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MRAC + H_∞ Fault Tolerant Control for Linear Parameter Varying Systems

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***Abstract*— Two different schemes for Fault Tolerant Control (FTC) based on Adaptive Control, Robust Control and Linear Parameter Varying (LPV) systems are proposed. These schemes include a Model Reference Adaptive Controller for an LPV system (MRAC-LPV) and a Model Reference Adaptive Controller with a H_∞ Gain Scheduling Controller for an LPV system (MRAC-H_∞GS-LPV). In order to compare the performance of these schemes, a Coupled-Tank system was used as testbed in which two different types of faults (abrupt and gradual) with different magnitudes and different operating points were simulated. Results showed that the use of a Robust Controller in combination with an Adaptive Controller for an LPV system improves the FTC schemes because this controller was Fault Tolerant against sensor fault and had an accommodation threshold for actuator fault magnitudes from 0 to 6.**

I. INTRODUCTION

Global markets have increased the demand for more and better products, which requires higher levels of plant availability and systems reliability. This issue has promoted that engineers and scientists give more attention to the design of methods and systems that can handle certain types of faults (i.e. Fault Tolerant Systems). On the other hand, global crisis creates more competition between industries and production losses and lack of presence in the markets are not an option. In addition, modern systems and challenging operating conditions increase the possibility of system failures which can cause loss of human lives and equipments. In these environments the use of automation and intelligent systems is fundamental to minimize the impact of faults. Therefore, Fault Tolerant Control methods have been proposed, in which the most important benefit is that the plant continues operating in spite of a fault; this strategy prevents that a fault develops into a more serious failure.

Although several applications have used LPV systems theory to develop FTC schemes ([1], [2], [3]) and also MRAC-based approaches for FTC have been explored ([4], [5], [6], [7], [8], [9]), none of them integrates the three methodologies proposed in this paper: MRAC, LPVs and H_∞.

The main intention of this work is to develop a passive structure of FTC able to deal with abrupt and gradual faults in actuators and sensors of nonlinear processes represented by LPV models. An MRAC controller was chosen as a FTC because guarantees asymptotic output tracking, it has a direct physical interpretation and it is easy to implement. The H_∞ Gain Scheduling Controller was also chosen because it increases the robust performance and stability of the close loop system.

Two different approaches for FTC based on Adaptive, Robust and LPV control are proposed. First, a Model Reference Adaptive Controller for an LPV system (MRAC-LPV) is considered and second a combination of a MRAC with a H_∞ Gain Scheduling controller for an LPV system (MRAC-H_∞GS-LPV) is also proposed. Results showed that MRAC-H_∞GS-LPV has a better performance than the MRAC-LPV approach, because was Fault Tolerant against sensor fault and had an accommodation threshold for actuator fault magnitudes from 0 to 6.

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II. BACKGROUND

A. LPV Control Theory

The Linear Parameter Varying (LPV) systems depend on a set of variant parameters over time. These systems can be represented in state space (continuous or discrete).

The principal characteristic of this type of system is the matrix representation function of one or more variable parameters over time. The continuous representation of an LPV system is:

$$\dot{x}=A(\varphi(t))x+B(\varphi(t))u \quad \textbf{(1)}$$
$$y=C(\varphi(t))x+D(\varphi(t))u \quad \textbf{(2)}$$

where $x \in \mathbb{R}^n$ represents the state space vector, $y \in \mathbb{R}^m$ is the measurement or output vector, $u \in \mathbb{R}^p$ is the input vector, φ represents the parameters variation over time and A(.), B(.), C(.) and D(.) are the continuous function of φ.

An LPV system can be obtained through different methodologies; if the physical representation of the nonlinear system is obtained, the Jacobian Linearization method, the State Transformation Method and the Substitution Function method can be used to obtain the LPV system. The main objective of these methodologies is to occult the nonlinearity of the system in any variable in order to get the LPV system. On the other hand, if the experimental data model is obtained, the LPV system can be created using the Least Square Estimation for different operating points of the system [10], [11].

