Consistent high-energy constraints in the anomalous QCD sector

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Pablo Roig [a,*], Juan José Sanz Cillero [b]

[a] *Instituto de Física, Universidad Nacional Autónoma de México, AP 20-364, México D.F. 01000, Mexico*
[b] *Departamento de Física Teórica and Instituto de Física Teórica, IFT-UAM/CSIC, Universidad Autónoma de Madrid, Cantoblanco, E-28049 Madrid, Spain*

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The anomalous $\langle VVP \rangle$ Green function and related form-factors ($\pi^0 \to \gamma^* \gamma^*$ and $\tau^- \to X^- \nu_\tau$ vector form-factors, with $X^- = (KK\pi)^-$, $\varphi^- \gamma$, $(\varphi V)^-$) are analyzed in this letter in the large-N_C limit. Within the single (vector and pseudoscalar) resonance approximation and the context of Resonance Chiral Theory, we show that all these observables overdetermine in a consistent way a unique set of compatible high-energy constraints for the resonance couplings. This result is in agreement with analogous relations found in the even intrinsic-parity sector of QCD like, e.g., $F_V^2 = 3F^2$. The antisymmetric tensor formalism is considered for the spin-one resonance fields. Finally, we have also worked out and provide here the relation between the two bases of odd intrinsic-parity Lagrangian operators commonly employed in the literature.

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1. Introduction

Chiral symmetry plays a crucial role in the structure of non-perturbative light-quark interactions in Quantum Chromodynamics (QCD). However, it becomes spontaneously broken, generating the corresponding chiral (pseudo) Goldstones φ^a. Its low-energy interaction can be then described through an effective field theory (EFT) which implements chiral symmetry and the discrete symmetries of QCD, denoted as Chiral Perturbation Theory (χPT) [1–5]. The Wess–Zumino–Witten (WZW) term reproduces the chiral anomaly of QCD [6,7] and will provide the leading contribution to the anomalous QCD amplitudes at low energies.

Resonance Chiral Theory (RχT) [8,9] describes the interactions between the light-quark resonances and the chiral pseudo-Goldstones based on chiral invariance and using the formal expansion in $1/N_C$ [10–12] as a guiding principle to sort out perturbative computations, with N_C the number of colours in QCD. In most applications, one is forced to truncate the resonance spectrum and just the lightest resonance multiplets are included.[1]

The RχT Lagrangian is built in such a way that it fulfills the discrete symmetries of QCD and chiral symmetry. It contains an even [8,9,15] and an odd intrinsic-parity sector [16,17], where the latter produces contributions to anomalous amplitudes in QCD.

Nonetheless, it should be pointed out that, apart from the WZW term, the RχT interactions are chiral invariant.

Although QCD fixes the hadronic couplings, these cannot be determined based on chiral symmetry considerations alone. *A priori*, they are free parameters of our RχT action and chiral symmetry just provides relations between particular processes. However, demanding a short-distance behaviour in accordance with QCD and its Operator Product Expansion (OPE) [18] will allow us to extract important constraints between couplings like, *e.g.*, the Weinberg Sum Rules (WSRs) [19].

Along the last years, there has been an extensive program of computation of Green functions and associated form-factors within the RχT framework [8,9,13,15–17,20–23], where a key ingredient has been the implementation of the high-energy QCD behaviour prescribed by the OPE [18,24] and the quark-counting rules for hadronic form-factors [25].

The difficulties to match the anomalous $\langle VVP \rangle$ Green function (between two vector currents and one pseudoscalar density) in the four-vector field representation for spin-one resonances [13,20,21] triggered the research reported in Ref. [16]. Therein, the authors included the chiral pseudo-Goldstones and the lightest vector multiplet and showed that these issues could be solved in the antisymmetric tensor field formalism for the vector resonances [8,9]. This will be the formalism used all along this letter.

