Accidental stability of dark matter

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(Dated: March 21, 2013)

We propose that dark matter is stable as a consequence of an *accidental* $\mathbb{Z}_2$ that results from a flavour symmetry group which is the double-cover group of the symmetry group of one of the regular geometric solids. Although model-dependent, the phenomenology resembles that of a generic "inert Higgs" dark matter scheme.

Keywords: flavour symmetry; dark matter; neutrino masses and mixing

I. INTRODUCTION

The Standard Model (SM) has so far provided a remarkably good description of Nature, except for its failure to account for neutrino oscillations [1, 2] and for the growing evidence for the existence of dark matter [3]. Perhaps these two seemingly unrelated problems constitute two sides of the same coin. Several attempts have been made [4–7] to account for the observed pattern of the neutrino mass and mixing parameters starting from a fundamental flavour symmetry [8]. Similarly, many models have been built to explain dark matter by invoking supersymmetry and imposing unbroken R-parity in an *ad hoc* fashion [9, 10]. Various papers in which a $\mathbb{Z}_2$ has been used to connect the neutrino masses to dark matter are given in Refs. [11–22].

There have been recently many attempts to link neutrino properties to dark matter by having the latter stabilized by a remnant $\mathbb{Z}_2$ arising from the underlying flavour symmetry [23–26].

Here we attempt a different but related approach, namely, we provide a class of models—with a flavour symmetry group which is the double-cover group of the symmetry group of one of the regular geometric solids—where dark matter is stabilized as a result of an *accidental* $\mathbb{Z}_2$. We present two examples of this class of models, namely, we upgrade two representative A_4-based models [5, 6] to endow them with an automatically stable dark-matter particle.

The plan of the paper is as follows. In Sec. II we present the basic mathematics. In Sec. III we describe our proposal. In Sec. IV we provide two realizations of the proposal, briefly describing the general aspects of dark-matter

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phenomenology in Sec. IV C. Relevant group character tables are collected in appendix A. Appendix B contains the derivation of the mass matrix of the scalars in the dark-matter sector of our first model.

II. SOME MATHEMATICS

Here we describe the relevant basic mathematics. Let

$$H = \vec{\pi} \cdot \vec{\sigma} = \begin{pmatrix} \pi_3 & \pi_1 - i\pi_2 \\ \pi_1 + i\pi_2 & -\pi_3 \end{pmatrix} \tag{1}$$

be a Hermitian matrix—the π_j $(j = 1, 2, 3)$ are real quantities and the σ_j are the Pauli matrices. Let M be a matrix of $SU(2)$. Then, the transformation

$$H \to MHM^\dagger = \vec{\pi}' \cdot \vec{\sigma} = \begin{pmatrix} \pi'_3 & \pi'_1 - i\pi'_2 \\ \pi'_1 + i\pi'_2 & -\pi'_3 \end{pmatrix} \tag{2}$$

is equivalent to a transformation

$$\begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix} \to \begin{pmatrix} \pi'_1 \\ \pi'_2 \\ \pi'_3 \end{pmatrix} = M' \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}, \tag{3}$$

where $M' \in SO(3)$. In this way, each matrix M' of $SO(3)$ may be mapped into two different matrices, M and $-M$, of $SU(2)$.[1] One says that $SU(2)$ is the *double cover* of $SO(3)$.[2]

One consequence of this fact is that $SU(2)$ has all the irreducible representations (irreps) as $SO(3)$, *viz.* the **1**, **3**, **5**, and so on,[3] plus some additional irreps of its own, *viz.* the **2**, **4**, **6**, and so on. Moreover, if we call the irreps of $SO(3)$ "vectorial" and the extra irreps of $SU(2)$ "spinorial", then the product either of two spinorial irreps or of two vectorial irreps has only vectorial irreps in its Clebsch–Gordan series, while the product of one spinorial irrep and one vectorial irrep has only spinorial irreps in the Clebsch–Gordan series; it all happens as if there were an *accidental* $\mathbb{Z}_2$ symmetry under which the spinorial irreps transformed into minus themselves.

Geometrically, $SO(3)$ may be interpreted as the group of rotations in three-dimensional space. It has three remarkable finite discrete subgroups, which are the symmetry groups of the five regular geometric solids. Those subgroups are A_4, which has 12 elements and is the symmetry group of the regular tetrahedron, S_4, which has 24 elements and is the symmetry group of the cube and of the regular octahedron, and A_5, which has 60 elements and is the symmetry group of the regular dodecahedron and of the regular icosahedron.

These three subgroups of $SO(3)$ each have a double cover in $SU(2)$. We shall adopt the convention of denoting the double cover of an $SO(3)$ subgroup by the name of that subgroup with a tilde. The double-cover groups have twice as many elements as the original group: $\tilde{A}_4$ has 24 elements,[4] $\tilde{S}_4$ has 48 elements,[5] and $\tilde{A}_5$ has 120 elements. The double-cover groups are produced by the same trick performed to obtain $SU(2)$ from $SO(3)$; one starts from an irrep **3** of the $SO(3)$ subgroup, one interprets each of its matrices M' as a transformation in Eq. (3), and one transforms that M' into two $SU(2)$ matrices M and $-M$ via Eq. (2); the matrices thus obtained constitute the defining two-dimensional irrep of the double-cover group. Remarkably, the double-cover groups also have vectorial and spinorial irreps; the vectorial irreps are identical to the irreps of the $SO(3)$ subgroup, while the spinorial irreps are extra irreps of the double-cover group.

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[1] More precisely, $SO(3)$ is isomorphic to $SU(2)/C$, where $C = \{\mathbb{1}_2, -\mathbb{1}_2\}$ is the centre of $SU(2)$, *i.e.* the $\mathbb{Z}_2$ group formed by the matrices of $SU(2)$ which commute with all the matrices of $SU(2)$.

