Proposal for a Phonon Laser Utilizing Quantum-Dot Spin States

A. Khaetskii,[1,*] V. N. Golovach,[2,3] X. Hu,[1] and I. Žutić[1]

[1]*Department of Physics, University at Buffalo, SUNY, Buffalo, New York 14260-1500*

[2]*Centro de Física de Materiales (CFM-MPC), Centro Mixto CSIC-UPV/EHU, Manuel de Lardizabal 5, E-20018 San Sebastián, Spain*

[3]*IKERBASQUE, Basque Foundation for Science, E-48011 Bilbao, Spain*

(Received 11 June 2013; revised manuscript received 29 August 2013; published 29 October 2013)

We propose a nanoscale realization of a phonon laser utilizing phonon-assisted spin flips in quantum dots to amplify sound. Owing to a long spin relaxation time, the device can be operated in a strong pumping regime, in which the population inversion is close to its maximal value allowed under Fermi statistics. In this regime, the threshold for stimulated emission is unaffected by spontaneous spin flips. Considering a nanowire with quantum dots defined along its length, we show that a further improvement arises from confining the phonons to one dimension, and thus reducing the number of phonon modes available for spontaneous emission. Our work calls for the development of nanowire-based, high-finesse phonon resonators.

DOI: 10.1103/PhysRevLett.111.186601 PACS numbers: 72.25.Dc, 71.70.Ej, 73.23.Hk, 73.63.Kv

Realizing acoustic analogues of active optical devices has been a long-standing challenge. Phonon lasers could provide versatile sources of coherent acoustic waves used for three-dimensional (3D) imaging of nanostructures or creating periodic strain of a material to rapidly modulate its optical or electronic properties. Recent experimental candidates include doped semiconductor superlattices and microcavity systems coupled to a radio-frequency mechanical mode [1–3], while many other possibilities have been considered theoretically [4–12]. Despite the obvious analogy between photons and (acoustic) phonons, a realization of the phonon laser is considerably more demanding. The key difficulty stems from the small value of the speed of sound [8], s, or, equivalently, from the high value of the phonon density of states (DOS), which makes the threshold for stimulated emission hard to overcome.

A class of highly controllable quantum systems emerging from the ideas of spintronics and spin-based quantum computing [13–16] may offer new regimes of physical parameters in which phonon lasing is feasible, despite the smallness of s. In particular, Zeeman sublevels of quantum dots (QDs) [17] have several desirable properties. They constitute reliable two-level systems with the spin relaxation rate $1/T_1$ [18–25] low compared to the electron tunneling rates, while the spin-selective tunneling [26–29] allows us to separately manipulate the populations of the spin-up and spin-down states.

The main requirements for the occurrence of stimulated emission are the following: (i) a population inversion for two levels, (ii) phonon emission must dominate over other relaxation channels, and, (iii) to overcome the threshold, the emission into the amplified mode should exceed the loss due to a finite phonon lifetime, τ_Q, in the resonator. Usually, the latter condition is difficult to fulfill. Because of the high DOS for phonons, spontaneous emission competes effectively with stimulated emission into the designated mode, hindering population inversion. Indeed, Chen and Khurgin [8] derive for the threshold pump rate (per unit volume),

$$R_{\rm th} = \frac{\pi\Gamma g(\omega_Q)}{\tau_Q} \sim \frac{\omega_Q^2\Gamma}{s^3\tau_Q}, \qquad (1)$$

where Γ is the width of the electronic level, $g(\omega)$ is the phonon DOS in three dimensions, and ω_Q is the frequency of the lasing mode ($\hbar = 1$ and $k_B = 1$). Remarkably, $R_{\rm th}$ in Eq. (1) does not depend on the interaction strength between the phonon field and the two-level system. Thus, the only realistic way of overcoming the threshold consists in using a small frequency ω_Q [8]. In the context of QDs discussed below, Eq. (1) describes the regime of weak pumping, which corresponds to a small value of the population inversion, and arises when the sequential-tunneling rate is small compared to $1/T_1$.