As a result of that $\langle VVP \rangle$ study [16], there were several subsequent analyses [22,23] where other Green functions that are order parameters of chiral symmetry breaking were evaluated in an analogous way. Finally, a global understanding of the short-distance constraints on these Green functions and related form-factors was

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presented in Ref. [15], together with an exhaustive evaluation of the resonance contributions to the χPT low-energy constants (LECs) of the even intrinsic-parity sector up to $\mathcal{O}(p^6)$.

However, the analysis of $\tau^- \to (K K \pi)^- \nu_\tau$ decays [26] revealed a set of high-energy constraints on the RχT couplings which was at odds with the findings of Ref. [16]. Furthermore, an inconsistent asymptotic behaviour for the $\pi^0 \to \gamma \gamma^*$ pion transition form-factor (TFF) was found [26] if the high-energy restrictions [16] were considered. After that, Ref. [17] revisited the $\langle V V P \rangle$ Green function. It extended the odd intrinsic-parity resonance Lagrangian, allowing for operators with multiple resonance fields, verified previous analyses of the saturation of odd intrinsic-parity $\mathcal{O}(p^6)$ LECs under integration of the resonances and performed a complete study of the saturation to the full set of odd $\mathcal{O}(p^6)$ LECs [4,5]. On the contrary to Ref. [16], the short-distance behaviours of the pion TFF and the $\langle V V P \rangle$ Green function were found to be compatible with QCD, provided the lightest pseudoscalar resonance multiplet was also taken into account (it was not included in the previous work [16]). Since the basis of Lagrangian operators employed in both Refs. [16,17] was different, it still remained unclear whether all inconsistencies among the referred high-energy constraints were fully solved by adding a pseudoscalar resonance multiplet to the action or not.

The aim of this letter is to answer this question. We will show that the odd intrinsic-parity RχT Lagrangian including the pseudo-Goldstones and the lightest multiplet of pseudoscalar and vector resonances [17] produces a unique consistent set of short-distance relations for the $\langle V V P \rangle$ Green function and the associated form-factors studied so far in the literature, in an analogous way to the agreement found in the even intrinsic-parity sector [15].

2. The odd intrinsic-parity resonance Lagrangian

The operators in the RχT Lagrangian can be classified according to the number of resonance fields:

$$\mathcal{L}_{R\chi T} = \mathcal{L}_G + \sum_R \mathcal{L}_R + \sum_{R,R'} \mathcal{L}_{RR'} + \dots \quad (1)$$

with $\mathcal{L}_G$ given by the operators with only pseudo-Goldstone fields and external vector and axial-vector sources. It contains the even-parity $\mathcal{O}(p^2)$ χPT Lagrangian [2,3,8] and the WZW term [6,7,16, 17].

At leading order in the $1/N_C$ expansion, the most general odd intrinsic-parity resonance chiral Lagrangian for processes involving one pseudo-Goldstone and two vector objects (two vector resonances, or one external source and one vector resonance) was derived in Ref. [16]:

$$\mathcal{L}_{R\chi T}^{\text{odd}} \supset \sum_{i=1}^{7} \frac{c_i}{M_V} \mathcal{O}_{\text{VJP}}^i + \sum_{i=1}^{4} d_i \mathcal{O}_{\text{VVP}}^i. \quad (2)$$

The monomials $\mathcal{O}_{\text{VJP}}^i$ and $\mathcal{O}_{\text{VVP}}^i$ are provided in Table 1. The antisymmetric tensor formulation is employed here to describe the spin-1 fields [8,9]. Analogous analyses with the spin-1 fields given in the four-vector (Proca) formalism can be found in Refs. [13,21], although the present work only studies the antisymmetric tensor representation.

We follow the notation and conventions of Ref. [15], where the chiral tensors entering $\mathcal{L}_{R\chi T}^{\text{odd}}$ are defined. M_V is the vector resonance multiplet mass in the chiral and large-N_C limits.