[2] Notice, however, that $SO(3)$ is *not* a subgroup of $SU(2)$.

[3] We adopt in this paper the standard practice of denoting an irrep by its dimension in boldface style.

[4] It is identified by the group-manipulation software GAP as $[24, 3]$ and it is named $SL(2, 3)$ in Ref. [27].

[5] Its GAP identifier is $[48, 28]$ and it is named $SL(2, 3) \to G \to C_2$ in Ref. [27].

Let us illustrate this firstly with the group $\tilde{A}_4$. In its defining irrep $\mathbf{2}_1$, it is generated by the $SU(2)$ matrices

$$M_1 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \text{ and } M_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sigma & \sigma \\ \sigma^3 & \sigma^7 \end{pmatrix}, \text{ where } \sigma = e^{i\pi/4} = \frac{1+i}{\sqrt{2}}. \tag{4}$$

These matrices produce, via the trick in Eqs. (2) and (3), the matrices

$$M_1' = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } M_2' = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \tag{5}$$

respectively, which belong to $SO(3)$ and generate the defining irrep $\mathbf{3}$ of A_4, which is also an irrep of $\tilde{A}_4$. The group $\tilde{A}_4$ has seven inequivalent irreps, four of which—the $\mathbf{3}$ and the $\mathbf{1}_j$—are also irreps of A_4 and are vectorial, while the extra three irreps $\mathbf{2}_j$ are not irreps of A_4 and are spinorial. The characters of the irreps of $\tilde{A}_4$ are given in appendix A. The $\mathbf{1}_1$ is the trivial irrep, in which all the elements of $\tilde{A}_4$ are represented by the number 1. The vectorial *vs.* spinorial character of the irreps is visible in the multiplication table given in table I.

$\otimes$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_1$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
$\mathbf{1}_2$		$\mathbf{1}_3$	$\mathbf{1}_1$	$\mathbf{3}$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_1$
$\mathbf{1}_3$			$\mathbf{1}_2$	$\mathbf{3}$	$\mathbf{2}_3$	$\mathbf{2}_1$	$\mathbf{2}_2$
$\mathbf{3}$				$\mathbf{3}, \mathbf{3}, \mathbf{1}_1, \mathbf{1}_2, \mathbf{1}_3$	$\mathbf{2}_1, \mathbf{2}_2, \mathbf{2}_3$	$\mathbf{2}_1, \mathbf{2}_2, \mathbf{2}_3$	$\mathbf{2}_1, \mathbf{2}_2, \mathbf{2}_3$
$\mathbf{2}_1$					$\mathbf{3}, \mathbf{1}_1$	$\mathbf{3}, \mathbf{1}_2$	$\mathbf{3}, \mathbf{1}_3$
$\mathbf{2}_2$						$\mathbf{3}, \mathbf{1}_3$	$\mathbf{3}, \mathbf{1}_1$
$\mathbf{2}_3$							$\mathbf{3}, \mathbf{1}_2$

TABLE I: Vectorial *vs.* spinorial character of the irreps of $\tilde{A}_4$.

One sees that spinorial irreps are exclusively obtained from the product of one vectorial and one spinorial irrep.

The same features apply to the double-cover groups of S_4 and A_5, the character tables of which are also given in appendix A. Note that the first five irreps of $\tilde{S}_4$ are vectorial and the latter three are spinorial. Similarly, the first five irreps of $\tilde{A}_5$ are vectorial and are also irreps of A_5, while the last four irreps of $\tilde{A}_5$ are spinorial.

Notice that $\tilde{S}_4$ has three inequivalent doublet irreps, one of which (the $\mathbf{2}_V$) is vectorial while the other two (the $\mathbf{2}_1$ and the $\mathbf{2}_2$) are spinorial. Similarly, $\tilde{A}_5$ has two inequivalent quadruplet irreps, one of which is vectorial (and is also an irrep of A_5) while the other one is spinorial.

As stressed above, the double-cover groups $\tilde{A}_4$, $\tilde{S}_4$, and $\tilde{A}_5$ are subgroups of $SU(2)$. The branching rules for the various irreps of $SU(2)$ in irreps of its subgroups are given in table II.

One sees that the $\mathbf{2}_V$ of $\tilde{S}_4$ and the $\mathbf{4}_V$ of $\tilde{A}_5$ are vectorial—they appear in the branching of the vectorial irreps $\mathbf{5}$ and $\mathbf{7}$, respectively, of $SU(2)$—in spite of having dimensions that one associates in the case of $SU(2)$ to spinorial irreps.

III. OUR PROPOSAL

The group A_4 has been used as horizontal-symmetry group for the leptonic sector in countless models and papers during the last decade [7, 8, 28]. It has been used to account for the predictions $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ [5] as well as

$SU(2)$	$\tilde{A}_4$	$\tilde{S}_4$	$\tilde{A}_5$
1	$\mathbf{1}_1$	$\mathbf{1}_1$	**1**
2	$\mathbf{2}_1$	$\mathbf{2}_1$	$\mathbf{2}_1$
3	**3**	$\mathbf{3}_1$	$\mathbf{3}_1$
4	$\mathbf{2}_2, \mathbf{2}_3$	**4**	$\mathbf{4}_\mathrm{S}$
5	$\mathbf{3}, \mathbf{1}_2, \mathbf{1}_3$	$\mathbf{3}_2, \mathbf{2}_\mathrm{V}$	**5**
6	$\mathbf{2}_1, \mathbf{2}_2, \mathbf{2}_3$	$\mathbf{4}, \mathbf{2}_2$	**6**
7	$\mathbf{3}, \mathbf{3}, \mathbf{1}_1$	$\mathbf{3}_1, \mathbf{3}_2, \mathbf{1}_2$	$\mathbf{4}_\mathrm{V}, \mathbf{3}_2$
8	$\mathbf{2}_1, \mathbf{2}_1, \mathbf{2}_2, \mathbf{2}_3$	$\mathbf{4}, \mathbf{2}_1, \mathbf{2}_2$	$\mathbf{6}, \mathbf{2}_2$

