PHYSICAL REVIEW B **86**, 060506(R) (2012)

Spin-polarized Josephson and quasiparticle currents in superconducting spin-filter tunnel junctions

F. Sebastian Bergeret,[1,2,*] Alvise Verso,[2] and Anatoly F. Volkov[2,3]
[1]*Centro de Física de Materiales (CFM-MPC), Centro Mixto CSIC-UPV/EHU, Manuel de Lardizabal 4, E-20018 San Sebastián, Spain*
[2]*Donostia International Physics Center (DIPC), Manuel de Lardizabal 5, E-20018 San Sebastián, Spain*
[3]*Theoretische Physik III, Ruhr-Universität Bochum, D-44780 Bochum, Germany*
(Received 15 June 2012; revised manuscript received 24 July 2012; published 8 August 2012)

We present a theoretical study of the effect of spin filtering on the Josephson and dissipative quasiparticle currents in a superconducting tunnel junction. By combining the quasiclassical Green's functions and the tunneling Hamiltonian method, we describe the transport properties of a generic junction consisting of two superconducting leads with an effective exchange field **h** separated by a spin-filter insulating barrier. We show that in addition to the tunneling of Cooper pairs with total spin projection $S_z = 0$ there is another contribution to the Josephson current due to triplet Cooper pairs with total spin projection $S_z \neq 0$. The latter is finite and not affected by the spin-filter effect provided that the fields **h** and the magnetization of the barrier are noncollinear. We also determine the quasiparticle current for a symmetric junction and show that the differential conductance may exhibit peaks at different values of the voltage depending on the polarization of the spin filter, and the relative angle between the exchange fields and the magnetization of the barrier. Our findings provide a plausible explanation for existing experiments on Josephson junctions with magnetic barriers, predict further effects, and show how spin-polarized supercurrents in hybrid structures can be created.

DOI: 10.1103/PhysRevB.86.060506 PACS number(s): 74.50.+r, 72.25.−b, 74.78.Fk

Introduction. The prediction of long-range triplet superconducting correlations in superconductor-ferromagnet (S-F) hybrid structures[1,2] has led to intense experimental activity in recent years.[3–7] These experiments have shown that a finite Josephson current can flow between two superconductors connected by a ferromagnetic layer whose thickness far exceeds the expected penetration length of singlet pairs. The Josephson current measured in these experiments is attributed to the flow of Cooper pairs in a triplet state. According to the theory, the appearance of triplet correlations occurs only in the presence of a magnetic inhomogeneity located in the vicinity of the S-F interface.[1,8–11] The inhomogeneity can be either artificially created[4] or can be an intrinsic property of the material, as for example the domain structure of the usual ferromagnets[6] or the spiral-like magnetization in certain rare-earth metals.[5,12]

The Josephson triplet current is nothing but a dissipationless spin-polarized current and therefore its control would be of great advantage in the field of spintronics.[13] Important building blocks of spintronic circuits are magnetic insulating barriers with spin-dependent transmission, so-called spin filters (I_{sf}), which have been studied in several experiments using, for example, europium chalcogenide tunnel barriers.[14–17] The question naturally arises whether one can use these spin-filter tunneling junctions to control and eventually to create a triplet Josephson current. We will address this question in the present Rapid Communication.

In spite of several studies of the transport properties of spin-filter tunneling barriers, the Josephson effect has only recently been explored through a S-I_{sf}-S structure.[18] The tunnel barrier used was a GdN film that reduced the value of the critical current I_c compared to a nonmagnetic barrier. In addition to the large reduction of I_c the authors of Ref. 18 also observed that the $I_c(T)$ curve deviates at low temperature from the expected tunneling behavior.[19] Theoretically, the effect of spin-dependent transmission on the Josephson current was first considered by Kulik[20] and Bulaevskii *et al.*[21] on the basis of the tunneling Hamiltonian. It was demonstrated that spin-selective tunneling always leads to a reduction of the critical current with respect to its value in the spin-independent case or even to the change of sign of the critical current. Later on it was shown that the magnetic barrier in an I_{sf}-S structure induces an effective exchange field in the superconductor.[22,23] Other theoretical works have addressed the Josephson effect through spin-active barriers in ballistic systems[24–26] and through ideally ballistic superconductor–ferromagnetic insulator–superconductor junctions.[27,28] Also the spin-polarized current through S-N-F junctions, where N is a normal non-magnetic metal, has been studied in Refs. 29 and 30. However, none of these works presented a comprehensive theoretical study of the Josephson effect by taking into account both the spin-filter effect and the presence of the exchange field in the superconducting electrodes, nor has the interplay between spin filtering and triplet supercurrents been investigated.

