Hybrid Quantization of Inflationary Universes

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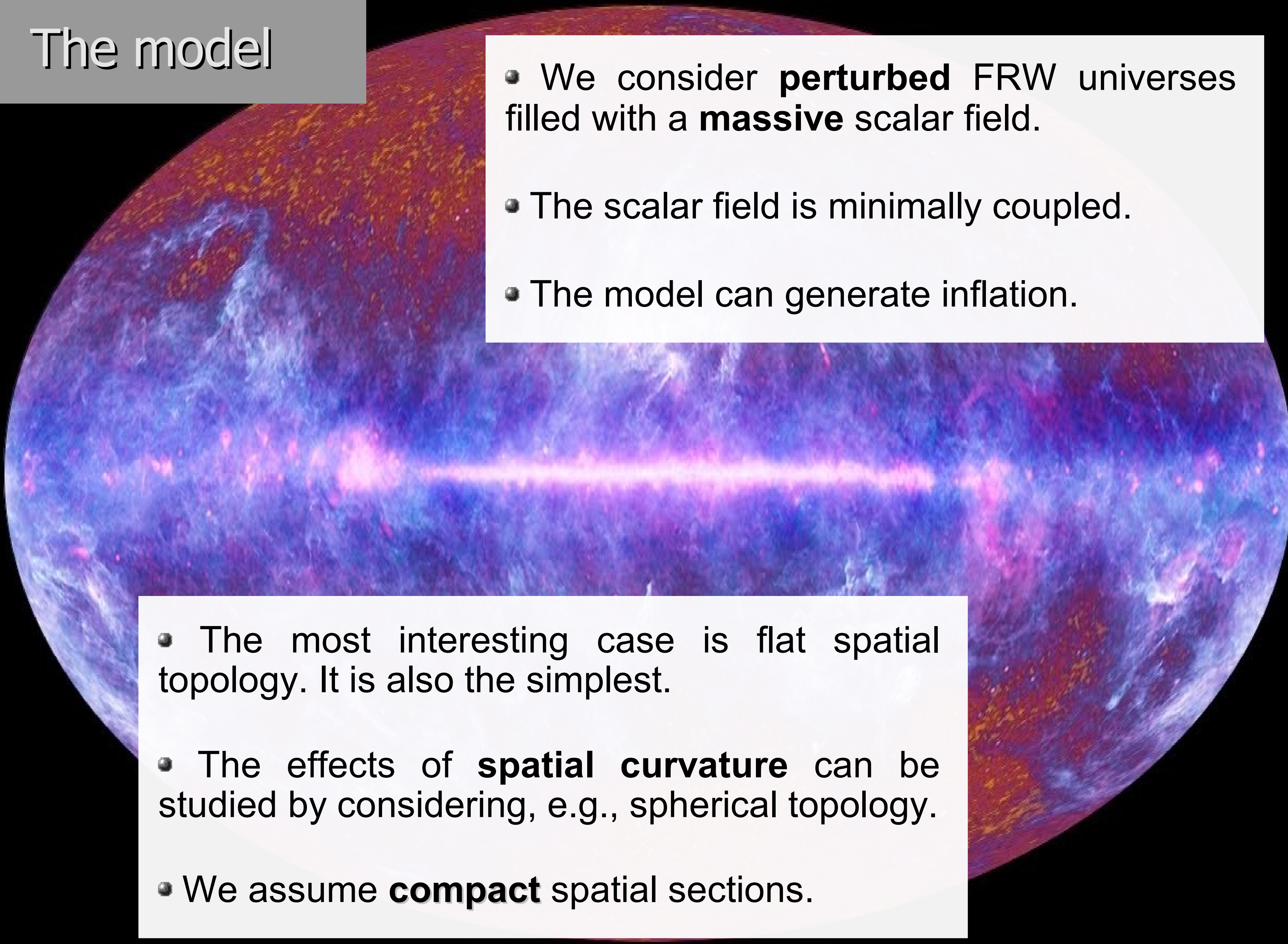
The model



- We consider **perturbed** FRW universes filled with a **massive** scalar field.

- The scalar field is minimally coupled.

- The model can generate inflation.

- The most interesting case is flat spatial topology. It is also the simplest.

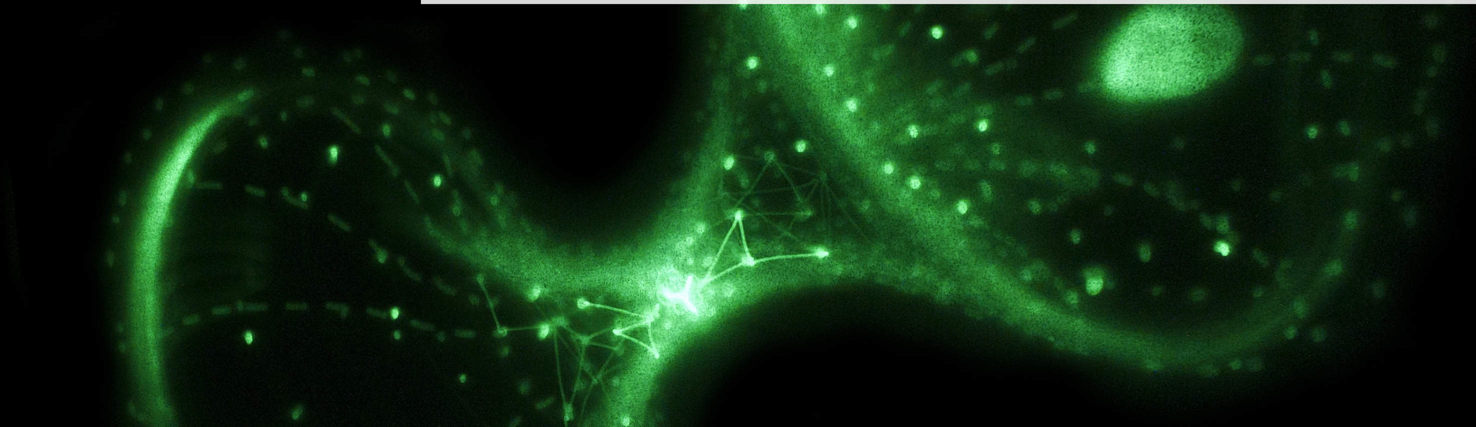
- The effects of **spatial curvature** can be studied by considering, e.g., spherical topology.

