

Nonlocal edge state transport in topological insulators

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We use the N -terminal scheme for studying the edge-state transport in two-dimensional topological insulators. We find the universal nonlocal response in the ballistic transport approach. This macroscopic exhibition of the topological order offers different areas for applications.

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I. INTRODUCTION

A topological insulator is a quantum phase of matter with gapless electron states on the surface and gapped ones in the bulk. In two-dimensional systems, conducting electron states propagate along the edge of the topological insulator. These states have a linear Dirac dispersion and their spin is locked to the momentum.¹⁻³ The edge or the surface electron states are topologically protected. The time-inversion symmetry leads to the topological protection of each Kramers partner and as a result to suppression of the backscattering. Therefore, the topological protection against moderate structural disorder is the base for many technological applications of topological insulators.

The features of topological insulators are presented in nanoscale samples. Actually, the topological order in topological insulators belongs to the phase state, which is called a short-range order created by the entangled quantum states.⁴ By this term we mean that this phase state is formed on the scale of the order of the lattice constant. The topological phase, which is formed on larger scales—the so-called long-range topological order—belongs to a fundamentally different class⁴ of the entangled quantum states. One of its features is nonlocality in the Hilbert space. The fact is that the total quantum state is not equal to the product of the local electron states. In other words, there are long-range correlations of topological excitations in this state. Usually the topological order affects the collision frequency.⁵ The geometrical Berry phase of quantum states in topological insulators resulting in interference phenomena leads to the weak-antilocalization effect.⁶

In this paper we consider the universality of the transport characteristics in two-dimensional topological insulators, which exhibit nonlocal topological order. The existence of nonlocality in the three-dimensional case follows already from the representation⁷ of the topological invariant. The nontrivial value of this Hopf invariant, being equal to unity, means that the two loops are linked. We focus on the significantly nonlocal response that was found in the recent experiments.^{8,9}

II. NONLOCAL RESPONSE

The features of electron states in two-dimensional topological insulators are exhibited in the quantum spin Hall effect.¹⁰⁻¹³

When studying the degree of nonlocality, we use the approach of Refs. 8 and 14. The main distinction here is that we employ the general N -terminal scheme. It is convenient to use the ballistic Landauer-Büttiker approach¹⁵ writing the current I_i , injected through the terminal i into a sample, as

$$I_i = \frac{e^2}{h} \sum_{j=1}^N (T_{ji} V_i - T_{ij} V_j). \quad (1)$$

Here V_j is the voltage on the terminal j ; e is the electron charge; h is the Planck constant; h/e^2 is the resistance unit, which we will equate to unity; T_{ij} is the matrix element of transmission from the terminal i to the terminal j ; and N is the total number of terminals.

The edge electron modes in topological insulators propagate in two directions. We take the transmission coefficients between neighboring terminals to be equal to unity $T_{i+1,i} = T_{i,i+1} = 1$ and other probabilities to be equal to zero. The N -terminal scheme implies the use of the periodic boundary conditions $T_{N+1,N} = T_{1,N}$ and $T_{N,N+1} = T_{N,1}$ in both directions of the edge-state propagation.

Let us consider the number of terminals N as a tuning parameter. The labels of terminals, whose voltages we will measure, will contain information about the influence of the edge-state current between the terminals, through which the current flows about the voltage distribution on other terminals. This distribution will define the degree of the response nonlocality. For example, in the case of $N = 4$,¹⁴ if the current flows from terminal 1 to terminal 4, we may be interested in the resistance between these terminals and the nonlocal response, expressed in terms of the resistance between terminals 1 and 2, 2 and 3, and 1 and 3. It is evident that with the increase in the number of terminals N , the resistance between terminals 1 and N , through which the current flows, will tend to unity. As regards the adjacent terminals such as 1 and 2, the measured resistance will tend to zero according to a certain law decreasing with the increasing number N .

Let the current I_{1N} flow through terminals 1 and N (see Fig. 1) and the voltage be measured on all other terminals. In this case, the equation that determines the voltage on the contacts has the form

$$AV = I, \quad (2)$$

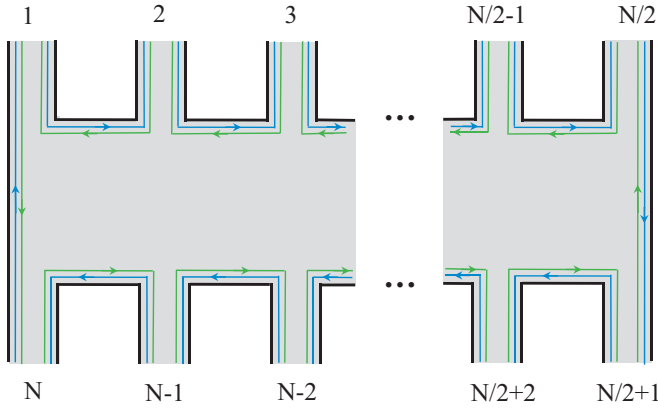


FIG. 1. (Color online) Distribution of edge states in a two-dimensional N -terminal insulator.

where the matrix A equals $A_{ij} = 2\delta_{ij} - \delta_{i,j+1} - \delta_{i,j-1} - \delta_{i,1}\delta_{j,N} - \delta_{i,N}\delta_{j,1}$, δ_{ij} is the Kronecker delta, $1 \leq i, j \leq N$, $V = (V_1, V_2, \dots, V_{N-1}, V_N)$, and $I = I_{1N}(1, 0, \dots, 0, -1)$. We are interested in the difference between the voltage on the terminals. Since the vector V is invariant with respect to the constant value shift, we may assume that $V_N = 0$.