In this Letter we show that Zeeman sublevels in semiconductor QDs are ideal two-level systems for use in phonon lasers. To create population inversion, spin-selective tunneling from the leads is used [Fig. 1(a)]. Spin flip is mediated by the spin-orbit interaction in the QD and accompanied by phonon emission [17,23–25]. We find a regime of strong pumping, when the upper level is occupied and the lower level is empty. This regime is accessible with QDs for realistic values of physical parameters because the characteristic tunneling rates that correspond to its onset are determined by small $1/T_1$ value. In this regime, the stronger the phonon field couples to the spin the lower the threshold for stimulated emission. Furthermore, the ability to tune the Zeeman splitting independently of the size of the QD allows one to control the strength of the spin-phonon coupling. The prescription for the frequency of an *optimal* phonon mode is $\omega_Q \sim s/a$, where a is the QD size along the phonon propagation.

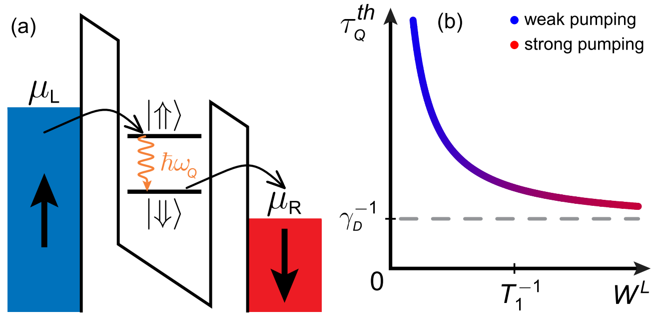

FIG. 1 (color online). (a) Phonon emitter: A QD in the sequential-tunneling regime with ferromagnetic leads of opposite polarizations. (b) The threshold value of the phonon lifetime τ_Q as a function of the tunnel rate W^L, showing the crossover between the weak ($W^L \ll 1/T_1$) and the strong ($W^L \gg 1/T_1$) pumping regimes. For $\tau_Q < \gamma_D^{-1}$ (below dashed line), no lasing is possible regardless of the pumping regime.

We also show that stimulated phonon emission can be envisioned even in the weak pumping regime for sufficiently small values of the Zeeman splitting Δ_Z. However, in three dimensions, the threshold value in Eq. (1) is too demanding because the rate of spontaneous emission is too high. By proceeding to a 1D situation in which the phonons are emitted only along a nanowire, we show that $R_{\rm th}$ can be strongly reduced. In this case all the "wrong" phonons which could have been emitted in the direction perpendicular to the propagation of the lasing mode are excluded.

We first consider an idealized situation depicted in Fig. 1(a), where a QD is tunnel coupled to two half-metal ferromagnets [30] at electrochemical potentials μ_L and μ_R. The electrons from the left lead can only tunnel into the higher-energy spin-up state. In order to proceed to the right lead, the electron should flip spin and transit to the lower, spin-down state by emitting a phonon of frequency $\omega_Q = \Delta_Z$ [31]. The QD Hamiltonian is

$$H_{\rm QD} = \sum_{s=\Uparrow,\Downarrow} \epsilon_s d_s^\dagger d_s + U n_\Uparrow n_\Downarrow, \tag{2}$$

where $\epsilon_\Uparrow = \Delta_Z/2$, $\epsilon_\Downarrow = -\Delta_Z/2$ ($\Delta_Z > 0$), $d_s^\dagger$ are the fermionic creation operators, and $n_s = d_s^\dagger d_s$. The on-site Coulomb energy U is assumed to be larger than the source-drain bias $eV = \mu_L - \mu_R$; therefore, the maximum dot occupation is 1. We keep only the empty-dot $|0\rangle$, spin-up $|\Uparrow\rangle = d_\Uparrow^\dagger|0\rangle$, and spin-down $|\Downarrow\rangle = d_\Downarrow^\dagger|0\rangle$ states. Further, we assume $eV > \Delta_Z$.