- We assume **compact** spatial sections.

The model

It's been well studied, even in LQC, though...

- Anomalies: Incorporate quantum effects, not the starting point for quantization.
- Effective dynamics: Needs a true derivation.



- **Approximations**: As few as possible. Should be derived or at least checked for consistency.
- In many cases these checks are only internal, within the approximated description.

Perturbations about flat FRW

- Truncation at **quadratic** order in the action.
- Includes **backreaction** at that order.
- Tests the validity of less refined truncations and provides the way to develop **approximation** methods, controlling their range of application.

Hybrid approach

Effects of quantum geometry are only accounted for in the background

- Succesfully applied in Gowdy cosmologies.
- In those cases there is no truncation. This is no drawback (think of the harmonic oscillator).
- In the present case, we only deal with the quadratically perturbed model.

Uniqueness of the Fock description

- Infinite **ambiguity** in selecting a Fock representation in QFT in curved space-times.
- This can be restricted by appealing to *background* **symmetries**.
- Typically this is not sufficent in non-stationarity.
- Proposal: demand the **UNITARITY** of the quantum evolution.

  The conventional interpretation of QM is guaranteed.
  This goes beyond the viewpoint of algebraic quantizations.
- There is a natural ambiguity in the **separation of the background** from the field.
  In cosmology, this introduces time-dependent canonical field transformations.
- Remarkably, symmetry invariance and dynamical unitarity select a **UNIQUE canonical pair** and a UNIQUE Fock representation for their CCR's.

Uniqueness of the Fock description

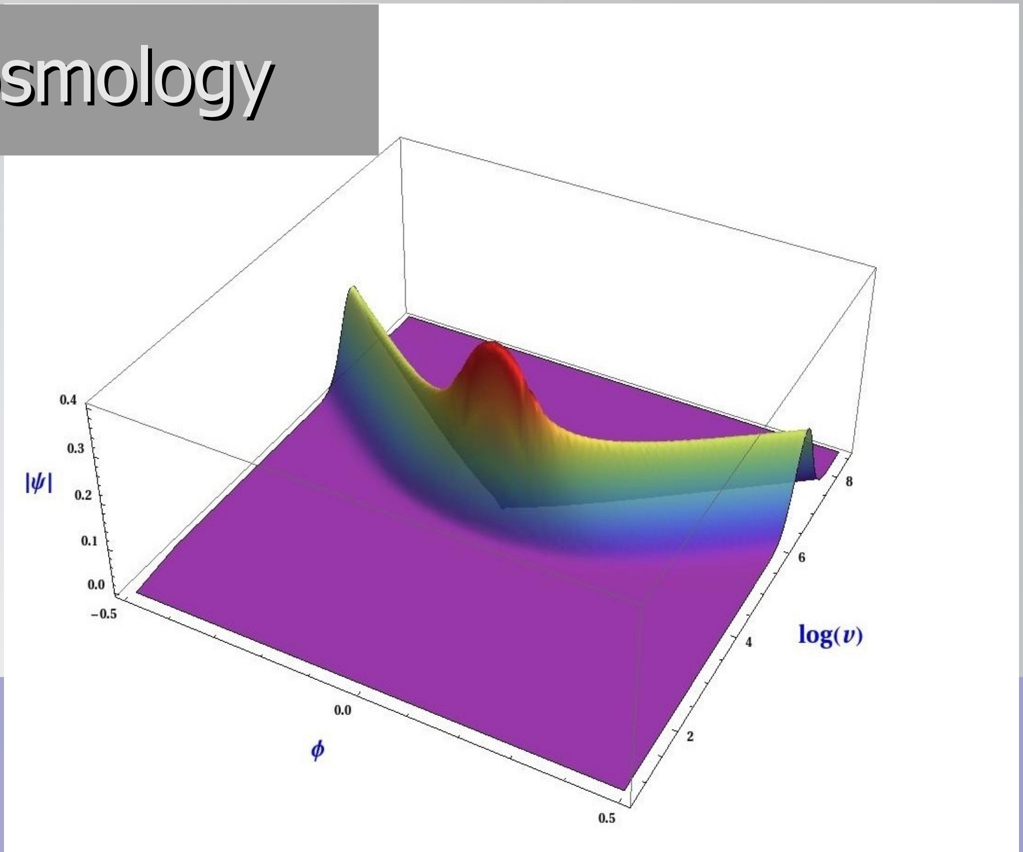
Uniqueness of the Fock description

- Recent works DO NOT incorporate the correct scaling (AA&N). This affects the quantum description, and in particular the *effective* approaches therein dereived.

- Moreover, one can even consider non-local canonical transformations, respecting the decoupling of field modes.

The **UNIQUENESS** of the quantization, up to unitary equivalence, is guaranteed.

Loop Quantum FRW Cosmology



- Avoids the Big Bang.
- Specific **proposal** such that:
  - Evolution can be defined even without ideal clocks (masless field).
  - The WdW limit is unambiguous in each superselection sector.
  - It is optimal for numerical computation.
- Control of changes of densitization in the scalar constraint.
  The lapse function is **not a function on phase space**.