The aim of the present Rapid Communication is to provide a complete description of the transport properties of Josephson junctions with spin filters. For that sake we introduce a simple model which allows us on the one hand to derive simple and useful expressions for the dc Josephson and quasiparticle currents in a S-I_{sf}-S and on the other hand to predict the conditions under which the creation of a spin-polarized supercurrent is possible. Our model considers the spin-filter effect of the I_{sf} barrier and a finite exchange field in the electrodes. We show that the contribution to the current from Cooper pairs in the singlet and triplet states with zero spin projection vanishes in the case of a fully spin-polarized barrier. However, the contribution to the Josephson current from tunneling of Cooper pairs in a triplet state with nonvanishing spin projection is independent of the strength of the spin filter. The latter contribution is finite provided that the exchange fields in the electrodes and the spin quantization axis of the barrier are noncollinear. This result explains how spin-polarized currents can be created and controlled by means of spin-filter barriers.

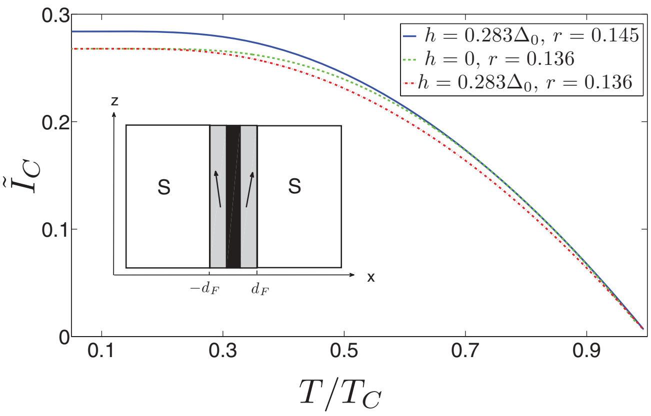

FIG. 1. (Color online) The temperature dependence of the critical current for different values of h and r. We assume that $\alpha = \beta = 0$. Inset: The structure described by our model Hamiltonian Eq. (1). The black region represents the spin-filter barrier while the gray regions are layers with a finite exchange field pointing in an arbitrary direction. d_F is the thickness of these layers and we have defined $\tilde{I}_c = 2I_c eR_N/(\Delta_0\pi)$, where Δ_0 is the value of the order parameter at $T = 0$ and $h = 0$.

We also calculate the differential tunneling conductance of the S-I_{sf}-S junction, and analyze how the Zeeman-split peaks depend on both the spin-filter parameter and the exchange field in the electrodes. Our model allows a quantitative description of existing transport experiments on S-I_{sf}-S junctions[14–16] and gives a possible explanation for the temperature dependence of the critical Josephson current observed in Ref. 18. We finally discuss the applicability of our model to real systems.

The model. We consider a tunnel junction between two superconductors (see the inset of Fig. 1). The tunneling barrier, the black area in Fig. 1, is a spin filter. The gray regions close to the barrier are thin ferromagnetic layers with a finite exchange field acting on the spin of the conducting electrons. The direction of these fields is arbitrary. We assume for simplicity that the thickness of the superconductors is smaller than the coherence length. In this case one can average the equations for the Green functions over the thickness and get a uniform superconductor with built-in exchange field.[31] Under these assumptions the system is described by a generic Hamiltonian which is homogeneous in space:

$$H = H_R + H_L + H_T, \tag{1}$$

where $H_{R(L)}$ describes the left and right electrodes consisting of a BCS superconductor with an intrinsic exchange field. For example, for the left electrode it reads

$$H_L = \sum_{k,s,s'} a^{\dagger}_{ks}[\xi_k\delta_{ss'} - (h_L\mathbf{n}_L \cdot \hat{\boldsymbol{\sigma}})_{ss'}]a_{ks} + \sum_k(\Delta_L a^{\dagger}_{k\uparrow}a^{\dagger}_{-k\downarrow} + \text{H.c.}), \tag{2}$$