Coupling between the QD and the leads is described by the tunneling Hamiltonian

$$H_T = \sum_{lk\sigma s} t_{\sigma s}^l c_{lk\sigma}^\dagger d_s + \text{H.c.}, \tag{3}$$

where $t_{\sigma s}^l$ is the matrix of tunneling amplitudes and $c_{lk\sigma}^\dagger$ creates an electron with momentum k and spin $\sigma = \uparrow, \downarrow$ in lead $l = L, R$. The relevant physical quantity here is the matrix of rates $\Gamma_{s's}^l = \pi\sum_\sigma (t_{\sigma s'}^l)^* \nu_\sigma^l t_{\sigma s}^l$, where ν_σ^l is the DOS of spin species σ in lead l. In our idealized situation the only nonzero sequential-tunneling rates, marked by arrows in Fig. 1(a), are

$$\begin{aligned} W^L &\equiv W_{\Uparrow 0}^L = 2\Gamma_{\Uparrow\Uparrow}^L f(\epsilon_\Uparrow - \mu_L), \\ W^R &\equiv W_{0\Downarrow}^R = 2\Gamma_{\Downarrow\Downarrow}^R [1 - f(\epsilon_\Downarrow - \mu_R)], \end{aligned} \tag{4}$$

where $f(\epsilon) = [1 + \exp(\epsilon/T)]^{-1}$ is the Fermi distribution function. The reverse rates, $W_{0\Uparrow}^L$ and $W_{\Downarrow 0}^R$, are obtained from Eq. (4) by replacing $f(\epsilon) \to 1 - f(\epsilon)$. However, these rates are suppressed at low temperatures, $T \ll eV - \Delta_Z$.

For the lasing mode, we write $H_Q = \omega_Q(N_Q + 1/2)$, where $N_Q = a^\dagger a$ is the phonon number operator, with $a^\dagger$ creating a phonon in the lasing mode. The coupling between the QD and the lasing mode is

$$H_a = \sum_{s's} M_{s's} d_{s'}^\dagger d_s a^\dagger + \text{H.c.}, \tag{5}$$

where $M_{s's}$ are matrix elements of the spin-phonon interaction, obtained by taking into account a combined effect of spin-orbit interaction and magnetic field [23–25,32]. In nanowires, such as InAs or InSb, the spin-orbit interaction is rather strong [33,34], facilitating an efficient spin-phonon coupling.

The coupling of the spin to the phonon continuum, i.e., to all modes except the lasing mode, is obtained from Eq. (5) by summing over the phonon modes. This coupling leads to spin relaxation [23–25,32] with the rate $1/T_1 = w_{\Downarrow\Uparrow} + w_{\Uparrow\Downarrow}$, where $w_{ss'}$ are rates for phonon-assisted transitions. One can estimate [23–25,32]

$$w_{\Downarrow\Uparrow} \simeq 2\pi |M_{\Downarrow\Uparrow}|^2 V g(\Delta_Z)[1 + N(\Delta_Z)], \tag{6}$$

where V is the sample volume in three dimensions (or length of nanowire in one dimension) and $N(\epsilon) = [\exp(\epsilon/T) - 1]^{-1}$ is the Bose-Einstein distribution function. The rate of phonon absorption, $w_{\Uparrow\Downarrow}$, is obtained from Eq. (6) by replacing $1 + N(\Delta_Z) \to N(\Delta_Z)$. For low temperatures ($T \ll \Delta_Z$), we set $w_{\Uparrow\Downarrow} = 0$.

We describe the QD by a density matrix $\hat{\rho}$, which includes diagonal and off-diagonal elements. The master equations can be derived in the standard way [35]. The key point is that we treat the laser mode as a classical field, assuming that its population is large $N_Q \gg 1$, $N_Q = |a|^2$, where a is a c number. Similar treatment for an electron coupled to an oscillating magnetic (ESR) field was used in Ref. [36]. Applying the rotating wave approximation (RWA), we obtain [37]

