

gr-qc/9911002

Quantum evolution in spacetime foam

Luis J. Garay

Instituto de Matemáticas y Física Fundamental, CSIC, C/ Serrano 121, 28006 Madrid, Spain
2 March 1999

Abstract

In this work, I review some aspects concerning the evolution of quantum low-energy fields in a foamlike spacetime, with involved topology at the Planck scale but with a smooth metric structure at large length scales, as follows. Quantum gravitational fluctuations may induce a minimum length thus introducing an additional source of uncertainty in physics. The existence of this resolution limit casts doubts on the metric structure of spacetime at the Planck scale and opens a doorway to nontrivial topologies, which may dominate Planck scale physics. This foamlike structure of spacetime may show up in low-energy physics through loss of quantum coherence and mode-dependent energy shifts, for instance, which might be observable. Spacetime foam introduces nonlocal interactions that can be modeled by a quantum bath, and low-energy fields evolve according to a master equation that displays such effects. Similar laws are also obtained for quantum mechanical systems evolving according to good real clocks, although the underlying Hamiltonian structure in this case establishes serious differences among both scenarios. *Contents.*—Quantum fluctuations of the gravitational field; Spacetime foam; Loss of quantum coherence; Quantum bath; Low-energy effective evolution; Real clocks; Conclusions.

Int. J. Mod. Phys. A14 (1999) 4079–4120

1 Quantum fluctuations of the gravitational field

Gravity deals with the frame in which everything takes place, i.e., with spacetime. We are used to putting everything into spacetime, so that we can name and handle events. General relativity made spacetime dynamical but the relations between different events were still sharply defined. Because of quantum mechanics, in such a dynamical frame, objects became fuzzy; exact locations were substituted by probability amplitudes of finding an object in a given region of space at a given instant of time. Spacetime undergoes the quantum fluctuations of the other interactions and, even more, introduces its own fluctuations, thus becoming an active agent in the theory. The quantum theory of gravity suffers from problems (see, e.g. Refs. [1, 2]) that have remained unsolved for many years and that are originated, in part, in this lack of a fixed immutable spacetime background.

A quantum uncertainty in the position of a particle implies an uncertainty in its momentum and, therefore, due to the gravity-energy universal interaction, would also imply an uncertainty in the geometry, which in turn would introduce an additional uncertainty in position of the particle. The geometry would thus be subject to quantum fluctuations that would constitute the spacetime foam and that should be of the same order as the geometry itself at the Planck scale. This would give rise to a minimum length [3] beyond which the geometrical properties of spacetime would be lost, while on larger scales it

would look smooth and with a well-defined metric structure. The key ingredients for the appearance of this minimum length are quantum mechanics, special relativity, which is essential for the unification of all kinds of energy via the finiteness of the speed of light, and a theory of gravity, i.e., a theory that accounts for the active response of spacetime to the presence of energy (general relativity, Sakharov's elasticity [4, 5], strings. . .). Thus, the existence of a lower bound to any output of a position measurement, seems to be a model-independent feature of quantum gravity. In fact, different approaches to this theory lead to this result [3].

Planck length ℓ_* might play a role analogous to the speed of light in special relativity. In this theory, there is no physics beyond this speed limit and its existence may be inferred through the relativistic corrections to the Newtonian behavior. This would mean that a quantum theory of gravity could be constructed only on "this side of Planck's border" as pointed out by Markov [6, 7] (as quoted in Ref. [8]). In fact, the analogy between quantum gravity and special relativity seems to be quite close: in the latter you can accelerate forever even though you will never reach the speed of light; in the former, given a coordinate frame, you can reduce the coordinate distance between two events as much as you want even though the proper distance between them will never decrease beyond Planck length (see Ref. [3], and references therein). This uncertainty relation $\Delta x \geq \ell_*$ also bears a close resemblance to the role of \hbar in quantum mechanics: no matter which variables are used, it is not possible to have an action S smaller than \hbar [9, 10, 11].

Based on the work by Bohr and Rosenfeld [12, 13, 14, 15] (see e.g. Ref. [16]) for the electromagnetic field, Peres and Rosen [17] and then DeWitt [18] carefully analyzed the measurement of the gravitational field and the possible sources of uncertainty (see also Refs. [8, 19, 20, 21]). Their analysis was carried out in the weak-field approximation (the magnitude of the Riemann tensor remains small for any finite domain) although the features under study can be seen to have more fundamental significance. This approximation imposes a limitation on the bodies that generate and suffer the gravitational field, which does not appear in the case of an electromagnetic field. The main reason for this is that, in this case, the relevant quantity that is involved in the uncertainty relations is the ratio between the charge and the mass of the test body, and this quantity can be made arbitrarily small. This is certainly not the case for gravitational interactions, since the equivalence principle precisely fixes the corresponding ratio between gravitational mass and inertial mass, and therefore it is not possible to make it arbitrarily small. Let us go into more detail in the comparison between the electromagnetic and the gravitational fields as far as uncertainties in the measurement are concerned and see how it naturally leads to a minimum volume of the measurement domain.

The measurement of the gravitational field can be studied from the point of view of continuous measurements [9, 10, 11], which we briefly summarize in what follows (throughout this work we set $\hbar = c = 1$, so that the only dimensional constant is Planck's length $\ell_* = \sqrt{G}$).

Continuous measurements.— Assume that we continuously measure an observable Q , within the framework of ordinary quantum mechanics. Let us call Δq the uncertainty of our measurement device. This means that, as a result of our measurement, we will obtain an output α that will consist of the result $q(t)$ and any other within the range $(q - \Delta q, q + \Delta q)$. The probability amplitude for an output α can be written in terms

of path integrals [9, 10] $A[\alpha] = \int_{\alpha} \mathcal{D}x e^{iS}$, where α denotes not only the output but also the set of trajectories in configuration space that lead to it. For a given uncertainty Δq , the set α is fully characterized by its central value q . We are particularly interested in studying the shape of the probability amplitude A . More precisely, we will pay special attention to its width $\Delta\alpha$ [9, 10, 11].

There are two different regimes of measurement, classical and quantum, depending on whether the uncertainty of the measuring device is large or small. The classical regime of measurement will be accomplished if Δq is large enough. In this regime, the width of the probability amplitude $\Delta\alpha$ can be seen to be proportional to the uncertainty Δq . Also, the uncertainty on the action can be estimated to be $\Delta S \gtrsim 1$. The quantum regime of measurement occurs when Δq is very small. Now the width of the probability amplitude is $\Delta\alpha \sim 1/\Delta q$. The uncertainty in the action is also greater than unity in this case.

Thus, in any regime of measurement, the action uncertainty will be greater than unity. In view of this discussion, the width $\Delta\alpha$ of the probability amplitude will achieve its minimum value, i.e., the measurement will be optimized, for uncertainties in the measurement device Δq that are neither too large nor too small. When this minimum nonvanishing value is achieved, the uncertainty in the action is also minimized and set equal to one. The limitation on the accuracy of any continuous measurement is, of course, an expression of Heisenberg's uncertainty principle. Since we are talking about measuring trajectories in some sense, a resolution limit should appear, expressing the fact that position and momentum cannot be measured simultaneously with infinite accuracy. In the classical regime of measurement, the accuracy is limited by the intrinsic uncertainty of the measuring device. On the other hand, when very accurate devices are employed, quantum fluctuations of the measuring apparatus affect the measured system and the final accuracy is also affected. The maximum accuracy is obtained when there is achieved a compromise between keeping the classical uncertainty low and keeping quantum fluctuations also small.

Measuring the gravitational field. — This discussion bears a close resemblance with the case of quantum gravity concerning the existence of a minimum length, where there exists a balance between the Heisenberg contribution $1/\Delta p$ to the uncertainty in the position and the active response of gravity to the presence of energy that produces an uncertainty $\Delta x \gtrsim \ell_*^2 \Delta p$. Actually, any measurement of the gravitational field is not only extended in time, but also extended in space. These measurements are made by determining the change in the momentum of a test body of a given size. That measurements of the gravitational field have to be extended in spacetime, i.e., that they have to be continuous, is due to the dynamical nature of this field. Before analyzing the gravitational field, let us first briefly discuss the electromagnetic field whose measurement can also be regarded as continuous.

In the case of an electromagnetic field, the action has the form $S = \int d^4x F^2$, where F is the electromagnetic field strength. Then, the action uncertainty principle $\Delta S \gtrsim 1$ implies that $\Delta(F^2) l^4 \gtrsim 1$, which can be conveniently written as $\Delta F l^3 \gtrsim q/(Flq)$, where l is the linear size of the test body and q is its electric charge. Here, we have already made the assumption that the quantum fluctuations of the test body are negligible, i.e., that its size l is larger than its Compton wave length $1/m$, where m is its rest mass. Flq is just the electromagnetic energy of the test body. If we impose the condition that the

electromagnetic energy of the test body be smaller than its rest mass m , the uncertainty relation above becomes in this case $\Delta F l^3 \gtrsim q/m$. The conditions $l \gtrsim 1/m$ and $l \gtrsim q^2/m$ that we have imposed on the test body and that can be summarized by saying that it must be classical from both the quantum and the relativistic point of view are the reflection of the following assumptions: the measurement of the field averaged over a spacetime region, whose linear dimensions and time duration are determined by l , is performed by determining the initial and final momentum of a uniformly charged test body; the time interval required for the momentum measurement is small compared to l ; any back-reaction can be neglected if the mass of the test body is sufficiently high; and finally, the borders of the test body are separated by a spacelike interval.

Let us now consider [11] a measurement of the scalar curvature averaged over a spacetime region of linear dimension l , given by the resolution of the measuring device (the test body). The action is $S = \ell_*^{-2} \int d^4x \sqrt{-g} R$, where the integral is extended over the spacetime region under consideration, so that it can be written as $S = \ell_*^{-2} R l^4$, R being now the average curvature. The action uncertainty principle $\Delta S \gtrsim 1$ gives the uncertainty relation for the curvature $\Delta R l^4 \gtrsim \ell_*^2$, which translates into the uncertainty relation $\Delta \Gamma l^3 \gtrsim \ell_*^2$ for the connection Γ , or in terms of the metric tensor, $\Delta g l^2 \gtrsim \ell_*^2$. The left hand side of this relation can be interpreted as the uncertainty in the proper separation between the borders of the region that we are measuring, so that it states the minimum position uncertainty relation $\Delta x \gtrsim \min(l, \ell_*^2/l) \gtrsim \ell_*$. It is worth noting that it is the concurrence of the three fundamental constants of nature \hbar , c (which have already been set equal to 1), and G that leads to a resolution limit. If any of them is dropped then this resolution limit disappears.

We see from the uncertainty relation for the electromagnetic field that an infinite accuracy can be achieved if an appropriate test body is used. This is not the case for the gravitational interaction. Indeed, the role of F is now played by Γ/ℓ_* , where Γ is the connection, and the role of q is played by $\ell_* m$. It is worth noting [17] that by virtue of the equivalence principle, active gravitational mass, passive gravitational mass and energy (rest mass in the Newtonian limit) are all equal, and hence, for the gravitational interaction, the ratio q/m is the universal constant ℓ_* . The two requirements of Bohr and Rosenfeld are now $l \gtrsim 1/m$ and $l \gtrsim \ell_*^2 m$ so that $l \gtrsim \ell_*$. This means that the test body should not be a black hole, i.e. its size should not exceed its gravitational radius, and that both its mass and linear dimensions should be larger than Planck's mass and length, respectively. As in the electromagnetic case, Bohr and Rosenfeld requirements can be simply stated as follows: the test body must behave classically from the points of view of quantum mechanics, special relativity and gravitation. Otherwise, the interactions between the test body and the object under study would make this distinction (the test body on the one hand and the system under study on the other) unclear as happens in ordinary quantum mechanics: the measurement device must be classical or it is useless as a measuring apparatus. In this sense, within the context of quantum gravity, Planck's scale establishes the border between the measuring device and the system that is being measured.

We can see that the problem of measuring the gravitational field, i.e., the structure of spacetime, can be traced back to the fact that any such measurement is nonlocal, i.e. the measurement device is aware of what is happening at different points of spacetime and takes them into account. In other words, the measurement device averages over a space-

time region. The equivalence principle also plays a fundamental role: the measurement device cannot decouple from the measured system and back reaction is unavoidable.

Vacuum fluctuations.— One should expect not only fluctuations of the gravitational field owing to the quantum nature of other fields and measuring devices but also owing to the quantum features of the gravitational field itself. As happens for any other field, in quantum gravity there will exist vacuum fluctuations that provide another piece of uncertainty to the gravitational field strength. It can also be computed by means of the action uncertainty principle. Indeed, in the above analyses, we have only considered first order terms in the uncertainty because it was assumed that there was a nonvanishing classical field that we wanted to measure. However, in the case of vacuum, the field vanishes and higher order terms are necessary. Let us discuss this issue for the electromagnetic case first. The uncertainty in the action can be calculated as $\Delta S = S[F + \Delta F] - S[F]$, so that $\Delta S = \int d^4x [2F\Delta F + (\Delta F)^2]$. The action uncertainty principle then yields the relation $\Delta F l^2 \gtrsim -Fl^2 + \sqrt{(Fl^2)^2 + 1}$. In the already studied limit of large electromagnetic field (or very large regions) $Fl^2 \gg 1$, the uncertainty relation for the field becomes $\Delta F l^3 \gtrsim 1/(Fl) \gtrsim q/m$ obtained above. On the other hand, the limit of vanishing electromagnetic field (or very small regions of observation) $Fl^2 \ll 1$ provides the vacuum fluctuations of the electromagnetic field $\Delta F l^2 \gtrsim 1$.

In the gravitational case, the situation is similar. The gravitational action can be qualitatively written in terms of the connection Γ as $S = \ell_*^{-2} \int d^4x (\partial\Gamma + \Gamma^2)$ so that the uncertainty in the action has the form

$$\Delta S \sim \ell_*^{-2} [\Delta\Gamma l^3 + \Gamma\Delta\Gamma l^4 + (\Delta\Gamma)^2 l^4]. \quad (1.1)$$

It is easy to argue that Γl must be at most of order 1 so that the contribution of the second term is qualitatively equivalent to that of the first one. Indeed, Γl is the gravitational potential which is given by $\Gamma l = \Gamma_{\text{ext}} l (1 - \Gamma l)$, Γ_{ext} being the external gravitational field. The last term is just an expression of the equivalence principle, according to which, any kind of energy, including the gravitational one, also generates a gravitational field. Thus, $\Gamma l = \Gamma_{\text{ext}} l / (1 + \Gamma_{\text{ext}} l)$ which is always smaller than one. The action uncertainty principle then implies that $\Delta\Gamma l^2 \gtrsim -l + \sqrt{l^2 + \ell_*^2}$ and that, in terms of the metric tensor,

$$\Delta g \gtrsim -1 + \sqrt{1 + \ell_*^2/l^2}. \quad (1.2)$$

For test bodies much larger than Planck size, i.e., for $l \gg \ell_*$, this uncertainty relation becomes the already obtained $\Delta g \gtrsim \ell_*^2/l^2$, valid for classical test bodies. However, for spacetime regions of very small size — closed to Planck length $l \gtrsim \ell_*$ — this uncertainty relation acquires the form $\Delta g \gtrsim \ell_*/l$. This uncertainty in the gravitational field comes from the vacuum fluctuations of spacetime itself and not from the disturbances introduced by measuring devices with $l \gg \ell_*$ [5, 22, 23, 24, 25, 26]. For alternative derivations of this uncertainty relation see, e.g., Refs. [5, 27].

We then see that proper distances have an uncertainty $\sqrt{\Delta g l^2}$ that approaches Planck length for very small (Planck scale) separations thus suggesting that Planck length represents a lower bound to any distance measurement. At the Planck scale, the gravitational field uncertainty is of order 1, i.e., the fluctuations are as large as the geometry itself. This is indicating that the low-energy theory that we have been using breaks down at the Planck scale and that a full theory of quantum gravity is necessary to study such regime.

2 Spacetime foam

In his work “On the hypotheses which lie at the basis of the geometry” [28], written more than a century ago, Riemann already noticed that “[. . .]. If this independence of bodies from position does not exist, we cannot draw conclusions from metric relations of the great, to those of the infinitely small; in that case the curvature at each point may have an arbitrary value in three directions, provided that the total curvature of every measurable portion of space does not differ sensibly from zero. Still more complicated relations may exist if we no longer suppose the linear element expressible as the square root of a quadratic differential. Now it seems that the empirical notions on which the metrical determinations of space are founded, the notion of a solid body and of a ray of light, cease to be valid for the infinitely small. We are therefore quite at liberty to suppose that the metric relations of space in the infinitely small do not conform to the hypotheses of geometry; and we ought in fact to suppose it, if we can thereby obtain a simpler explanation of phenomena.”

