \end{aligned} \quad (\text{A.1})$$

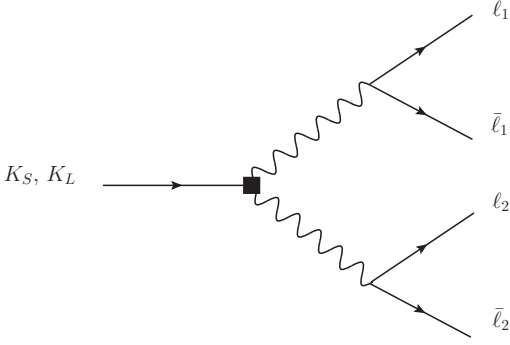


Fig. 7 Amplitudes of $K_{S,L}$ in four leptons.

and

$$\begin{aligned} \mathcal{M}_B &= -e^2 \frac{F_S((p_3 + p_2)^2, (p_1 + p_4)^2)}{(p_3 + p_2)^2 (p_1 + p_4)^2} \\ &\times [(p_2 + p_3) \cdot (p_1 + p_4) g^{\mu\nu} - (p_2 + p_3)^\nu (p_1 + p_4)^\mu] \\ &\times [\bar{u}(p_3) \gamma_\mu v(p_2)] [\bar{u}(p_1) \gamma_\nu v(p_4)]. \end{aligned} \quad (\text{A.2})$$

Thus the total squared amplitude is given by (under symmetries considerations, $|\mathcal{M}_A|^2 = |\mathcal{M}_B|^2$),

$$|\mathcal{M}|^2 = \frac{1}{2} (|\mathcal{M}_A|^2 + \mathcal{M}_A \mathcal{M}_B^*). \quad (\text{A.3})$$

In the mixed case, $\ell_1 = \mu$ and $\ell_2 = e$, there are only two diagrams, and we have $|\mathcal{M}|^2 = |\mathcal{M}_A|^2$.

Appendix A.2: The K_L decays amplitudes

The calculations of the amplitudes involving the K_L are identical in procedure that the ones for the K_S , we have to distinguish two kinds of amplitudes

$$\begin{aligned} \mathcal{M}_A &= e^2 \frac{F_L((p_1 + p_2)^2, (p_3 + p_4)^2)}{(p_1 + p_2)^2 (p_3 + p_4)^2} \\ &\times \varepsilon_{\mu\nu\rho\sigma} (p_1 + p_2)^\rho (p_3 + p_4)^\sigma \\ &\times [\bar{u}(p_1) \gamma^\mu v(p_2)] [\bar{u}(p_3) \gamma^\nu v(p_4)] \end{aligned} \quad (\text{A.4})$$

and

$$\begin{aligned} \mathcal{M}_B &= -e^2 \frac{F_L((p_3 + p_2)^2, (p_1 + p_4)^2)}{(p_3 + p_2)^2 (p_1 + p_4)^2} \\ &\times \varepsilon_{\mu\nu\rho\sigma} (p_2 + p_3)^\rho (p_1 + p_4)^\sigma \\ &\times [\bar{u}(p_3) \gamma^\mu v(p_2)] [\bar{u}(p_1) \gamma^\nu v(p_4)]. \end{aligned} \quad (\text{A.5})$$

The total amplitude is given by (under symmetries considerations, $|\mathcal{M}_A|^2 = |\mathcal{M}_B|^2$),

$$|\mathcal{M}|^2 = \frac{1}{2} (|\mathcal{M}_A|^2 + \mathcal{M}_A \mathcal{M}_B^*). \quad (\text{A.6})$$

As before, in the mixed case, $\ell_1 = \mu$ and $\ell_2 = e$, there are only two diagrams, and we have $|\mathcal{M}|^2 = |\mathcal{M}_A|^2$.

Appendix B: Bremsstrahlung CP-violating part

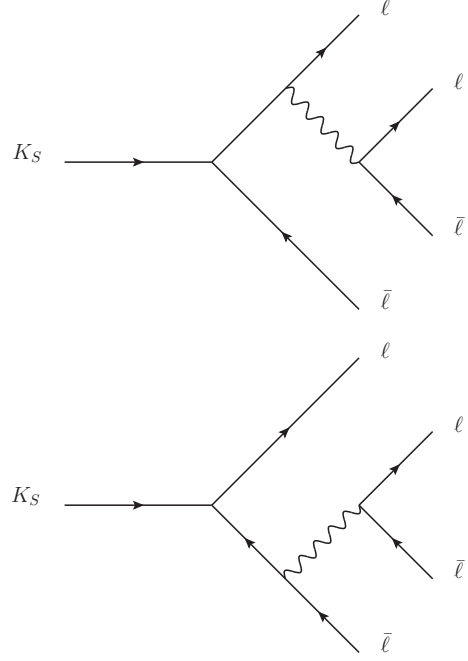


Fig. 8 Bremsstrahlung amplitudes for K_S in four leptons.