TABLE II: Branching rules for the lowest-dimensional irreps of $SU(2)$.

to explain the full tri-bimaximal mixing (TBM), namely the fact that the lepton mixing matrix U is rather close to

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \end{pmatrix}. \tag{6}$$

We can take U_{TBM} as a first-order approximation to the true lepton mixing matrix: $U \approx U_{\mathrm{TBM}}$.[6]

The group A_5 has also been used as a flavour group for the leptonic sector, namely in a model [31] that predicts $\cos\theta_{12} = \varphi$, where θ_{12} is the solar-neutrino mixing angle and $\varphi = \left(1 + \sqrt{5}\right)/2$ is the so-called "golden ratio" [32]. Finally, the flavour group S_4 has also been used in a few papers [28, 29, 33, 34].

Our proposal consists in the following. In any of the flavour models using either an A_4, S_4, or A_5 flavour-symmetry group, one may use instead their double covers $\tilde{A}_4$, $\tilde{S}_4$, and $\tilde{A}_5$, respectively. This is so because the vectorial irreps and their respective Clebsch–Gordan series and coefficients are identical for any group and its double cover. When one does that, one obtains a model in which all the 'matter', *i.e.* all the fermion and scalar fields, are in vectorial irreps. We propose *to add to any such model some 'dark matter' in spinorial irreps of the flavour group, viz.* of $\tilde{A}_4$, $\tilde{S}_4$, or $\tilde{A}_5$.

It is furthermore crucial that no field of the 'dark matter' sector acquires a vacuum expectation value (VEV). Indeed, let H denote a generic field in the 'matter' sector and η a generic field in the 'dark matter' sector. Since η has a spinorial character—even if it is an integer-spin field!—under the flavour group, it will only have $\eta\eta$ and $\eta\eta\eta\eta$ self-interactions, plus $\eta\eta H$ and $\eta\eta HH$ interactions with the 'matter' sector. The latter interactions, however, cannot cause the lightest η field to decay, they can only cause it to co-annihilate. It follows that the lightest η field is stable, and therefore, if electrically neutral, it constitutes a potentially viable dark-matter candidate. The vectorial *vs.* spinorial character of the various irreps of the flavour group effectively acts as an (accidental) $\mathbb{Z}_2$ symmetry preventing ηHH and ηHHH couplings, which would cause η to decay into matter. It is also crucial that no η field acquires a VEV $\langle\eta\rangle_0$, lest the $\eta\eta HH$ interaction produces a $\langle\eta\rangle_0 \eta HH$ interaction which would cause η to decay.

Notice that the fact that eventually the whole flavour symmetry group ends up being spontaneously broken is immaterial for the above reasoning. Indeed, the one thing that matters is that the flavour symmetry group leads to *an accidental $\mathbb{Z}_2$ symmetry which remains unbroken* and prevents the appearance in the Lagrangian—even upon renormalization—of terms of the form ηHH or ηHHH, which would cause the fields η in the 'dark matter' sector to decay into fields H of the 'matter' sector.

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[6] Even in a model in which $U = U_{\mathrm{TBM}}$, this prediction is in general approximate, since it may only hold at a high-energy scale and it will then be corrected by the renormalization-group evolution [30] down to low-energy scale. It may also be corrected by other effects, for instance a non-diagonal charged-lepton mass matrix.

IV. MODEL BUILDING

We consider in this section, as explicit examples, two models based on A_4, which we extend to $\tilde{A}_4$ in order to include inert dark matter "accidentally" stabilized, as described in the previous section. In this section we shall use the more usual notation T' to denote the double-cover group $\tilde{A}_4$ of A_4.

A. Model 1

Consider the model defined by table III. This model is a generalization of the model in Ref. [6]. In table III, $SU(2)$

	L	l_R	ν_R	H	H_T	h	ϕ	η
$SU(2)$	**2**	**1**	**1**	**2**	**2**	**2**	**1**	**2**
T'	**3**	**3**	**3**	$\mathbf{1_1}$	**3**	$\mathbf{1_1}$	**3**	$\mathbf{2_1}$
$\mathbb{Z}_2$	$+$	$+$	$-$	$+$	$+$	$-$	$+$	$+$

TABLE III: Matter assignment of model 1.

is the Standard Model (SM) gauge group and there is a flavour symmetry T' and an additional $\mathbb{Z}_2$ symmetry. Note that that additional $\mathbb{Z}_2$ is *not* the accidental $\mathbb{Z}_2$ that stabilizes the dark matter; it has been introduced only to obtain the TBM limit.

We observe that the scalar field η is the only one that has spinorial character under T', namely, it is a doublet of T' and therefore it cannot interact directly with the SM fermions through Yukawa couplings. It can couple to SM particles only through the "Higgs portal", namely via terms like $\eta^\dagger \eta H^\dagger H$ or $\eta^\dagger \eta H_T^\dagger H_T$, and so on. The neutral component of η is a good dark matter candidate since it can be produced in the early universe through the Higgs portal and since its spinorial character ensures its stability, as described in Sec. III.