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B. Model Reference Adaptive Control (MRAC)

The MRAC, shown in Figure 1, implements a close loop controller where the adaptation mechanism adjusts the controller parameters to match the process output with the reference model output. The reference model is specified as the ideal model behavior that the system is expected to follow. This type of controller behaves as a close loop controller because the actuating error signal (difference between the input and the feedback signal) is fed to the controller in order to reduce the error to achieve the desired output value. The controller error is calculated as follows:

$$e = y - y_m \quad \textbf{(3)}$$

where y is the process output and y_m is the reference output.

To reduce the error, a cost function was used in the form of:

$$J(\theta)=1/2\, e^2(\theta) \quad \textbf{(4)}$$

where θ is the adaptive parameter inside the controller.

The function above can be minimized if the parameters θ change in the negative direction of the gradient J, this is called the gradient descent method and is represented by:

$$d\theta/_{dt} = -\gamma\frac{\partial J}{\partial\theta} = -\gamma\frac{\partial e}{\partial\theta}e \quad \textbf{(5)}$$

where γ is the speed of learning. The implemented MRAC used in this experiment is a second order system and has two adaptation parameters: adaptive feed forward gain (θ_1) and adaptive feedback gain(θ_2). These parameters will be updated to follow the reference model.

$$\frac{\partial e}{\partial\theta_1}=\left(\frac{a_{1r}s+a_{0r}}{s^2+a_{1r}s+a_{0r}}\right)u_c \rightarrow \frac{d\theta_1}{dt}=-\gamma\frac{\partial e}{\partial\theta_1}e=-\gamma\left(\frac{a_{1r}s+a_{0r}}{s^2+a_{1r}s+a_{0r}}u_c\right)e \quad \textbf{(6)}$$

$$\frac{\partial e}{\partial\theta_2} = -\left(\frac{a_{1r}s+a_{0r}}{s^2+a_{1r}s+a_{0r}}\right)y \rightarrow \frac{d\theta_2}{dt}= -\gamma\frac{\partial e}{\partial\theta_2}e = \gamma\left(\frac{a_{1r}s+a_{0r}}{s^2+a_{1r}s+a_{0r}}y\right)e \quad \textbf{(7)}$$

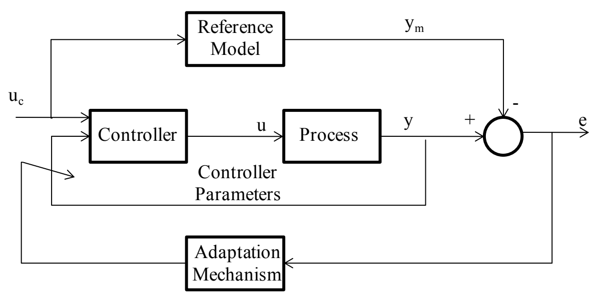

Fig. 1. Model Reference Adaptive Controller (MRAC) general scheme [12].

III. Proposed Schemes

Two different FTC schemes were developed in this work: a MRAC-LPV scheme and a MRAC-H∞GS-LPV scheme. To test these approaches, a second order coupled two-tank system was chosen. This coupled-tank system is composed by two cylindrical tanks (see Figure 2): an upper and a lower tank (tank 1 and tank 2). A pump is used to transport water from the water reservoir to tank 1. Then, the outlet flow of tank 1 flows to tank 2. Finally, the outlet flow of tanks 2 ends in the water reservoir [4]. The water levels of the tanks are measured using pressure sensors located at the bottom of each tank. The differential dynamic model of this system is [13]:

$$\dot{h}_1(t)= -{}^{a_1}/_{A_1}\sqrt{2g}\sqrt{h_1(t)}+{}^{k_p}/_{A_1}u(t) \quad \textbf{(8)}$$

$$\dot{h}_2(t)= -{}^{a_1}/_{A_2}\sqrt{2g}\sqrt{h_1(t)} - {}^{a_2}/_{A_2}\sqrt{2g}\sqrt{h_2(t)} \quad \textbf{(9)}$$

$$y(t)=h_2(t) \quad \textbf{(10)}$$

In Table 1, the variables definition involves in the above system are explained.