Classical system: FRW

- **Massive scalar field minimally coupled to a compact, flat FRW universe.**

Geometry:

$$A^i_a = c\,{}^0e^i_a(2\pi)^{-1}; \qquad E^a_i = p\sqrt{{}^0e}\,{}^0e^a_i(2\pi)^{-2}.$$

$$\{c, p\} = 8\pi G\gamma/3.$$

$$a^2 = e^{2\alpha} = |p|(2\pi\sigma)^{-2}; \qquad \pi_\alpha = -pc(\gamma 8\pi^3\sigma^2)^{-1}.$$

$$\sigma^2 = G(6\pi^2)^{-1}.$$

Matter:

$$\varphi = (2\pi)^{3/2}\sigma\phi; \qquad \pi_\varphi = (2\pi)^{-3/2}\sigma^{-1}\pi_\phi.$$

Hamiltonian constraint:

$$C_0 = -\frac{6}{\gamma^2}\sqrt{|p|}\,c^2 + \frac{8\pi G}{V}(\pi_\phi^2 + m^2V^2\phi^2).$$

$$V = |p|^{3/2}.$$

Classical system: Modes

- We expand inhomogeneities in a (real) **Fourier basis**:

$$Q_{\vec{n},+} = \frac{1}{2\pi^{3/2}} \cos \vec{n}\cdot\vec{\theta}, \qquad Q_{\vec{n},-} = \frac{1}{2\pi^{3/2}} \sin \vec{n}\cdot\vec{\theta}. \qquad \vec{n} \in \mathbb{Z}^3, \qquad n_1 \geq 0.$$

- The basis is **orthonormal**, and we exclude the zero mode in the expansions.

- These functions are eigenmodes of the Laplace-Beltrami operator of the standard flat metric on the three-torus, with eigenvalue

$$-\omega_n^2 = -\vec{n}\cdot\vec{n}.$$

- We only consider **scalar perturbations**: decoupled from vector and tensor perturbations at dominant order.

Classical system: Inhomogeneities

- Mode expansion of the inhomogeneities:

$$h_{ij}=(\sigma e^{\alpha})^{2}\left[{}^{0}h_{ij}+2\epsilon(2\pi)^{3/2}\sum\left\{a_{\vec{n},\pm}(t)Q_{\vec{n},\pm}\,{}^{0}h_{ij}+b_{\vec{n},\pm}(t)\left(\frac{3}{\omega_n^{2}}(Q_{\vec{n};\pm})_{,ij}+Q_{\vec{n},\pm}\,{}^{0}h_{ij}\right)\right\}\right],$$

$$N=\sigma N_{0}(t)\left[1+\epsilon(2\pi)^{3/2}\sum g_{\vec{n},\pm}(t)Q_{\vec{n},\pm}\right],$$

$$N_{i}=\epsilon(2\pi)^{3/2}\sigma^{2}e^{\alpha}\sum\frac{k_{\vec{n},\pm}(t)}{\omega_n}(Q_{\vec{n},\pm})_{;i},$$

$$\Phi=\frac{1}{\sigma}\left[\frac{\varphi(t)}{(2\pi)^{3/2}}+\epsilon\sum f_{\vec{n},\pm}(t)Q_{\vec{n},\pm}\right].$$

The corrections *include* in principle higher-order perturbations.

Classical system: Action

- Truncating the action at **quadratic** order in perturbations, one obtains:

$$H=\frac{N_0\sigma}{16\pi G}C_0+\epsilon^2\sum\left(N_0 H_2^{\vec{n},\pm}+N_0 g_{\vec{n},\pm}H_1^{\vec{n},\pm}+k_{\vec{n},\pm}\widetilde{H}_1^{\vec{n},\pm}\right),$$

$$\begin{aligned}H_2^{\vec{n},\pm}2e^{3\alpha}=&-\pi^2_{a_{\vec{n},\pm}}+\pi^2_{b_{\vec{n},\pm}}+\pi^2_{f_{\vec{n},\pm}}+2\pi_\alpha\left(a_{\vec{n},\pm}\pi_{a_{\vec{n},\pm}}+4b_{\vec{n},\pm}\pi_{b_{\vec{n},\pm}}\right)-6\pi_\varphi a_{\vec{n},\pm}\pi_{f_{\vec{n},\pm}}\\&+\pi_\alpha^2\left(\frac{1}{2}a^2_{\vec{n},\pm}+10b^2_{\vec{n},\pm}\right)+\pi_\varphi^2\left(\frac{15}{2}a^2_{\vec{n},\pm}+6b^2_{\vec{n},\pm}\right)-\frac{e^{4\alpha}}{3}\left[\omega_n^2a^2_{\vec{n},\pm}+(\omega_n^2-18)b^2_{\vec{n},\pm}\right]\\&+e^{4\alpha}\omega_n^2\left[f^2_{\vec{n},\pm}-\frac{2}{3}a_{\vec{n},\pm}b_{\vec{n},\pm}\right]+e^{6\alpha}m^2\sigma^2\left[\varphi^2\left(\frac{3}{2}a^2_{\vec{n},\pm}+6b^2_{\vec{n},\pm}\right)+6\varphi a_{\vec{n},\pm}f_{\vec{n},\pm}+f^2_{\vec{n},\pm}\right],\end{aligned}$$

$$\begin{aligned}H_1^{\vec{n},\pm}2e^{3\alpha}=&2\pi_\varphi\pi_{f_{\vec{n},\pm}}-2\pi_\alpha\pi_{a_{\vec{n},\pm}}-(\pi_\alpha^2+3\pi_\varphi^2)a_{\vec{n},\pm}-\frac{2}{3}e^{4\alpha}\omega_n^2(a_{\vec{n},\pm}+b_{\vec{n},\pm})\\&+e^{6\alpha}m^2\sigma^2\varphi(3\varphi a_{\vec{n},\pm}+2f_{\vec{n},\pm})\end{aligned}$$

$$\widetilde{H}_1^{\vec{n},\pm}3e^{\alpha}=\pi_{b_{\vec{n},\pm}}-\pi_{a_{\vec{n},\pm}}+\pi_\alpha(a_{\vec{n},\pm}+4b_{\vec{n},\pm})+3\pi_\varphi f_{\vec{n},\pm}.$$