The Lagrangian invariant under the SM gauge group and under the $T' \times \mathbb{Z}_2$ flavour symmetry is

$$
\begin{aligned}
\mathcal{L} = {} & y_H \left(\overline{L}l_R\right)_{\mathbf{1}_1} H + y_s \left(\overline{L}l_R\right)_{\mathbf{3}_s} H_T + y_a \left(\overline{L}l_R\right)_{\mathbf{3}_a} H_T \\
& + y_h \left(\overline{L}\nu_R\right)_{\mathbf{1}_1} h + m \left(\nu_R^T C^{-1} \nu_R\right)_{\mathbf{1}_1} + y_\nu \left(\nu_R^T C^{-1} \nu_R\right)_{\mathbf{3}_s} \phi + \text{H.c.}
\end{aligned}
\tag{7}
$$

We assume that the scalar T' triplets have VEVs aligned along the directions

$$
\langle H_T \rangle_0 = v_T \left(1,1,1\right), \quad \langle \phi \rangle_0 = v \left(1,0,0\right).
\tag{8}
$$

Then the charged-lepton mass matrix is given by

$$
M_\ell = \begin{pmatrix}
y_H v_H & (y_s + y_a)\, v_T & (y_s - y_a)\, v_T \\
(y_s - y_a)\, v_T & y_H v_H & (y_s + y_a)\, v_T \\
(y_s + y_a)\, v_T & (y_s - y_a)\, v_T & y_H v_H
\end{pmatrix},
\tag{9}
$$

which is of the form

$$
\begin{pmatrix}
\alpha & \beta & \gamma \\
\gamma & \alpha & \beta \\
\beta & \gamma & \alpha
\end{pmatrix}
$$

and is diagonalized as

$$
U_\omega^\dagger M_\ell U_\omega = \begin{pmatrix}
\alpha + \beta + \gamma & 0 & 0 \\
0 & \alpha + \omega\beta + \omega^2\gamma & 0 \\
0 & 0 & \alpha + \omega^2\beta + \omega\gamma
\end{pmatrix},
\tag{10}
$$

where

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{11}$$

We note that in order to have $m_e \ll m_\mu \ll m_\tau$ a fine-tuning among α, β, and γ is required: we need $\alpha \approx \omega^2 \beta \approx \omega \gamma \sim m_\tau/3$. Assuming $v_H \sim 100$ GeV, the Yukawa couplings must be of order 10^{-2} so as to give $y_H v_H \sim m_\tau$. Thus, assuming $y_{a,s} \sim \mathcal{O}(1)$, v_T must be $\mathcal{O}(\text{GeV})$ to give $y_{a,s} v_T \sim m_\tau$.

In the model there are five Higgs doublets—H, three H_T, and h—that acquire VEVs, respectively v_H, v_T, and v_h. These are the VEVs that contribute to the masses of the gauge bosons $W^\pm$ and Z^0. However, we assume $v_T, v_h \ll v_H$, and then H is, to a good approximation, the SM Higgs doublet.

We assume that no component of η acquires a VEV. We also assume that the lightest component of η is neutral; that is our dark matter candidate. The differences among the squared masses of the various neutral components of η are almost of order $\mathcal{O}(v_H v_T)$, and then coannihilation is not too strong. In appendix B the form of the mass matrix of the neutral components of η is explicitly computed.

The Dirac neutrino mass matrix is proportional to the unit matrix:

$$M_D \propto \mathcal{I}, \tag{12}$$

while the right-handed-neutrino Majorana mass matrix is

$$M_R = \begin{pmatrix} m & 0 & 0 \\ 0 & m & y_\nu v \\ 0 & y_\nu v & m \end{pmatrix}. \tag{13}$$

The light-neutrino Majorana mass matrix arises from the type-I seesaw mechanism [35],

$$M_\nu = -M_D M_R^{-1} M_D^T \propto M_R^{-1}. \tag{14}$$

In the basis where the charged-lepton mass matrix is diagonal, $\hat{M}_\nu$ is diagonalized by U_{TBM} and the mass eigenvalues satisfy the sum rule [36, 37]

$$\frac{2}{m_2} = \frac{1}{m_1} + \frac{1}{m_3}, \tag{15}$$

where the eigenvalues m_j should be understood as being complex, *viz.* the neutrino masses are the $|m_j|$.

In our model we should assign quarks to invariants of T'. In other words, quarks are flavour-blind and couple only to the SM Higgs doublet H, which is also T'-invariant.[7] With the above matter assignment, one may show that the quark mixing matrix is predicted to be the identity matrix, which is a good first approximation. In order to generate both the Cabibbo angle and the reactor-neutrino mixing angle we must extend the model in some way. Note that the data from the recent reactor experiments Double Chooz [38], Daya Bay [39], and RENO [40] seem to indicate that the reactor angle and the Cabibbo angle are of the same order of magnitude. Since minimal $SU(5)$ has $M_\ell = M_d^T$, one may have deviations for lepton mixing (through the charged-lepton mass matrix M_ℓ) and for quark mixing (through the down-type-quark mass matrix M_d) of the same order — see for instance Refs. [41, 42]. Another possibility is by assuming two extra flavon fields $\phi' \sim \mathbf{1}_2$ and $\phi'' \sim \mathbf{1}_3$ as in Ref. [43], where it is shown that this leads to a sufficiently large reactor angle.

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[7] If, however, we want to embed the model in a Grand Unified Theory (GUT), then a possibility is to assign the quarks to triplets of T' just like the charged leptons. Then we can in principle implement either an $SU(5)$ or an $SO(10)$ GUT framework.