Table I
Variables Definition

Variable	Definition	Value
h_1	water level of tank 1	-
h_2	water level of tank 2	-
A_1	cross-section area of tank 1	15.5179 cm^2
A_2	cross-section area of tank 2	15.5179 cm^2
a_1	cross-section area of the outflow orifice of tank 1	0.1781 cm^2
a_2	cross-section area of the outflow orifice of tank 2	0.1781 cm^2
U	pump voltage	-
k_p	pump gain	3.3 cm^3/ V s
G	gravitational constant	981 cm/s^2
α_4	approximation constant	2.981 x 10^{-7}
α_3	approximation constant	-3.659 x 10^{-5}
α_2	approximation constant	1.73 x 10^{-3}
α_1	approximation constant	-4.036 x 10^{-2}
α_0	approximation constant	0.583

An LPV model of the above system is computed by a polynomial fitting technique that approximates $\sqrt{h_i}$ for $0\le h_i\le 30$ cm with $\varphi_i h_i$, where [14]:

$$\varphi_i=\alpha_4h_i^4+\alpha_3h_i^3+\alpha_2h_i^2+\alpha_1h_i+\alpha_0 \quad \textbf{(11)}$$

The parameters φ_1 and φ_2 are bounded with the following values:

$$0.1=\underline{\varphi}_1 \le \varphi_1\le\overline{\varphi}_1=0.6 \quad \textbf{(12)}$$

$$0.1=\underline{\varphi}_2\le \varphi_2\le\overline{\varphi}_2=0.6 \quad \textbf{(13)}$$

The LPV ends in:

$$\dot{x}=A(\varphi)x+Bu \quad \textbf{(14)}$$

$$y=Cx \quad \textbf{(15)}$$

where:

$$x=\begin{bmatrix}h_1\\h_2\end{bmatrix} \quad \textbf{(16)}$$

$$y=\begin{bmatrix}y_1\\y_2\end{bmatrix} \quad \textbf{(17)}$$

$$A(\varphi)=\begin{bmatrix}-0.5085\varphi_1 & 0\\0.5085\varphi_1 & -0.5085\varphi_2\end{bmatrix} \quad \textbf{(18)}$$

$$B=\begin{bmatrix}0.2127\\0\end{bmatrix} \quad \textbf{(19)}$$

$$C=[0 \quad 1] \quad \textbf{(20)}$$

$$D=\begin{bmatrix}0\\0\end{bmatrix} \quad \textbf{(21)}$$

A. MRAC-LPV Controller

A Model Reference Adaptive Controller of the LPV system was designed (MRAC-LPV). First, the state-space LPV model was transformed to a continuous version:

$$G_{LPV}(s)=C(sI-A)^{-1}B+D \quad \textbf{(22)}$$

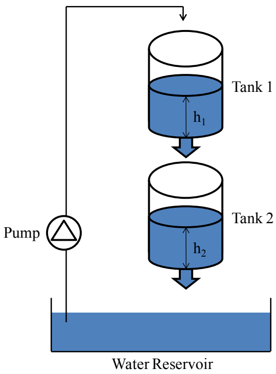

Fig. 2. Coupled-tank system designed by [15].