Within the same framework, and motivated by the analogous work on the even-intrinsic parity resonance Lagrangian accomplished in Ref. [15], the authors of Ref. [17] constructed the most

Table 1

Monomials with one vector resonance field ($\mathcal{O}_{\text{VJP}}^i$) and two vector fields ($\mathcal{O}_{VVP}^i$) in the basis of Ref. [16].

i	$\mathcal{O}_{\text{VJP}}^i$	i	$\mathcal{O}_{\text{VVP}}^i$
1	$\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle$	1	$\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle$
2	$\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\alpha}, f_+^{\rho\sigma}\}\nabla_\alpha u^\nu\rangle$	2	$i\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\sigma}\}\chi_-\rangle$
3	$i\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\sigma}\}\chi_-\rangle$	3	$\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, V^{\rho\alpha}\}u^\sigma\rangle$
4	$i\varepsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}[f_-^{\rho\sigma}, \chi_+]\rangle$	4	$\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\}u_\alpha\rangle$
5	$\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, f_+^{\rho\sigma}\}u^\sigma\rangle$		
6	$\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\alpha}, f_+^{\rho\sigma}\}u^\nu\rangle$		
7	$\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, f_+^{\rho\alpha}\}u_\alpha\rangle$		

Table 2

Monomials with one vector resonance field in the basis of Ref. [17].

i	$\widehat{\mathcal{O}}_{i\mu\nu\alpha\beta}^V$	i	$\widehat{\mathcal{O}}_{i\mu\nu\alpha\beta}^V$
1	$i\langle V_{\mu\nu}(h_{\alpha\sigma}u^\sigma u_\beta - u_\beta u^\sigma h_{\alpha\sigma})\rangle$	11	$\langle V_{\mu\nu}\{f_{+\alpha\rho}, f_{-\beta\sigma}\}\rangle g^{\rho\sigma}$
2	$i\langle V_{\mu\nu}(u^\sigma h_{\alpha\sigma}u_\beta - u_\beta h_{\alpha\sigma}u^\sigma)\rangle$	12	$\langle V_{\mu\nu}\{f_{+\alpha\rho}, h_{\beta\sigma}\}\rangle g^{\rho\sigma}$
3	$i\langle V_{\mu\nu}(u^\sigma u_\beta h_{\alpha\sigma} - h_{\alpha\sigma}u_\beta u^\sigma)\rangle$	13	$i\langle V_{\mu\nu}f_{+\alpha\rho}\rangle\langle\chi_-\rangle$
4	$i\langle[V_{\mu\nu}, \nabla_\alpha\chi_+]u_\beta\rangle$	14	$i\langle V_{\mu\nu}\{f_{-\alpha\beta}, \chi_-\}\rangle$
5	$i\langle V_{\mu\nu}[f_{-\alpha\beta}, u_\sigma u^\sigma]\rangle$	15	$i\langle V_{\mu\nu}[f_{-\alpha\beta}, \chi_+]\rangle$
6	$i\langle V_{\mu\nu}(f_{-\alpha\sigma}u_\beta u^\sigma - u^\sigma u_\beta f_{-\alpha\sigma})\rangle$	16	$\langle V_{\mu\nu}\{\nabla_\sigma f_{+\beta\sigma}, u^\sigma\}\rangle$
7	$i\langle V_{\mu\nu}(u^\sigma f_{-\alpha\sigma}u_\beta - u_\beta f_{-\alpha\sigma}u^\sigma)\rangle$	17	$i\langle V_{\mu\nu}\{\nabla^\sigma f_{+\alpha\sigma}, u_\beta\}\rangle$
8	$i\langle V_{\mu\nu}(f_{-\alpha\sigma}u^\sigma u_\beta - u_\beta u^\sigma f_{-\alpha\sigma})\rangle$	18	$\langle V_{\mu\nu}u_\alpha u_\beta\rangle\langle\chi_-\rangle$
9	$\langle V_{\mu\nu}\{\chi_-, u_\alpha u_\beta\}\rangle$		
10	$\langle V_{\mu\nu}u_\alpha\chi_-u_\beta\rangle$		