B. Model 2

Another A_4 model that may be extended to T' in order to accommodate naturally stable inert dark matter is the model proposed in Refs. [5, 44]. It is described by the field representation content (in a supersymmetric notation) in table IV. Once again, the model has a discrete Abelian symmetry, in this case $\mathbb{Z}_3$, which has nothing to do with the

	$\hat{Q}$	$\hat{L}$	$\hat{u}_1^c,\ \hat{d}_1^c,\ \hat{e}_1^c$	$\hat{u}_2^c,\ \hat{d}_2^c,\ \hat{e}_2^c$	$\hat{u}_3^c,\ \hat{d}_3^c,\ \hat{e}_3^c$	$\hat{\phi}_{1,2}$	$\hat{\eta}_{1,2}$
$SU(2)$	**2**	**2**	**1**	**1**	**1**	**2**	**2**
T'	**3**	**3**	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{1}_1$	$\mathbf{2}_1$
$\mathbb{Z}_3$	1	1	ω	ω	ω	1	1

TABLE IV: Matter assignment of model 2.

accidental symmetry that stabilizes dark matter. Notice that the only field that has spinorial character under T' is η, which is a doublet of T' and for that reason has no Yukawa couplings to the SM fermions. The stability of η is ensured by its spinorial character under T' and the lightest neutral component of η is a dark matter candidate.[8]

In addition to the fields in table IV, the model of Refs. [5, 44] contains the heavy quark, heavy lepton, and Higgs superfields in table V, which are all gauge-$SU(2)$ singlets.

	$\hat{U}$	$\hat{U}^c$	$\hat{D}$	$\hat{D}^c$	$\hat{E}$	$\hat{E}^c$	$\hat{N}^c$	$\hat{\chi}$
A_4	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**
$\mathbb{Z}_3$	1	1	1	1	1	1	1	ω^2

TABLE V: Extra matter assignment of the model 2.

The superpotential is given by

$$
\begin{aligned}
\hat{W} \;=\; & M_U \hat{U}_i \hat{U}_i^c + f_u \hat{Q}_i \hat{U}_i^c \hat{\phi}_2 + h_{ijk}^u \hat{U}_i \hat{u}_j^c \hat{\chi}_k \\
& + M_D \hat{D}_i \hat{D}_i^c + f_d \hat{Q}_i \hat{D}_i^c \hat{\phi}_1 + h_{ijk}^d \hat{D}_i \hat{d}_j^c \hat{\chi}_k \\
& + M_E \hat{E}_i \hat{E}_i^c + f_e \hat{L}_i \hat{E}_i^c \hat{\phi}_1 + h_{ijk}^e \hat{E}_i \hat{e}_j^c \hat{\chi}_k \\
& + M_N \hat{N}_i^c \hat{N}_i^c + f_N \hat{L}_i \hat{N}_i^c \hat{\phi}_2 + \mu \hat{\phi}_1 \hat{\phi}_2 \\
& + M_\chi \hat{\chi}_i \hat{\chi}_i + h_\chi \hat{\chi}_1 \hat{\chi}_2 \hat{\chi}_3 .
\end{aligned}
\tag{16}
$$

One can show that the scalar field χ may acquire VEV along the T' direction

$$
\langle \chi \rangle_0 \sim (1,1,1) .
\tag{17}
$$

The charged-lepton masses are generated after integrating out the heavy E and E^c fields. The result is given as

$$
\mathcal{M}_e = U_\omega \begin{pmatrix} h_1^e & 0 & 0 \\ 0 & h_2^e & 0 \\ 0 & 0 & h_3^e \end{pmatrix} \frac{\sqrt{3} f_e v_1 u}{M_E} .
\tag{18}
$$

The right-handed-neutrino Majorana mass matrix is proportional to the identity matrix, $M_N \propto \mathcal{I}$. Hence, after a type-I seesaw, the light-neutrino Majorana mass matrix is given by

$$
\mathcal{M}_\nu = \frac{f_N^2 v_2^2}{M_N} U_L^T U_L \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \equiv \lambda .
\tag{19}
$$

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[8] This model may already possess, besides η, the usual supersymmetric dark matter candidates.

This shows that, at this stage, neutrinos are degenerate and the atmospheric neutrino mixing angle is maximal.

As we run down to the electroweak scale, Eq. (19) is corrected by the wavefunction renormalizations of ν_e, ν_μ, and ν_τ, as well as by the corresponding vertex renormalizations. One can then obtain the neutrino squared-mass differences as well as the solar mixing angle. In contrast to the previous example, here the lepton mixing is not predicted to be tri-bimaximal since the solar angle is left unpredicted. Given the structure of λ at the high scale, its form at the low scale is fixed to first order as

$$\lambda_{\text{low scale}} = \begin{pmatrix} 1 + 2\delta_{ee} & \delta_{e\mu} + \delta_{e\tau} & \delta_{e\mu} + \delta_{e\tau} \\ \delta_{e\mu} + \delta_{e\tau} & 2\delta_{\mu\tau} & 1 + \delta_{\mu\mu} + \delta_{\tau\tau} \\ \delta_{e\mu} + \delta_{e\tau} & 1 + \delta_{\mu\mu} + \delta_{\tau\tau} & 2\delta_{\mu\tau} \end{pmatrix}, \tag{20}$$

where we have assumed all the parameters to be real. The matrix in Eq. (20) is obtained by multiplying the matrix of Eq. (19) on the left and on the right by all possible $\nu_i \to \nu_j$ transitions. The mass matrix in Eq. (20) is manifestly μ–τ symmetric, yielding maximal atmospheric mixing angle and a solar mixing angle that can be fitted to the measured value. As shown in Ref. [5], assuming the parameters δ_{ij} to be complex a deviation of the reactor angle from zero can be obtained.