$$G_{LPV}(s)=[0 \quad 1]\times$$
$$\left(\begin{bmatrix}s & 0\\0 & s\end{bmatrix}-\begin{bmatrix}-0.5085\ \varphi_1 & 0\\0.5085\ \varphi_1 & -0.5085\ \varphi_2\end{bmatrix}\right)^{-1}\begin{bmatrix}0.2127\\0\end{bmatrix} \quad \textbf{(23)}$$

$$G_{LPV}(s)=\frac{0.108158\ \varphi_1}{(s+0.5085\ \varphi_2)(s+0.5085\ \varphi_1)} \quad \textbf{(24)}$$

$$G_{LPV}(s)=\frac{0.108158\ \varphi_1}{s^2+0.5085(\varphi_1+\varphi_2)s+0.258572\ \varphi_1\varphi_2} \quad \textbf{(25)}$$

The reference model is:

$$\text{Reference Model}=\frac{0.108158\ \varphi_1}{s^2+0.5085(\varphi_1+\varphi_2)s+0.258572\ \varphi_1\varphi_2} \quad \textbf{(26)}$$

This model is the same as the process model when with no faults.

$$\text{Process Model}=\frac{0.108158\ \varphi_1}{s^2+0.5085(\varphi_1+\varphi_2)s+0.258572\ \varphi_1\varphi_2} \quad \textbf{(27)}$$

The adaptive feed forward update rule (θ_1) is:

$$\frac{d\theta_1}{dt}=-\gamma\frac{\partial e}{\partial\theta_1}e=-\gamma\left(\frac{0.5085(\varphi_1+\varphi_2)s+0.258572\ \varphi_1\varphi_2}{s^2+0.5085(\varphi_1+\varphi_2)s+0.258572\ \varphi_1\varphi_2}\right)e \quad \textbf{(28)}$$

The adaptive feedback update rule (θ_2) is:

$$\frac{d\theta_2}{dt}=-\gamma\frac{\partial e}{\partial\theta_2}e=\gamma\left(\frac{0.5085(\varphi_1+\varphi_2)s+0.258572\ \varphi_1\varphi_2}{s^2+0.5085(\varphi_1+\varphi_2)s+0.258572\ \varphi_1\varphi_2}\right)e \quad \textbf{(29)}$$

Figure 3 represents the MRAC-LPV scheme, in this figure the Reference Model, the Process Model, the feed forward update rule (bottom left) and the feedback update rule (bottom right) are represented as LPV systems. The feed forward and the feedback update rule change in order to follow the reference model.

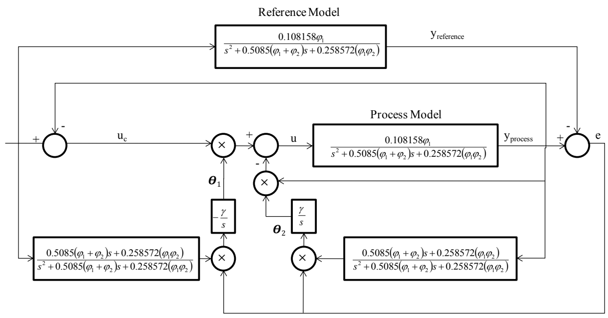

Fig. 3. MRAC-LPV Controller Structure.

B. MRAC-H$_\infty$GS-LPV Controller

In order to design the H_∞ Gain Scheduling LPV Controller for the MRAC-H$_\infty$GS-LPV Controller (Figure 4), two weighting functions were established (W_{mi} and W_{ai}). To obtain W_{mi} and W_{ai} the next procedure was realized: First, 4 plants were calculated using the extreme operation points ($\varphi_1=0.1$, $\varphi_2=0.1$; $\varphi_1=0.1$, $\varphi_2=0.6$; $\varphi_1=0.6$, $\varphi_2=0.1$; $\varphi_1=0.6$, $\varphi_2=0.6$) and a nominal plant ($\varphi_1=0.35$, $\varphi_2=0.35$) were obtained using an average of the operation points.