Longitudinal gauge

- We can adopt **longitudinal gauge** by imposing:

$$\pi_{a_{\vec{n},\pm}} - \pi_\alpha a_{\vec{n},\pm} - 3\pi_\varphi f_{\vec{n},\pm} = 0, \qquad b_{\vec{n},\pm} = 0.$$

- This removes the constraints *linear* in perturbations.

$$\pi_{b_{\vec{n},\pm}} = 0, \qquad a_{\vec{n},\pm} = 3\frac{\pi_\varphi \pi_{f_{\vec{n},\pm}} + (e^{6\alpha} m^2 \sigma^2 \varphi - 3\pi_\alpha \pi_\varphi) f_{\vec{n},\pm}}{9\pi_\varphi^2 + \omega_n^2 e^{4\alpha}}.$$

- Together with dynamical stability, this fixes $g_{\vec{n},\pm} = -a_{\vec{n},\pm}$, $k_{\vec{n},\pm} = 0$.

The shift vanishes, and the spatial metric is proportional to ${}^0h_{ij}$.

Longitudinal gauge: Reduction

- After **REDUCTION**, a canonical set is:

$$\bar{\varphi} = \varphi + 3\sum a_{\vec{n},\pm} f_{\vec{n},\pm}, \qquad \pi_{\bar{\varphi}} = \pi_{\varphi},$$

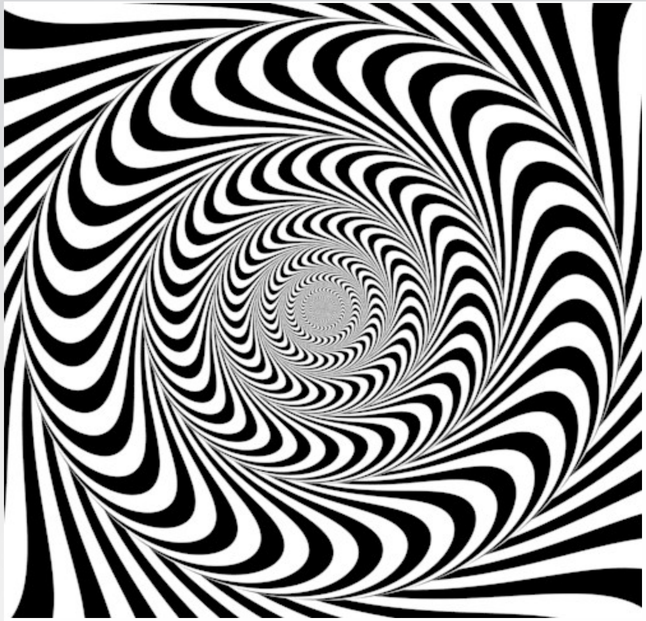
$$\bar{\alpha} = \alpha + \frac{1}{2}\sum \left(a^2_{\vec{n},\pm} + f^2_{\vec{n},\pm}\right), \qquad \pi_{\bar{\alpha}} = \pi_{\alpha} - \sum f_{\vec{n},\pm}\left(\pi_{f_{\vec{n},\pm}} - 3\pi_{\varphi} a_{\vec{n},\pm} - \pi_{\alpha} f_{\vec{n},\pm}\right),$$

$$\bar{f}_{\vec{n},\pm} = e^{\alpha} f_{\vec{n},\pm}, \qquad \pi_{\bar{f}_{\vec{n},\pm}} = e^{-\alpha}\left(\pi_{f_{\vec{n},\pm}} - 3\pi_{\varphi} a_{\vec{n},\pm} - \pi_{\alpha} f_{\vec{n},\pm}\right).$$

The genuine background variables are corrected with **quadratic** perturbations.

We have already **scaled** the matter field variables.

Longitudinal gauge: Dynamics



- The modes of the scaled matter field satisfy a quasi-KG equation with time-dependent mass:

$$\ddot{\bar{f}}_{\vec{n},\pm} + r_n \dot{\bar{f}}_{\vec{n},\pm} + (\omega_n^2 + s + s_n)\bar{f}_{\vec{n},\pm} = 0,$$
$$\pi_{\bar{f}_{\vec{n},\pm}} = (1 + p_n)\dot{\bar{f}}_{\vec{n},\pm} + q_n \bar{f}_{\vec{n},\pm},$$

$$s = m^2\sigma^2 e^{2\bar{\alpha}} - \frac{e^{-4\bar{\alpha}}}{2}\left(\pi_{\bar{\alpha}}^2 + 21\,\pi_{\bar{\varphi}}^2 + 3\,e^{6\bar{\alpha}} m^2\sigma^2\bar{\varphi}^2\right).$$

$r_n,\ s_n,\ p_n,\ q_n$ are of order ω_n^{-2}.

- For any given background, there exists a **UNIQUE** Fock quantization with the symmetry of the three-torus and unitary dynamics.

- The system can be put in the form of a KG field with time-dependent mass by means of a **mode-dependent** canonical quantization, varying in time.