C. Dark matter

In model 1 above there are two gauge-$SU(2)$ doublets placed in the T' doublet η. In model 2, which is supersymmetric, there are instead four gauge-$SU(2)$ doublets placed in the T' doublets $\hat\eta_1$ and $\hat\eta_2$. The models 1 and 2 are just two simple examples realizing our idea in Sec. III. In contrast with the inert dark matter scenarios [45], here the dark matter is stabilized accidentally in the context of flavour symmetry-based models, as already mentioned in Sec. III. On the other hand, just as in the inert dark matter models, in our models above a "Higgs portal" exists, namely terms of the type $\eta\eta HH$ which connect the dark matter to normal matter, so dark matter can be produced with a relic abundance $0.09 \leq \Omega h^2 \leq 0.13$ consistent with the WMAP measurements [46].

It is possible to perform a detailed study of the parameter space of either of the above models, but that goes way beyond the scope of the present paper. A calculation has been performed for the discrete dark matter scenario in Ref. [25]. In that scenario, dark matter belongs to a triplet representation of $\tilde{A}_4$, instead of the spinorial $\mathbf{2}_1$ representation of $\tilde{A}_4$ of the models above. However, we do not expect substantial differences from the phenomenological point of view. By analogy with Refs. [47–49], we expect our dark matter candidate η^0 to be viable within a mass range of 40 to 80 GeV.

V. SUMMARY

In summary, we have proposed that dark matter is stable because of an accidental $\mathbb{Z}_2$ symmetry which results from a flavour group which is the double-cover group of the symmetry group of one of the regular geometric solids. The phenomenology is similar to that of generic inert dark matter scenarios with a Higgs portal, except that it appears here in the framework of discrete flavour symmetry schemes.

Appendix A: Character tables

Here for completeness we present the character tables of the groups discussed in the text.

Class	n	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$
C_1	1	1	1	1	3	2	2	2
C_2	1	1	1	1	3	-2	-2	-2
C_3	6	1	1	1	-1	0	0	0
C_4	4	1	ω	ω^2	0	1	ω	ω^2
C_5	4	1	ω^2	ω	0	1	ω^2	ω
C_6	4	1	ω^2	ω	0	-1	$-\omega^2$	$-\omega$
C_7	4	1	ω	ω^2	0	-1	$-\omega$	$-\omega^2$

TABLE VI: Character table of $\tilde{A}_4$. Here, n is the number of elements in each class and $\omega \equiv \exp\left(2i\pi/3\right) = \left(-1 + i\sqrt{3}\right)/2$.

Class	n	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{2}_{\rm V}$	$\mathbf{3}_1$	$\mathbf{3}_2$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{4}$
C_1	1	1	1	2	3	3	2	2	4
C_2	1	1	1	2	3	3	-2	-2	-4
C_3	6	1	-1	0	1	-1	$\sqrt{2}$	$-\sqrt{2}$	0
C_4	6	1	-1	0	1	-1	$-\sqrt{2}$	$\sqrt{2}$	0
C_5	6	1	1	2	-1	-1	0	0	0
C_6	12	1	-1	0	-1	1	0	0	0
C_7	8	1	1	-1	0	0	1	1	-1
C_8	8	1	1	-1	0	0	-1	-1	1

TABLE VII: Character table of $\tilde{S}_4$.

Class	n	$\mathbf{1}$	$\mathbf{3}_1$	$\mathbf{3}_2$	$\mathbf{4}_{\rm V}$	$\mathbf{5}$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{4}_{\rm S}$	$\mathbf{6}$
C_1	1	1	3	3	4	5	2	2	4	6
C_2	1	1	3	3	4	5	-2	-2	-4	-6
C_3	30	1	-1	-1	0	1	0	0	0	0
C_4	20	1	0	0	1	-1	1	1	-1	0
C_5	20	1	0	0	1	-1	-1	-1	1	0
C_6	12	1	$-b$	$-d$	-1	0	b	d	-1	1
C_7	12	1	$-b$	$-d$	-1	0	$-b$	$-d$	1	-1
C_8	12	1	$-d$	$-b$	-1	0	d	b	-1	1
C_9	12	1	$-d$	$-b$	-1	0	$-d$	$-b$	1	-1

TABLE VIII: Character table of $\tilde{A}_5$. Here, $b = 2\cos\dfrac{4\pi}{5} = \dfrac{-1-\sqrt{5}}{2}$ and $d = 2\cos\dfrac{2\pi}{5} = \dfrac{-1+\sqrt{5}}{2}$.

Appendix B: The neutral-scalar squared-mass matrix in model 1

Let U_1 and U_2 be the generators of T' (the double-covering group of A_4). The irreps of T' may be given thus:

$$
\mathbf{1}_k : \quad U_1 \rightarrow 1, \qquad\qquad U_2 \rightarrow \omega^{k-1};
$$

$$
\mathbf{2}_k : \quad U_1 \rightarrow \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \qquad U_2 \rightarrow \frac{\omega^{k-1}}{\sqrt{2}} \begin{pmatrix} \sigma & \sigma \\ \sigma^3 & \sigma^7 \end{pmatrix};
$$

$$
\mathbf{3} : \quad U_1 \rightarrow \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_2 \rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},
$$

(B1)

for $k = 1, 2, 3$, where $\omega = \exp(i2\pi/3)$ and $\sigma = \exp(i\pi/4)$. Let (a, b) be a $\mathbf{2}_1$ and (x, y, z) be a $\mathbf{3}$ of T'. Then,

$$
\begin{pmatrix} az + bx - iby \\ -bz + ax + iay \end{pmatrix} \text{ is a } \mathbf{2}_1, \quad \begin{pmatrix} az + \omega^2 bx - i\omega by \\ -bz + \omega^2 ax + i\omega ay \end{pmatrix} \text{ is a } \mathbf{2}_2, \quad \begin{pmatrix} az + \omega bx - i\omega^2 by \\ -bz + \omega ax + i\omega^2 ay \end{pmatrix} \text{ is a } \mathbf{2}_3.
$$