Using the Low's theorem [24], the amplitude $K_S(q) \rightarrow \mu(p_-) \bar{\mu}(p_+) \gamma^*(k)$ is the product of the $K_S(q) \rightarrow \mu(p_-) \bar{\mu}(p_+)$ amplitude times the contribution of the soft photon radiated:

$$\begin{aligned} \mathcal{M}(K_S \rightarrow \mu \bar{\mu} \gamma^*) \\ = k^2 F_B^\mu(k, p_-, p_+) \epsilon_\mu(k) \mathcal{M}(K_S \rightarrow \mu \bar{\mu}), \end{aligned} \quad (\text{B.7})$$

where $\mathcal{M}(K_S \rightarrow \mu \bar{\mu})$ is the decay amplitude of K_S into two muons

$$\mathcal{M}(K_S \rightarrow \mu \bar{\mu}) = \mathcal{A}_{SD} \bar{u} \gamma_5 v \quad (\text{B.8})$$

and

$$\begin{aligned} F_B^\mu(k, p_-, p_+) \\ = \frac{2e}{k^2} \left[\frac{p_-^\mu}{k^2 + 2k \cdot p_-} + \frac{p_+^\mu}{k^2 - 2k \cdot p_+} \right]. \end{aligned} \quad (\text{B.9})$$

Now, we just have to contract with the muonic current $-ie\bar{u}\gamma^\nu v \epsilon_\nu^*$ to obtain the Bremsstrahlung contri-

bution

$\mathcal{M}_{\text{Brems.}}$

$$\begin{aligned}
&= \mathcal{A}_{SD} \left\{ F_B^\mu(q_1, p_3, p_4) [\bar{u}(p_1)\gamma_\mu v(p_2)] [\bar{u}(p_3)\gamma_5 v(p_4)] \right. \\
&+ F_B^\mu(q_2, p_1, p_2) [\bar{u}(p_3)\gamma_\mu v(p_4)] [\bar{u}(p_1)\gamma_5 v(p_2)] \\
&- F_B^\mu(p_2 + p_3, p_1, p_4) [\bar{u}(p_3)\gamma_5 v(p_2)] [\bar{u}(p_1)\gamma_\mu v(p_4)] \\
&\left. - F_B^\mu(p_1 + p_4, p_3, p_2) [\bar{u}(p_1)\gamma_\mu v(p_4)] [\bar{u}(p_3)\gamma_5 v(p_2)] \right\}. \tag{B.10}
\end{aligned}$$

In our case, $\text{Re}\mathcal{A}_{SD}$ can be neglected, all the short-distance information is contained in $\text{Im}\mathcal{A}_{SD}$ and we have [10]:

$\text{Im}\mathcal{A}_{SD}$

$$= -\frac{G_F \alpha_{\text{em}}(M_Z)}{\pi \sin^2 \theta_W} \sqrt{2} m_\mu F_K \text{Im}(V_{ts}^* V_{td}) Y(x_t), \tag{B.11}$$

with $x_t = m_t^2/M_W^2$ and $Y(x)$ is the Inami-Lin function:

$$Y(x) = \frac{x}{8} \left[\frac{4-x}{1-x} + \frac{3x}{(1-x)^2} \ln x \right]. \tag{B.12}$$

References

1. R. Aaij *et al.* [LHCb Collaboration], JHEP **1301**, 090 (2013)
2. R. Aaij *et al.* [LHCb Collaboration], Eur. Phys. J. C **73**, 2373 (2013)
3. T. Miyazaki and E. Takasugi, Phys. Rev. D **8**, 2051-2062 (1973).
4. G. Ecker and A. Pich, Nucl. Phys. B **366**, 189 (1991).
5. L. Cappiello, G. D'Ambrosio and M. Miragliuolo, Phys. Lett. B **298** 423 (1993); A. G. Cohen, G. Ecker and A. Pich, Phys. Lett. B **304** 347 (1993)
6. J.L. Goity and L. Zhang, Phys. Rev. D **57**, 7031-7033 (1998).
7. W. Birkfellner, Diploma Thesis, Univ. of Vienna (1996).
8. V. Cirigliano, G. Ecker, H. Neufeld, A. Pich and J. Portoles, Rev. Mod. Phys. **84**, 399 (2012)
9. G. D'Ambrosio, G. Isidori and J. Portolés, Phys. Rev. B **423**, 385-394 (1998).
10. G. Isidori and R. Unterdorfer, JHEP **01**, 009 (2004).
11. J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
12. G. D'Ambrosio and D. Espriu, Phys. Lett. B **175** 237 (1986); J.L. Goity, Z. Phys. C **34**, (1987) 341.
13. G. Buchalla, G. D'Ambrosio and G. Isidori, Nucl. Phys. B **672**, 387 (2003)
14. N. Cabibbo and A. Maksymowicz, Phys. Rev. B **137**, 438-443 (1965). Erratum, Phys. Rev. B **168**, 1926 (1968).
15. G. D'Ambrosio, G. Isidori, A. Pugliese and N. Paver, Phys. Rev. D **50**, 5767-5774 (1994).
16. P. Heiliger and L. M. Sehgal, Phys. Rev. D **48** (1993) 4146 [Erratum-ibid. D **60** (1999) 079902].
17. G. D'Ambrosio, G. Ecker, G. Isidori and H. Neufeld, hep-ph/9411439.
18. G. D'Ambrosio and J. Portolés, Nucl. Phys. B **492** 417 (1997).
19. J. -M. Gerard, C. Smith and S. Trine, Nucl. Phys. B **730** (2005)
20. D. Gomez-Dumm and A. Pich, Phys. Rev. Lett **80**, 4663 (1998).
21. M. Knecht, S. Peris, M. Perrottet and E. de Rafael, Phys. Rev. Lett. **83** (1999) 5230
22. L. M. Sehgal and J. van Leusen, Phys. Rev. Lett. **83**, 4933 (1999)
23. L. Cappiello, O. Cata, G. D'Ambrosio and D.-N. Gao, Eur. Phys. J. C **72**, 1872 (2012) [Erratum-ibid. C **72**, 2208 (2012)]
24. F.E. Low, Phys. Rev. **110**, 974 (1958).