In the middle of this century, Weyl [29] took these ideas a bit further and envisaged (multiply connected) topological structures of ever-increasing complexity as possible constituents of the physical description of surfaces. He wrote in this respect [29] “A more detailed scrutiny of a surface might disclose that what we had considered an elementary piece in reality has tiny handles attached to it which change the connectivity character of the piece, and that a microscope of ever greater magnification would reveal ever new topological complications of this type, *ad infinitum*.”

Few years later, Wheeler described this topological complexity of spacetime at small length scales as the foamlike structure of spacetime [23]. According to Wheeler [5, 22, 23, 24, 25, 30], at the Planck scale, the fluctuations of the geometry are so large and involve so large energy densities that gravitational collapse should be continuously being done and undone at that scale. Because of this perpetuity and ubiquity of Planck scale gravitational collapse, it should dominate Planck scale physics. In this continuously changing scenario, there is no reason to believe that spacetime topology remains fixed and predetermined. Rather, it seems natural to accept that the topology of spacetime is also subject to quantum fluctuations that change all its properties. Therefore, this scenario, in which spacetime is endowed with a foamlike structure at the Planck scale, seems to be a natural ingredient of the yet-to-be-built quantum theory of gravity. Furthermore, from the functional integration point of view [31], in quantum gravity all histories contribute and, among them, there seems unnatural not to consider nontrivial topologies as one considers not trivial geometries [22, 23, 32] (see, however, Ref. [33]). On the other hand, it has been shown [34] that there exist solutions to the equations of general relativity on manifolds that present topology changes. In these solutions, the metric is degenerate on a set of measure zero but the curvature remains finite. This means that allowing degenerate metrics amounts to open a doorway to classical topology change. Furthermore, despite the difficulties of finding an appropriate interpretation for these degenerate metrics in the classical Lorentzian theory, they will naturally enter the path integral formulation of quantum gravity. This is therefore an indication that topology change should be taken into account in any quantum theory of gravity [34] (for an alternative description of topology change within the framework of noncommutative geometry, see Ref. [35, 36]).

Adopting a picture in which spacetime topology depends on the scale on which one

performs the observations, we would conclude that there would be a trivial topology on large length scales but more and more complicated topologies as we approach the Planck scale.

Spacetime foam may have important effects in low-energy physics. Indeed, the complicated topological structure may provide mechanisms for explaining the vanishing of the cosmological constant [37, 38, 39] and for fixing all the constants of nature [37, 40] (for a recent proposal for deriving the electroweak coupling constant from spacetime foam see Ref. [41]). Spacetime foam may also induce loss of quantum coherence [42] and may well imply the existence of an additional source of uncertainty. Related to this, it might produce frequency-dependent energy shifts [43, 44, 45] that would slightly alter the dispersion relations for the different low-energy fields. Finally, spacetime foam has been proposed as a mechanism for regulating both the ultraviolet [46] (see also Refs. [47, 48, 49, 50]) and the infrared [51] behavior of quantum field theory.

It is well-known that it is not possible to classify all four-dimensional topologies [52, 53] and, consequently, all the possible components of spacetime foam. With the purpose of exemplifying the richness and complexity of the vacuum of quantum gravity, in what follows, we will briefly discuss a few different kinds of fluctuations encompassed by spacetime foam, where the word fluctuations will just denote spacetime configurations that contribute most to the gravitational path integral [23]: simply connected nontrivial topologies, multiply connected topologies with trivial second homology group (i.e. with vanishing second Betti number), spacetimes with a nontrivial causal structure, i.e. with closed timelike curves, in a bounded region, and, finally, nonorientable tunnels.

Hawking [53] argued that the dominant contribution to the quantum gravitational path integral over metrics and topologies should come from topologies whose Euler characteristic χ_E was approximately given by the spacetime volume in Planck units, i.e., from topologies with $\chi_E \sim (l/l_*)^4$. In this analysis, he restricted to compact simply-connected manifolds with negative cosmological constant λ . The choice of compact manifolds obeys to a normalization condition similar to introducing a box of finite volume in nongravitational physics. The cosmological constant is introduced for this purpose and it being negative is because saddle-point Euclidean metrics with high Euler characteristic and positive λ do not seem to exist, so that positive- λ configurations will not contribute significantly to the Euclidean path integral. Finally, simple connectedness can be justified by noting that multiply-connected compact manifolds can be unwrapped by going to the universal covering manifold that, although will be noncompact, can be made compact with little cost in the action. He then concluded that, among these manifolds, the dominant topology is $S^2 \times S^2$ [54] which has an associated second Betti number $B_2 = \chi_E - 2 = 2$. These results are based on the semiclassical approximation and, as such, should be treated with some caution.

Compact simply-connected bubbles with the topology $S^2 \times S^2$ can be interpreted as closed loops of virtual black holes [54] if one realizes [55] that the process of creation of a pair of real charged black holes accelerating away from each other in a spacetime which is asymptotic to \mathbb{R}^4 is provided by the Ernst solution [56]. This solution has the topology $S^2 \times S^2$ minus a point (which is sent to infinity) and this topology is the topological sum of the bubble $S^2 \times S^2$ plus \mathbb{R}^4 . Virtual black holes will not obey classical equations of motion but will appear as quantum fluctuations of spacetime and thus will become part of the spacetime foam. As a consequence, one can conclude that the dominant contribution

to the path integral over compact simply-connected topologies would be given by a gas of virtual black holes with a density of the order of one virtual black hole per Planck volume.

A similar analysis within the context of quantum conformal gravity has been performed by Strominger [57] with the conclusion that the quantum gravitational vacuum indeed has a very involved structure at the Planck scale, with a proliferation of nontrivial compact topologies.

Carlip [38, 39] has studied the influence of the cosmological constant λ on the sum over topologies. It should be stressed that this cosmological constant is not related to the observed cosmological constant [53]. Rather, it is introduced as a source term of the form $\ell_*^{-2}\lambda V$, where V is the spacetime volume, added to the vacuum gravitational action $\ell_*^{-2} \int R\sqrt{g}$. In the semiclassical approximation, this sum is dominated by the saddle-points, which are Einstein metrics. The classical Euclidean action for these metrics has the form $\tilde{v}/(\ell_*^2\lambda)$, where, up to irrelevant numerical factors, $\tilde{v} = \lambda^2 V$ is the normalized spacetime volume of the manifold and is independent of λ . In fact, \tilde{v} characterizes the topology of the manifold. For instance, for hyperbolic manifolds it can be identified with the Euler characteristic. Carlip has shown that, in the semiclassical approximation, the behavior of the density of topologies, which counts the number of manifolds with a given value for \tilde{v} , crucially depends on the sign of the cosmological constant.

For negative values of λ , the partition function receives relevant contributions from spacetimes with arbitrarily complicated topology, so that processes that could be expected to contribute to the vacuum energy might produce more and more complicated spacetime topologies, as we briefly discuss in what follows, thus providing a mechanism for the vanishing of the cosmological constant. The Euclidean path integral in the semiclassical approximation can be written as

$$Z[\lambda] = \sum_{\tilde{v}} \rho(\tilde{v}) e^{\tilde{v}/(\ell_*^2\lambda)}, \quad (2.1)$$

where $\rho(\tilde{v})$ is a density of topologies. It can be argued that for negative λ , the density of topologies $\rho(\tilde{v})$ grows with the topological complexity \tilde{v} at least as $\rho(\tilde{v}) \gtrsim \exp(\tilde{v} \ln \tilde{v})$, i.e., it is superexponential [38, 39]. Then, after introducing an infrared cutoff to ensure the convergence of the sum above, the topologies that will contribute most to $Z[\lambda]$ will lie around some maximum value of the topological complexity \tilde{v}_{\max} . The true cosmological constant Λ , obtained from the microcanonical ensemble, is in this case

$$-\frac{1}{\Lambda\ell_*^2} = \left. \frac{\partial \ln \rho(\tilde{v})}{\partial \tilde{v}} \right|_{\tilde{v}_{\max}} \gtrsim 1 + \ln \tilde{v}_{\max} \quad (2.2)$$

and the ‘‘topological capacity’’

$$c_V = -\left. \frac{1}{\Lambda^2\ell_*^4} \left(\frac{\partial^2 \ln \rho(\tilde{v})}{\partial \tilde{v}^2} \right)^{-1} \right|_{\tilde{v}_{\max}} = -\ell_*^{-2} \left(\frac{\partial \Lambda}{\partial \tilde{v}_{\max}} \right)^{-1} \lesssim -\tilde{v}_{\max} (1 + \ln \tilde{v}_{\max}), \quad (2.3)$$

where these quantities have been defined by analogy with the thermodynamical temperature and heat capacity, respectively. In this analogy, $-\Lambda$ plays the role of temperature while the topological complexity \tilde{v} is analogous to the energy. According to this picture, the behavior of spacetime foam would be analogous to a thermodynamical system with

negative heat capacity, in which, as we put energy into the system, a greater and greater proportion of it is employed in the exponential production of new states rather than in increasing the energy of already existing states. Similarly, since the topological capacity is negative, which is a consequence of the superexponential density of topologies, the microcanonical cosmological constant will approach a vanishing value as the maximum topological complexity \tilde{v}_{\max} approaches infinity. We then see that this process, which could be expected to increase the vacuum energy $|\Lambda|$, actually contributes to decrease it, until it approaches the smallest value $|\Lambda| = 0$. The case of positive λ presents a different behavior. The topological complexity has a finite maximum value, namely, that of the four-sphere $\tilde{v}_{\max} = \chi_E^{\max} = 2$ and the density of topologies $\rho(\tilde{v})$ increases as \tilde{v} decreases.

The superexponential lower bound to the density of topologies given above receives the main contribution from multiply connected manifolds, among which, Euclidean wormholes [58, 59] have deserved much attention during the last decade (see, e.g., Ref. [60]). Wormholes are four-dimensional spacetime handles that have vanishing second Betti number, while the first Betti number provides the number of handles. They were regarded as a possible mechanism for the complete evaporation of black holes [61, 62, 63, 64]. An evaporating black hole would have a wormhole attached to it and this wormhole would transport the information that had fallen into the black hole to another, quite possibly far away, region of spacetime. More recently, Hawking [54] has proposed an alternative scenario in which black holes, at the end of their evaporation process, will have a very small size and will eventually dilute in the sea of virtual black holes that form part of spacetime foam. Wormholes also constitute the main ingredient in Coleman's proposal for explaining the vanishing of the cosmological constant and for fixing all the constants of nature [37, 40] (see also Ref. [65]). Wormholes have been studied in the so-called dilute gas approximation in which wormhole ends are far apart from each other. It should be noted, however, that, although the semiclassical approximation probably ceases to be valid at the Planck scale, it gives a clear indication that one should expect a topological density of one wormhole per unit four-volume, i.e., the first Betti number B_1 should be approximately equal to the spacetime volume $B_1 \sim V$ at the Planck scale. Multiply connected topology fluctuations may suffer instabilities against uncontrolled growth both in Euclidean quantum gravity [66, 67, 68] (see however Ref. [69]) and in the Lorentzian sector [70, 71]. These instabilities might put serious limitations to the kind of multiply connected topologies encompassed by spacetime foam.

One should also expect other configurations with nontrivial causal structure to contribute to spacetime foam. For instance, quantum time machines [72, 73], have been recently proposed as possible components of spacetime foam. From the semiclassical point of view, most of the hitherto proposed time machines [74, 75] are unstable because quantum vacuum fluctuations generate divergences in the stress-energy tensor, i.e., are subject to the chronology protection conjecture [76, 77] (for a beautiful and detailed report on time machines see Ref. [27]). However, quantum time machines [72, 73] confined to small spacetime regions, for which the chronology protection conjecture does not apply [78], are likely to occur within the realm of spacetime foam, where strong causality violations or even the absence of a causal structure are expected. We have in fact argued that the spacetime metric undergoes quantum fluctuations of order 1 at the Planck scale. Since the slope of the light cone is determined by the speed of light obtained from $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = 0$, the uncertainty in the metric will also introduce an uncertainty in

the slope of the light cone of order 1 at the Planck scale so that the notion of causality is completely lost.

As happens with the causal structure, orientability is likely to be lost at the Planck scale [79, 80], where the lack of an arbitrarily high resolution would blur the distinction between the two sides of any surface. Therefore, nonorientable topologies can be regarded as additional configurations that may well be present in spacetime foam and thus contribute to the vacuum structure of quantum gravity. Indeed, quantum mechanically stable nonorientable spacetime tunnels that connect two asymptotically flat regions with the topology of a Klein bottle can be constructed [80] as a generalization of modified Misner space [72, 73].

The presence of quantum time machines or nonorientable tunnels in spacetime amounts to the existence of Planck-size regions in which violations of the weak energy condition occur. Although from the classical point of view, the weak energy condition seems to be preserved, it is well-known (see, e.g., Ref. [27]) that quantum effects may well involve such exotic types of energy.

3 Loss of quantum coherence

The quantum structure of spacetime would be relevant at energies close to Planck scale and one could expect that the quantum gravitational virtual processes that constitute the spacetime foam could not be described without knowing the details of the theory of quantum gravity. However, the gravitational nature of spacetime fluctuations provides a mechanism for studying the effects of these virtual processes in the low-energy physics. Indeed, virtual gravitational collapse and topology change would forbid a proper definition of time at the Planck scale. More explicitly, in the presence of horizons, closed timelike curves, topology changes, etc., any Hamiltonian vector field that represents time evolution outside the fluctuation would vanish at points inside the fluctuation. This means that it would not be possible to describe the evolution by means of a Hamiltonian unitary flow from an initial to a final state and, consequently, quantum coherence would be lost. These effects and their order of magnitude would not depend on the detailed structure of the fluctuations but rather on their existence and global properties. In general, the regions in which the asymptotically timelike Hamiltonian vector fields vanish are associated with infinite redshift surfaces and, consequently, these small spacetime regions would behave as magnifiers of Planck length scales transforming them into low-energy modes as seen from outside the fluctuations [81, 82]. Therefore, spacetime foam and the related lower bound to spacetime uncertainties would leave their imprint, which may be not too small, in low-energy physics and low-energy experiments would effectively suffer a nonvanishing uncertainty coming from this lack of resolution in spacetime measurements. In this situation, loss of quantum coherence would be almost unavoidable [42].

The idea that the quantum gravitational fluctuations contained in spacetime foam could lead to a loss of quantum coherence was put forward by Hawking and collaborators [42, 83, 84]. This proposal was based in part on the thermal character of the emission predicted for evaporating black holes [85, 86, 87]. If loss of coherence occurs in macroscopic black holes, it seems reasonable to conclude that the small black holes that are continuously being created and annihilated everywhere within spacetime foam will also induce loss

of quantum coherence [42, 87]. On the other hand, scattering amplitudes of low-energy fields by topologically nontrivial configurations ($S^2 \times S^2$, K^3 and CP^2 bubbles) lead to the conclusion that pure states turn into a partly incoherent mixture upon evolution in these nontrivial backgrounds under certain simplifying assumptions and, consequently, that quantum coherence is lost.

They made explicit calculations for specific asymptotically flat spacetimes with nontrivial simply-connected topologies [42, 54, 83, 84, 88, 89, 90] or causal structure [91] which showed that it was not possible to separate the complex-time graphs for the obtained Lorentzian Green functions into two disconnected parts. More explicitly, the Euclidean Green functions obtained in these backgrounds mix positive and negative frequencies when the analytic continuation to Lorentzian signature is performed, since the Green functions develop extra acausal singularities. This situation is analogous to that in black hole physics where Lorentzian Green functions show periodic poles in imaginary time [84]. Although these calculations were performed in a finite dimensional approximation to metrics of given topology, the contributions of these extra singularities can be determined by dimensional analysis and therefore they seem to be characteristic of each topology and hold for any metric in them [84]. In contrast, Gross [92] calculated scattering amplitudes in specific four-dimensional solutions that can be interpreted as three-dimensional Kaluza-Klein instantons and concluded that there was no loss of quantum coherence in such models. Hawking [88] in turn replied to this criticism that the solutions used by Gross were special cases in the sense that the associated three-dimensional Kaluza-Klein instantons were flat and therefore topologically trivial. He further argued, with examples, that solutions with topologically nontrivial three-dimensional instantons can be constructed and that lead to a nonunitary evolution.