- This transformation is **unitarily** implementable in the privileged quantization.

Longitudinal gauge: Hamiltonian

- The remaining **Hamiltonian constraint** reads:

$$H=\frac{N_0\sigma}{16\pi G}C_0+\epsilon^2 N_0\sum H_2^{\vec{n},\pm}, \qquad H_2^{\vec{n},\pm}\,2e^{\bar{\alpha}}=\bar{E}_{\bar{f}\bar{f}}\bar{f}^2_{\vec{n},\pm}+\bar{E}_{\bar{f}\pi}\bar{f}_{\vec{n},\pm}\pi_{\bar{f}_{\vec{n},\pm}}+\bar{E}_{\pi\pi}\pi^2_{\bar{f}_{\vec{n},\pm}},$$

$$\bar{E}^n_{\bar{f}\bar{f}}=\omega_n^2+e^{2\bar{\alpha}}m^2\sigma^2-\frac{e^{-4\bar{\alpha}}}{2}\left(\pi_{\bar{\alpha}}^2+15\pi_{\bar{\varphi}}^2+3e^{6\bar{\alpha}}m^2\sigma^2\bar{\varphi}^2\right)-\frac{3}{\omega_n^2}e^{-8\bar{\alpha}}\left(e^{6\bar{\alpha}}m^2\sigma^2\bar{\varphi}-2\pi_{\bar{\alpha}}\pi_{\bar{\varphi}}\right)^2.$$

$$\bar{E}^n_{\bar{f}\pi}=-\frac{3}{\omega_n^2}e^{-6\bar{\alpha}}\pi_{\bar{\varphi}}\left(e^{6\bar{\alpha}}m^2\sigma^2\bar{\varphi}-2\pi_{\bar{\alpha}}\pi_{\bar{\varphi}}\right), \qquad \bar{E}^n_{\pi\pi}=1-\frac{3}{\omega_n^2}e^{-4\bar{\alpha}}\pi_{\bar{\varphi}}^{2.}$$

The corrections in cyan are of order ω_n^{-2}.

Longitudinal gauge: Metric (at linear order)

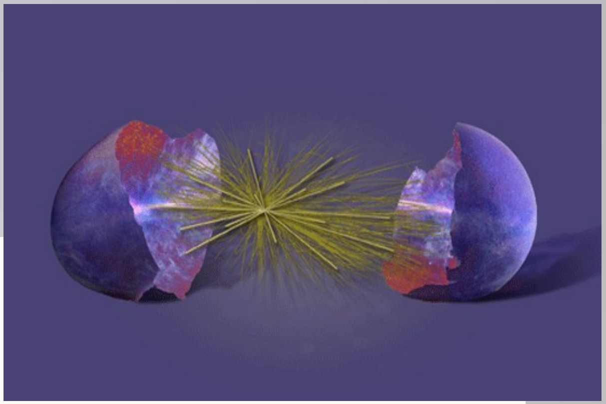
$$h_{ij}=\left(\sigma e^{\bar{\alpha}}\right)^{0}h_{ij}\left[1+\epsilon\, 2(2\pi)^{3/2}\sum a_{\vec{n},\pm}Q_{\vec{n},\pm}\right],$$

$$N=\sigma N_{0}\left(1-\epsilon(2\pi)^{3/2}\sum a_{\vec{n},\pm}Q_{\vec{n},\pm}\right),\qquad N_{i}=0,$$

$$a_{\vec{n},\pm}=\frac{3}{\omega_{n}^{2}}e^{-3\bar{\alpha}}\left[\pi_{\bar{\varphi}}\pi_{\bar{f}_{\vec{n},\pm}}+e^{-2\bar{\alpha}}\left(e^{6\bar{\alpha}}m^{2}\sigma^{2}\bar{\varphi}-2\pi_{\bar{\alpha}}\pi_{\bar{\varphi}}\right)f_{\vec{n},\pm}\right],$$

$$\Phi=\frac{1}{\sigma}\left(\frac{\bar{\varphi}}{(2\pi)^{3/2}}+\epsilon\, e^{-\bar{\alpha}}\sum \bar{f}_{\vec{n},\pm}Q_{\vec{n},\pm}\right).$$

Gauge invariants



- The **Mukhanov-Sasaki** modes and their momenta have the expression:

$$v_{\vec{n},\pm} = A_n \bar{f}_{\vec{n},\pm} + B_n \pi_{\bar{f}_{\vec{n},\pm}}, \qquad \pi_{v_{\vec{n},\pm}} = \dot{v}_{\vec{n},\pm} = F_n \bar{f}_{\vec{n},\pm} + G_n \pi_{\bar{f}_{\vec{n},\pm}},$$

$$A_n = 1 + \frac{3e^{-4\bar{\alpha}}\pi_{\bar{\varphi}}}{\omega_n^2 \pi_{\bar{\alpha}}}\left(e^{6\bar{\alpha}} m^2 \sigma^2 \bar{\varphi} - 2\pi_{\bar{\alpha}}\pi_{\bar{\varphi}}\right), \qquad B_n = \frac{3e^{-2\bar{\alpha}}\pi_{\bar{\varphi}}^2}{\omega_n^2 \pi_{\bar{\alpha}}},$$

$$F_n = -\frac{3e^{-2\bar{\alpha}}\pi_{\bar{\varphi}}^2}{\pi_{\bar{\alpha}}} - \frac{3e^{-6\bar{\alpha}}}{\omega_n^2 \pi_{\bar{\alpha}}}\left[e^{12\bar{\alpha}} m^4 \sigma^4 \bar{\varphi}^2 - \frac{e^{6\bar{\alpha}}\pi_{\varphi}}{2\pi_{\bar{\alpha}}} m^2 \sigma^2 \bar{\varphi}\left(5\pi_{\bar{\alpha}}^2 - 3\pi_{\bar{\varphi}}^2 + 3e^{6\bar{\alpha}} m^2 \sigma^2 \bar{\varphi}^2\right)\right]$$