Table 3

Monomials with two vector resonance fields (left hand side) and a pseudoscalar resonance and a vector resonance field (right hand side) in the basis of Ref. [17].

i	$\widehat{\mathcal{O}}_{i\mu\nu\alpha\beta}^{VV}$	i	$\widehat{\mathcal{O}}_{i\mu\nu\alpha\beta}^{PV}$
1	$i\langle V_{\mu\nu}V_{\alpha\beta}\rangle\langle\chi_-\rangle$	1	$i\langle\{V_{\mu\nu}, P\}u_\alpha u_\beta\rangle$
2	$i\langle\{V_{\mu\nu}, V_{\alpha\beta}\}\chi_-\rangle$	2	$i\langle V_{\mu\nu}u_\alpha P u_\beta\rangle$
3	$\langle\{\nabla^\sigma V_{\mu\nu}, V_{\alpha\sigma}\}u_\beta\rangle$	3	$\langle\{V_{\mu\nu}, P\}f_{+\alpha\beta}\rangle$
4	$\langle\{\nabla_\beta V_{\mu\nu}, V_{\alpha\sigma}\}u^\sigma\rangle$		

general resonance chiral Lagrangian in the odd intrinsic-parity sector that can generate chiral low-energy constants up to $\mathcal{O}(p^6)$ [4,5]. They considered the contribution from the lightest resonance multiplets, in particular vector (V) and pseudoscalar (P) resonances. The latter was absent in the previous treatment in Ref. [16]. The part of the odd intrinsic-parity Lagrangian relevant for the $\langle V V P \rangle$ Green function and the related form-factors studied in this article is given by [17]

$$\tilde{\mathcal{L}}_{R\chi T}^{\text{odd}} \supset \sum_i \sum_X \kappa_i^X \varepsilon^{\mu\nu\alpha\beta} \widehat{\mathcal{O}}_{i\mu\nu\alpha\beta}^X, \quad X = V, VV, PV, \quad (3)$$

where the corresponding operators can be read in Tables 2 and 3.

It is possible to rewrite the resonance Lagrangian from Ref. [16] (Table 1) in terms of the basis of monomials in Ref. [17] (Tables 2 and 3) by means of partial integration and the Bianchi and Schouten identities [5]. This exercise yields the following relations between the $\tilde{\mathcal{L}}_{R\chi T}^{\text{odd}}$ and the $\mathcal{L}_{R\chi T}^{\text{odd}}$ couplings:

$$\kappa_1^{VV} = \frac{-d_1}{8n_f}, \qquad \kappa_2^{VV} = \frac{d_1}{8} + d_2, \qquad \kappa_3^{VV} = d_3, \qquad \kappa_4^{VV} = d_4,$$

$$-2M_V\kappa_5^V = M_V\kappa_6^V = M_V\kappa_7^V = \frac{c_6}{2},$$

$$M_V\kappa_{11}^V = \frac{c_1 - c_2 - c_5 + c_6 + c_7}{2},$$

$$M_V\kappa_{12}^V = \frac{c_1 - c_2 - c_5 + c_6 - c_7}{2}, \qquad n_f M_V\kappa_{13}^V = \frac{-c_2 + c_6}{4},$$

$$M_V\kappa_{14}^V = \frac{c_2 + 4c_3 - c_6}{4},$$

$$M_V \kappa_{15}^V = c_4, \qquad M_V \kappa_{16}^V = c_6 + c_7,$$

$$M_V \kappa_{17}^V = -c_5 + c_6. \tag{4}$$

No high-energy constraint is considered for the derivation of these relations.