Superscattering operator.— Let us consider a scattering process in an asymptotically flat spacetime with nontrivial topology. If we denote the density matrices at the far past and far future by ρ_- and ρ_+ , respectively, there will be a superscattering operator $\$$ that relates both of them $\rho_+ = \$ \cdot \rho_-$, i.e., that provides the evolution between the two asymptotically flat regions across the nontrivial topology fluctuation [42]. Let $|0_\pm\rangle$ represent the vacuum at each region and $\{|A_\pm\rangle\}$ a basis of the Fock space, so that we can write $|A_\pm\rangle = \Upsilon_{\pm A}^\dagger |0_\pm\rangle$, where Υ^A is a string of annihilation operators and, consequently, Υ_A^\dagger is a string of creation operators. The density matrices ρ_\pm can then be written as

$$\rho_\pm = \sum_{AB} \rho_{\pm B}^A |A_\pm\rangle \langle B_\pm| = \rho_{\pm B}^A \Upsilon_{\pm A}^\dagger |0_\pm\rangle \langle 0_\pm| \Upsilon_\pm^B, \quad (3.1)$$

where a sum over repeated indices is assumed.

The density matrices at both asymptotic regions can then be related by noting that the density matrix at the far future ρ_+ is given by the expectation values in the far-past state ρ_- of a complete set of future operators built out of creation and annihilation operators, namely,

$$\rho_{+D}^C = \text{tr}(\Upsilon_{+D}^\dagger \Upsilon_+^C \rho_-) = \rho_{-B}^A \langle 0_- | \Upsilon_-^B \Upsilon_{+D}^\dagger \Upsilon_+^C \Upsilon_{-A}^\dagger | 0_- \rangle. \quad (3.2)$$

Therefore, the superscattering matrix $\$_{DA}^C{}^B \equiv \langle 0_- | \Upsilon_-^B \Upsilon_{+D}^\dagger \Upsilon_+^C \Upsilon_{-A}^\dagger | 0_- \rangle$, relates the density matrices in both asymptotic regions, i.e., $\rho_{+D}^C = \$_{DA}^C{}^B \rho_{-B}^A$. Note that the superscattering matrix $\$_{DA}^C{}^B$ is Hermitian in both pairs of indices CD and AB to ensure that

the Hermiticity of the density matrix is preserved. Also, the conservation of probability, i.e., $\text{tr}(\rho_{\pm}) = 1$, implies that $\mathcal{S}_{CA}^C{}^B = \delta_A^B$.

The relation between this superscattering operator and the Green functions discussed above is easily obtained if we write the annihilation operators $a_{\pm}(k)$ that form Υ_{\pm}^A at each asymptotic region in terms of the corresponding field operators. For instance, in the case of a complex scalar field, this expression (up to numerical normalization factors) has the well-known form

$$a_{\pm}(k) = -i \int_{\Sigma_{\pm}} d\Sigma^{\mu}(x) e^{-kx} \overleftrightarrow{\nabla}_{\mu} \phi(x), \quad (3.3)$$

where Σ_{\pm} represent spacelike surfaces in the infinite past and future.

We now introduce the identity operator $1 = \sum_n |n\rangle\langle n|$, with $|n\rangle$ being energy eigenstates, in the expression for \mathcal{S}

$$\mathcal{S}_{DA}^C{}^B = \sum_n \langle 0_- | \Upsilon_-^B \Upsilon_{+D}^{\dagger} | n \rangle \langle n | \Upsilon_+^C \Upsilon_{-A}^{\dagger} | 0_- \rangle \quad (3.4)$$

and note that the only state that can contribute is that with zero energy, i.e., $n = 0$ for energy to be conserved. If spacetime is globally hyperbolic, so that asymptotic completeness holds, there is a one-to-one map between states at any spacetime region, in particular, between the vacua $|0\rangle$ and $|0_{\pm}\rangle$. Therefore, the only contribution from $1 = \sum_n |n\rangle\langle n|$ can be regarded as coming from $|0_{+}\rangle\langle 0_{+}|$:

$$\mathcal{S}_{DA}^C{}^B = \langle 0_- | \Upsilon_-^B \Upsilon_{+D}^{\dagger} | 0_{+} \rangle \langle 0_{+} | \Upsilon_+^C \Upsilon_{-A}^{\dagger} | 0_- \rangle. \quad (3.5)$$

In this case, the superscattering operator factorizes into two unitary factors:

$$\mathcal{S}_{DA}^C{}^B = S_A^C S_D^{*B}, \quad (3.6)$$

with $S_A^C = \langle 0_{+} | \Upsilon_+^C \Upsilon_{-A}^{\dagger} | 0_- \rangle = \langle C_{+} | A_{-} \rangle$. Note that the scattering matrix S is indeed unitary, i.e., $S_A^C S_C^{*B} = \sum_C \langle B_{-} | C_{+} \rangle \langle C_{+} | A_{-} \rangle = \delta_A^B$ by virtue of the condition of conservation of probability. The factorizability of the superscattering operator \mathcal{S} always implies unitary evolution for the density matrix. Indeed, if the superscattering operator can be factorized as $\mathcal{S} \cdot \rho = S \rho S^{\dagger}$ for some scattering operator S , then conservation of probability, which amounts to require that $\text{tr}(\mathcal{S} \cdot \rho) = 1$ provided that $\text{tr}(\rho) = 1$, implies that

$$1 = \text{tr}(\mathcal{S} \cdot \rho) = \text{tr}(S \rho S^{\dagger}) = \text{tr}(\rho S^{\dagger} S) \quad (3.7)$$

and therefore $S^{\dagger} S = 1$, i.e., the scattering operator S is unitary. In this case, the operator \mathcal{S} also implies a unitary evolution for the density matrix since it preserves $\text{tr}(\rho^2)$:

$$\text{tr}(\rho_{+}^2) = \text{tr}[(\mathcal{S} \cdot \rho_{-})(\mathcal{S} \cdot \rho_{-})] = \text{tr}(S \rho_{-} S^{\dagger} S \rho_{-} S^{\dagger}) = \text{tr}(S \rho_{-}^2 S^{\dagger}) = \text{tr}(\rho_{-}^2). \quad (3.8)$$

If, on the other hand, we cannot guarantee that states at different spacetime regions are one-to-one related, then the zero energy state $|0\rangle$ will not correspond in general to the zero energy state $|0_{+}\rangle$ and the superscattering operator will not admit a factorized form: $\mathcal{S} \cdot \rho \neq S \rho S^{\dagger}$. When the superscattering operator does not satisfy the factorization condition, the evolution does not preserve $\text{tr}(\rho^2)$ in general and quantum coherence is lost. This can be seen explicitly in the analysis below.

Quasilocal superscattering. — Let us assume that the dynamics that underlies a superscattering operator $\$$ is quasilocal. By quasilocal we mean that any possible effect leading to a nonfactorizable superscattering operator is confined to a spacetime region whose size r is much smaller than the characteristic spacetime size l of the low-energy fields, i.e., we will assume that $r/l \ll 1$. Then, the superscattering equation $\rho_+ = \$ \cdot \rho_-$ can be obtained by integrating a differential equation of the form $\dot{\rho}(t) = L(t) \cdot \rho(t)$, where $L(t)$ is a linear operator [93]. Furthermore, it can be shown that $L(t)$ can be generally written as [94]

$$\begin{aligned} L \cdot \rho &= -i[H_0, \rho] - \frac{1}{2}h_{\alpha\beta}(Q^\beta Q^\alpha \rho + \rho Q^\beta Q^\alpha - 2Q^\alpha \rho Q^\beta) \\ &= -i[H_0, \rho] - \frac{i}{2}\text{Im}(h_{\alpha\beta})[Q^\alpha, [Q^\beta, \rho]_+] - \frac{1}{2}\text{Re}(h_{\alpha\beta})[Q^\alpha, [Q^\beta, \rho]], \end{aligned} \quad (3.9)$$

where H_0 and Q^α form a complete set of Hermitian matrices, Q^α have been chosen to be orthogonal, i.e., $\text{tr}(Q^\alpha Q^\beta) = \delta^{\alpha\beta}$, and $h_{\alpha\beta}$ is a Hermitian matrix. A sufficient, but not necessary, condition for having a decreasing value of $\text{tr}(\rho^2)$ and, consequently, loss of coherence is that $h_{\alpha\beta}$ be real and positive. As a simple example, we can consider the case in which we have only one operator Q . Then,

$$\frac{d}{dt}\text{tr}(\rho^2) = -\text{tr}(\rho^2 Q^2 - \rho Q \rho Q). \quad (3.10)$$

If we diagonalize the density matrix and call $\{|i\rangle\}$ to the preferred basis in which ρ is diagonal, so that $\rho = \sum_i p_i |i\rangle\langle i|$, this equation becomes

$$\frac{d}{dt}\text{tr}(\rho^2) = -\sum_{ij} p_i |Q_{ij}|^2 (p_i - p_j) = -\sum_{i>j} |Q_{ij}|^2 (p_i - p_j)^2, \quad (3.11)$$

where $Q_{ij} = \langle i|Q|j\rangle$. We then see that provided that Q is not diagonal in the basis $\{|i\rangle\}$, $\frac{d}{dt}\text{tr}(\rho^2) < 0$, except for very specific states, such as the obvious $p_i = p_j$, which has maximum entropy.

There has been an interesting debate on the possible violations of energy and momentum conservation or locality in processes that do not lead to a factorizable $\$$ matrix. According to Gross [92] and Ellis *et al.* [93], a nonfactorizable $\$$ matrix allows for continuous symmetries whose associated generators are not conserved. In other words, “invariance principles are no longer equivalent to conservation laws” [93].

Let us illustrate this issue with the simple example [93] of two spin-1/2 particles in a state described by the density matrix $\rho_- = \frac{1}{4}(1 - \vec{s}_1 \vec{s}_2)$, where $\vec{s}_{1,2}$ are the spin vectors of the particles 1 and 2, respectively. This density matrix represents a pure state since $\text{tr}(\rho_-^2) = 1$. In fact, the two particles are in a rotationally invariant pure state with vanishing total spin. Assume that the final state can be obtained by a superscattering operator $\$$. Then, $\rho_+ = \$ \cdot \rho_-$ must have the form $\rho_+ = \frac{1}{4}(1 - \beta \vec{s}_1 \vec{s}_2)$, for it to conserve probability $\text{tr}(\rho_+) = 1$ and be rotationally invariant. Furthermore, since $\text{tr}(\rho_+^2) = (1 + 3\beta^2)/4 \leq 1$, we must have $\beta \leq 1$, the equality holding only when ρ_+ is a pure state. The initial state is such that $\text{tr}[(\vec{s}_1 + \vec{s}_2)^2 \rho_-] = -1$, which means that, in any given direction, there is initially a perfect anticorrelation between the spin of the two particles, so that the total spin vanishes. However, for the final state, $\text{tr}[(\vec{s}_1 + \vec{s}_2)^2 \rho_+] = 1 - \beta$. We then see that,

despite the rotational invariance of the states and the evolution, we will not obtain total anticorrelation in the final state and, hence, spin conservation, unless $\beta = 1$, i.e., unless quantum coherence is preserved.

In particular, these authors [92, 93] argued that energy and momentum conservation does not follow from Poincaré invariance. However, energy and momentum conservation is a consequence of the field equations in the asymptotic regions [88]. This issue also arises when the evolution of the density matrix is obtained by a differential equation whose integral leads to a nonfactorizable S operator. If this equation is assumed to be local on scales a bit larger than Planck length, then there appears a conflict between this pretended locality on the one hand and energy and momentum conservation on the other [94]. This violation of energy and momentum conservation comes from the high-energy modes, whose characteristic evolution times is of the same order as the size of the non-trivial topology region. Again, the existence of asymptotic regions would enforce this conservation and this can be effectively achieved if the propagating fields are regarded as low-energy ones and, therefore, with characteristic size l much larger the size r of the fluctuation. Furthermore, Unruh and Wald [95] analyzed simple non-Markovian toy models that lose quantum coherence and argued that conservation of energy and momentum need not be in conflict with causality and locality, in contrast with the claims of Ref. [94] (see also Ref. [96, 97]). Therefore, these topology fluctuations can be regarded as nonlocal in the length scale r , since, within this scale, the unitary S -matrix diagrams will be mixed (thus leading to a nonfactorizable S matrix), while from the low-energy point of view, the fluctuations are confined in a very small region so that they can be described as local effective interactions in a master differential equation as above. This relation will be the subject of the next two sections.

4 Quantum bath

Spacetime foam contains, according to the scenario above, highly nontrivial topological or causal configurations, which will introduce additional features in the description of the evolution of low-energy fields as compared with topologically trivial, globally hyperbolic manifolds. The analogy with fields propagating in a finite-temperature environment is compelling. Actually, despite the different conceptual and physical origin of the fluctuations, we will see that the effects of these two systems are not that different.

In order to build an effective theory that accounts for the propagation of low-energy fields in a foamlike spacetime, we will substitute the spacetime foam, in which we possibly have a minimum length because the notion of distance is not valid at such scale, by a fixed background with low-energy fields living on it. We will perform a 3+1 foliation of the effective spacetime that, for simplicity, will be regarded as flat, t denoting the time parameter and x the spatial coordinates. The gravitational fluctuations and the minimum length present in the original spacetime foam will be modeled by means of nonlocal interactions that relate spacetime points that are sufficiently close in the effective background, where a well-defined notion of distance exists [43, 44, 45] (for related ideas see also Refs. [98, 99] and for a review on stochastic gravity see Ref. [100]). Furthermore, these nonlocal interactions will be described in terms of local interactions as follows. Let $\{h_i[\phi; t]\}$ be a basis of local gauge-invariant interactions at the spacetime point (x, t) made

out of factors of the form $\ell_*^{2n(1+s)-4} [\phi(x, t)]^{2n}$, ϕ being the low-energy field strength of spin s . As a notational convention, each index i implies a dependence on the spatial position x by default; whenever the index i does not carry an implicit spatial dependence, it will appear underlined \underline{i} . Also, any contraction of indices (except for underlined ones) will entail an integral over spatial positions.

Influence functional.— The low-energy density $\rho[\phi, \varphi; t]$ at the time t in the field representation can be generally related to the density matrix at $t = 0$

$$\rho[\phi, \varphi; t] = \int D\phi' D\varphi' \$[\phi, \varphi; t | \phi', \varphi'; 0] \rho[\phi', \varphi'; 0], \quad (4.1)$$

which we will write in the compact form $\rho(t) = \$ (t) \cdot \rho(0)$. Here $\$(t)$ is the propagator for the density matrix and $D\phi \equiv \prod_x \phi(x, t)$. This propagator has the form

$$\$[\phi, \varphi; t | \phi', \varphi'; 0] = \int \mathcal{D}\phi \mathcal{D}\varphi e^{i\{S_0[\phi; t] - S_0[\varphi; t]\}} \mathcal{F}[\phi, \varphi; t], \quad (4.2)$$

where $\mathcal{F}[\phi, \varphi; t]$ is the so-called influence functional [101, 102, 103], $\mathcal{D}\phi \equiv \prod_{x,s} \phi(x, s)$ and these path integrals are performed over paths $\phi(s)$, $\varphi(s)$ such that at the end points match the values ϕ, φ at t and ϕ', φ' at $s = 0$. The influence functional $\mathcal{F}[\phi, \varphi; t]$ contains all the information about the interaction of the low-energy fields with spacetime foam. Let us now introduce another functional $\mathcal{W}[\phi, \varphi; t]$ that we will call influence action and such that $\mathcal{F}[\phi, \varphi; t] = \exp \mathcal{W}[\phi, \varphi; t]$. If the influence action $\mathcal{W}[\phi, \varphi; t]$ were equal to the zero, then we would have unitary evolution provided by a factorized superscattering matrix. However, \mathcal{W} does not vanish in the presence of gravitational fluctuations and, in fact, the nonlocal effective interactions will be modeled by terms in \mathcal{W} that follow the pattern

$$\int dt_1 \cdots dt_N v^{i_1 \cdots i_N}(t_1 \dots t_N) h_{i_1}[\phi; t_1] \cdots h_{i_N}[\phi; t_N]. \quad (4.3)$$

Here, $v^{i_1 \cdots i_N}(t_1 \dots t_N)$ are dimensionless complex functions that vanish for relative space-time distances larger than the length scale r of the gravitational fluctuations. If the gravitational fluctuations are smooth in the sense that they only involve trivial topologies or contain no horizons, the coefficients $v^{i_1 \cdots i_N}(t_1 \dots t_N)$ will be N -point propagators which, as such, will have infinitely long tails and the size of the gravitational fluctuations will be effectively infinite. In other words, we would be dealing with a local theory written in a nonstandard way. The gravitational origin of these fluctuations eliminate these long tails because of the presence of gravitational collapse and topology change. This means that, for instance, virtual black holes [54] will appear and disappear and horizons will be present throughout. As Padmanabhan [81, 82] has also argued, horizons induce nonlocal interactions of finite range since the Planckian degrees of freedom will be magnified by the horizon (because of an infinite redshift factor) thus giving rise to low-energy interactions as seen from outside the gravitational fluctuation. Virtual black holes represent a kind of components of spacetime foam that because of the horizons and their nontrivial topology will induce nonlocal interactions but, most probably, other fluctuations with complicated topology will warp spacetime in a similar way and the same magnification process will also take place.