$$-\frac{3e^{-6\bar{\alpha}}\pi_{\bar{\varphi}}^2}{2\omega_n^2 \pi_{\bar{\alpha}}}\left(11\pi_{\bar{\alpha}} - 15\pi_{\bar{\varphi}}^2 - 3e^{6\bar{\alpha}} m^2 \sigma^2 \bar{\varphi}^2\right),$$

$$G_n = 1 + \frac{3e^{-4\bar{\alpha}}\pi_{\bar{\varphi}}}{2\omega_n^2 \pi_{\bar{\alpha}}}\left[-2e^{6\bar{\alpha}} m^2 \sigma^2 \bar{\varphi} + \frac{\pi_{\bar{\varphi}}}{\pi_{\bar{\alpha}}}\left(\pi_{\bar{\alpha}}^2 - 3\pi_{\bar{\varphi}}^2 + 3e^{6\bar{\alpha}} m^2 \sigma^2 \bar{\varphi}^2\right)\right].$$

- If we construct annihilation and creation variables with these invariants (for zero mass), the Bogoliubov transformation, which is **mode dependent**, is **UNITARY** in the privileged Fock quantization .

Robustness under gauge fixing

- Similar results are obtained in the gauge of flat spatial sections $a_{\vec{n}\pm} = b_{\vec{n},\pm} = 0.$

- Moreover, the same **symplectic** structure for **gauge invariants** is obtained.

Quantization: Homogeneous sector

- We quantize the homogeneous sector with standard loop techniques, using improved dynamics and the MMO proposal.

- In the volume basis $\{|v\rangle; v\in\mathbb{R}\}$, with $\hat{V}=|\hat{p}|^{3/2}$,

$$\hat{N}_{\bar{\mu}}|v\rangle=|v+1\rangle, \qquad \hat{p}|v\rangle=sgn(v)(2\pi\gamma G\hbar\sqrt{\Delta}|v|)^{2/3}|v\rangle.$$

- The kinematic Hilbert space is $H_{kin}^{FRW-LQC}\otimes H_{kin}^{matt}$.

- The inverse volume is regularized as usual.

$$\widehat{\left[\frac{1}{V}\right]}=\widehat{\left[\frac{1}{\sqrt{|p|}}\right]}^{3}, \qquad \widehat{\left[\frac{1}{\sqrt{|p|}}\right]}=\frac{3}{4\pi\gamma G\hbar\sqrt{\Delta}}\widehat{sgn(p)}\sqrt{|\hat{p}|}\left(\hat{N}_{-\bar{\mu}}\sqrt{|\hat{p}|}\hat{N}_{\bar{\mu}}-\hat{N}_{\bar{\mu}}\sqrt{|\hat{p}|}\hat{N}_{-\bar{\mu}}\right).$$

Quantization: Homogeneous Hamiltonian

- After **decoupling the zero-volume** state, we change densitization for the *FRW* constraint:

$$\hat{C}_0 = \left[\widehat{\frac{1}{V}}\right]^{1/2} \hat{\mathcal{C}}_0 \left[\widehat{\frac{1}{V}}\right]^{1/2}.$$

$$\hat{\mathcal{C}}_0 = -\frac{6}{\gamma^2}\hat{\Omega}_0^2 + 8\pi G\left(\hat{\pi}_\phi^2 + m^2\hat{\phi}^2\hat{V}^2\right).$$

- The gravitational part, with the **MMO proposal**, is:

$$\hat{\Omega}_0 = \frac{1}{4i\sqrt{\Delta}}\hat{V}^{1/2}\left[\widehat{sgn(p)}\left(\hat{N}_{2\bar{\mu}} - \hat{N}_{-2\bar{\mu}}\right) + \left(\hat{N}_{2\bar{\mu}} - \hat{N}_{-2\bar{\mu}}\right)\widehat{sgn(p)}\right]\hat{V}^{1/2}.$$

Takes into account the triad orientation (manifest in anisotropic scenarios).

- This operator has the generic form

$$\widehat{\Omega_0^2}|v\rangle = f_+(v)|v+4\rangle + f(v)|v\rangle + f_-(v)|v-4\rangle.$$

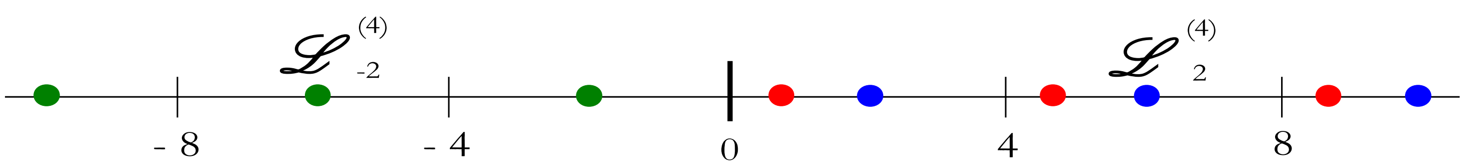
Quantization: Superselection

- $\hat{\Omega}_0^2$ can be seen as a difference operator.

$$\hat{\Omega}_0^2|v\rangle = f_+(v)|v+4\rangle + f(v)|v\rangle + f_-(v)|v-4\rangle.$$

- The real function $f_+(v)$ $\left(f_-(v)\right)$ vanishes in the **interval** $[-4,0]$ $\left([0,4]\right)$.