$$\frac{d\rho_\Uparrow}{dt} = W^L_{\Uparrow 0}\rho_0 - W^L_{0\Uparrow}\rho_\Uparrow + w_{\Uparrow\Downarrow}\rho_\Downarrow - w_{\Downarrow\Uparrow}\rho_\Uparrow - \gamma N_Q(\rho_\Uparrow - \rho_\Downarrow),$$
$$\frac{d\rho_\Downarrow}{dt} = W^R_{\Downarrow 0}\rho_0 - W^R_{0\Downarrow}\rho_\Downarrow + w_{\Downarrow\Uparrow}\rho_\Uparrow - w_{\Uparrow\Downarrow}\rho_\Downarrow + \gamma N_Q(\rho_\Uparrow - \rho_\Downarrow),$$
$$\frac{d\rho_0}{dt} = W^L_{0\Uparrow}\rho_\Uparrow + W^R_{0\Downarrow}\rho_\Downarrow - (W^L_{\Uparrow 0} + W^R_{\Downarrow 0})\rho_0,$$
$$\frac{dN_Q}{dt} = \gamma N_Q(\rho_\Uparrow - \rho_\Downarrow) - \frac{N_Q}{\tau_Q}, \quad (7)$$

where $\rho_\Uparrow + \rho_\Downarrow + \rho_0 = 1$ due to the Coulomb blockade and $\gamma = 2|M_{\Downarrow\Uparrow}|^2/\Gamma$, with $\Gamma = (W^L_{0\Uparrow} + W^R_{0\Downarrow})/2 + 1/T_2$ being the decay rate of $\rho_{\Uparrow\Downarrow}$ due to tunneling as well as due to a spin decoherence time T_2. The phonon loss is described by the rate $1/\tau_Q$, which accounts for intrinsic phonon decay and escape through the mirrors. The Rabi flips occur at rate γN_Q.

Equation (7) is written for a single QD in the system. When there are N_D identical QDs and the interdot distance is much larger than the phonon wavelength, Eq. (7) assumes the change $\gamma \to \gamma N_D$ and $N_Q \to N_Q/N_D$. Hereof, we introduce $\gamma_D = \gamma N_D$, for which the normalization volume of the phonon wave function (appearing in $M_{\Downarrow\Uparrow} \propto 1/\sqrt{V}$) effectively becomes $n^{-1} = V/N_D$, i.e., the volume per one QD.

Next, we seek the stationary solution of Eq. (7) and study the onset of stimulated emission. For nontrivial solutions, for which N_Q does not vanish identically, we obtain the population inversion

$$\rho_\Uparrow - \rho_\Downarrow = 1/(\gamma_D\tau_Q), \quad (8)$$

which is interpreted as follows: In the stationary regime, the incoming rate to the lasing mode equals the decay rate $1/\tau_Q$. The number of phonons per QD reads

$$N_Q/N_D = \frac{(W^R - 1/T_1)(\tau_Q - 1/\gamma_D)}{2 + W^R/W^L} - \frac{1}{\gamma_D T_1}. \quad (9)$$

From the condition $N_Q > 0$ one gets

$$\frac{1}{\gamma_D\tau_Q} < \frac{T_1 W^R - 1}{T_1 W^R + 1 + W^R/W^L}, \quad (10)$$

defining the threshold relation for the onset of stimulated emission [see Fig. 1(b)]. The quantity $T_1 W^R$ should be larger than unity (we assume $T_1 W^R \gg 1$). In Fig. 2, N_Q is plotted versus W^L for realistic parameters, see further. Along with the classical solution, which has $N_Q > 0$ only above the threshold, we plot also its quantum counterpart (which has $N_Q > 0$ for all $W^L > 0$). The quantum solution was computed numerically for a single dot (see Supplemental Material [38]). At the threshold, N_Q corresponding to the quantum curve is of order unity. Above the threshold, the quantum and classical curves rapidly converge.

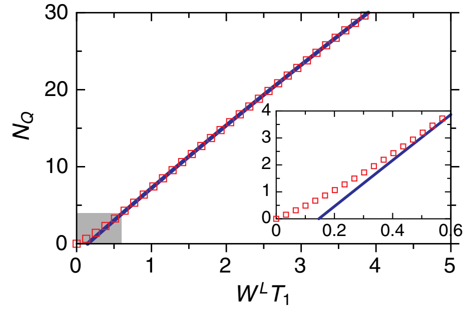

FIG. 2 (color online). Phonon population N_Q as a function of W^L, obtained from Eq. (9) for a single dot (solid line). The squares represent N_Q obtained from a quantum treatment (see Supplemental Material [38]). The parameters used are $W^L T_1 = 100$, $\gamma T_1 = 0.8$, $\tau_Q/T_1 = 10$. Inset: shaded area shown in a larger scale.