The coefficients $v^{i_1 \dots i_N}(t_1 \dots t_N)$ can depend only on relative positions and not on the location of the gravitational fluctuation itself. The physical reason for this is conservation of energy and momentum: the fluctuations do not carry energy, momentum, or gauge charges. Thus, diffeomorphism invariance is preserved, at least at low-energy scales. One should not expect that at the Planck scale this invariance still holds. However, this violation of energy-momentum conservation is safely kept within Planck scale limits [95], where the processes will no longer be Markovian.

Finally, the coefficients $v^{i_1 \dots i_N}(t_1 \dots t_N)$ will contain a factor $[e^{-S(r)/2}]^N$, $S(r)$ being the Euclidean action of the gravitational fluctuation, which is of the order $(r/\ell_*)^2$. This is just an expression of the idea that inside large fluctuations, interactions that involve a large number of spacetime points are strongly suppressed. As the size of the fluctuation decreases, the probability for events in which three or more spacetime points are correlated increases, in close analogy with the kinetic theory of gases: the higher the density of molecules in the gas, the more probable is that a large number of molecules collide at the same point. The expansion parameter in this example is typically the density of molecules. In our case, the natural expansion parameter is the transition amplitude. It is given by the square root of the two-point transition probability which in the semiclassical approximation is of the form $e^{-S(r)}$.

Thus the N -local interaction term in \mathcal{W} will be of order $[e^{-S(r)/2}]^N$. In the weak-coupling approximation, i.e., up to second order in the expansion parameter, the triloal and higher effective interactions do not contribute. The terms corresponding to $N = 0, 1$ are local and can be absorbed in the bare action (note that the coefficient v is constant and that the coefficients $v^{i_1}(t_1)$ cannot depend on spacetime positions because of diffeomorphism invariance). Consequently, we can write the action functional \mathcal{W} as a bilocal whose most general form is [102]

$$\begin{aligned} \mathcal{W}[\phi, \varphi; t] = & -\frac{1}{2} \int_0^t ds \int_0^s ds' \{h_i[\phi; s] - h_i[\varphi; s]\} \\ & \times \{v^{ij}(s-s')h_j[\phi; s'] - v^{ij}(s-s')^*h_j[\varphi; s']\}, \end{aligned} \quad (4.4)$$

where we have renamed $v^{ij}(s, s')$ as $v^{ij}(s-s')$, and without loss of generality we have set $s > s'$. This complex coefficient is Hermitian in the pair of indices ij and depends on the spatial positions $x_{\underline{i}}$ and $x_{\underline{j}}$ only through the relative distance $|x_{\underline{i}} - x_{\underline{j}}|$. It is of order $e^{-S(r)}$ and is concentrated within a spacetime region of size r .

Let us now decompose $v^{ij}(\tau)$ in terms of its real and imaginary parts as

$$v^{ij}(\tau) = c^{ij}(\tau) + i\dot{f}^{ij}(\tau), \quad (4.5)$$

where $c^{ij}(\tau)$ and $f^{ij}(\tau)$ are real and symmetric, and the overdot denotes time derivative. The imaginary part is antisymmetric in the exchange of i, τ and $j, -\tau$ and has been written as a time derivative for convenience, since this choice does not involve any restriction. The f term can then be integrated by parts to obtain

$$\begin{aligned} \mathcal{W}[\phi, \varphi; t] = & -\frac{1}{2} \int_0^t ds \int_0^s ds' c^{ij}(s-s') \{h_i[\phi; s] - h_i[\varphi; s]\} \{h_j[\phi; s'] - h_j[\varphi; s']\} \\ & -\frac{i}{2} \int_0^t ds \int_0^s ds' \dot{f}^{ij}(s-s') \{h_i[\phi; s] - h_i[\varphi; s]\} \{\dot{h}_j[\phi; s'] + \dot{h}_j[\varphi; s']\}. \end{aligned} \quad (4.6)$$

In this integration, we have ignored surface terms that contribute, at most, to a finite renormalization of the bare low-energy Hamiltonian.

The functions $f^{ij}(\tau)$ and $c^{ij}(\tau)$ characterize spacetime foam in our effective description but, under fairly general assumptions, the characterization can be carried out by a smaller set of independent functions. In what follows we will simplify this set. With this aim, we first write $f^{ij}(\tau)$ and $c^{ij}(\tau)$ in terms of their spectral counterparts $\tilde{f}^{ij}(\omega)$ and $\tilde{c}^{ij}(\omega)$. Lorentz invariance and spatial homogeneity implies that $f^{ij}(\tau)$ and $c^{ij}(\tau)$ must have the form

$$f^{ij}(\tau) = \int_0^\infty d\omega \tilde{f}^{ij}(\omega) 8\pi \frac{\sin(\omega|x_i - x_j|)}{\omega|x_i - x_j|} \cos(\omega\tau), \quad (4.7)$$

$$c^{ij}(\tau) = \int_0^\infty d\omega \tilde{c}^{ij}(\omega) 8\pi \frac{\sin(\omega|x_i - x_j|)}{\omega|x_i - x_j|} \cos(\omega\tau), \quad (4.8)$$

for some real functions $\tilde{f}^{ij}(\omega)$ and $\tilde{c}^{ij}(\omega)$. It seems reasonable to assume a kind of equanimity principle by which spacetime foam produces interactions whose intensity does not depend on the pair of interactions h_i itself but on its independent components for each mode, i.e., that the spectral interaction is given by products of functions $\chi^i(\omega)$:

$$\tilde{f}^{ij}(\omega) = \chi^i(\omega)\chi^j(\omega), \quad (4.9)$$

$$\tilde{c}^{ij}(\omega) = g(\omega)\chi^i(\omega)\chi^j(\omega), \quad (4.10)$$

where $g(\omega)$ is a function that, together with $\chi^i(\omega)$, fully characterize spacetime foam under these assumptions.

Then, $f^{ij}(\tau)$ and $c^{ij}(\tau)$ can be written as

$$f^{ij}(\tau) = \int_0^\infty d\omega G^{ij}(\omega) \cos(\omega\tau), \quad (4.11)$$

$$c^{ij}(\tau) = \int_0^\infty d\omega g(\omega) G^{ij}(\omega) \cos(\omega\tau), \quad (4.12)$$

with

$$G^{ij}(\omega) = 8\pi \frac{\sin(\omega|x_i - x_j|)}{\omega|x_i - x_j|} \chi^i(\omega)\chi^j(\omega). \quad (4.13)$$

The functions $\chi^i(\omega)$ can be interpreted as the spectral effective couplings between spacetime foam and low-energy fields. Since $v^{ij}(\tau)$ is of order $e^{-S(r)}$ and is concentrated in a region of linear size r , the couplings $\chi^i(\omega)$ will have dimensions of length, will be of order $e^{-S(r)/2}r$, and will induce a significant interaction for all frequencies ω up to the natural cutoff r^{-1} . On the other hand, the function $g(\omega)$ has dimensions of inverse length and must be of order r^{-1} . Actually, this function must be almost flat in the frequency range $(0, r^{-1})$ to ensure that all the modes contribute significantly to all bilocal interactions. As we will see, the function $g(\omega)$ also admits a straightforward interpretation in terms of the mean occupation number for the mode of frequency ω .

Once we have computed the influence functional \mathcal{F} , it is possible to obtain the master equation that governs the evolution of the density of low-energy fields, although we will not follow this procedure here. We postpone the derivation of the full master equation until next section.

The bilocal effective interaction does not lead to a unitary evolution. The reason for this is that it is not sufficient to know the fields and their time derivatives at an instant of time in order to know their values at a later time: we need to know the history of the system, at least for a time r . There exist different trajectories that arrive at a given configuration $(\phi, \dot{\phi})$. The future evolution depends on these past trajectories and not only on the values of ϕ and $\dot{\phi}$ at that instant of time. Therefore, the system cannot possess a well-defined Hamiltonian vector field and suffers from an intrinsic loss of predictability [104].

This can be easily seen if we restrict to the case in which $f^{ij}(\tau)$ vanishes, i.e., $v^{ij}(\tau) = c^{ij}(\tau)$. Then, the influence functional \mathcal{F}_c is the characteristic functional of a Gaussian probability functional distribution, i.e., it can be written as

$$\mathcal{F}_c[\phi, \varphi; t] = \int \mathcal{D}\alpha e^{-\frac{1}{2} \int_0^t ds \int_0^s ds' \gamma_{ij}(s-s') \alpha^i(s) \alpha^j(s')} e^{i \int_0^t ds \alpha^i(s) \{h_i[\phi; s] - h_i[\varphi; s]\}}. \quad (4.14)$$

Here, the continuous matrix $\gamma_{ij}(s-s')$ is the inverse of $c^{ij}(s-s')$, i.e.,

$$\int ds'' \gamma_{ik}(s-s'') c^{kj}(s''-s') = \delta_i^j \delta(s-s'). \quad (4.15)$$

Then, in this case, the propagator $\mathcal{S}(t)$ has the form

$$\mathcal{S}(t) = \int \mathcal{D}\alpha P[\alpha] \mathcal{S}_\alpha(t), \quad (4.16)$$

where $\mathcal{S}_\alpha(t)$ is just a factorizable propagator associated with unitary evolution governed by the action $S_0 + \int \alpha^i h_i$ and

$$P[\alpha] = e^{-\frac{1}{2} \int_0^t ds \int_0^s ds' \gamma_{ij}(s-s') \alpha^i(s) \alpha^j(s')}. \quad (4.17)$$

Therefore, $\mathcal{S}(t)$ is just the average with Gaussian weight $P[\alpha]$ of the unitary propagator $\mathcal{S}_\alpha(t)$.

Note that the quadratic character of the distribution for the fields α^i is a consequence of the weak-coupling approximation, which keeps only the bilocal term in the action. Higher-order terms would introduce deviations from this noise distribution. The nonunitary nature of the bilocal interaction has been encoded inside the fields α^i , so that, when insisting on writing the system in terms of unitary evolution operators, an additional sum over the part of the system that is unknown naturally appears. Note also that we have a different field α^i for each kind of interaction h_i . Thus, we have transferred the nonlocality of the low-energy field ϕ to the set of fields α^i , which are nontrivially coupled to it and that represent spacetime foam.

Semiclassical diffusion.— We can see that the limit of vanishing $f^{ij}(\tau)$, with nonzero $c^{ij}(\tau)$ (and therefore real $v^{ij}(\tau)$), is a kind of semiclassical approximation since, in this limit, one ignores the quantum nature of the gravitational fluctuations. Indeed, the fields α^i represent spacetime foam but, as we have seen, the path integral for the whole system does not contain any trace of the dynamical character of the fields α^i . It just contains a Gaussian probability distribution for them. The path integral above can then

be interpreted as a Gaussian average over the classical noise sources α^i . Classicality here means that we can keep the sources α^i fixed, ignoring the noise commutation relations, and, at the end of the calculations, we just average over them.

The low-energy density matrix ρ then satisfies the following master equation [43, 44, 45]

$$\dot{\rho} = -i[H_0, \rho] - \int_0^\infty d\tau c^{ij}(\tau)[h_i, [h_j^1(-\tau), \rho]], \quad (4.18)$$

where $h_j^1(-\tau) = e^{-iH_0\tau}h_j e^{iH_0\tau}$. Since $e^{iH_0\tau} = 1 + O(\tau/l)$, the final form of the master equation for a low-energy system subject to gravitational fluctuations treated as a classical environment and at zeroth order in r/l (the effect of higher order terms in r/l will be thoroughly studied together with the quantum effects) is

$$\dot{\rho} = -i[H_0, \rho] - \int_0^\infty d\tau c^{ij}(\tau)[h_i, [h_j, \rho]] \quad (4.19)$$

(for similar approaches yielding this type of master equation see also Refs. [94, 105, 106]).

The first term gives the low-energy Hamiltonian evolution that would also be present in the absence of fluctuations. The second term is a diffusion term which will be responsible for the loss of coherence (and the subsequent increase of entropy). It is a direct consequence of the foamlike structure of spacetime and the related existence of a minimum length. Note there is no dissipation term. This term is usually present in order to preserve the commutation relations under time evolution. However, we have considered the classical noise limit, i.e., the noise α has been considered as a classical source and the commutation relations are automatically preserved. We will see that the dissipation term, apart from being of quantum origin, is r/l times smaller than the diffusion term and we have only considered the zeroth order approximation in r/l .

The characteristic decoherence time τ_d induced by the diffusion term can be easily calculated. Indeed, the interaction Hamiltonian density h_i is of order $\ell_*^{-4}(\ell_*/l)^{2n_{\underline{i}}(1+s_{\underline{i}})}$ and $c^{ij}(\tau)$ is of order $e^{-S(r)}$. Furthermore, the diffusion term contains one integral over time and two integrals over spatial positions. The integral over time and the one over relative spatial positions provide a factor r^4 , since $c^{ij}(\tau)$ is different from zero only in a spacetime region of size r^4 , and the remaining integral over global spatial positions provides a factor l^3 , the typical low-energy spatial volume. Putting everything together, we see that the diffusion term is of order $l^{-1}\epsilon^2 \sum_{\underline{i}\underline{j}} (\ell_*/l)^{\eta_{\underline{i}}+\eta_{\underline{j}}}$, with $\eta_{\underline{i}} = 2n_{\underline{i}}(1+s_{\underline{i}}) - 2$ and $\epsilon = e^{-S(r)/2}(r/\ell_*)^2$. This quantity defines the inverse of the decoherence time τ_d . Therefore, the ratio between the decoherence time τ_d and the low-energy length scale l is

$$\tau_d/l \sim \epsilon^{-2} \left[\sum_{\underline{i}\underline{j}} (\ell_*/l)^{\eta_{\underline{i}}+\eta_{\underline{j}}} \right]^{-1}. \quad (4.20)$$

Because of the exponential factor in ϵ , only the gravitational fluctuations whose size is very close to Planck length will give a sufficiently small decoherence time. Slightly larger fluctuations will have a very small effect on the unitarity of the effective theory. For the interaction term that corresponds to the mass of a scalar field, the parameter η vanishes and, consequently, $\tau_d/l \sim \epsilon^{-2}$. Thus, the scalar mass term will lose coherence faster than any other interaction. Indeed, for higher spins and/or powers of the field strength, $\eta \geq 1$

and therefore τ_d/l increases by powers of l/ℓ_* . For instance, the next relevant decoherence time corresponds to the scalar-fermion interaction term $\phi^2\bar{\psi}\psi$, which has an associated decoherence ratio $\tau_d/l \sim \epsilon^{-2}l/\ell_*$. We see that the decoherence time for the mass of scalars is independent of the low-energy length scale and, for gravitational fluctuations of size close to Planck length, ϵ may be not too small so that scalar masses may lose coherence fairly fast, maybe in a few times the typical evolution scale. Hawking has argued [54] that this might be the reason for not observing the Higgs particle. Higher power and/or spin interactions will lose coherence much slower but for sufficiently high energies l^{-1} , although much smaller than the gravitational fluctuations energy r^{-1} , the decoherence time may be small enough. This means that quantum fields will lose coherence faster for higher-energy regimes. Hawking has also suggested that loss of quantum coherence might be responsible for the vanishing of the θ angle in quantum chromodynamics [54].

Spacetime foam as a quantum bath.— As we have briefly mentioned before, considering that the coefficients v^{ij} are real amounts to ignore the quantum dynamical nature of spacetime foam, paying attention only to its statistical properties. In what follows, we will study these quantum effects and show that spacetime foam can be effectively described in terms of a quantum thermal bath with a nearly Planckian temperature that has a weak interaction with low-energy fields. As a consequence, other effects, apart from loss of coherence, such as Lamb and Stark transition-frequency shifts, and quantum damping, characteristic of systems in a quantum environment [107, 108], naturally appear as low-energy predictions of this model [43, 44, 45].