For arbitrary N , the solution of Eq. (2) has the form $V_i = I_{1N}(1 - \frac{i}{N})$. Therefore, the resistance $(V_1 - V_N)/I_{1N}$ between terminals 1 and N equals $R_{1N,1N} = (N - 1)/N$. The nonlocal resistance $R_{kl,ij} = (V_i - V_j)/I_{kl}$ at $k = 1$ and $l = N$ when measuring the voltage between the terminals i and j is

$$R_{1N,ij} = \frac{j - i}{N}. \quad (3)$$

In Eq. (2) we implied that the current was conducted through the first and the last terminals. Thus Eq. (3) is valid only in this case. We may also conduct the current through terminals 1 and 4 or terminals 1 and 3, for example, in the same way it was realized in the experiment^{8,9} at $N = 6$. To find the voltage distribution in the situation when the current flows from terminal 1 to terminal k , on the right-hand side of Eq. (2) we have to use the equation for the current in the form of $I = I_{1k}(1, 0, \dots, -1, \dots, 0)$. Here -1 is located in the k place. The exact solution of Eq. (2) in this general case has the form

$$V_i = I_{1k} \left(1 - \frac{i}{N}(1 - k + N) \right) \quad (4)$$

if $1 \leq i \leq k$ and

$$V_i = I_{1k} \left(1 - \frac{i}{N} \right) (1 - k) \quad (5)$$

if $k \leq i \leq N$.

The values of the resistance $R_{14,14} = 3/2$ and $R_{14,23} = 1/2$ calculated with the use of Eq. (4) coincide with the experimental results.⁸ When the current flows from terminal 1 to terminal 3, the experimental resistance⁸ equals $R_{13,13} = 4/3$ and $R_{13,56} = 1/3$. This can be obtained from Eq. (5) at $k = 3$ and $N = 6$.

III. DISCUSSION

The chiral edge states in the quantum Hall effect propagate only in one of the two possible directions. The matrix elements T_{ij} of the transmissions for such states in Eq. (1) are not equal to zero for terminals with indices $j > i$: $T_{i,i+1} = 1$ and $T_{i+1,i} = 0$. Therefore, the resistance $R_{1N,ij}$ appears to be equal to zero. In other words, the nonlocal resistance in systems with broken time-reversal symmetry is absent. We should emphasize that the main feature of the considered situation, which governs the obtained results, is the existence of the time-reversal symmetry. This symmetry manifests at the macroscopic level in the form of nonlocal effects, caused by the existence of the helical edge states. The form of responses and the universal character of the obtained numbers depend on the experimental conditions, for example, on temperature, which determines the contribution of the inelastic backscattering processes to the conductivity. The experimental data⁸ show a high degree of universality. The universality of $R_{1N,1N} = (N - 1)/N$, $R_{1N,ij} = (i - j)/N$, and Eqs. (4) and (5) can be verified in the experimental setups appropriate for studying the quantum spin Hall effect by varying the total number of terminals and indices of current-carrying terminals.

If the time-reversal symmetry is broken by magnetic impurities and the condition of the absence of the backscattering is weakened, the matrix elements $T_{i,j}$ can be written as $T_{i+1,i} = 1 + k_1$ and $T_{i,i+1} = k_2$.¹⁶ Here $k_1 < 1$ and $k_2 < 1$ are constants and the unity in the matrix element $T_{i+1,i}$ means the existence of the chiral edge mode. In this quasi-helical edge state,¹⁶ the voltage distribution on the terminals when the current flows through the voltage leads 1 and N has the form

$$V_i = \left(\frac{1 - r^{N-i}}{1 - r^N} \right) \frac{I_{1N}}{1 + k_1}, \quad (6)$$

where $r = k_2/(1 + k_1)$.

In this paper we focus on the universal exhibition of topological order in the transport properties of ideal two-dimensional topological insulators in the most straightforward and representative form. The study of the transport characteristics of the so-called ideal topological insulator SmB_6 revealed⁹ that in the three-dimensional case the transport properties significantly depend on the geometry of samples and terminal assignments. A deviation from the universal behavior takes place also in two-dimensional systems. It occurs due to metal droplets inside real contacts. This phenomenon can be described in terms of an additional terminal. The effect of this and other factors such as the finite width of the terminal, reflections from the internal interfaces, and other conditions on amplitudes of the transitions between current and voltage terminals was studied in Ref. 17.