Two pumping regimes can be distinguished in Eq. (10) [see also Fig. 1(b)]. In the weak pumping regime ($T_1 W^L \ll 1$), the QD is almost empty, $\rho_0 \approx 1$, and the Coulomb blockade is unimportant. From Eq. (10), the threshold value of the pump rate is

$$W^L_{\rm th} = \frac{1}{T_1}\frac{1}{\gamma_D\tau_Q} \sim \frac{g\Gamma}{\tau_Q n}, \quad (11)$$

where we used $1/T_1 \sim |M_{\Downarrow\Uparrow}|^2 gV$ and g is the phonon DOS evaluated at the Zeeman energy. Equation (11) holds for any dimensionality, provided n is understood as the corresponding concentration of QDs. Note that Eq. (11) is similar to Eq. (1), and the coupling constant drops out.

Equation (11) can as well be obtained from the following consideration. At the threshold, the incoming rate to the lasing mode should exceed the decay rate $\rho_\Uparrow\gamma_D > 1/\tau_Q$. On the other hand, the pump rate at the threshold equals the spontaneous emission rate, $W^L_{\rm th} = \rho_\Uparrow/T_1$. Excluding $\rho_\Uparrow$, one obtains Eq. (11).

Comparing the threshold pump rates in three and one dimension, we obtain

$$\frac{W^L_{\rm th,3D}}{W^L_{\rm th,1D}} \sim \frac{g_3 n_1}{g_1 n_3} \sim \frac{A}{\lambda^2_{\rm ph}} \gg 1, \quad (12)$$

where g_1 (g_3) is the 1D (3D) phonon DOS, $\lambda_{\rm ph}$ is the phonon wave length, and A is the mirror area in the 3D case. The ratio in Eq. (12) represents the number of phonon modes in the transverse direction which are useless for lasing. Therefore, one can greatly reduce the threshold pump value by proceeding to a set up shown in Fig. 3(a) [39].

In the opposite regime of strong pumping, $T_1 W^L \gg 1$, the effective threshold pump rate $W^L_{\rm th}\rho_0$ of the upper level saturates at the $1/T_1$ value. In this regime $\rho_\Uparrow \approx 1$, and the Pauli principle forbids an electron from the left lead to tunnel into the QD until the electron inside the QD flips its spin and leaves the QD. Thus, the spin-flip rate $1/T_1$ determines the effective threshold pump rate in this case.

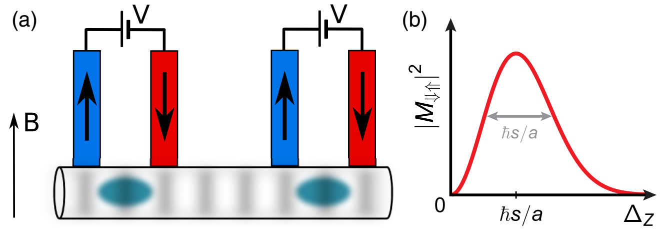

FIG. 3 (color online). (a) Phonon nanolaser: QDs defined along a nanowire and contacted by ferromagnetic fingers. Phonons are emitted along the nanowire and are reflected at its ends. (b) Coupling constant $|M_{\Downarrow\Uparrow}|^2$ versus the Zeeman splitting Δ_Z; maximal coupling is achieved at $\Delta_Z \simeq \hbar s/a$.

From the condition that the incoming rate to the lasing mode exceeds the decay rate, we obtain for the threshold value of τ_Q the following inequality: $1/\gamma_D \tau_Q^{\rm th} < 1$. This inequality follows also from Eq. (10). Note that the phonon DOS does not enter this inequality, and $\tau_Q^{\rm th}$ is determined only by the coupling constant $|M_{\Downarrow\Uparrow}|^2$, shown as a function of Δ_Z in Fig. 3(b). The dependence of $\tau_Q^{\rm th}$ on the tunneling rate W^L is shown in Fig. 1(b). The plateau value of $\tau_Q^{\rm th} \propto 1/\gamma_D$ can be reduced by choosing a material with strong spin-orbit interaction such as InAs.