Let us consider a Hamiltonian of the form

$$H = H_0 + H_{\text{int}} + H_{\text{b}}. \quad (4.21)$$

H_0 is the bare Hamiltonian that represents the low-energy fields and H_{b} is the Hamiltonian of a bath that, for simplicity, will be represented by a real massless scalar field. The interaction Hamiltonian will be of the form $H_{\text{int}} = \xi^i h_i$, where the noise operators ξ^i are given by

$$\xi^i(x, t) = \int dx' \chi^i(x - x') p(x', t). \quad (4.22)$$

Here, $p(x, t)$ is the momentum of the bath scalar field whose mode decomposition has the form

$$p(x, t) = i \int dk \sqrt{\omega} [a^\dagger(k) e^{i(\omega t - kx)} - a(k) e^{-i(\omega t - kx)}], \quad (4.23)$$

$\omega = \sqrt{k^2}$, and a and a^\dagger are, respectively, the annihilation and creation operators associated with the bath; $\chi^i(y)$ represent the couplings between the low-energy field and the bath in the position representation. Since we are trying to construct a model for spacetime foam, we will assume that the couplings $\chi^i(y)$ will be concentrated on a region of radius r and that they are determined by the spectral couplings $\chi^i(\omega)$ introduced before:

$$\chi^i(y) = \int \frac{dk}{\omega} \chi^i(\omega) \cos(ky). \quad (4.24)$$

The influence functional in this case has the form [102]

$$\mathcal{F}[\phi, \varphi; t] = \int Dq' DQ' \rho_{\text{b}}[q', Q'; 0] \int Dq DQ e^{i\{S_{\text{b}}[q; t] - S_{\text{b}}[Q; t]\}} e^{i\{S_{\text{int}}[\phi, q; t] - S_{\text{int}}[\varphi, Q; t]\}}, \quad (4.25)$$

where these path integrals are performed over paths $q(s)$ and $Q(s)$ such that at the initial time match the values q' and Q' and S_b is the action of the bath.

If we assume that the bath is in a stationary, homogeneous, and isotropic state, this influence functional can be computed to yield an influence action \mathcal{W} of the form discussed above. Furthermore, for a thermal state with temperature $T \sim 1/r$, the function $g(\omega)$ has the form

$$g(\omega) = \omega[N(\omega) + 1/2], \quad (4.26)$$

where $N(\omega) = [\exp(\omega/T) - 1]^{-1}$ is the mean occupation number of the quantum thermal bath corresponding to the frequency ω . Recall that the functions $G^{ij}(\omega)$ and, hence, $f^{ij}(\tau)$ are uniquely determined by the couplings $\chi^{\dot{z}}(\omega)$. In particular, they are completely independent of the state of the bath or the system. All the relevant information about the bath is encoded in the function $g(\omega)$.

With this procedure, we see that spacetime foam can be represented by a quantum bath determined by $g(\omega)$ that interacts with the low-energy fields by means of the couplings $\chi^{\dot{z}}(\omega)$ which characterize spacetime foam, in the sense that both systems produce the same low-energy effects.

This model that we have proposed is particularly suited to the study of low-energy effects produced by simply connected topology fluctuations such as closed loops of virtual black holes [54]. Virtual black holes will not obey classical equations of motion but will appear as quantum fluctuations of spacetime and thus will become part of the spacetime foam as we have discussed. Particles could fall into these black holes and be re-emitted. The scattering amplitudes of these processes [54, 90] could be interpreted as being produced by nonlocal effective interactions that would take place inside the fluctuations and the influence functional obtained above could then be interpreted as providing the evolution of the low-energy density matrix in the presence of a bath of ubiquitous quantum topological fluctuations of the virtual-black-hole type.

Wormholes and coherence. — Euclidean solutions of the wormhole type were obtained for a variety of matter contents (see, e.g., [109, 110, 111]). Quantum solutions to the Wheeler-DeWitt equation that represent wormholes can be found in Refs. [112, 113, 114, 115, 116, 117]. These solutions allowed the calculation of the effective interactions that they introduce in low-energy physics [112, 114, 115, 116, 118, 119, 120].

Wormholes do not seem to induce loss of coherence despite the fact that they render spacetime multiply connected [121, 122]. The reason why they seem to preserve coherence is that, in the dilute gas approximation, they join spacetime regions that may be far apart from each other and therefore, both wormhole mouths must be delocalized, i.e., the multiply-connectedness requires energy and momentum conservation in both spacetime regions separately. In this way, wormholes can be described as bilocal interactions whose coefficients v^{ij} do not depend on spacetime positions. Diffeomorphism invariance on each spacetime region also requires the spacetime independence of v^{ij} . This can also be seen by analyzing these wormholes from the point of view of the universal covering manifold, which is, by definition, simply connected. Here, each wormhole is represented by two boundaries located at infinity and suitably identified. This identification is equivalent to introducing coefficients v^{ij} that relate the bases of the Hilbert space of wormholes in both regions of the universal covering manifold. Since v^{ij} are just the coefficients in a change of basis, they will be constant. As a direct consequence, the correlation time for the fields

α^i is infinite. This means that the fields α^i cannot be interpreted as noise sources that are Gaussian distributed at each spacetime point independently. Rather, they are infinitely coherent thus giving rise to superselection sectors. The Gaussian distribution to which they are subject is therefore global, spacetime independent. The only effect of wormholes is thus introducing local interactions with unknown but spacetime independent coupling constants [121, 122, 123]. The spacetime independence implies that, once an experiment to determine one such constant is performed, it will remain with the obtained value forever, in sharp contrast with those induced by simply-connected topological fluctuations such as virtual black holes. In this way and because of the infinite-range correlations induced by wormholes, which forbid the existence of asymptotic regions necessary to analyze scattering processes, the loss of coherence produced by these fluctuations should actually be ascribed to the lack of knowledge of the initial state or, in other words, to the impossibility of preparing arbitrarily pure quantum states [121].

One could also expect some effects originated in their quantum nature. However, the coefficients v^{ij} are spacetime independent. This means that c^{ij} are constant and, consequently, $\tilde{c}^{ij}(\omega) \sim \delta(\omega)$. As we have argued, $\tilde{c}^{ij}(\omega)$ and $\tilde{f}^{ij}(\omega)$ are related by a nearly flat function so that $\tilde{f}^{ij}(\omega) \sim \delta(\omega)$ as well. This in turn implies that f^{ij} is also constant and $\dot{f}^{ij} = 0$, therefore concluding that v^{ij} is real. We have already argued that in the case of real v^{ij} , no quantum effects will show up.

Wormhole spacetimes do not lead, strictly speaking, to loss of quantum coherence although global hyperbolicity does not hold. On the other hand, the difficulties in quantum gravity with unitary propagation mainly come from the quantum field theory axiom of asymptotic completeness [42, 124], which is closely related to global hyperbolicity. Indeed, in order to guarantee asymptotic completeness, it is necessary that the expectation value of the fields at any spacetime position be determined by their values at a Cauchy surface at infinity. Topologically nontrivial spacetimes however are not globally hyperbolic in general and therefore do not admit a foliation in Cauchy surfaces. Let us have a closer look at this issue.

Gravitational entropy, which is closely related to the loss of quantum coherence, has its origin in the existence of two-dimensional spheres in Euclidean space that cannot be homotopically contracted to a point, i.e., with nonvanishing second Betti number. These two-dimensional surfaces become fixed points of the timelike Killing vector, so that global hyperbolicity is lost. A well-known example (for other more sophisticated examples see Ref. [125]) is a Schwarzschild black hole whose Euclidean sector is described by the metric

$$ds^2 = f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega_2^2, \quad (4.27)$$

with $f(r) = 1 - 2\ell_*^2 m/r$, m being the black hole mass. In order to make this solution regular, we consider the region $r \geq 2\ell_*^2 m$ and set t to be periodic with period $\beta = 8\pi\ell_*^2 m$. The surface defined by $r = 2\ell_*^2 m$ is a fixed point of the Killing vector ∂_t . Thus, we have a spacetime with the topology of $\mathbb{R}^2 \times S^2$, so that $B_2 = 1$. As we will see below, it is the existence of this surface that accounts for the entropy of this spacetime. This does not mean that it is localized in the surface itself. Rather, it is a global quantity characteristic of the whole spacetime manifold. The Euclidean action of this solution is given by the sum of the contributions I_{fp} , I_∞ of the two surface terms at $r = 2\ell_*^2 m$ and $r = \infty$. In the semiclassical approximation, the partition function is given by $Z = e^{-I_{\text{fp}} - I_\infty}$. Taking into account that the entropy is $S = \ln Z - \beta E$ and that βE is precisely the surface

term at infinity $\beta E = -I_\infty$, we conclude that the entropy is given by the surface term at $r = 2\ell_*^2 m$, $S = -I_{\text{fp}} = 4\pi\ell_*^2 m^2$, as is well-known.

In the wormhole case, the second Betti number is zero and the first and third Betti numbers are equal. For a spacetime with a single wormhole, $B_1 = B_3 = 1$. This means that there exists one circle that cannot be homotopically contracted to a point and that there also exists one three-sphere that is not homotopic to a point, but all two-spheres are contractible. Regular solutions of this sort can be identified with the wormhole throat. The only contributions to the Euclidean action in this case come from the asymptotic regions, which is precisely the term that we have to subtract from $\ln Z$, in the semiclassical approximation, in order to calculate the gravitational entropy. Thus, wormholes have vanishing entropy despite the fact that they are not globally hyperbolic. From the point of view of their universal covering manifold, a wormhole is represented by two three-surfaces whose contribution to the action are equal in absolute value but with opposite sign because of their reverse orientations, thus leaving only the asymptotic contribution, irrelevant as far as the entropy is concerned (for a different approach see Ref. [126]). The striking difference between wormholes and virtual black holes is precisely the formation of horizons which has no counterpart in the wormhole case. This is closely related to the issue of the infinite-range spacetime correlations established by wormholes versus the finite size of the regions occupied by virtual black holes or quantum time machines, for instance.

5 Low-energy effective evolution

As we have already mentioned, from the influence functional obtained in the previous section, we can obtain the master equation satisfied by the low-energy density matrix, although here we will follow a different procedure: We will derive the master equation in the canonical formalism from von Neumann equation for the joint system of the low-energy fields plus the effective quantum bath coupled to them that accounts for the effects of spacetime foam.

Master equation.— It is easy to see that the function $f^{ij}(\tau)$ given in Eq. (4.11) determines the commutation relations at different times of the noise variables. Indeed, taking into account the commutation relations for the annihilation and creation operators a and a^\dagger , i.e.,

$$[a(k), a(k')] = 0, \quad [a(k), a^\dagger(k')] = \delta(k - k'), \quad (5.1)$$

we obtain by direct calculation the relation

$$[\xi^i(t), \xi^j(t')] = i \frac{d}{dt} f^{ij}(t - t'), \quad (5.2)$$

Similarly, the function $c^{ij}(\tau)$ of Eq. (4.12) determines the average of the anticommutator of the noise variables,

$$\frac{1}{2} \langle [\xi^i(t), \xi^j(t')]_+ \rangle = c^{ij}(t - t'), \quad (5.3)$$

where the average of any operator Q has been defined as $\langle Q \rangle \equiv \text{tr}_b(Q\rho_b)$, provided that the bath is in a stationary, homogeneous, and isotropic state determined by $g(\omega)$, i.e.,

$$\langle a(k) \rangle = 0, \quad \langle a(k)a(k') \rangle = 0, \quad \langle a^\dagger(k)a(k') \rangle = [g(\omega)/\omega - 1/2]\delta(k - k'). \quad (5.4)$$

We are now ready to write down the master equation for the low-energy density matrix. We will describe the whole system (low-energy field and bath) by a density matrix $\rho_T(t)$. We will assume that, initially, the low energy fields and the bath are independent, i.e., that at the time $t = 0$

$$\rho_T(0) = \rho(0) \otimes \rho_b. \quad (5.5)$$

If the low-energy fields and the bath do not decouple at any time, an extra renormalization term should be added to the Hamiltonian. In the interaction picture, the density matrix has the form

$$\rho_T^I(t) = U^\dagger(t)\rho_T(t)U(t), \quad (5.6)$$

with $U(t) = U_0(t)U_b(t)$, where $U_0(t) = e^{-iH_0t}$ and $U_b(t) = e^{-iH_bt}$. It obeys the equation of motion

$$\dot{\rho}_T^I(t) = -i[\xi^i(t)h_i^I(t), \rho_T^I(t)]. \quad (5.7)$$

Here,

$$\xi^i(t) = U^\dagger(t)\xi^i U(t) = U_b^\dagger(t)\xi^i U_b(t), \quad (5.8)$$

$$h_i^I(t) = U^\dagger(t)h_i U(t) = U_0^\dagger(t)h_i U_0(t). \quad (5.9)$$

Integrating this evolution equation and introducing the result back into it, tracing over the variables of the bath, defining $\rho^I(t) \equiv \text{tr}_b[\rho_T^I(t)]$, and noting that $\text{tr}_b[\xi^i(t)h_i^I(t)\rho_T^I(t_0)] = 0$, we obtain

$$\dot{\rho}^I(t) = - \int_{t_0}^t dt' \text{tr}_b \{ [\xi^i(t)h_i^I(t), [\xi^j(t')h_j^I(t'), \rho^I(t')]] \}. \quad (5.10)$$

In the weak-coupling approximation, which implies that $\xi^i h_i$ is much smaller than H_0 and H_b (this is justified since it is of order ϵ), we assume that the bath density matrix does not change because of the interaction, so that $\rho_T^I(t) = \rho^I(t) \otimes \rho_b$. The error introduced by this substitution is of order ϵ and ignoring it in the master equation amounts to keep terms only up to second order in this parameter. Since $[\xi^i(t), h_j^I(t')] = 0$ because $[\xi^i, h_j] = 0$, the right hand side of this equation can be written in the following way

$$- \int_0^t dt' \{ c^{ij}(t-t')[h_i^I(t), [h_j^I(t'), \rho^I(t')]] + \frac{i}{2} f^{ij}(t-t')[h_i^I(t), [h_j^I(t'), \rho^I(t')]_+ \}. \quad (5.11)$$

The Markov approximation allows the substitution of $\rho^I(t')$ by $\rho^I(t)$ in the master equation because the integral over t' will get a significant contribution from times t' that are close to t due to the factors $f^{ij}(t-t')$ and $c^{ij}(t-t')$ and because, in this interval of time, the density matrix ρ^I will not change significantly. Indeed, the typical evolution time of ρ^I is the low-energy time scale l , which will be much larger than the time scale r associated with the bath. If we perform a change of the integration variable from t' to $\tau = t - t'$, write

$$\rho^I(t') = \rho^I(t - \tau) = \rho^I(t) - \tau \dot{\rho}^I(t) + O(\tau^2), \quad (5.12)$$

and introduce this expression in the master equation above, we easily see that the error introduced by the Markovian approximation is of order ϵ^2 , i.e., it amounts ignore a term of order ϵ^4 . The upper integration limit t in both integrals can be substituted by ∞ for evolution times t much larger than the correlation time r , because of the factors $f^{ij}(\tau)$ and $c^{ij}(\tau)$ that vanish for $\tau > r$.

Then, after an integration by parts of the f term, and transforming the resulting master equation back to the Schrödinger picture we obtain

$$\dot{\rho} = -i[H'_0, \rho] - \frac{i}{2} \int_0^\infty d\tau f^{ij}(\tau) [h_i, [\dot{h}_j^1(-\tau), \rho]_+] - \int_0^\infty d\tau c^{ij}(\tau) [h_i, [h_j^1(-\tau), \rho]], \quad (5.13)$$

where $H'_0 = H_0 - \frac{1}{2}f^{ij}(0)h_i h_j$ is just the original low-energy Hamiltonian plus a finite renormalization originated in the integration by parts of the f term. It can be checked that the low-energy density matrix $\rho(t)$ obtained by means of the influence functional \mathcal{F} is indeed a solution of this master equation.