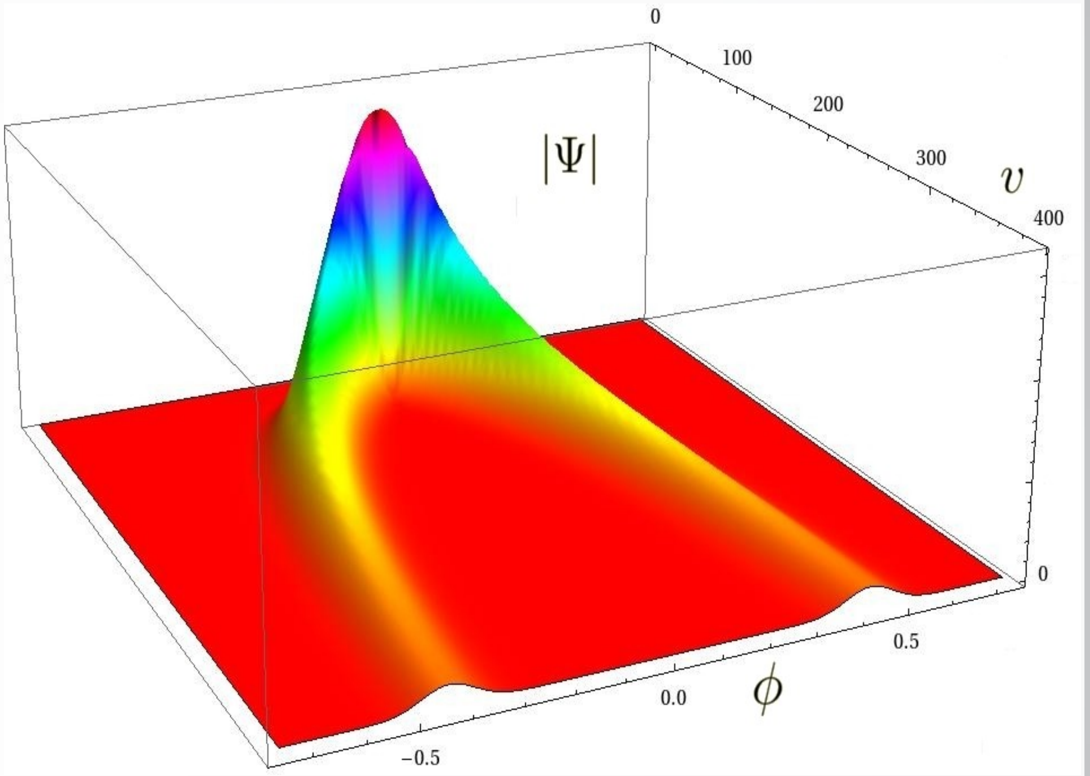
- The operator preserves the **superselection** sectors $\mathscr{L}^{(4)}_{\pm\epsilon} := \{\pm(\epsilon+4n),\ n \in \mathbb{N}\}$



- This operator is selfadjoint in those sectors. Its eigenfunctions are real, and determined by their value at the **minimum volume** $\epsilon \in (0,4]$.

Quantization: Homogeneous states

- **Solutions** to the constraint are determined, e.g., by their initial values at minimum volume.
- If the scalar field serves as a clock, an alternate possibility is to give the value at a section of constant field. This is not always possible.
- The space of *physical* states can be identified, e. g., with $L^2(\mathbb{R}, d\phi)$.



Fock and hybrid quantizations

- We quantize the **rescaled inhomogeneous modes** using annihilation and creation variables constructed from our canonical variables and zero mass.
- We obtain a **Fock space** $\mathscr{F}$, with basis of n-*particle* states:

$$\left\{ \left| N \right\rangle = \left| N_{(1,0,0),\,+}\,, N_{(1,0,0),\,-}\,, \ldots \right\rangle ; \quad N_{\vec{n},\pm} \in \mathbb{N},\ \sum N_{\vec{n},\pm} < \infty \right\}.$$

- We proceed to a hybrid quantization, with Hilbert space

$$H_{kin}^{FRW-LQC} \otimes H_{kin}^{matt} \otimes \mathscr{F}\,.$$

- The Hamiltonian constraint is **not trivial.**

Quantum Hamiltonian of the perturbations

- We quantize the quadratic contribution of the perturbations to the Hamiltonian adapting the quantization **proposals of the homogeneous sector** and using a symmetric factor ordering:

  - We **symmetrize** products of the type $\hat{\phi}\,\hat{\pi}_{\phi}$.
  - We take a **symmetric geometric** factor ordering $V^k A \to \hat{V}^{k/2}\hat{A}\,\hat{V}^{k/2}$.
  - We adopt the **LQC** representation $(cp)^{2\mathrm{m}} \to \left[\hat{\Omega}_0^2\right]^m$.
  - In order to **preserve the FRW superselection sectors**, we adopt the prescription $(cp)^{2\mathrm{m}+1} \to \left[\hat{\Omega}_0^2\right]^{m/2}\hat{\Lambda}_0\left[\hat{\Omega}_0^2\right]^{m/2}$, where

$$\hat{\Lambda}_0 = -\frac{i}{8\sqrt{\Delta}}\,\hat{V}^{1/2}\left[\widehat{sgn(p)}\left(\hat{N}_{4\bar{\mu}}-\hat{N}_{-4\bar{\mu}}\right)+\left(\hat{N}_{4\bar{\mu}}-\hat{N}_{-4\bar{\mu}}\right)\widehat{sgn(p)}\right]\hat{V}^{1/2}.$$

The situation is similar to that found with the Hubble parameter in LQC.