For strong pumping the threshold condition can be rewritten in terms of the phonon mean free path ($l_{\rm ph}^{\rm th} = s\tau_Q^{\rm th}$). Specifically, for one dimension,

$$N_D \frac{l_{\rm ph}^{\rm th}}{L_z} = \Gamma T_1, \qquad (13)$$

where L_z is the distance between the mirrors. Assuming also that $\gamma_D \tau_Q \gg 1$, i.e., well above the threshold, we obtain for the number of phonons in the strong tunneling regime

$$N_Q \approx \frac{W^L W^R}{2W^L + W^R} \tau_Q N_D. \qquad (14)$$

Our consideration so far did not take into account the leakage current, i.e., when the rates $W_{\Downarrow 0}^L$ and $W_{0\Uparrow}^R$ are present. Such processes are possible for minority carriers in ferromagnets and because with the spin-orbit interaction the spin-up and spin-down directions are not exactly collinear. Adding the corresponding terms into Eq. (7), we derive a new threshold condition, which in the case of relatively strong leakage ($W_{0\Uparrow}^R \gg 1/T_1$) takes the form

$$\frac{1}{\gamma_D \tau_Q} < \frac{W^L W^R - \tilde{W}^L \tilde{W}^R}{W^L W^R + \tilde{W}^L \tilde{W}^R + W^R \tilde{W}^R}, \qquad (15)$$

where $\tilde{W}^L \equiv W_{\Downarrow 0}^L$ and $\tilde{W}^R \equiv W_{0\Uparrow}^R$. We see that even in the case of strong spin-orbit coupling, when $\tilde{W}^R$ and W^R are of the same order of magnitude, but not very close to each other ($\tilde{W}^R < W^R$, $\tilde{W}^L < W^L$), the threshold condition is similar to what we had before in the strong tunneling case without leakage, when the right-hand side of Eq. (15) was unity. Note that when the polarization degree in the ferromagnet tends to zero, then the right-hand side of Eq. (15) also tends to zero. Keeping in mind that $\eta_i \propto W^i - \tilde{W}^i$, where η_i is the degree of polarization of lead $i = L, R$, we obtain that the right-hand side of Eq. (15) is a linear function of the sum $\eta_R + \eta_L$, when these degrees are small.

Finally, the power output of the phonon laser reads

$$P = \hbar\omega_Q N_Q/\tau_Q. \qquad (16)$$

We next estimate the relevant parameters. Since the typical length of the QDs is $a \approx (0.3\text{–}1) \times 10^{-5}$ cm, in order to have reasonably strong coupling to the phonons one needs to choose not a very large Zeeman gap $\Delta_Z \simeq \hbar s/a$, 1 K $< \Delta_Z <$ 5 K [see Fig. 3(b)]. Therefore, the temperature is also restricted to these values. To have not a very high value of the threshold phonon mean free path, we take the tunneling rate $\Gamma = (10^9\text{–}10^{10})\ {\rm s}^{-1}$, which corresponds to the current $I = e\Gamma \approx 1$ nA. Then, assuming for InAs QDs relatively short $T_1 \simeq (10^{-7}\text{–}10^{-8})$ s, and taking $N_D = 10$, we obtain from Eq. (13) the ratio $l_{\rm ph}^{\rm th}/L_z = 10$. To find how realistic is that, we take $L_z = 1\ \mu$m, and $\tau_Q \simeq 10^{-7}$ s, which corresponds to a phonon mean free path $(10^{-2}\text{–}10^{-1})$ cm. Then we obtain $l_{\rm ph}/L_z \sim (10^2\text{–}10^3)$. The indicated values for $l_{\rm ph}$ were experimentally observed [40–42] for the THz acoustic phonons in 3D semiconductors. For the number of phonons above the threshold, see Eq. (14), we get $N_Q \simeq N_D \Gamma \tau_Q \simeq 10^3$. For the power, see Eq. (16), one gets $P \approx 4.2 \times 10^{-6}$ erg/ sec for $\Delta_Z = 3$ K. Taking the diameter of a wire 10^{-6} cm, we obtain for the power density $\approx 1\ {\rm W/cm^2}$.