Before discussing this equation in full detail, let us first study the classical noise limit. With this aim, let us introduce the parameter

$$\sigma = \int dk' [a(k), a^\dagger(k')], \quad (5.14)$$

which is equal to 1 for quantum noise and 0 for classical noise. Then, the f term is proportional to σ and therefore vanishes in the classical noise limit. On the other hand, the function $g(\omega)$ becomes $g(\sigma\omega)$ when introducing the parameter σ . In the limit $\sigma \rightarrow 0$, it acquires the value $g(0)$ which is a constant of order $1/r$. Therefore, $c^{ij}(\tau)$ becomes in this limit $c_{\text{class}}^{ij}(\tau) = g(0)f^{ij}(\tau)$. Also, the renormalization term of the low-energy Hamiltonian vanishes in this limit. In this way, we have arrived at the same master equation that we obtained in the previous section. This is not surprising because the origin of the f term is precisely the noncommutativity of the noise operators, i.e., its quantum nature, while the c_{class} term actually contains the information about the state of the bath. In the case of a thermal bath, $g(0)$ is precisely the temperature of the bath. At zeroth order in r/l , the master equation for classical noise then acquires the form

$$\dot{\rho} = -i[H_0, \rho] - \int_0^\infty d\tau c_{\text{class}}^{ij}(\tau) [h_i, [h_j, \rho]]. \quad (5.15)$$

Low-energy effects.— Let us now analyze the general master equation, valid up to second order in ϵ , that takes into account the quantum nature of the gravitational fluctuations. These contributions will be fairly small in the low-energy regime, but may provide interesting information about the higher-energy regimes in which l may be of the order of a few Planck lengths and for which the weak-coupling approximation is still valid. In order to see these contributions explicitly, let us further elaborate the master equation. In terms of the operator L_0 defined as $L_0 \cdot A = [H_0, A]$ acting of any low-energy operator A , the time dependent interaction $h_j^1(-\tau)$ can be written as

$$h_j^1(-\tau) = e^{-iL_0\tau} h_j. \quad (5.16)$$

The interaction h_j can be expanded in eigenoperators $h_{j\Omega}^\pm$ of the operator L_0 , i.e.,

$$h_j = \int d\mu_\Omega (h_{j\Omega}^+ + h_{j\Omega}^-), \quad (5.17)$$

with $L_0 \cdot h_{j\Omega}^\pm = \pm \Omega h_{j\Omega}^\pm$ and $d\mu_\Omega$ being an appropriate spectral measure, which is naturally cut off around the low-energy scale l^{-1} . This expansion always exists provided that the eigenstates of H_0 form a complete set. Then, $h_j^1(-\tau)$ can be written as

$$h_j^1(-\tau) = \int d\mu_\Omega (e^{-i\Omega\tau} h_{j\Omega}^+ + e^{i\Omega\tau} h_{j\Omega}^-). \quad (5.18)$$

It is also convenient to define the new interaction operators for each low-energy frequency Ω

$$h_{j\Omega}^1 = h_{j\Omega}^+ - h_{j\Omega}^-, \quad h_{j\Omega}^2 = h_{j\Omega}^+ + h_{j\Omega}^-. \quad (5.19)$$

The quantum noise effects are reflected in the master equation through the term proportional to $f^{ij}(\tau)$ and the term proportional to $c^{ij}(\tau)$, both of them integrated over $\tau \in (0, \infty)$. Because of these incomplete integrals, each term provides two different kinds of contributions whose origin can be traced back to the well-know formula

$$\int_0^\infty d\tau e^{i\omega\tau} = \pi\delta(\omega) + \mathcal{P}(i/\omega), \quad (5.20)$$

where \mathcal{P} is the Cauchy principal part [127].

The master equation can then be written in the following form

$$\dot{\rho} = -(iL'_0 + L_{\text{diss}} + L_{\text{diff}} + iL_{s-1}) \cdot \rho, \quad (5.21)$$

where the meaning of the different terms are explained in what follows.

The first term $-iL'_0 \cdot \rho$, with $L'_0 \cdot \rho = [H'_0, \rho]$, is responsible for the renormalized low-energy Hamiltonian evolution. The renormalization term is of order ε^2 as compared with the low-energy Hamiltonian H_0 , where $\varepsilon^2 = \epsilon^2 \sum_{\underline{i}\underline{j}} (\ell_*/l)^{\eta_{\underline{i}} + \eta_{\underline{j}}}$ and, remember, $\eta_{\underline{i}} = 2n_{\underline{i}}(1 + s_{\underline{i}}) - 2$ is a parameter specific to each kind of interaction term h_i .

The dissipation term

$$L_{\text{diss}} \cdot \rho = \frac{\pi}{4} \int d\mu_\Omega \Omega G^{ij}(\Omega) [h_i, [h_{j\Omega}^1, \rho]_+] \quad (5.22)$$

is necessary for the preservation in time of the low-energy commutators in the presence of quantum noise. As we have seen, it is proportional to the commutator between the noise creation and annihilation operators and, therefore, vanishes in the classical noise limit. Its size is of order $\varepsilon^2 r/l^2$.

The diffusion process is governed by

$$L_{\text{diff}} \cdot \rho = \frac{\pi}{2} \int d\mu_\Omega g(\Omega) G^{ij}(\Omega) [h_i, [h_{j\Omega}^2, \rho]], \quad (5.23)$$

which is of order ε^2/l .

The next term provides an energy shift which can be interpreted as a mixture of a gravitational ac Stark effect and a Lamb shift by comparison with its quantum optics analog [107, 108]. Its expression is

$$L_{s-1} = - \int d\mu_\Omega \mathcal{P} \int_0^\infty d\omega \frac{\Omega}{\omega^2 - \Omega^2} G^{ij}(\omega) \left\{ g(\omega) [h_i, [h_{j\Omega}^1, \rho]] + \frac{\Omega}{2} [h_i, [h_{j\Omega}^2, \rho]_+] \right\}. \quad (5.24)$$

The second term is of order $\varepsilon^2 r^2/l^3$, which is fairly small. However, the first term will provide a significant contribution of order $\varepsilon^2 r/l^2[\ln(l/r)+1]$. This logarithmic dependence on the relative scale is indeed characteristic of the Lamb shift [107, 108, 128]. As we have argued the function $g(\omega)$ must be fairly flat in the whole range of frequencies up to the cutoff $1/r$ and be of order $1/r$ in order to reproduce the appropriate correlations $c^{ij}(\tau)$. A thermal bath, for instance, produces a function $g(\omega)$ with the desired characteristics, at least at the level of approximation that we are considering. In this specific case, it can be seen that the logarithmic contribution to the energy shift is not present and it would only appear in the zero temperature limit. However, since we are modeling spacetime foam with this thermal bath, the effective temperature is $1/r$, which is close to Planck scale and certainly far from zero. From the practical point of view, the presence or not of this logarithmic contribution is at most an order of magnitude larger than the standard one and therefore it does not significantly affect the results. Almost any other state of the bath with a more or less uniform frequency distribution will contain such logarithmic contribution.

As a summary, the f term provides a dissipation part, necessary for the preservation of commutators, and a fairly small contribution to what can be interpreted as a gravitational Lamb shift. On the other hand, the c term gives rise to a diffusion term and a shift in the oscillation frequencies of the low-energy fields that can be interpreted as a mixture of a gravitational Stark effect and a Lamb shift. The size of these effects, compared with the bare evolution, are the following: the diffusion term is of order ε^2 (see, however, Refs. [129, 130]); the damping term is smaller by a factor r/l , and the combined effect of the Stark and Lamb shifts is of order $(r/l)[\ln(l/r)+1]$ as compared with the diffusion term. Note that the quantum effects induced by spacetime foam become relevant as the low-energy length scale l decreases, as we see from the fact that these effects depend on the ratio r/l , while, in this situation, the diffusion process becomes faster, except for the mass of scalars, which always decoheres in a time scale which is close to the low-energy evolution time.

Observational and experimental prospects.— These quantum gravitational effects are just energy shifts and decoherence effects similar to those appearing in other areas of physics, where fairly well established experimental procedures and results exist, and which can indeed be applied here, provided that sufficiently high accuracy can be achieved.

Neutral kaon beams have been proposed as experimental systems for measuring loss of coherence owing to quantum gravitational fluctuations [93, 131, 132, 133]. In these systems, the main experimental consequence of the diffusion term (together with the dissipative one necessary for reaching a stationary regime) is violation of CPT [42, 134] because of the nonlocal origin of the effective interactions (see also Refs. [135, 136]). The estimates for this violation are very close to the values accessible by current experiments with neutral kaons and will be within the range of near-future experiments. Macroscopic neutron interferometry [93, 137] provides another kind of experimental systems in which the effects of the diffusion term may have measurable consequences since they may cause the disappearance of the interference fringes [93, 137].

As for the gravitational Lamb and Stark effects, they are energy shifts that depend on the frequency, so that different low-energy modes will undergo different shifts. This translates into a modification of the dispersion relations, which makes the velocity of prop-

agation frequency-dependent, as if low-energy fields propagated in a “medium”. Therefore, upon arrival at the detector, low-energy modes will experience different time delays (depending on their frequency) as compared to what could be expected in the absence of quantum gravitational fluctuations. These time delays in the detected signals will be very small in general. However, it might still be possible to measure them if we make the low-energy particles travel large (cosmological) distances. In fact, γ -ray bursts provide such a situation as has been recently pointed out [138] (see also Refs. [139, 140, 141]), thus opening a new doorway to possible observations of these quantum gravitational effects. These authors assume that the dispersion relation for photons has a linear dependence on r/l because of quantum gravitational fluctuations, i.e., that the speed of light is of the form $v \sim 1 + \zeta r/l$, with ζ being an unknown parameter of order 1 (see also Ref. [142]). In this situation, photons that travel a distance L will show a frequency-dependent time delay $\Delta t \sim \zeta L r/l$. Using data from a γ -ray flare associated with the active galaxy Markarian 421 [141, 143] which give $l^{-1} \sim 1\text{TeV}$, $L \sim 1.1 \times 10^{16}$ light-seconds, and a variability time scale δt less than 280 seconds, it can be obtained the upper bound $\zeta r/\ell_* < 250$. If ζ is indeed of order 1, this inequality implies an upper limit on the scale r of the gravitational fluctuations of a few hundred Planck lengths. One would then expect that the presence of the gravitational Lamb and Stark shifts predicted above could be observationally tested. However, in spacetime foam the role of the parameter ζ is played by ε^2 and this quantity is much smaller than 1, since it contains two factors which are smaller than 1 for different reasons. The first one is $e^{-S(r)}(r/\ell_*)^2$. In the semiclassical approximation to nonperturbative quantum gravity, this exponential can be interpreted as the density of topological fluctuations of size r , which decreases with r fairly fast. The second factor is, for the electromagnetic field, of the form $(\ell_*/l)^4$; it comes from the spin dependence of the effective interactions and is closely related to the existence of a length scale in quantum gravity. Then, ε^2 in this case maybe so small that might render any bound on the size of quantum spacetime foam effects on the electromagnetic field nonrestrictive at all.

6 Real clocks

In previous sections, we have analyzed the evolution of low-energy fields in the bath of quantum gravitational fluctuations that constitute spacetime foam. Here we will briefly discuss the evolution of physical systems when measured by real clocks, which are generally subject to errors and fluctuations, in contrast with ideal clocks which, although would accurately measure the time parameter that appears in the Schrödinger equation, do not exist in nature (see, e.g., Refs. [144, 145, 146, 147, 148, 149]). The evolution according to real clocks bears a close resemblance with low-energy fields propagating in spacetime foam, although there also exist important differences which will be discussed at the end of this section.

Quantum real clocks inevitably introduce uncertainties in the equations of motion, as has been widely discussed in the literature from various points of view (see, e.g., Refs. [144, 145, 146, 147, 148, 149]). Actually, real clocks are not only subject to quantum fluctuations. They are also subject to classical imperfections, small errors, that can only be dealt with statistically. For instance, an unavoidable classical source of stochasticity is temperature, which will introduce thermal fluctuations in the behavior of real clocks.

Thus, the existence of ideal clocks is also forbidden by the third law of thermodynamics. Even at zero-temperature, the quantum vacuum fluctuations of quantum field theory make propagating physical systems (real clocks among them) suffer a cold-diffusion and consequently a need for a stochastic description of their evolution [150].

Let us study, within the context of the standard quantum theory, the evolution of an arbitrary system according to a real clock [151].

Good real clocks.— A real clock will be a system with a degree of freedom t that closely follows the ideal time parameter t_i , i.e., $t_i = t + \Delta(t)$, where $\Delta(t)$ is the error at the real-clock time t . Given any real clock, its characteristics will be encoded in the probability functional distribution for the continuous stochastic processes $\Delta(t)$ [152, 153] of clock errors, $\mathcal{P}[\Delta(t)]$, which must satisfy appropriate conditions, so that it can be regarded as a good clock.

A first property is that Galilean causality should be preserved, i.e., that causally related events should always be properly ordered in clock time as well, which implies that $t_i(t') > t_i(t)$ for every $t' > t$. In terms of the derivative $\alpha(t) = d\Delta(t)/dt$ of the stochastic process $\Delta(t)$, we can state this condition as requiring that, for any realization of the stochastic sequence, $\alpha(t) > -1$.

A second condition that we would require good clocks to fulfill is that the expectation value of relative errors, determined by the stochastic process $\alpha(t)$, be zero, i.e., $\langle \alpha(t) \rangle = 0$ for all t . Furthermore, a good clock should always behave in the same way (in a statistical sense). We can say that the clock behaves consistently in time as a good one if those relative errors $\alpha(t)$ are statistically stationary, i.e., the probability functional distribution $\mathcal{P}[\alpha(t)]$ for the process of relative errors $\alpha(t)$ (which can be obtained from $\mathcal{P}[\Delta(t)]$, and vice versa) must not be affected by global shifts $t \rightarrow t + t_0$ of the readout of the clock. Note that the stochastic process $\Delta(t)$ need not be stationary, despite the stationarity of the process $\alpha(t)$.

The one-point probability distribution function for the variables $\alpha(t)$ should be highly concentrated around the zero mean, if the clock is to behave nicely, i.e.,

$$\langle \alpha(t)\alpha(t - \tau) \rangle \equiv c(\tau) \leq c(0) \ll 1, \quad (6.1)$$

where $c(\tau) = c(-\tau)$.

The correlation time ϑ for the stochastic process $\alpha(t)$ is given by

$$\vartheta = \int_0^\infty c(\tau)/c(0). \quad (6.2)$$

For convenience, let us introduce a new parameter κ with dimensions of time, defined as $\kappa^2 = c(0)\vartheta^2$ and for which the good-clock conditions imply $\kappa \ll \vartheta$. As we shall see, ϑ cannot be arbitrarily large, and, therefore, the ideal clock limit is given by $\kappa \rightarrow 0$.

In addition to these properties, a good clock must have enough precision in order to measure the evolution of the specific system, which imposes further restrictions on the clock. On the one hand, the characteristic evolution time l of the system must be much larger than the correlation time ϑ of the clock. On the other hand, the leading term in the asymptotic expansion of the variance $\langle \Delta(t)^2 \rangle$ for large t is of the form $\kappa^2 t / \vartheta$ which means that, after a certain period of time, the absolute errors can be too large. The maximum

admissible standard deviation in $\Delta(t)$ must be at most of the same order as l . Then the period of applicability of the clock to the system under study, i.e., the period of clock time during which the errors of the clock are smaller than the characteristic evolution time of the system is approximately equal to $l^2\vartheta/\kappa^2$. For a good clock, $\kappa \ll \vartheta \ll l$, as we have seen, so that the period of applicability is much larger than the characteristic evolution time l .

Evolution laws.— We shall now obtain the evolution equation for the density matrix of an arbitrary quantum system in terms of the clock time t . Let H be the time-independent Hamiltonian of the system and S its action in the absence of errors.

For any given realization of the stochastic process $\alpha(t)$ that characterizes a good clock, we can write the density matrix at the time t , $\rho_\alpha(t)$, in terms of the initial density matrix $\rho(0)$ as

$$\rho_\alpha(t) = \mathcal{S}_\alpha(t) \cdot \rho(0), \quad (6.3)$$

where the density matrix propagator $\mathcal{S}_\alpha(t)$ has the form

$$\mathcal{S}_\alpha(t) = \int \mathcal{D}q \mathcal{D}q' e^{iS_\alpha[q;t] - iS_\alpha[q';t]}. \quad (6.4)$$

Here, $S_\alpha[q; t] = S[q; t] - \int_0^t ds \alpha(s) H[q(s)]$ is the action of the system for the given realization of the stochastic process $\alpha(t)$.