Then, the multiplicative uncertainty (W_{mi}) and additive uncertainty (W_{ai}) were calculated for each plant as follows:

$$W_{mi}=\frac{(\text{Plant i-Nominal Plant})}{\text{Nominal Plant}} \quad \textbf{(30)}$$

$$W_{ai}=\text{Plant i-Nominal Plant} \quad \textbf{(31)}$$

The next step is to plot a Bode diagram of the above uncertainties and find a weighting function that includes all the individual plant uncertainties.

With the Bode diagrams, the multiplicative and additive uncertainties functions that include all the plants were computed:

$$W_{mt}=\frac{0.75s^4+0.33\ s^3+0.02\ s^2-0.00882\ s}{s^4+0.568837\ s^3+0.091878\ s^2+0.003019\ s} \quad \textbf{(32)}$$

$$W_{at}=\frac{0.02839\ s^4+0.01249\ s^3+0.000757\ s^2-0.0003338\ s}{s^6+0.9247\ s^5+0.326\ s^4+0.05373\ s^3+0.003984\ s^2+9.561e^{-5}s} \quad \textbf{(33)}$$

After calculating W_{mt} and W_{at} the following procedure was realized:

1. The value of the learning rate γ and the specific desired operation points were established as φ_1 and φ_2.

2. W_{mt} and W_{at} have to be transformed into a Linear Time Invariant (LTI) system.
3. The parameter range has to be specified in order to obtain the variation range of values of a time-varying parameter or uncertain vector. In this experiment there are 2 dependent parameters, this means that the range of values of these parameters form a multi-dimensional box.

$$0.1=\underline{\varphi}_1\leq\varphi_1\leq\overline{\varphi}_1=0.6 \quad \textbf{(34)}$$

$$0.1=\underline{\varphi}_2\leq\varphi_2\leq\overline{\varphi}_2=0.6 \quad \textbf{(35)}$$

4. The state space LPV model is transformed into an LTI system and then the parameter dependent system is specified.
5. The loop shaping structure of the LPV system is specified.
6. The augmented plant is formed.
7. The H$_\infty$ Gain Scheduling Controller was calculated with the *hinfgs* Matlab® function. This function calculates an H$_\infty$ gain scheduled control for parameter dependent system with an affine dependence on the time varying parameters. The parameters are assumed to be measured in real time. To calculate the controller the function implements the quadratic H$_\infty$ performance approach.
8. The desired operating points are specified in order to return the convex decomposition of the parameters set of box corners.
9. The evaluation of the desired operating points in the polytopic representation of the gain-scheduled controller is realized. From here, the state space matrices are extracted and then transformed into a continuous time space.

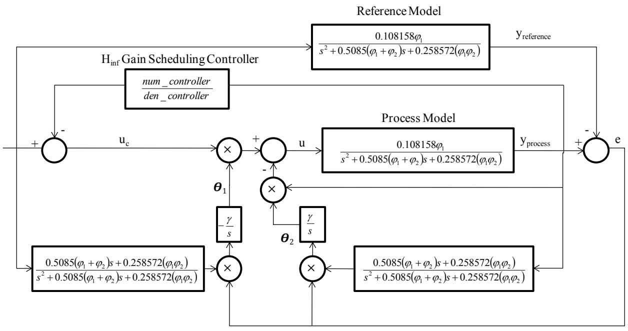

Fig. 4. MRAC H$_\infty$ Gain Scheduling LPV Controller arquitecture.

The input of both controllers must be persistently exciting in order to converge to the desired output value.

IV. RESULTS

Two different types of faults were simulated in the implemented schemes: abrupt and gradual faults.

Abrupt faults in actuators represent for instance a pump stuck and in sensors a constant bias in measurement. A gradual fault could be a progressive loss of electrical power in pump, and a drift in the measurement for sensors.

For each of the two proposed schemes: MRAC-LPV Controller and MRAC H$_\infty$ Gain Scheduling LPV Controller (MRAC- H$_\infty$GS-LPV) both faults were tested obtaining the results shown in Table II. These results explain the range of fault size in which the methodologies are robust, fault tolerant or unstable against the fault.