In conclusion, we propose to use the Zeeman sublevels of the ground orbital state of the QD for phonon lasing. Because of a low value of the spin-relaxation rate, a large value of the population inversion can be easily achieved. We show that a promising practical implementation is a system of elongated QDs embedded into a nanowire.

A. K. acknowledges financial support from the SPINMET Project (FP7-PEOPLE-2009-IRSES). The work was supported by the ONR, DOE-BES DE-SC0004890, and NSF-ECCS. X. Hu also acknowledges support by the U.S. ARO and NSF-PIF. The work of V. N. G. was supported by the Spanish Ministry of Economy and Competitiveness under Project No. FIS2011-28851-C02-02.

*On leave from Institute of Microelectronics Technology, Russian Academy of Sciences, 142432 Chernogolovka, Moscow District, Russia.
akhaetsk@buffalo.edu

[1] R. P. Beardsley, A. V. Akimov, M. Henini, and A. J. Kent, Phys. Rev. Lett. **104**, 085501 (2010).
[2] I. S. Grudinin, H. Lee, O. Painter, and K. J. Vahala, Phys. Rev. Lett. **104**, 083901 (2010).

[3] J. B. Khurgin, Physics **3**, 16 (2010).
[4] I. M. Asher and M. O. Scully, Phys. Rev. A **8**, 1988 (1973).
[5] S. T. Zavtrak, Phys. Rev. E **51**, 2480 (1995).
[6] S. S. Makler, D. E. Tuyarot, E. V. Anda, and M. I. Vasilevskiy, Surf. Sci. **361–362**, 239 (1996).
[7] J. Y. Prieur, M. Devaud, J. Joffrin, C. Barre, M. Stenger, and M. Chapellier, Physica (Amsterdam) **219B–220B**, 235 (1996).
[8] J. Chen and J. B. Khurgin, IEEE J. Quantum Electron. **39**, 600 (2003).
[9] I. Bargatin and M. L. Roukes, Phys. Rev. Lett. **91**, 138302 (2003).
[10] E. M. Chudnovsky and D. A. Garanin, Phys. Rev. Lett. **93**, 257205 (2004).
[11] J.-T. Lü, P. Hedegard, and M. Brandbyge, Phys. Rev. Lett. **107**, 046801 (2011).
[12] R. Okuyama, M. Eto, and T. Brandes, J. Phys. Soc. Jpn. **82**, 013704 (2013).
[13] S. Datta and B. Das, Appl. Phys. Lett. **56**, 665 (1990).
[14] I. Žutić, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. **76**, 323 (2004).
[15] D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998).
[16] *Semiconductor Spintronics and Quantum Computing*, edited by D. D. Awschalom, D. Loss, and N. Samarth (Springer, New York, 2002).
[17] R. Hanson, L. P. Kouwenhoven, J. R. Petta, S. Tarucha, and L. M. K. Vandersypen, Rev. Mod. Phys. **79**, 1217 (2007).
[18] J. M. Elzerman, R. Hanson, L. H. Willems van Beveren, B. Witkamp, L. M. K. Vandersypen, and L. P. Kouwenhoven, Nature (London) **430**, 431 (2004).
[19] M. Kroutvar, Y. Ducommun, D. Heiss, M. Bichler, D. Schuh, G. Abstreiter, and J. J. Finley, Nature (London) **432**, 81 (2004).
[20] R. Hanson, L. H. Willems van Beveren, I. T. Vink, J. M. Elzerman, W. J. M. Naber, F. H. L. Koppens, L. P. Kouwenhoven, and L. M. K. Vandersypen, Phys. Rev. Lett. **94**, 196802 (2005).
[21] A. C. Johnson, J. R. Petta, J. M. Taylor, A. Yacoby, M. D. Lukin, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Nature (London) **435**, 925 (2005).
[22] S. Amasha, K. MacLean, I. P. Radu, D. M. Zumbühl, M. A. Kastner, M. P. Hanson, and A. C. Gossard, Phys. Rev. Lett. **100**, 046803 (2008).
[23] A. V. Khaetskii and Yu. V. Nazarov, Phys. Rev. B **61**, 12639 (2000).
[24] A. V. Khaetskii and Y. V. Nazarov, Phys. Rev. B **64**, 125316 (2001).
[25] V. N. Golovach, A. Khaetskii, and D. Loss, Phys. Rev. Lett. **93**, 016601 (2004).
[26] S. Amasha, K. MacLean, I. P. Radu, D. M. Zumbühl, M. A. Kastner, M. P. Hanson, and A. C. Gossard, Phys. Rev. B **78**, 041306 (2008).
[27] P. Stano and P. Jacquod, Phys. Rev. B **82**, 125309 (2010).
[28] J. Paaske, A. Andersen, and K. Flensberg, Phys. Rev. B **82**, 081309 (2010).
[29] G. Katsaros, V. N. Golovach, P. Spathis, N. Ares, M. Stoffel, F. Fournel, O. G. Schmidt, L. I. Glazman, and S. De Franceschi, Phys. Rev. Lett. **107**, 246601 (2011).
[30] In practice, normal-metal leads can also be used, since quantum dots with spin-orbit interaction exhibit the phenomenon of spin-selective tunneling [26–29] in a magnetic field.
[31] The case of ferromagnetic leads containing both spin species, and the role of the leakage current due to the spin-orbit interaction are discussed later.
[32] M. Trif, V. N. Golovach, and D. Loss, Phys. Rev. B **77**, 045434 (2008).
[33] C. Fasth, A. Fuhrer, L. Samuelson, V. N. Golovach, and D. Loss, Phys. Rev. Lett. **98**, 266801 (2007).
[34] S. Nadj-Perge, V. S. Pribiag, J. W. G. van den Berg, K. Zuo, S. R. Plissard, E. P. A. M. Bakkers, S. M. Frolov, and L. P. Kouwenhoven, Phys. Rev. Lett. **108**, 166801 (2012).
[35] K. Blum, *Density Matrix Theory and Applications* (Plenum, New York, 1996), Chap. 8.
[36] H.-A. Engel and D. Loss, Phys. Rev. Lett. **86**, 4648 (2001).
[37] Following a standard RWA approach, we solve the time dependent equations for the nondiagonal components of the density matrix. At long times $t \gg \Gamma^{-1}$ the solution is