The present study of the anomalous sector at high energies also requires the following pieces of the even-intrinsic parity Lagrangian [8]:

$$\mathcal{L}_{R\chi T}^{even} \supset \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + i d_m \langle P \chi_- \rangle. \tag{5}$$

3. High-energy constraints

3.1. $\langle VVP \rangle$ Green function

We consider first the Green function $\langle VVP \rangle$ between two vector currents $J_V^{\mu,a}(x)$ and $J_V^{\nu,b}(y)$ and one pseudoscalar density $J_P^c(z)$. In momentum space, the OPE prescribes a very precise short-distance behaviour for $\Pi_{VVP}^{\mu\nu}(\lambda p, \lambda q, \lambda r)$ when $\lambda \to \infty$ [24]. Matching the $\langle VVP \rangle$ Green function prediction from the resonance chiral Lagrangian $\mathcal{L}_{R\chi T}^{odd}$ in Eq. (2) and the previously referred OPE asymptotic behaviour yields [16]

$$4c_3 + c_1 = 0, \tag{6}$$

$$c_1 - c_2 + c_5 = 0, \tag{7}$$

$$c_5 - c_6 = \frac{N_C}{64\pi^2} \frac{M_V}{\sqrt{2} F_V}, \tag{8}$$

$$d_1 + 8d_2 = -\frac{N_C}{64\pi^2} \frac{M_V^2}{F_V^2} + \frac{F^2}{4F_V^2}, \tag{9}$$

$$d_3 = -\frac{N_C}{64\pi^2} \frac{M_V^2}{F_V^2} + \frac{F^2}{8F_V^2}. \tag{10}$$

If one incorporates pseudoscalar resonances and the $\hat{\mathcal{O}}_i^{PV}$ operators from Lagrangian $\tilde{\mathcal{L}}_{R\chi T}^{odd}$ in Eq. (3), one now obtains [17][2]

$$M_V \left(2\kappa_{12}^V + 4\kappa_{14}^V + \kappa_{16}^V - \kappa_{17}^V\right) = 4c_3 + c_1 = 0, \tag{11}$$

$$M_V \left(2\kappa_{12}^V + \kappa_{16}^V - 2\kappa_{17}^V\right) = c_1 - c_2 + c_5 = 0, \tag{12}$$

$$-M_V \kappa_{17}^V = c_5 - c_6 = \frac{N_C M_V}{64\sqrt{2}\pi^2 F_V}, \tag{13}$$

$$8\kappa_2^{VV} = d_1 + 8d_2 = -\frac{N_C}{64\pi^2} \frac{M_V^2}{F_V^2} + \frac{F^2}{4F_V^2} + \frac{4\sqrt{2} d_m \kappa_3^{PV}}{F_V}, \tag{14}$$

$$\kappa_3^{VV} = d_3 = -\frac{N_C}{64\pi^2} \frac{M_V^2}{F_V^2} + \frac{F^2}{8F_V^2} + \frac{4\sqrt{2} d_m \kappa_3^{PV}}{F_V}. \tag{15}$$

The five constraints derived in Ref. [17] for κ_{14}^V, $(2\kappa_{12}^V + \kappa_{16}^V)$, κ_{17}^V, κ_2^{VV} and $(8\kappa_2^{VV} - \kappa_3^{VV})$ have been recast in the equivalent form in Eqs. (11)–(15). The κ_l^V and κ_m^{VV} couplings have been rewritten in terms of the c_i and d_j couplings of the $\mathcal{L}_{R\chi T}^{odd}$ Lagrangian by means of the relations in Eq. (4). This reproduces the first three constraints derived from $\mathcal{L}_{R\chi T}^{odd}$ in Eqs. (6)–(8). Notice, however, that the inclusion of the lightest pseudoscalar resonance multiplet modifies the constraints (9) and (10).