where a ($a^{\dagger}$) is the annihilation (creation) operator of a particle with momentum k and spin s, ξ_k is the quasiparticle energy, Δ is the superconducting gap, $\boldsymbol{\sigma} = (\sigma_1,\sigma_2,\sigma_3)$ is the vector of Pauli matrices, h_L is the amplitude of the effective exchange field, and $\mathbf{n}$ is a unit vector pointing in its direction. The H_T term in Eq. (1) describes the spin-selective tunneling through the spin filter and is given by

$$H_T = \sum_{s,s'}(\mathcal{T}\hat{\sigma}_0 + \mathcal{U}\hat{\sigma}_z)_{ss'}a^{\dagger}_s b_{s'} + \text{H.c.}, \tag{3}$$

where a and b are the field operators in the left and right electrodes, respectively. $\mathcal{T}$ and $\mathcal{U}$ are the spin-independent and spin-dependent tunneling matrix elements. We neglect their momentum dependence. The tunneling amplitude for spin up (down) is then given by $T_{\uparrow(\downarrow)} = \mathcal{T} \pm \mathcal{U}$. We assume that the origin of the different tunneling amplitudes is the conduction-band splitting in the ferromagnetic insulating barrier, which leads to different tunnel barrier heights for spin-up and spin-down electrons.[16,17]

In order to calculate the current through the junction it is convenient to introduce the quasiclassical Green functions $\check{g}_{R(L)}$ for the left and right electrodes. An expression for the current in terms of $\check{g}_{R(L)}$ can be obtained straightforwardly from the equations of motions for the Green functions after integration over the quasiparticle energy. In the lowest order in tunneling the current is given by

$$I = \frac{1}{32eR_N(T^2_{\uparrow} + T^2_{\downarrow})}\int d\epsilon \text{Tr}\{\hat{\tau}_3[\check{\Gamma}\check{g}_{L\alpha}\check{\Gamma},\check{g}_{R\beta}]^K\}, \tag{4}$$

where $R_N = 1/[4\pi eN(0)(T^2_{\uparrow} + T^2_{\downarrow})]$ is the resistance of the barrier in the normal state, $N(0)$ is the normal density of states at the Fermi level, thehacek denotes 8×8 matrices in the Gor'kov-Nambu (τ_i)–spin (σ_i)–Keldysh space, $\check{\Gamma} = \mathcal{T}\hat{\tau}_0 \otimes \hat{\sigma}_0 + \mathcal{U}\hat{\tau}_3 \otimes \hat{\sigma}_3$, α and β are the angles between the exchange field of the L and R electrodes with respect to the z axis (see the inset in Fig. 1), and $\check{g}_{\alpha}$ is the bulk Green's function which can be obtained by solving the quasiclassical equations. The matrix $\check{g}_{L\alpha}$ (and by analogy $\check{g}_{R\beta}$) can be written as $\check{g}_{L\alpha} = \check{R}_{\alpha} \cdot \check{g}_{L0} \cdot \check{R}^{\dagger}_{\alpha}$, where $\check{g}_{L0}$ is the known solution for the case of an exchange field along the z axis, and $\check{R}_{\alpha} = \cos(\alpha/2) + i\hat{\tau}_3 \otimes \hat{\sigma}_1 \sin(\alpha/2)$.

Results. We first proceed to determine the Josephson critical current through the spin filter. We assume that $\Delta_L = \Delta_R = \Delta$ and $h_L = h_R = h$ and that the exchange field in the left (right) electrode forms an angle α (β) with the magnetization of the I_{sf} barrier which points in the z direction. From Eq. (4) we find the Josephson current $I_J = I_c \sin\varphi$, where φ is the phase difference between the superconductors and the critical current I_c is given by the general expression

$$eR_N I_c = 2\pi T \sum_{\omega_n>0}\{r[f^2_s + f^2_t \cos\alpha\cos\beta] + f^2_t \sin\alpha\sin\beta\}, \tag{5}$$

where $r = 2T_{\downarrow}T_{\uparrow}/(T^2_{\downarrow} + T^2_{\uparrow})$ is a parameter describing the efficiency of the spin filtering ($r = 0$ denotes full polarization and $r = 1$ a nonmagnetic barrier[32]). $f_{s(t)} = (f_+ \pm f_-)/2$ are the anomalous Green's functions where $f_{\pm} = \Delta/\sqrt{(\omega_n \pm ih)^2 + \Delta^2}$ and ω_n is the Matsubara frequency. The amplitude of the singlet component is determined by f_s whereas the amplitude of the triplet component is given by f_t. Equation (5) is one of the main results of our work. If $h = 0$ it reproduces the expression presented in Refs. 20 and 21, which is the well-known Ambegaokar-Baratoff (AB) formula for the critical current[19] multiplied by a factor $r < 1$.