$$\rho_{\Uparrow\Downarrow} = \frac{iM^*_{\Downarrow\Uparrow} a e^{-i\omega_Q t}}{\Gamma + i(\Delta_Z - \omega_Q)}(\rho_\Uparrow - \rho_\Downarrow),$$

where $\omega_Q = \Delta_Z$ in our case. The component which describes the wave with opposite frequency has a small magnitude with respect to the parameter $\Gamma/\Delta_Z \ll 1$, and has been omitted. This solution describes the situation when the system reaches a steady state, when all the time derivatives on the left-hand side of Eq. (7) are actually meant to be zero. This solution is valid under the condition $\Delta_Z \gg \Gamma \gg M_{\Downarrow\Uparrow}\sqrt{N_Q}$.
[38] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.111.186601 for the master equations for the quantum case and a description of the numerical calculation of N_Q.
[39] The dots can be created by different methods, for example, by depletion gates, or by the growth of different layers. The mirrors can be just represented by the ends of the wire. The end surface should be flat enough in order to prevent an excitation of spurious (localized) modes. If the curvature radius of the end surface is larger than the wire diameter, then the excitation to the next mode of phonon transverse quantization will be strongly suppressed.
[40] A. J. Kent, N. M. Stanton, L. J. Challis, and M. Henini, Appl. Phys. Lett. **81**, 3497 (2002).
[41] U. Zeitler, A. M. Devitt, J. E. Digby, C. J. Mellor, A. J. Kent, K. A. Benedict, and T. Cheng, Phys. Rev. Lett. **82**, 5333 (1999).
[42] H. Kittel, E. Held, W. Klein, and R. P. Huebener, Z. Phys. B **77**, 79 (1989).