The next Table and Figures show the implementation of the faults in the above methodologies. These operation points were selected to demonstrate the capabilities of both controllers, but any operation point between the range of φ_1 and φ_2 can be chosen.

TABLE II
RESULTS OF EXPERIMENTS OF THE MRAC-LPV AND THE MRAC-H∞GS-LPV APPROACHES

Approach	**Sensor Faults**		**Actuator Faults**	
	Abrupt Faults	**Gradual Faults**	**Abrupt Faults**	**Gradual Faults**
MRAC-LPV	0 < f < 1 → FT	+/-0 < f <+/-1 → FT	0 < f < 6 → FT	+/-0 < f <+/-6 → FT
	f> 1 → U	f>+/- 1 → U	f> 6 → U	f>+/- 6 → U
MRAC-H$_\infty$-LPV	FT	FT	0 < f < 6 → FT	+/-0 < f <+/-6 → FT
	-	-	f> 6 → U	f>+/- 6 → U

f=1 → 10% deviation from nom. value, f=2 → 20 % deviation, and so on
FT = Fault Tolerant, U = Unstable.

In Table II the accommodation (Fault Tolerant) and the unstable ranges for the MRAC-LPV and the MRAC-H$_\infty$-LPV are shown. For example for abrupt sensor faults the MRAC-LPV has a Fault Tolerant threshold for fault from magnitude 0 to 1. On the other hand, the MRAC-H$_\infty$-LPV was Fault Tolerant for all magnitudes of this specific faults type.

Figure 5 shows for abrupt faults case, the best scheme is the MRAC-H$_\infty$-LPV because is robust against sensor faults of magnitude 1 (10 % deviation from nominal value) and is fault tolerant to actuator faults of magnitude 6 (60 % deviation from nominal value), for the actuator fault the real deviation from the nominal system performed by the controller was of 10% from the nominal value at the time of the fault. On the other hand, the MRAC-LPV resulted to be fault tolerant for sensor and abrupt faults, for example for sensor fault the real deviation from the nominal value was 80% and for the actuator fault the real deviation was of 90% from the nominal value. Both controllers are working in the operating point $\varphi_1=0.3$ and $\varphi_2=0.5$, the abrupt-sensor fault was introduced at time 5,000 s and an abrupt-actuator fault was introduced at time 15,000 s. In addition, a change in the operating point was performed at time 10,000 s.

Figure 6 shows for abrupt faults case, the best scheme is the MRAC-H$_\infty$-LPV because is fault tolerant against sensor faults of magnitude 10 and is fault tolerant to actuator faults of magnitude 6. The above means that for the sensor fault the system has a real deviation of 0.8% and for the actuator fault the system has a real deviation of 10% from the nominal value. On the other hand, the MRAC-LPV resulted to be unstable for sensor faults of magnitude 10 and fault tolerant for abrupt faults of magnitude 6 because the real deviation from the nominal system was more than -300%. It is important to mention that the fault was accommodated after 10,000 s because the MRAC controller continues to minimize the error over the time; this is one of the advantages of this controller. In this example both controller are working in the operating point $\varphi_1=0.3$ and $\varphi_2=0.5$, the abrupt-sensor was introduced at time 5,000 s and the abrupt-actuator fault was introduced at time 15,000 s. In addition a change in the operating point was performed at time 10,000 s.