In the case of a fully spin-polarized barrier ($r = 0$), i.e., if either $T_\downarrow$ or $T_\uparrow$ is zero, Eq. (5) shows that the singlet Cooper pairs do not contribute to the Josephson current. The contribution to the current is due only to the second term on the right-hand side (RHS) which is independent of r and proportional to the amplitude of the triplet component f_t. This term does not vanish provided that neither α nor β is equal to 0 or π. This important result shows that even though in the electrodes only the triplet component with (locally) zero spin projection exists, the noncollinearity between h and the magnetization of the barrier induces a coupling between them and leads to the creation of a spin-polarized supercurrent. In our model the parameters r and h are independent. However, for a ferromagnetic insulator/superconductor system they might be related to each other.[22] We assume next that the exchange fields in the left and right electrodes are parallel to the magnetization of the barrier ($\alpha = \beta = 0$) and compute the temperature dependence of the critical current using Eq. (5). In Fig. 1 we show this dependence for different sets of parameters (h,r). Throughout this Rapid Communication the order parameter $\Delta(T)$ is determined self-consistently and the temperature in the figures is normalized with respect to the critical temperature which depends on h. The $I_c(T)$ curve was measured in Ref. 18 for a Josephson junction with a spin filter as tunneling barrier. If we assume, as the authors of Ref. 18 did, a finite spin-filtering effect ($r < 1$) but neglect the exchange field in the superconductor ($h = 0$) we obtain the dashed curve in Fig. 1, which is nothing but the AB curve multiplied by a prefactor $r \approx 0.27$. If we assume now a finite value of the effective exchange field in the S layers (dot-dashed curve in Fig. 1) for the same value of r one obtains that the critical current is smaller than the AB curve for all values of temperatures. If we now keep the same value for the finite exchange field but slightly change the value of r (solid line in Fig. 1), one can see that for lower temperatures the $I_c(T)$ curve exceeds is higher than the AB curve. This behavior, which is in qualitative agreement with the results of Ref. 18, shows that the interplay between h and r is crucial to understand the transport properties of the junction. We cannot conclusively say, though, that the experiment can be fully explained by these results. Indeed, measurements of the tunneling conductance in junctions with GdN barriers suggest a finite exchange field inside the S electrodes.[33] However, GdN barriers may also exhibit a complicated temperature-dependent magnetic domain structure that could also modify the $I_c(T)$ behavior.[18] This hypothetical effect is beyond the scope of the present work. Let us now assume that the exchange fields in the S layers and the magnetization of the I_{sf} barrier are noncollinear (we set $\alpha = \beta = \pi/2$ in order to maximize the contribution of the triplet supercurrent). In Fig. 2(a) we show the temperature dependence of the critical current for different values of the spin-filter parameter r corresponding to highly polarized barriers. For large values of r the critical current is positive for all temperatures (0 junction). However, if r is small enough the second term in the RHS of (5) starts to dominate and at certain interval of temperature $I_c < 0$ (π junction), i.e., our model predicts a zero-π transition for large enough spin-filter efficiency. Thus, it is more likely to observe the 0-π transition in systems containing europium chalcogenide tunnel barriers with almost 100% spin polarization[17] than by using GdN

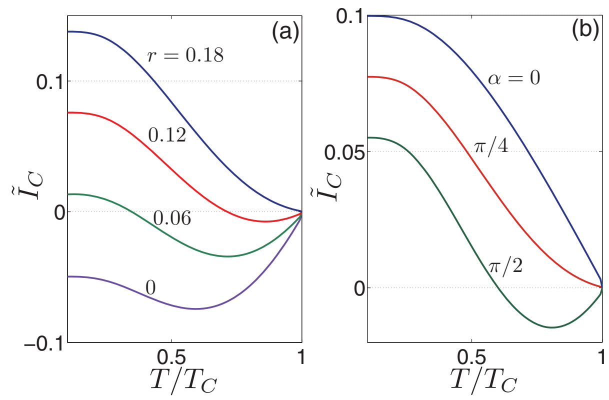

FIG. 2. (Color online) The temperature dependence of $\tilde{I}_c$ for different values of r and $\alpha = \pi/2$ (a); and for different values of α and $r = 0.1$ (b). In both panels $h = 0.567\Delta_0$.