(B2)

Let (a, b) be a $\mathbf{2}_p$ and (a', b') be a $\mathbf{2}_q$ of T'. Then,

$$
\begin{pmatrix} \omega^{p+q-2}(aa' - bb') \\ i\omega^{2(p+q)-2}(aa' + bb') \\ -ab' - ba' \end{pmatrix} \text{ is a } \mathbf{3}.
$$

(B3)

We consider a simplified version of our model 1 by neglecting the scalars h and ϕ in table III. We then have a six-Higgs-doublet model, where the Higgs doublets are in a $\mathbf{1}_1$, a $\mathbf{3}$, and a $\mathbf{2}_1$ of T', denoted respectively H, H_T, and η in table III. Let then

$$
\phi_0 : \ \mathbf{1}_1 \text{ of } T', \quad \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} : \ \mathbf{2}_1 \text{ of } T', \quad \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} : \ \mathbf{3} \text{ of } T'
$$

(B4)

be Higgs doublets. Then,

$$
\phi_0^\dagger : \ \mathbf{1}_1 \text{ of } T', \quad \begin{pmatrix} \eta_2^\dagger \\ -\eta_1^\dagger \end{pmatrix} : \ \mathbf{2}_1 \text{ of } T', \quad \begin{pmatrix} \phi_1^\dagger \\ \phi_2^\dagger \\ \phi_3^\dagger \end{pmatrix} : \ \mathbf{3} \text{ of } T'.
$$

(B5)

Making the products of these, one obtains the following irreps of T:

$$
\mathbf{1}_1 : \quad \phi_0^\dagger \phi_0, \quad \eta_1^\dagger \eta_1 + \eta_2^\dagger \eta_2, \quad \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3;
$$

(B6)

$$
\mathbf{1}_2 : \quad \phi_1^\dagger \phi_1 + \omega^2 \phi_2^\dagger \phi_2 + \omega \phi_3^\dagger \phi_3;
$$

(B7)

$$
\mathbf{1}_3 : \quad \phi_1^\dagger \phi_1 + \omega \phi_2^\dagger \phi_2 + \omega^2 \phi_3^\dagger \phi_3;
$$

(B8)

$$
\mathbf{2}_1 : \quad \begin{pmatrix} \eta_2^\dagger \phi_0 \\ -\eta_1^\dagger \phi_0 \end{pmatrix}, \quad \begin{pmatrix} \phi_0^\dagger \eta_1 \\ \phi_0^\dagger \eta_2 \end{pmatrix}, \quad \begin{pmatrix} \eta_2^\dagger \phi_3 - \eta_1^\dagger \phi_1 + i\eta_1^\dagger \phi_2 \\ \eta_1^\dagger \phi_3 + \eta_2^\dagger \phi_1 + i\eta_2^\dagger \phi_2 \end{pmatrix}, \quad \begin{pmatrix} \phi_3^\dagger \eta_1 + \phi_1^\dagger \eta_2 - i\phi_2^\dagger \eta_2 \\ -\phi_3^\dagger \eta_2 + \phi_1^\dagger \eta_1 + i\phi_2^\dagger \eta_1 \end{pmatrix};
$$

(B9)

$$
\mathbf{2}_2 : \quad \begin{pmatrix} \eta_2^\dagger \phi_3 - \omega^2 \eta_1^\dagger \phi_1 + i\omega \eta_1^\dagger \phi_2 \\ \eta_1^\dagger \phi_3 + \omega^2 \eta_2^\dagger \phi_1 + i\omega \eta_2^\dagger \phi_2 \end{pmatrix}, \quad \begin{pmatrix} \phi_3^\dagger \eta_1 + \omega^2 \phi_1^\dagger \eta_2 - i\omega \phi_2^\dagger \eta_2 \\ -\phi_3^\dagger \eta_2 + \omega^2 \phi_1^\dagger \eta_1 + i\omega \phi_2^\dagger \eta_1 \end{pmatrix};
$$

(B10)

$$\mathbf{2_3}: \quad \begin{pmatrix} \eta_2^\dagger\phi_3 - \omega\eta_1^\dagger\phi_1 + i\omega^2\eta_1^\dagger\phi_2 \\ \eta_1^\dagger\phi_3 + \omega\eta_2^\dagger\phi_1 + i\omega^2\eta_2^\dagger\phi_2 \end{pmatrix}, \quad \begin{pmatrix} \phi_3^\dagger\eta_1 + \omega\phi_1^\dagger\eta_2 - i\omega^2\phi_2^\dagger\eta_2 \\ -\phi_3^\dagger\eta_2 + \omega\phi_1^\dagger\eta_1 + i\omega^2\phi_2^\dagger\eta_1 \end{pmatrix}; \tag{B11}$$

$$\mathbf{3}: \quad \begin{pmatrix} \eta_2^\dagger\eta_1 + \eta_1^\dagger\eta_2 \\ i\omega^2\left(\eta_2^\dagger\eta_1 - \eta_1^\dagger\eta_2\right) \\ \eta_1^\dagger\eta_1 - \eta_2^\dagger\eta_2 \end{pmatrix}, \quad \begin{pmatrix} \phi_2^\dagger\phi_3 \\ \phi_3^\dagger\phi_1 \\ \phi_1^\dagger\phi_2 \end{pmatrix}, \quad \begin{pmatrix} \phi_3^\dagger\phi_2 \\ \phi_1^\dagger\phi_3 \\ \phi_2^\dagger\phi_1 \end{pmatrix}, \quad \begin{pmatrix} \phi_0^\dagger\phi_1 \\ \phi_0^\dagger\phi_2 \\ \phi_0^\dagger\phi_3 \end{pmatrix}, \quad \begin{pmatrix} \phi_1^\dagger\phi_0 \\ \phi_2^\dagger\phi_0 \\ \phi_3^\dagger\phi_0 \end{pmatrix}. \tag{B12}$$