Quantum Hamiltonian of the perturbations

- With the FRW **densitization**:

$$\hat{H}_2^{\vec{n},\pm} = \frac{\sigma}{16\pi G}\,\widehat{\left[\frac{1}{V}\right]}^{1/2}\,\hat{C}_2^{\vec{n},\pm}\,\widehat{\left[\frac{1}{V}\right]}^{1/2}.$$

$$\hat{C}_2^{\vec{n},\pm} = 6(2\pi)^4\sigma^2\left[2\omega_n\widehat{\left[\frac{1}{V}\right]}^{-2/3} + \frac{\hat{Y}^-}{\omega_n} + \frac{\hat{Z}}{\omega_n^3}\right]\hat{N}_{\vec{n},\pm} + 4\pi G\left[\left(\frac{\hat{Y}^+}{\omega_n} + \frac{\hat{Z}}{\omega_n^3}\right)\hat{X}^+_{\vec{n},\pm} + \frac{3i\sigma^2\hat{W}}{\omega_n^2}\hat{X}^-_{\vec{n},\pm}\right],$$

$$\hat{N}_{\vec{n},\pm} = \hat{a}^\dagger_{f^-_{\vec{n},\pm}}\hat{a}_{f^-_{\vec{n},\pm}}, \qquad \hat{X}^\pm_{\vec{n},\pm} = \left(\hat{a}^\dagger_{f^-_{\vec{n},\pm}}\right)^2 \pm \left(\hat{a}_{f^-_{\vec{n},\pm}}\right)^2,$$

$$\hat{Y}^\pm = \frac{m^2}{(2\pi)^2} - \pi\sigma^2\widehat{\left[\frac{1}{V}\right]}^{1/3}\left(\frac{1}{\gamma^2(2\pi)^3\sigma^2}\hat{\Omega}_0^2 + 3(5\pm 2)\hat{\pi}_\phi^2 + 3m^2\hat{V}^2\hat{\phi}^2\right)\widehat{\left[\frac{1}{V}\right]}^{1/3},$$

$$\hat{Z} = -\frac{3\sigma^2}{2\pi}\widehat{\left[\frac{1}{V}\right]}\left(\frac{2}{\gamma}\hat{\Lambda}_0\hat{\pi}_\phi + m^2\hat{V}^2\hat{\phi}\right)^2\widehat{\left[\frac{1}{V}\right]},$$

$$\hat{W} = -\widehat{\left[\frac{1}{V}\right]}^{2/3}\left(\frac{4}{\gamma}\hat{\Lambda}_0\hat{\pi}_\phi^2 + m^2\hat{V}^2(\hat{\phi}\hat{\pi}_\phi + \hat{\pi}_\phi\hat{\phi})\right)\widehat{\left[\frac{1}{V}\right]}^{2/3}.$$

Solutions to the constraint

- If the matter field serves as a **clock**:

$$\hat{C}_0 + \epsilon^2\left(\sum \hat{C}_2^{\vec{n},\pm}\right) = 0.$$

$$(\Psi|\hat{\pi}_\phi = \frac{1}{\sqrt{8\pi G}}(\Psi|\left[\hat{\Theta}_0^2 - \epsilon^2\left(\sum \hat{C}_2^{\vec{n},\pm}\right)^\dagger\right]^{1/2} \approx \frac{1}{\sqrt{8\pi G}}(\Psi|\left[\hat{\Theta}_0 - \frac{\epsilon^2}{2}\hat{\Theta}_0^{-1}\left(\sum \hat{C}_2^{\vec{n},\pm}\right)^\dagger\right],$$

$$\hat{\Theta}_0^2 = \boldsymbol{P}\left(8\pi G\,\hat{\pi}_\phi^2 - \hat{C}_0\right).$$

- We can pass to an **interaction picture** and use a Born-Oppenheimer-like approximation.

- This can be done even without the above *perturbative expansion*.

- This leads to a sort of ***effective*** QFT for the inhomogeneities.

Physical states

- An alternate **perturbative** scheme:

$$(\Psi| = (\Psi|^{(0)} + \epsilon^2 (\Psi|^{(2)} \ldots$$

- **FRW solution:** $(\Psi|^{(0)} \hat{C}_0 = 0,$

$$\hat{C}_0 = -\frac{6}{\gamma^2}\hat{\Omega}_0^2 + 8\pi G\left(\hat{\pi}_\phi^2 + m^2\hat{\phi}^2\hat{V}^2\right).$$

- ***Evolution*** **of the perturbations:**

$$(\Psi|^{(2)}\hat{C}_0 = -(\Psi|^{(0)}\left(\sum \hat{C}_2^{\vec{n},\pm}\right)^{\dagger}.$$

- Solutions are characterized by their initial data at **minimum volume.**
- From these data we arrive, e.g., at the **physical Hilbert space** $H_{kin}^{matt} \otimes \mathscr{F}$.

Conclusions

- We have considered a perturbed FRW universe with a **massive** scalar field.

- Two **approximations**:
  - ☆ The action has been truncated to second order in the perturbations.
  - ☆ A hybrid quantization scheme has been adopted.

- **First** complete **quantization** of a model with inflation within LQC *(k=1)*.

- **Backreaction** has been included.

Conclusion

- For quantum simulations, the FRW prescription is **optimal**.
- Opposite to the situation in other analyses, the inhomogeneities have **UNITARY** dynamics in an (*effective*) QFT approximation.
- **No internal time** (matter clock) is needed. If a matter clock is available, one can obtain the inhomogeneities evolution adopting an **interaction picture**.
- Generally, one can construct quantum states perturbatively from data at **minimum volume**. This allows one to get a **physical Hilbert space**.