Let us clarify the role played by the contacts in the edge-state transport following the approach and the main conclusions of Refs. 8, 17, and 18 in more detail. First of all, we note that a contact is not a time-reversal symmetry-breaking potential that mixes counterpropagating edge states with opposite spins. Contacts are finally an electron degree of freedom reservoir that incoherently populates both edge-state channels. An ideal contact populates both edge-state channels with equal weight by injecting spin-up and spin-down electrons with equal probability.⁸ This is the origin of the

additional resistance produced by the contacts. A contribution of such a dephasing reservoir into an additional longitudinal resistance can be negligible under the condition $L < L_c \sim 1/\eta$. Here L is the characteristic linear size of a contact, L_c is the dephasing length, and η is the dephasing strength of the self-energy part.⁸ Note that the self-energy should not break the time-reversal symmetry. Decoherent behavior arises due to the existence of the dephasing reservoir with the distribution function included in the so-called lesser self-energy of the leads.⁸

However, there are sharp dips in the conductance even for small values of ηL . They can be strong enough to completely block coherent transport at one of the edges.⁸ Therefore, even a small dephasing region can equally affect a probe terminal. The experimental value of the maximal resistance for the six-terminal device is $1/7$ instead of the theoretical prediction $1/6$. Such a result is consistent with the existence of the additional dephasing region. Dephasing regions can also exist due to an inhomogeneity of the sample. The experimental results have shown⁸ that a change in the gate voltage also affects the homogeneity of the device due to trap state charging at the semiconductor-insulator interface. This leads to an inhomogeneous potential in the gated area and to the creation of the metallic islands that exist when most of the gated regions are insulating ones. In other words, a metallic island can lead to the effect similar to an additional probe. The experimental situations when the coherent transport is observable in the whole sample are discussed in Refs. 8 and 17.

There are two different methods to suppress the nonlocal transport. The first approach is to make the device scale sufficiently small so as to induce backscattering in the channels of the edge states. Backscattering occurs when the wave functions for opposite spin orientations overlap.¹⁹ This happens for a device width of about 200 nm. Therefore, if the width W_1 of the central device strip is rather large, the deviation T'_{1N} from the ideal value $T_{1N} = 1$ is negligibly small.¹⁷ The same condition $W > W_2$ for the absence of the tunneling between the edges of the individual terminal is valid for the terminal width W_2 . Measurements of the nonlocal resistance¹⁷ in devices when they are in the quantum spin Hall effect states show the values expected for the nonperturbative nonlocal edge-state transport. The numerical simulations of the scattering matrix at the metal-topological-insulator interface has confirmed the negligibly small value of T'_{1N} for the employed samples. The second method to suppress the edge-state contribution to nonlocal transport is to choose such nonlocal configurations that imply the edge channel transport over distances longer than the inelastic length.¹⁸ This means that the maximal number of terminals $N < N_c = L_1/(W_2 + L_2)$ can be roughly

estimated as 10 for the real experimental parameters. Here L_1 is the characteristic sample size and L_2 is the distance between terminals.

Let us consider the regime of the quantum spin Hall state when the renormalized voltage $V_g^* = 0$ and the resistance is maximal. The Fermi level in the bulk is located now at the center of the energy gap. In this case, the assignments of the current and voltage terminals determine the following values of the resistance:

$$R_{1k,ij} = \begin{cases} \frac{j-i}{N}(1-k+N), & 1 \leq i, j \leq k \\ \frac{j-i}{N}(1-k), & k \leq i, j \leq N \\ \frac{j-i}{N}(1-k) + (k-i), & 1 \leq i \leq k, k \leq j \leq N. \end{cases} \quad (7)$$

One can easily verify that after the interchange of the current ($1k$) and voltage (ij) probe indices and the shifts $k \rightarrow j - i + 1$, $j \rightarrow k - i + 1$, and $i \rightarrow N - i + 2$ these equations satisfy the Onsager-Casimir symmetry relations $R_{mn,kl} = R_{kl,mn}$ (Ref. 20) for the nonlocal resistances $R_{mn,kl}$. We would also like to mention the fact that topological universality of the ballistic transport due to the edge states under ideal conditions is determined by the topological properties of the electron quantum bulk states. Therefore, the considered phenomenon in a trivial insulator is absent.

IV. CONCLUSION

We have described here the universal distribution of the resistances studying the edge-state transport in the two-dimensional topological insulators in the ballistic transport regime. It is of interest to extend the problem of macroscopic manifestation of the topological order to similar topologically ordered systems. Another way of copying the letter H in the vertical direction as it is done in the ladderlike structures²¹ corresponds to the four-terminal case. This is because the distribution of edge degrees of freedom will be equivalent to the distribution taking place in the system with one letter H. To model the distribution of the degrees of freedom in the systems with a long-range topological order, we have to use Y-shape contacts as building blocks.

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