Making $\eta_{1,2} \to \eta_{1,2}^0$, $\phi_0 \to v_H$, and $\phi_{1,2,3} \to v_T$, one obtains

$$\mathbf{1_1}: \quad |v_H|^2, \quad |v_T|^2, \quad \left|\eta_1^0\right|^2 + \left|\eta_2^0\right|^2; \tag{B13}$$

$$\mathbf{2_1}: \quad v_H\begin{pmatrix} \eta_2^{0*} \\ -\eta_1^{0*} \end{pmatrix}, \quad v_H^*\begin{pmatrix} \eta_1^0 \\ \eta_2^0 \end{pmatrix}, \quad v_T\begin{pmatrix} \eta_2^{0*} + (i-1)\,\eta_1^{0*} \\ \eta_1^{0*} + (i+1)\,\eta_2^{0*} \end{pmatrix}, \quad v_T^*\begin{pmatrix} \eta_1^0 + (1-i)\,\eta_2^0 \\ -\eta_2^0 + (1+i)\,\eta_1^0 \end{pmatrix}; \tag{B14}$$

$$\mathbf{2_2}: \quad v_T\begin{pmatrix} \eta_2^{0*} + \left(i\omega - \omega^2\right)\eta_1^{0*} \\ \eta_1^{0*} + \left(i\omega + \omega^2\right)\eta_2^{0*} \end{pmatrix}, \quad v_T^*\begin{pmatrix} \eta_1^0 + \left(\omega^2 - i\omega\right)\eta_2^0 \\ -\eta_2^0 + \left(\omega^2 + i\omega\right)\eta_1^0 \end{pmatrix}; \tag{B15}$$

$$\mathbf{2_3}: \quad v_T\begin{pmatrix} \eta_2^{0*} + \left(i\omega^2 - \omega\right)\eta_1^{0*} \\ \eta_1^{0*} + \left(i\omega^2 + \omega\right)\eta_2^{0*} \end{pmatrix}, \quad v_T^*\begin{pmatrix} \eta_1^0 + \left(\omega - i\omega^2\right)\eta_2^0 \\ -\eta_2^0 + \left(\omega + i\omega^2\right)\eta_1^0 \end{pmatrix}; \tag{B16}$$

$$\mathbf{3}: \quad \begin{pmatrix} \eta_2^{0*}\eta_1^0 + \eta_1^{0*}\eta_2^0 \\ i\left(\eta_2^{0*}\eta_1^0 - \eta_1^{0*}\eta_2^0\right) \\ \left|\eta_1^0\right|^2 - \left|\eta_2^0\right|^2 \end{pmatrix}, \quad \begin{pmatrix} |v_T|^2 \\ |v_T|^2 \\ |v_T|^2 \end{pmatrix}, \quad \begin{pmatrix} v_H^* v_T \\ v_H^* v_T \\ v_H^* v_T \end{pmatrix}, \quad \begin{pmatrix} v_H v_T^* \\ v_H v_T^* \\ v_H v_T^* \end{pmatrix}. \tag{B17}$$

Therefore, the quartic terms in the T'-invariant scalar potential yield *only* the following mass terms for η_1^0 and η_2^0 when ϕ_0^0 acquires VEV v_H and $\phi_{1,2,3}^0$ acquire identical VEVs v_T:

$$a\left(\left|\eta_1^0\right|^2 + \left|\eta_2^0\right|^2\right), \tag{B18}$$

$$b\left[\left|\eta_1^0\right|^2 - \left|\eta_2^0\right|^2 + (1+i)\,\eta_1^0\eta_2^{0*} + (1-i)\,\eta_1^{0*}\eta_2^0\right], \tag{B19}$$

$$\frac{c+id}{2}\left[2\eta_1^0\eta_2^0 + (-1-i)\left(\eta_1^0\right)^2 + (1-i)\left(\eta_2^0\right)^2\right], \tag{B20}$$

$$\frac{c-id}{2}\left[2\eta_1^{0*}\eta_2^{0*} + (-1+i)\left(\eta_1^{0*}\right)^2 + (1+i)\left(\eta_2^{0*}\right)^2\right], \tag{B21}$$

where a, b, c, and d are real quantities with mass-squared dimension. We may write the mass terms in equations (B18)–(B21) in the form

$$\begin{pmatrix} \Re\eta_1^0 & \Re\eta_2^0 & \Im\eta_1^0 & \Im\eta_2^0 \end{pmatrix} \begin{pmatrix} a+b-c+d & b+c & c+d & b-d \\ b+c & a-b+c+d & -b-d & -c+d \\ c+d & -b-d & a+b+c-d & b-c \\ b-d & -c+d & b-c & a-b-c-d \end{pmatrix} \begin{pmatrix} \Re\eta_1^0 \\ \Re\eta_2^0 \\ \Im\eta_1^0 \\ \Im\eta_2^0 \end{pmatrix}. \tag{B22}$$

It is easy to convince oneself that the squared-mass matrix in Eq. (B22), even though quite restrictive, still allows the four neutral components of η to be non-degenerate. In our model, we should allow the term in Eq. (B18) to be dominant, while b, c, and d in Eqs. (B19)–(B21) are $\mathcal{O}\left(v_H v_T\right)$ and provide the non-degeneracy.

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