films with a spin-filter efficiency of around 75%.[18] Note that in the fully polarized limit $r = 0$ the critical current is negative for all temperatures. In Fig. 2(b) we show the $I_c(T)$ dependence for $r = 0.1$ and different values of α. Negative values of the current appear if α is close to $\pi/2$. The origin of the π-junction behavior described here is different from the one studied in Ref. 20. The π-junction behavior shown in Fig. 2 is caused by the noncollinearity of the exchange fields and the magnetization of the I_{sf}, i.e., it is determined by the second term in the RHS of Eq. (5). In contrast, in Ref. 21 there is no such term and the π-junction behavior was obtained by assuming that $\mathcal{T} < \mathcal{U}$ (i.e., by choosing $r < 0$). For completeness we note that the Josephson current in metallic multilayered SFFS junctions also depends on the angle between magnetization orientations in different F layers. This problem (without spin-filter barriers) has been studied in numerous papers on the basis of the Usadel, Eilenberger, or Bogoliubov–de Gennes equations (see, for example, Refs. 8,9,34–41 and references in the review articles Refs. 2 and 13).

Let us now calculate the quasiparticle current I_{qp} from Eq. (4). For the normalized current $j_{qp} = I_{qp}(V)/I_N(V)$ [$I_N(V) = V/R_N$ is the current through the junction in the normal state] we get

$$j_{\alpha\beta} = \frac{1}{eV}\int d\epsilon\, F_V Y_{\alpha\beta}(\epsilon,h,V), \tag{6}$$

where $Y_{\alpha\beta}(\epsilon,h,V)$ is the spectral conductance and $F_V = 0.5\{\tanh[(\epsilon + eV/2)/2T] - \tanh[(\epsilon - eV/2)/2T]\}$. We present here the expression for a symmetric junction, i.e., $\nu_r = \nu_l$ and $\alpha = \beta$, although similar expressions hold for arbitrary angles α and β. It reads

$$Y_{\alpha\alpha} = \nu_{0+}\nu_{0-} + \nu_{3+}\nu_{3-} - (1 - r)\nu_{3+}\nu_{3-}\sin^2\alpha. \tag{7}$$

We have defined the density of states $\nu_{0,3}(\epsilon) = [\nu(\epsilon + h) \pm \nu(\epsilon - h)]/2$ with $\nu(\epsilon) = \epsilon/\sqrt{\epsilon^2 - \Delta^2}$ and $\nu_{0\pm} = \nu_0(\epsilon \pm eV/2)$.

The left panel of Fig. 3 shows the voltage dependence of the normalized differential conductance G_{qp} for zero temperature in the symmetric case $\alpha = \beta = \pi/4$. In the absence of the spin-filter effect ($r = 1$) the differential conductance is an even function of V, showing a peak at $eV = 2\Delta$ and no signature

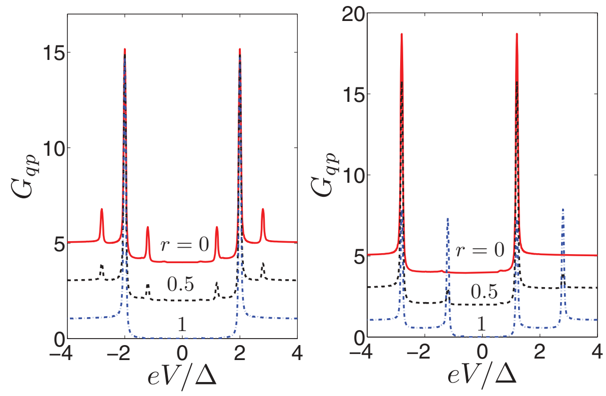

FIG. 3. (Color online) The zero-temperature normalized differential conductance $G_{qp} = R_N dI_{qp}/dV$ for $h = 0.4\Delta_0$ and $r = 0,0.5,1$. Left panel: $\alpha = \beta = \pi/4$; right panel: $\alpha = 0$, $\beta = \pi$. In calculating the curves we have added a small $\eta = 0.01\Delta_0$ damping factor. Note that the curves are shifted vertically for clarity.