The average of the density matrix $\rho_\alpha(t)$ can be regarded as the density matrix of the system $\rho(t)$ at the clock time t :

$$\rho(t) = \int \mathcal{D}\alpha \mathcal{P}[\alpha] \mathcal{S}_\alpha(t) \cdot \rho(0). \quad (6.5)$$

In the good-clock approximation, only the two-point correlation function $c(\tau)$ is relevant, so that we can write the probability functional as a Gaussian distribution. The integration over $\alpha(t)$ is then easily performed to obtain the influence action \mathcal{W}

$$\mathcal{W}[q, q'; t] = -\frac{1}{2} \int_0^t ds \int_0^s ds' \{H[q(s)] - H[q'(s)]\} c(s - s') \{H[q(s')] - H[q'(s')]\}. \quad (6.6)$$

We see that there is no dissipative term there as could be expected from the fact that the noise source is classical [101, 102]. Moreover, as the interaction term is proportional to H , there is no response of the system to the outside noise, which means that the associated impedance is infinite [154, 107, 155].

Therefore, we see that the effect of using good real clocks for studying the evolution of a quantum system is the appearance of an effective interaction term in the action integral which is bilocal in time. This can be understood as the first term in a multilocal expansion, which corresponds to the weak-field expansion of the probability functional around the Gaussian term. This nonlocality in time admits a simple interpretation: correlations between relative errors at different instants of clock time can be understood as correlations between clock-time flows at those clock instants. The clock-time flow of the system is governed by the Hamiltonian and, therefore, the correlation of relative errors induces an effective interaction term, generically multilocal, that relates the Hamiltonians at different clock instants.

From the form of the influence action, it is not difficult to see that, in the Markov approximation and provided that the system evolves for a time smaller than the period of applicability of the clock, the density matrix $\rho(t)$ satisfies the master equation

$$\dot{\rho}(t) = -i[H, \rho(t)] - (\kappa^2/\vartheta)[H, [H, \rho(t)]], \quad (6.7)$$

where the overdot denotes derivative with respect to the clock time t . Notice that, in the ideal clock limit, $\kappa \rightarrow 0$, the unitary von Neumann equation is recovered. We should also point out that irreversibility appears because the errors of the clock cannot be eliminated once we have started using it.

From a different point of view, the clock can be effectively modeled by a thermal bath, with temperature T_b to be determined, coupled to the system. Let $H + H_{\text{int}} + H_b$ be the total Hamiltonian, where H is the free Hamiltonian of the system and H_b is the Hamiltonian of a bath that will be represented by a collection of harmonic oscillators [107, 155]. The interaction Hamiltonian will be of the form $H_{\text{int}} = \xi H$, where the noise operator ξ is given by

$$\xi(t) = \frac{i}{\sqrt{2\pi}} \int_0^\infty d\omega \chi(\omega) [a^\dagger(\omega) e^{i\omega t} - a(\omega) e^{-i\omega t}]. \quad (6.8)$$

In this expression, a and a^\dagger are, respectively, the annihilation and creation operators associated with the bath, and $\chi(\omega)$ is a real function, to be determined, that represents the coupling between the system and the bath for each frequency ω .

Identifying, in the classical noise limit, the classical correlation function of the bath with $c(\tau)$, the suitable coupling between the system and the bath is given by the spectral density of fluctuations of the clock:

$$T_b \chi(\omega)^2 = \int_0^\infty d\tau c(\tau) \cos(\omega\tau). \quad (6.9)$$

With this choice, the master equation for evolution according to real clocks is identical to the master equation for the system obtained by tracing over the effective bath.

Loss of coherence.— The master equation contains a diffusion term and will therefore lead to loss of coherence. However, this loss depends on the initial state. In other words, there exists a pointer basis [156, 157, 158], so that any density matrix which is diagonal in this specific basis will not be affected by the diffusion term, while any other will approach a diagonal density matrix. The stochastic perturbation $\alpha(t)H$ is obviously diagonal in the basis of eigenstates of the Hamiltonian, which is therefore the pointer basis: the interaction term cannot induce any transition between different energy levels. The smallest energy difference provides the inverse of the characteristic time for the evolution of the system l and, therefore, the decay constant is $\kappa^2/\vartheta l^2$, equal to the inverse of the period of applicability of the clock. By the end of this period, the density matrix will have been reduced to the diagonal terms and a much diminished remnant of those off-diagonal terms with slow evolution. In any case, the von Neumann entropy grows if the density matrix is not initially diagonal in the energy basis.

The effect of decoherence due to errors of real clocks does not only turn up in the quantum context. Consider for instance a classical particle with a definite energy moving

under a time-independent Hamiltonian H . Because of the errors of the clock, we cannot be positive about the location of the particle in its trajectory on phase space at our clock time t . Therefore we have an increasing spread in the coordinate and conjugate momentum over the trajectory. For a generic system, this effect is codified in the classical master equation

$$\dot{\rho} = \{H, \rho\} + (\kappa^2/\vartheta)\{H, \{H, \rho\}\}, \quad (6.10)$$

where $\rho(t)$ is the probability distribution on phase space in clock time. Finally, it should be observed that the mechanism of decoherence is neither tracing over degrees of freedom, nor coarse graining, nor dephasing [159, 160]. Even though there is no integration over time introduced here by fiat, as happens in dephasing in quantum mechanics, the spread in time due to the errors of the clock has a similar effect, and produces decoherence.

Real clocks and spacetime foam.— As we have seen, there exist strong similarities between the evolution in spacetime foam and that in quantum mechanics with real clocks. In both cases, the fluctuations are described statistically and induce loss of coherence. However, there are some major differences. In the case of real clocks, the diffusion term contains only the Hamiltonian of the system while, in the spacetime foam analysis, a plethora of interactions appeared. Closely related to this, fluctuations of the real clock affect in very similar ways to both classical and quantum evolution; this is not the case in spacetime foam. The origin of these differences is the nature of the fluctuations that we are considering and, more specifically, the existence or not of horizons. Indeed, when studying real clocks, we have ensured that they satisfied Galilean causality, i.e., that the real-time parameter always grows as compared with the ideal time, so that no closed timelike curves are allowed in Galilean spacetime, whichever clock we are using. This requirement is in sharp contrast with the situation that we find in spacetime foam, where we have to consider topological fluctuations that contain horizons (virtual black holes, time machines, etc.). Scattering processes in a spacetime with horizons are necessarily of quantum nature. A classical scattering process in the presence of these horizons would inevitably lead to loss of probability because of the particles that would fall inside the horizons and would never come out to the asymptotic region.

In other words, the underlying dynamics is completely different in both cases. Spacetime foam provides a non-Hamiltonian dynamics since the underlying manifold is not globally hyperbolic. On the other hand, in the case of quantum mechanics according to clocks subject to small errors, the underlying evolution is purely Hamiltonian, although the effective one is an average over all possible Hamiltonian evolutions and becomes nonunitary.

7 Conclusions

Quantum fluctuations of the gravitational field may well give rise to the existence of a minimum length in the Planck scale [3]. This can be seen, for instance, by making use of the fact that measurements and vacuum fluctuations of the gravitational field are extended both in space and time and can therefore be treated with the techniques employed for continuous measurements, in particular the action uncertainty principle [11]. The existence of this resolution limit spoils the metric structure of spacetime at

the Planck scales and opens a doorway to nontrivial topologies [5], which will not only contribute to the path integral formulation but will also dominate the Planck scale physics thus endowing spacetime with a foamlike structure [23] with very complicated topology. Indeed, at the Planck scale, both the partition function and the density of topologies seem to receive the dominant contribution from topological configurations with very high Betti numbers [53, 38].

Spacetime foam may leave its imprint in the low-energy physics. For instance, it can play the role of a universal regulator for both the ultraviolet [46] and infrared [51] divergences of quantum field theory. It has also been proposed as the key ingredient in mechanisms for the vanishing of the cosmological constant [38, 37]. Furthermore, it seems to induce loss of coherence [42] in the low-energy quantum fields that propagate on it as well as mode-dependent energy shifts [43]. In order to study some of these effects in more detail, we have built an effective theory in which spacetime foam has been substituted by a fixed classical background plus nonlocal interactions between the low-energy fields confined to bounded spacetime regions of nearly Planck size [43]. In the weak-coupling approximation, these nonlocal interactions become bilocal. The low-energy evolution is nonunitary because of the absence of a nonvanishing timelike Hamiltonian vector field. The nonunitarity of the bilocal interaction can be encoded in a quantum noise source locally coupled to the low-energy fields. From the form of the influence functional that accounts for the interaction with spacetime foam, we have derived a master equation for the evolution of the low-energy fields which contains a diffusion term, a damping term, and energy shifts that can be interpreted as gravitational Lamb and Stark effects. We have also discussed the size of these effects as well as the possibility of observing them in the near future.

We have seen that the evolution of quantum systems according to good real clocks [151] is quite similar to that in spacetime foam. Indeed, we have argued that good classical clocks, which are naturally subject to fluctuations, can be described in statistical terms and we have obtained the master equation that governs the evolution of quantum systems according to these clocks. This master equation is diffusive and produces loss of coherence. Moreover, real clocks can be described in terms of effective interactions that are nonlocal in time. Alternatively, they can be modeled by an effective thermal bath coupled to the system. In view of this analysis, we have seen that, although there exist strong similarities between propagation in spacetime foam and according to real clocks, there are also important differences that come from the fact that the underlying evolution laws for spacetime foam are nonunitary because of the presence of horizons while, in the case of real clocks, the underlying evolution is unitary and the loss of coherence is due to an average over such Hamiltonian evolutions.

Acknowledgments

I am grateful to C. Barceló and P.F. González-Díaz for helpful discussions and reading the manuscript. I was supported by funds provided by DGICYT and MEC (Spain) under Projects PB93-0139, PB94-0107, and PB97-1218.

References

- [1] C.J. Isham, in *Canonical Gravity: from Classical to Quantum* eds. J. Ehlers and H. Friedrich (Springer-Verlag, Berlin, 1994). *Prima facie questions in quantum gravity*.
- [2] C.J. Isham, in *Proceedings of the 14th International Conference on General Relativity and Gravitation* (World Scientific, Singapore, 1997). *Structural issues in quantum gravity*.
- [3] L.J. Garay, *Int. J. Mod. Phys. A***10**, 145 (1995). *Quantum gravity and minimum length*.
- [4] A.D. Sakharov, *Doklady Akad. Nauk S.S.S.R.* **177**, 70 (1967); *Sov. Phys. Doklady* **12**, 1040 (1968). *Vacuum fluctuations in curved spacetime and the theory of gravitation*.
- [5] C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
- [6] M.A. Markov, preprint P-0187 of Inst. Nucl. Res., Moscow (1980). *On quantum violation of topology in small spatial regions*.
- [7] M.A. Markov, preprint P-0208 of Inst. Nucl. Res., Moscow (1981). *On the maximon and the concept of elementary particle*.
- [8] H.-H. Borzeszkowski and H.-J. Treder, *The Meaning of Quantum Gravity* (Reidel, Dordrecht, 1988).
- [9] M.B. Mensky, *Phys. Lett. A***155**, 229 (1991). *The action uncertainty principle in continuous quantum measurements*.
- [10] M.B. Mensky, *Physics-Uspokhi* **41**, 923 (1998); quant-ph/9812017. *Decoherence and the theory of continuous measurements*.
- [11] M.B. Mensky, *Phys. Lett. A***162**, 219 (1992). *The action uncertainty principle and quantum gravity*.
- [12] N. Bohr and L. Rosenfeld, *Det. Kgl. Danske Videnskabernes Selskab., Math.-fys. Medd.* **12**, 8 (1933); in *Selected Papers of Lèon Rosenfeld*, eds. Cohen and Stachel (Reidel, Dordrecht, 1979); in *Quantum Theory and Measurement*, eds. J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, 1983). *On the question of measurability of electromagnetic field quantities*.
- [13] L. Rosenfeld, *Nucl. Phys.* **40**, 353 (1963). *On quantization of fields*.
- [14] N. Bohr and L. Rosenfeld, *Phys. Rev.* **D78**, 794 (1950). *Field and charge measurements in quantum electrodynamics*.
- [15] L. Rosenfeld, in *Niels Bohr and the Development of Physics*, ed. W. Pauli (Pergamon, London, 1955). *On quantum electrodynamics*.

- [16] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1954) 3rd ed.
- [17] A. Peres and N. Rosen, *Phys. Rev.* **118**, 335 (1960). *Quantum limitations on the measurements of the gravitational field.*
- [18] B.S. DeWitt, in *Gravitation: An Introduction to Current Research*, ed. L. Witten (Wiley, New York, 1962). *The quantization of gravity.*
- [19] T. Regge, *Nuovo Cimento* **7**, 215 (1958). *Gravitational fields and quantum mechanics.*
- [20] H.-H. Borzeszkowski and H.-J. Treder, *Found. Phys.* **12**, 1113 (1982). *Quantum theory and Einstein's general relativity.*
- [21] H.-J. Treder, *Found. Phys.* **15**, 161 (1985). *The Planckions as the largest elementary particles and the smallest test bodies.*
- [22] J.A. Wheeler, *Phys. Rev.* **97**, 511 (1955). *Geons.*
- [23] J.A. Wheeler, *Ann. Phys.* **2**, 604 (1957). *On the nature of quantum geometrodynamics.*
- [24] J.A. Wheeler, *Geometrodynamics* (Academic Press, London, 1962)
- [25] J.A. Wheeler, in *Relativity, Groups and Topology*, eds. B.S. and C.M. DeWitt (Gordon and Breach, New York, 1964). *Geometrodynamics and the issue of the final state.*
- [26] J.A. Wheeler, in *Battelle Rencontres: 1967 Lectures on Mathematics and Physics*, eds. C. DeWitt and J.A. Wheeler (W. Benjamin and Co., New York, 1968). *Super-space and the nature of quantum geometrodynamics.*
- [27] M. Visser, *Lorentzian wormholes* (American Institute of Physics, Woodbury, 1996).
- [28] B. Riemann, *Nature* **8**, 14 (1873) (translated into English by W.K. Clifford). *On the hypotheses which lie at the bases of geometry.*
- [29] H. Weyl, *Philosophy of mathematics and natural science*, Princeton University Press, Princeton (1949).
- [30] C.W. Misner and J.A. Wheeler, *Ann. Phys.* **2**, 525 (1957). *Classical physics as geometry: gravitation, electromagnetism, unquantized charge, and mass properties of curved empty space.*
- [31] C.W. Misner, *Rev. Mod. Phys.* **29**, 497 (1957). *Feynman quantization of general relativity.*
- [32] C.W. Misner, *Phys. Rev.* **118**, 1110 (1960). *Wormhole initial conditions.*
- [33] B.S. DeWitt, in *Relativity, Groups, and Topology II*, eds. B.S. DeWitt and R. Stora (North Holland, Amsterdam, 1984). *Spacetime approach to quantum field theory.*

- [34] G.T. Horowitz, *Class. Quantum Grav.* **8**, 587 (1991). *Topology change in classical and quantum gravity.*
- [35] J. Madore and L.A. Saeger, *Class. Quantum Grav.* **15**, 811 (1998). *Topology at the Planck length.*
- [36] G. Mangano, hep-th/9810174. *Spacetime at the Planck scale and path integral.*
- [37] S. Coleman, *Nucl. Phys.* **B310**, 643 (1988). *Why there is nothing rather than something: a theory of the cosmological constant.*
- [38] S. Carlip, *Phys. Rev. Lett.* **79**, 4071 (1997). *Spacetime foam and the cosmological constant.*
- [39] S. Carlip, *Class. Quantum Grav.* **15**, 2629 (1998). *Dominant topologies in Euclidean quantum gravity.*
- [40] S.W. Hawking, *Nucl. Phys.* **B335**, 155 (1990). *Do wormholes fix the constants of nature?*
- [41] J.L. Rosales, gr-qc/9902049. *Space time foam, thermodynamics, and the electroweak coupling constant.*
- [42] S.W. Hawking, *Commun. Math. Phys.* **87**, 395 (1982). *The unpredictability of quantum gravity.*
- [43] L.J. Garay, *Phys. Rev. Lett.* **80**, 2508 (1998). *Spacetime foam as a quantum thermal bath.*
- [44] L.J. Garay, in *Proceedings of the International Seminar on Mathematical Cosmology*, eds. M. Rainer and H.-J. Schmidt (World Scientific, Singapore, 1998). *The thermal properties of topology fluctuations.*
- [45] L.J. Garay, *Phys. Rev.* **D58**, 124015 (1998). *Thermal properties of spacetime foam.*
- [46] L. Crane and L. Smolin, *Nucl. Phys.* **B267**, 714 (1986). *Renormalizability of general relativity on a background of spacetime foam.*
- [47] S. Deser, *Rev. Mod. Phys.* **29**, 417 (1957). *General relativity and the divergence problem in quantum field theory.*
- [48] T.D. Lee, *Phys. Lett.* **B122**, 217 (1983). *Can time be a discrete variable?*
- [49] T. Padmanabhan, *Ann. Phys. (NY)* **165**, 38 (1985). *Physical significance of Planck's length.*
- [50] T. Padmanabhan, *Phys. Rev. Lett.* **60**, 2229 (1988). *Acceptable density perturbations from inflation due to quantum gravitational damping.*
- [51] A.M.R. Magnon, *Class. Quantum Grav.* **5**, 299 (1988). *Primordial foam and absorption of infrared divergences in quantum gravity.*