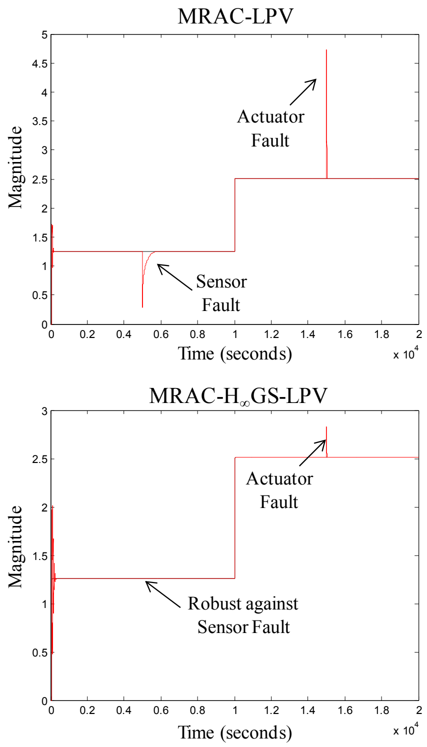

Fig. 5. Comparisons between MRAC-H∞-LPV and MRAC-LPV Controllers with an abrupt-sensor fault of magnitude 1 and an abrupt-actuator fault of magnitude 6, the operating points are $\varphi_1=0.3$ and $\varphi_2=0.5$.

Figure 7 presents for gradual faults case, the best scheme is the MRAC-H_∞-LPV because is robust against sensor faults of magnitude 1 (maximum deviation of 10% from nominal value with a 1 %/sec ramp) and is fault tolerant to actuator faults of magnitude 6 (maximum deviation of 60% from nominal value with a 1 %/sec ramp) change); the real system deviation at the time of the fault for the actuator time was of 4.5% and was accommodated immediately. On the other hand, the MRAC-LPV resulted to be fault tolerant for sensor and actuator faults of magnitude 1 and 6, respectively; in which the real deviation from the nominal value for the sensor fault was of 77% and for the actuator fault was of 82%. Both controllers are working in the operating point $\varphi_1=0.6$ and $\varphi_2=0.6$, the gradual-sensor fault was introduced in time 5,000 s and the gradual-actuator fault was introduced at time 15,000 s. In addition a change in the operating point was performed at time 10,000 s.

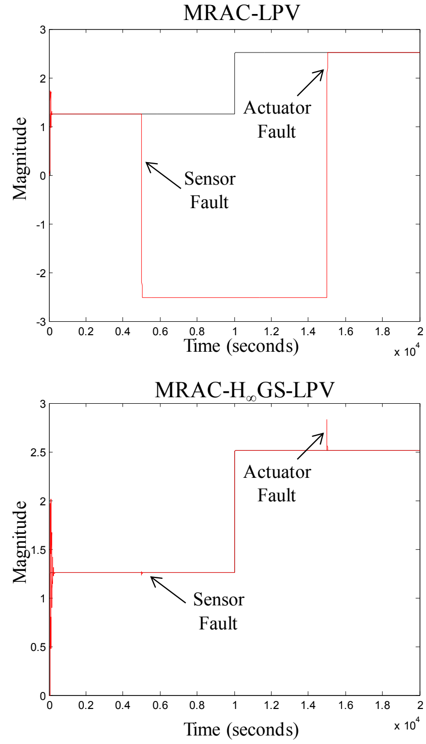

Fig. 6. Comparisons between MRAC-H∞-LPV and MRAC-LPV Controllers with an abrupt-sensor fault of magnitude 10 and an abrupt-actuator fault of magnitude 6, the operating points are $\varphi_1=0.3$ and $\varphi_2=0.5$.