of the exchange splitting[42] (dash-dotted line in the left panel of Fig. 3). However, for $r < 1$ and $0 < \alpha < \pi$ two additional peaks appear at $eV = 2(\Delta \pm h)$. Notice that the height of these peaks increases with decreasing r. Thus, by measuring the differential conductance one can extract information about the model parameters α, r, and h. From Eqs. (6) and (7) one can also show that in order to obtain an asymmetric $G_{qp}(V)$ dependence one should set $h_L \neq h_R$ as discussed in Ref. 16. In the antiparallel case, where $\alpha = 0$, $\beta = \pi$ (right panel of Fig. 3) and for $r < 1$ the differential conductance has peaks at $eV = 2(\Delta \pm h)$ (but not at $eV = 2\Delta$). These peaks have the same size for $r = 1$. However, as r decreases towards zero, the difference between the peak sizes increases. In the fully spin-polarized case ($r = 0$) one of these peaks vanishes.

Discussion and conclusions. Our model can describe different systems. First, the model applies to junctions made of two magnetic superconductors separated by a spin-filter barrier I_{sf}. But it can also describe a S-F-I_{sf}-F-S junction with the widths of the F-S electrodes smaller than the characteristic length over which the Green functions vary. We have also verified that our results are qualitatively valid for long S electrodes. These results will be discussed in more detail elsewhere. Finally, our model can also describe a simple S-I_{sf}-S structure, assuming that the I_{sf} barrier induces an effective exchange field in the superconductor over distances of the order of the superconducting coherence length as predicted in Ref. 22.

In conclusion, by combining the quasiclassical Green functions and the tunneling Hamiltonian approach we have studied the effect of spin filtering on the Josephson and quasiparticle currents in tunneling junctions. We have shown that for fully polarized barriers the singlet component does not contribute to the supercurrent I_J. However, if the direction of the exchange field **h** in both electrodes is not parallel to the quantization axis of the barrier, a nonzero I_J current is observed due to the triplet component. In this case the current is 100% spin polarized. We have also calculated the differential conductance and shown its dependence on the spin-filter parameter r and the misalignment angle. By measuring the differential conductance one can extract information about the magnetic structure of the spin-filter junction. Our findings are relevant for the creation, control, and manipulation of spin-polarized supercurrents as well as for the characterization of S-I_{sf}-S junctions.

Acknowledgments. We thank Mark Blamire, Jason Robinson, Avradeep Pal, Ilya Tokatly, and Alberto Rojo for useful discussions and David Pickup for careful reading of our manuscript. The work of F.S.B and A.V. was supported by the Spanish Ministry of Economy and Competitiveness under Project No. FIS2011-28851-C02-02 and the Basque Government under UPV/EHU Project No. IT-366- 07. A.F.V. is grateful to the DIPC for hospitality and financial support.

*Corresponding author: sebastian_bergeret@ehu.es

[1] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. Lett. **86**, 4096 (2001).

[2] F. Bergeret, A. Volkov, and K. Efetov, Rev. Mod. Phys. **77**, 1321 (2005).

[3] R. S. Keizer, S. T. B. Goennenwein, T. M. Klapwijk, G. Miao, G. Xiao, and A. Gupta, Nature (London) **439**, 825 (2006).

[4] T. S. Khaire, M. A. Khasawneh, W. P. Pratt, and N. O. Birge, Phys. Rev. Lett. **104**, 137002 (2010).

[5] J. W. A. Robinson, J. D. S. Witt, and M. G. Blamire, Science **329**, 59 (2010).

[6] C. Klose, T. Khaire, Y. Wang, W. Pratt, N. Birge, B. McMorran, T. Ginley, J. Borchers, B. Kirby, B. Maranville, and J. Unguris, Phys. Rev. Lett. **108**, 127002 (2012).

[7] M. S. Anwar, F. Czeschka, M. Hesselberth, M. Porcu, and J. Aarts, Phys. Rev. B **82**, 100501(R) (2010).

[8] A. F. Volkov, F. S. Bergeret, and K. B. Efetov, Phys. Rev. Lett. **90**, 117006 (2003).

[9] M. Houzet and A. I. Buzdin, Phys. Rev. B **76**, 060504(R) (2007).

[10] If the magnetization inhomogeneity occurs on a scale of the order of the Fermi wavelength, one can talk about a spin-active interface (Ref. 11).