- [52] A.A. Markov, in *Proceedings of the International Congress of Mathematicians 1958* (Cambridge University Press, Cambridge, 1960) in Russian. *The problem in homeomorphy*.
- [53] S.W. Hawking, *Nucl. Phys.* **B144**, 349 (1978). *Spacetime foam*.
- [54] S.W. Hawking, *Phys. Rev.* **D53**, 3099 (1996). *Virtual black holes*.
- [55] G.W. Gibbons, in *Fields and Geometry 1986*, ed. A. Jadczyk (World Scientific, Singapore, 1986). *Quantised flux-tubes in Einstein-Maxwell theory and non-compact internal spaces*.
- [56] F.J. Ernst, *J. Math. Phys.* **17**, 515 (1976). *Removal of the nodal singularity of the C-metric*.
- [57] A. Strominger, *Phys. Rev. Lett.* **52**, 1733 (1984). *Vacuum topology and incoherence in quantum gravity*.
- [58] S.W. Hawking, *Mod. Phys. Lett.* **A5**, 145 (1990). *Baby universes*.
- [59] S.W. Hawking, *Mod. Phys. Lett.* **A5**, 453 (1990). *Baby universes II*.
- [60] C. Barceló, *Ph. D. Thesis* (Universidad de Granada, Granada, 1998). *Agujeros de gusano máximamente siméricos*.
- [61] S.W. Hawking, *Phys. Lett.* **B195**, 337 (1987). *Quantum coherence down the wormhole*.
- [62] J. Polchinski and A. Strominger, *Phys. Rev.* **D50**, 7403 (1994). *A possible resolution of the black hole information puzzle*.
- [63] P.F. González-Díaz, *Phys. Lett.* **B261**, 357 (1991). *Baby universe metric equivalent to an interior black hole metric*.
- [64] M. Cavaglià, V. de Alfaro, and A.T. Filippov, *Int. J. Mod. Phys.* **D5**, 227 (1996). *Hamiltonian formalism for black holes and quantization. 2*.
- [65] W.G. Unruh, *Phys. Rev.* **D40**, 1053 (1989). *Quantum coherence, wormholes and the cosmological constant*.
- [66] I. Klebanov, L. Susskind and T. Banks, *Nucl. Phys.* **B317**, 665 (1989). *Wormholes and the cosmological constant*.
- [67] W. Fischler and L. Susskind, *Phys. Lett.* **B217**, 48 (1989). *A wormhole catastrophe*.
- [68] J. Polchinski, *Nucl. Phys.* **B325**, 619 (1989). *Decoupling versus excluded volume or the return of the giant wormholes*.
- [69] S. Coleman and K. Lee, *Phys. Lett.* **B221**, 242 (1989). *Escape from the menace of the giant wormholes*.

- [70] I.H. Redmount and W.-M. Suen, *Phys. Rev.* **D47**, R2163 (1993). *Is quantum spacetime foam unstable?*
- [71] I.H. Redmount and W.-M. Suen, *Phys. Rev.* **D49**, 5199 (1994). *Quantum dynamics of Lorentzian spacetime foam.*
- [72] P.F. González-Díaz, *Phys. Rev.* **D58**, 124011 (1998). *Quantum time machine.*
- [73] P.F. González-Díaz, in *Proceedings of the International Seminar on Current Topics in Mathematical Cosmology*, eds. M. Rainer and H.-J. Schmidt (World Scientific, Singapore, 1998). *CTC's in quantum gravity.*
- [74] M.J. Morris, K.S. Thorne, and U. Yurtsever, *Phys. Rev. Lett.* **61**, 1446 (1988). *Wormholes, time machines, and the weak energy condition.*
- [75] M.J. Morris and K.S. Thorne, *Am. J. Phys.* **56**, 395 (1988). *Wormholes in spacetime and their use for interstellar travel: a tool for teaching general relativity.*
- [76] S.W. Hawking, *Phys. Rev.* **D46**, 603 (1992). *The chronology protection conjecture.*
- [77] M.J. Cassidy and S.W. Hawking, *Phys. Rev.* **D57**, 2372 (1998). *Models for chronology protection.*
- [78] Li-Xin Li and J.R. Gott, III, *Phys. Rev. Lett.* **80**, 2980 (1998). *Selfconsistent vacuum for Misner space and the chronology protection conjecture.*
- [79] J.L. Friedman *et al.*, *Nucl. Phys.* **B309**, 533 (1988). *Non-orientable foam and an effective Planck mass for point-like fermions.*
- [80] P.F. González-Díaz and L.J. Garay, *Phys. Rev.* **D59**, 064020 (1999). *Nonorientable spacetime tunneling.*
- [81] T. Padmanabhan, *Phys. Rev. Lett.* **81**, 4297 (1998). *Quantum structure of spacetime and black hole entropy.*
- [82] T. Padmanabhan, hep-th/9801138. *Event horizon: magnifying glass for Planck length physics.*
- [83] S.W. Hawking, D.N. Page, and C.N. Pope, *Phys. Lett.* **B86**, 175 (1979). *The propagation of particles in spacetime foam.*
- [84] S.W. Hawking, D.N. Page, and C.N. Pope, *Nucl. Phys.* **B170**, 283 (1980). *Quantum gravitational bubbles.*
- [85] S.W. Hawking, *Commun. Math. Phys.* **43**, 199 (1975). *Particle creation by black holes.*
- [86] R.M. Wald, *Commun. Math. Phys.* **45**, 9 (1975). *On particle creation by black holes.*
- [87] S.W. Hawking, *Phys. Rev.* **D14**, 2460 (1976). *Breakdown of predictability in gravitational collapse.*

- [88] S.W. Hawking, *Nucl. Phys.* **B244**, 135 (1984). *Non-trivial topologies in quantum gravity.*
- [89] N.P. Warner, *Commun. Math. Phys.* **86**, 419 (1982). *The scattering of spin-1 particles by quantum gravitational bubbles.*
- [90] S.W. Hawking and S.F. Ross, *Phys. Rev.* **D56**, 6403 (1997). *Loss of quantum coherence through scattering off virtual black holes.*
- [91] S.W. Hawking, *Phys. Rev.* **D52**, 5681 (1995). *Quantum coherence and closed time-like curves.*
- [92] D.J. Gross, *Nucl. Phys.* **B236**, 349 (1984). *Is quantum gravity predictable?*
- [93] J. Ellis *et al.*, *Nucl. Phys.* **B241**, 381 (1984). *Search for violations of quantum mechanics.*
- [94] T. Banks, L. Susskind, and M.E. Peskin, *Nucl. Phys.* **B244**, 125 (1984). *Difficulties for the evolution of pure states into mixed states.*
- [95] W.G. Unruh and R.M. Wald, *Phys. Rev.* **D52**, 2176 (1995). *On evolution laws taking pure states to mixed states in quantum field theory.*
- [96] M. Srednicki, *Nucl. Phys.* **B410**, 143 (1993). *Is purity eternal?*
- [97] J. Liu, hep-th/9301082. *Evolution of pure states into mixed ones.*
- [98] R. Martín and E. Verdaguer, gr-qc/9811070. *On the semiclassical Einstein-Langevin equation.*
- [99] R. Martín and E. Verdaguer, gr-qc/9812063. *An effective stochastic semiclassical theory for the gravitational field.*
- [100] B.L. Hu, gr-qc/9902064. *Stochastic gravity.*
- [101] R.P. Feynman and F.L. Vernon, *Ann. Phys.* **24**, 118 (1963). *The theory of a general quantum system interacting with a linear dissipative system.*
- [102] R.P. Feynman and A.R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965).
- [103] A.O. Caldeira and A.J. Leggett, *Physica* **121A**, 587 (1983). *Path integral approach to quantum Brownian motion.*
- [104] D.A. Eliezer and R.P. Woodard, *Nucl. Phys.* **B325**, 389 (1989). *The problem of nonlocality in string theory.*
- [105] L. Diósi, *Phys. Lett.* **A120**, 377 (1987). *A universal master equation for the gravitational violation of quantum mechanics.*
- [106] I.C. Percival, *Proc. Roy. Soc.* **A451**, 503 (1995). *Quantum space-time fluctuations and primary state diffusion.*

- [107] C.W. Gardiner, *Quantum Noise* (Springer Verlag, Berlin, 1991).
- [108] H. Carmichel, *An Open Systems Approach to Quantum Optics* (Springer Verlag, Berlin, 1993).
- [109] S.B. Giddings and A. Strominger, *Nucl. Phys.* **B306**, 890 (1988). *Axion-induced topology change in quantum gravity and string theory.*
- [110] J.J. Halliwell and R. Laflamme, *Class. Quantum Grav.* **6**, 1839 (1989). *Conformal scalar field wormholes.*
- [111] B.J. Keay and R. Laflamme, *Phys. Rev.***D40**, 2118 (1989). $S^1 \times S^2$ wormholes.
- [112] S.W. Hawking, *Phys. Rev.* **D37**, 904 (1988). *Wormholes in spacetime.*
- [113] S.W. Hawking and D.N. Page, *Phys. Rev.* **D42** (1990) 2655. *Spectrum of wormholes.*
- [114] A. Lyons, *Nucl. Phys.* **B324**, 253 (1989). *Fermions in wormholes.*
- [115] H.F. Dowker, *Nucl. Phys.* **B331**, 194 (1990). *Electromagnetic wormhole vertex.*
- [116] H.F. Dowker and R. Laflamme, *Nucl. Phys.* **B366**, 209 (1991). *Wormholes and gravitons.*
- [117] C. Barceló *et al.*, *Phys. Rev.* **D53**, 3162 (1996). *Asymptotically anti-de Sitter wormholes.*
- [118] C. Barceló and L.J. Garay, *Phys. Rev.* **D57**, 5291 (1998). *Quantum fields in anti-de Sitter wormholes.*
- [119] C. Barceló and L.J. Garay, *Int. J. Mod. Phys.* **D7**, 623 (1998). *Wormhole effective interactions in anti-de Sitter spacetime.*
- [120] A. Lukas, *Nucl. Phys.* **B442**, 533 (1995). *Wormhole effects on Yang-Mills theory.*
- [121] S. Coleman, *Nucl. Phys.* **B307**, 867 (1988). *Black holes as red herrings: topological fluctuations and the loss of quantum coherence.*
- [122] S.B. Giddings and A. Strominger, *Nucl. Phys.* **B307**, 854 (1988). *Loss of incoherence and determination of coupling constants in quantum gravity.*
- [123] S.W. Hawking and R. Laflamme, *Phys. Lett.* **B209**, 39 (1988). *Baby universes and the non-renormalizability of gravity.*
- [124] L. Álvarez-Gaumé and C. Gómez, *Commun. Math. Phys.* **89**, 235 (1983). *Remarks on quantum gravity.*
- [125] S.W. Hawking and C.J. Hunter, hep-th/9808085. *Gravitational entropy and global structure.*
- [126] P.F. González-Díaz, *Nucl. Phys.* **B351**, 767 (1991). *The density matrix of wormholes.*

- [127] M. Reed and B. Simon, *Methods of Modern Mathematical Physics I. Functional Analysis* (Academic Press, New York, 1972).
- [128] C. Itzykson and J.B. Zuber, *Quantum Field Theory* (McGraw-Hill, Singapore, 1985).
- [129] J. Ellis, N.E. Mavromatos, and D.V. Nanopoulos, *Mod. Phys. Lett.* **A12**, 1759 (1997). *Quantum decoherence in a D-foam background.*
- [130] J. Ellis *et al.*, *Mod. Phys. Lett.* **A12**, 243 (1997). *Quantum decoherence in a four-dimensional black hole background.*
- [131] P. Huet and M.E. Peskin, *Nucl. Phys.* **B434**, 3 (1995). *Violation of CPT and quantum mechanics in the K0-anti-K0 system.*
- [132] P. Huet, hep-ph/9607435. *CPT violation from Planck scale physics.*
- [133] F. Benatti and R. Floreanini, *Nucl. Phys.* **B511**, 550 (1998). *Completely positive dynamics of correlated neutral kaons.*
- [134] D.N. Page, *Gen. Rel. Grav.* **14**, 299 (1982). *Is quantum gravity deterministic and/or time symmetric?*
- [135] D.I. Fivel, quant-ph/9710042. *An indication from the magnitude of CP violations that gravitation is a possible cause of wave-function collapse.*
- [136] D.V. Ahluwalia, hep-ph/9901274. *Non-locality and gravity-induced CP violation.*
- [137] A. Zeilinger, M.A. Horne, and C.G. Shull, in *Proceedings of the International Symposium on the Foundations of Quantum Mechanics*, ed. S. Kamefuchi *et al.* (Physical Society of Japan, Tokyo, 1984).
- [138] G. Amelino-Camelia *et al.*, *Nature* **393**, 763 (1998). *Potential sensitivity of gamma ray burster observations to wave dispersion in vacuo.*
- [139] G. Amelino-Camelia *et al.*, astro-ph/9810483. *Sensitivity of astrophysical observations to gravity-induced wave dispersion in vacuo.*
- [140] B.E. Schaefer, astro-ph/9810479. *Severe limitations on variations of the speed of light with frequency.*
- [141] S.D. Biller *et al.*, gr-qc/9810044. *Limits to quantum gravity effects from observation of TeV flares in active galaxies.*
- [142] R. Gambini and J. Pullin, gr-qc/9809038. *Nonstandard optics from quantum spacetime.*
- [143] J. Gaidos *et al.*, *Nature* **383**, 319 (1996). *Extremely rapid burst of TeV photons from the active galaxy Markarian 421.*
- [144] E.P. Wigner, *Rev. Mod. Phys.* **29**, 255 (1957). *Relativistic invariance and quantum phenomena.*

- [145] H. Salecker and E.P. Wigner, *Phys. Rev.* **109**, 571 (1958). *Quantum limitations of the measurement of space-time distances.*
- [146] A. Peres, *Am. J. Phys.* **48**, 552 (1980). *Measurement of time by quantum clocks.*
- [147] D.N. Page and W.K. Wootters, *Phys. Rev.* **D27**, 2885 (1983). *Evolution without evolution: Dynamics described by stationary observables.*
- [148] J.B. Hartle, *Phys. Rev.* **D38**, 2985 (1988). *Quantum kinematics of space-time. 2: A model quantum cosmology with real clocks.*
- [149] W.G. Unruh and R.M. Wald, *Phys. Rev.* **D40**, 2598 (1989). *Time and the interpretation of canonical quantum gravity.*
- [150] G. Gour and L. Sriramkumar, gr-qc/9808032. *Will small particles exhibit Brownian motion in the quantum vacuum?*
- [151] I.L. Egusquiza, L.J. Garay, and J.M. Raya, quant-ph/9811009. *Quantum evolution according to real clocks.*
- [152] N.G. Van Kampen, *Stochastic Processes in Physics and Chemistry* (North-Holland, Amsterdam, 1981).
- [153] C.W. Gardiner, *Handbook of Stochastic Methods for Physics, Chemistry and the Natural Sciences* (Springer-Verlag, Berlin, 1985).
- [154] H.B. Callen and T.A. Welton, *Phys. Rev.* **83**, 34 (1951). *Irreversibility and generalized noise.*
- [155] L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995).
- [156] W.H. Zurek, *Phys. Rev.* **D24**, 1516 (1981). *Pointer basis of quantum apparatus: Into what mixture does the wave packet collapse?*
- [157] W.H. Zurek, *Phys. Rev.* **D26**, 1862 (1982). *Environment induced superselection rules.*
- [158] W.H. Zurek, in *The Physical Origins of Time Asymmetry*, eds. J.J. Halliwell, J. Pérez-Mercader, and W.H. Zurek (Cambridge University Press, Cambridge, 1994). *Preferred set of states, predictability, classicality, and environment-induced decoherence.*
- [159] D. Giulini *et al.*, *Decoherence and the Appearance of the Classical World in Quantum Theory* (Springer-Verlag, Berlin, 1996).
- [160] F. Cooper *et al.*, *Phys. Rev. D* **55**, 6471 (1997). *Nonequilibrium dynamics of symmetry breaking in $\lambda\phi^4$ field theory.*