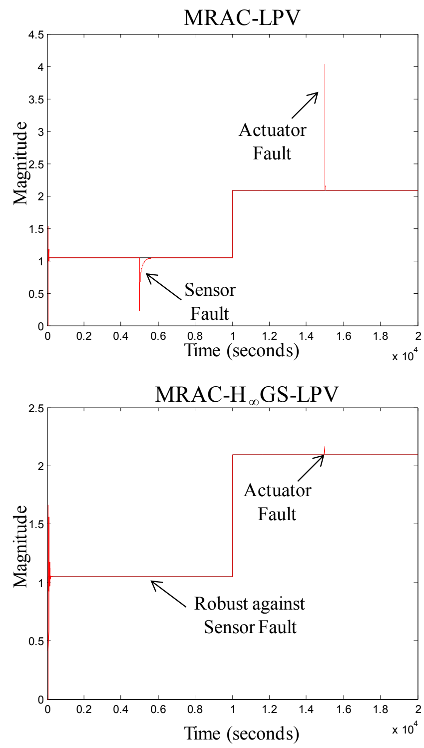

Fig. 7. Comparisons between MRAC-H∞-LPV and MRAC-LPV Controllers with a gradual-sensor fault of magnitude 1 and a gradual-actuator fault of magnitude 6, the operating points are $\varphi_1=0.6$ and $\varphi_2=0.6$.

Figure 8 describes that for gradual faults, the MRAC-H$_\infty$-LPV scheme is fault tolerant against sensor fault of magnitude 10 and it is fault tolerant to actuator faults of magnitude 6. The above resulted in a real deviation from the nominal value of 2% and 4.5% for sensor and actuator faults, respectively. Also, the MRAC-LPV resulted to be fault tolerant to sensor and actuator faults of magnitude 10 and 6, respectively; with a real deviation of 81% for sensor fault and of 91% for actuator fault from the nominal value. Even though, both schemes are fault tolerant against sensor and actuator faults, the best scheme is the MRAC-H$_\infty$-LPV because the deviation of the process model from the reference model of this scheme is smaller than the deviation of the MRAC-LPV scheme. Both controllers are working in the operating point $\varphi_1=0.6$ and $\varphi_2=0.6$, the gradual-sensor fault was introduced at time 5,000 s and the gradual-actuator fault was introduce at time 15,000 s. In addition a change in the operating point was performed at time 10,000 s.

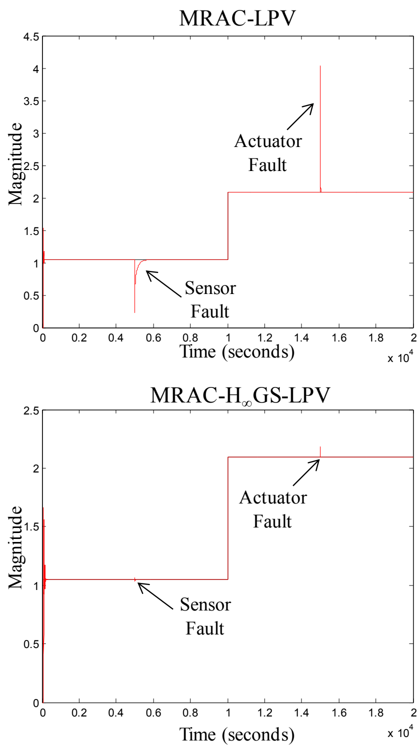

Fig. 8. Comparisons between MRAC-H∞-LPV and MRAC-LPV Controllers with a gradual-sensor fault of magnitude 10 and a gradual-actuator fault of magnitude 6, the operating points are $\varphi_1=0.6$ and $\varphi_2=0.6$.

V. CONCLUSIONS

In the experiments, the MRAC-H$_\infty$GS-LPV methodology behaved better than the MRAC-LPV scheme because was fault tolerant against sensor faults of any magnitude (f=1 and f=10). The MRAC-H$_\infty$GS-LPV showed better results because is a combination of two type of controllers, one is a Model Reference Adaptive Controller (MRAC) and the other one is a H$_\infty$ Gain Scheduling Controller, both controllers were designed for an LPV system giving them the possibility of controlling any desired operating condition between the operation range of the dependent variables (φ_1 and φ_2). On the other hand, the MRAC-LPV methodology resulted to be fault tolerant for sensor faults magnitudes between 0 and 1 and it was fault tolerant for actuator fault magnitudes between 0 and 6 (the MRAC-H$_\infty$GS-LPV approach had the same fault tolerant threshold for actuator faults).

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