[11] M. Eschrig, J. Kopu, J. C. Cuevas, and Gerd Schön, Phys. Rev. Lett. **90**, 137003 (2003).

[12] I. Sosnin, H. Cho, V. T. Petrashov, and A. F. Volkov, Phys. Rev. Lett. **96**, 157002 (2006).

[13] M. Eschrig, Phys. Today **64**(1), 43 (2011).

[14] P. M. Tedrow, J. E. Tkaczyk, and A. Kumar, Phys. Rev. Lett. **56**, 1746 (1986).

[15] J. S. Moodera, X. Hao, G. A. Gibson, and R. Meservey, Phys. Rev. Lett. **61**, 637 (1988).

[16] X. Hao, J. S. Moodera, and R. Meservey, Phys. Rev. B **42**, 8235 (1990).

[17] T. S. Santos, J. S. Moodera, K. V. Raman, E. Negusse, J. Holroyd, J. Dvorak, M. Liberati, Y. Idzerda, and E. Arenholz, Phys. Rev. Lett. **101**, 147201 (2008).

[18] K. Senapati, M. G. Blamire, and Z. H. Barber, Nat. Mater. **10**, 1 (2011).

[19] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963).

[20] I. O. Kulik, Sov. Phys. JETP **22**, 841 (1966).

[21] L. N. Bulaevskii, V. V. Kuzii, and A. A. Sobyanin, JETP Lett. **25**, 290 (1977).

[22] T. Tokuyasu, J. A. Sauls, and D. Rainer, Phys. Rev. B **38**, 8823 (1988).

[23] A. Cottet, D. Huertas-Hernando, W. Belzig, and Yl. V. Nazarov, Phys. Rev. B **80**, 184511 (2009).

[24] M. Fogelström, Phys. Rev. B **62**, 11812 (2000),

[25] A. Cottet and W. Belzig, Phys. Rev. B **72**, 180503(R) (2005),

[26] M. S. Kalenkov, A. V. Galaktionov, and A. D. Zaikin, Phys. Rev. B **79**, 014521 (2009),

[27] Y. Tanaka and S. Kashiwaya, Physica C **274**, 357 (1997).

[28] S. Kawabata, Y. Asano, Y. Tanaka, A. A. Golubov, and S. Kashiwaya, Phys. Rev. Lett. **104**, 117002 (2010).

[29] D. Huertas-Hernando, Y. V. Nazarov, and W. Belzig, Phys. Rev. Lett. **88**, 047003 (2002).

[30] F. Giazotto and F. Taddei, Phys. Rev. B **77**, 132501 (2008).

[31] F. S. Bergeret, A. F. Volkov, and K. B. Efetov, Phys. Rev. Lett. **86**, 3140 (2001).

[32] Throughout this paper we take values for r such that $0 < r < 1$ (i.e., for $\mathcal{T} \geqslant \mathcal{U}$). However, all the expressions in the present work are also valid for the case $\mathcal{T} < \mathcal{U}$. The spin-filter efficiency is usually defined as $P = T_\uparrow^2 - T_\downarrow^2/(T_\uparrow^2 + T_\downarrow^2)$ (Ref. 18); thus the relation between the parameter r and P is given by $P^2 = 1 - r^2$.

[33] A. Pal and M. G. Blamire (private communication).

[34] Z. Pajović, M. Božović, Z. Radović, J. Cayssol, and A. Buzdin, Phys. Rev. B **74**, 184509 (2006).

[35] V. Braude and Yu. V. Nazarov, Phys. Rev. Lett. **98**, 077003 (2007).

[36] B. Crouzy, S. Tollis, and D. A. Ivanov, Phys. Rev. B **75**, 054503 (2007).

[37] K. Halterman, P. H. Barsic, and O. T. Valls, Phys. Rev. Lett. **99**, 127002 (2007).

[38] I. B. Sperstad, J. Linder, and A. Sudbo, Phys. Rev. B **78**, 104509 (2008).

[39] A. F. Volkov, and K. B. Efetov, Phys. Rev. B **81**, 144522 (2010).

[40] J. Linder and A. Sudbø, Phys. Rev. B **82**, 020512(R) (2010).

[41] L. Trifunovic, Zorica Popović, and Zoran Radović, Phys. Rev. B **84**, 064511 (2011).

[42] R. Mersevey and P. M. Tedrow, Phys. Rep. **238**, 173 (1994).