3.2. $\tau^- \to (KK\pi)^- \nu_\tau$ form-factors

A series of high-energy constraints were extracted from the analysis of the $\tau^- \to (KK\pi)^- \nu_\tau$ decays [26] after demanding that the corresponding contribution to the spectral function of the vector–vector correlator vanished asymptotically[3]:

$$M_V \left(2\kappa_{12}^V + \kappa_{16}^V - 2\kappa_{17}^V\right) = c_1 - c_2 + c_5 = 0, \tag{16}$$

$$-M_V \kappa_{17}^V = c_5 - c_6 = \frac{N_C}{192\pi^2} \frac{F_V M_V}{\sqrt{2} F^2}, \tag{17}$$

$$\kappa_3^{VV} = d_3 = -\frac{N_C}{192\pi^2} \frac{M_V^2}{F^2}. \tag{18}$$

In order to write the left-hand side of these equations we have employed the relations (4). The relations involving the $V\varphi\varphi\varphi$ couplings (one vector field and three Goldstone fields) are omitted since they are irrelevant for our discussion [26].

3.3. $\tau^- \to \varphi^- \gamma \nu_\tau$ and $\pi^0 \to \gamma^* \gamma^*$ form-factors

The $\tau^- \to \varphi^- \gamma \nu_\tau$ decay ($\varphi^- = \pi^-, K^-$) is described by a vector and an axial-vector form-factors $F_V(t)$ and $F_A(t)$, respectively, which were computed in the RχT framework in Ref. [28]. The requirement that $F_V(t)$ vanishes at high momentum transfer ($t \to \infty$) [25] produces the constraints

$$M_V \left(2\kappa_{12}^V + \kappa_{16}^V - 2\kappa_{17}^V\right) = c_1 - c_2 + c_5 = 0, \tag{19}$$

$$-M_V \kappa_{17}^V = c_5 - c_6 = \frac{N_C}{32\pi^2} \frac{M_V}{\sqrt{2} F_V} + \frac{F_V}{\sqrt{2} M_V} d_3, \tag{20}$$

with $d_3 = \kappa_3^{VV}$ in $\tilde{\mathcal{L}}_{R\chi T}^{odd}$ notation.

The $\pi^0(r) \to \gamma^*(p)\gamma^*(q)$ TFF, $\mathcal{F}_{\pi\gamma^*\gamma^*}(p^2, q^2)$, was studied in Ref. [17] by means of the $\tilde{\mathcal{L}}_{R\chi T}^{odd}$ Lagrangian. Requiring that $\mathcal{F}_{\pi\gamma^*\gamma^*}(0, q^2)$, with one on-shell photon, vanishes at high momentum transfer [25], yields precisely the two previous constraints in Eqs. (19) and (20). One reaches this result if no further short-distance constraints are applied (like, for instance, those from the $\langle VVP \rangle$). Moreover, the requirement that $\mathcal{F}_{\pi\gamma^*\gamma^*}(q^2, q^2)$, with both photons off-shell, vanishes when $q^2 \to \infty$ [29] yields the additional relation

$$-M_V \kappa_{17}^V = c_5 - c_6 = \frac{N_C M_V}{64\sqrt{2}\pi^2 F_V}. \tag{21}$$

Remarkably, although just the $\pi^0 \to \gamma^* \gamma^*$ TFF was constrained to achieve this equation, it reproduces exactly the short-distance $\langle VVP \rangle$ relation in Eq. (13).

Ref. [17], on the other hand, substituted the $\langle VVP \rangle$ relations (13)–(15) and expressed the $\mathcal{F}_{\pi\gamma^*\gamma^*}(0, q^2)$ constraint in the form

$$1 + \frac{32\sqrt{2} F_V d_m \kappa_3^{PV}}{F^2} = 0. \tag{22}$$

3.4. $\tau^- \to (\varphi V)^- \nu_\tau$ vector form-factor

The transition $\tau^- \to (\varphi V)^- \nu_\tau$ (with $\varphi = \pi^{-,0}, K^{-,0}$ and $V = \rho^{0,-}, \omega, \overline{K}^{*0}, K^{*0,-}$) is parametrized by one vector form-factor $V(t)$ and three axial-vector form-factors $A_{1,2,3}(t)$. It was computed

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in Ref. [30] by means of RχT for the case with two vector resonance multiplets. High-energy constraints where extracted after demanding that these form-factors vanished for large momentum transfer [25]. Restricting ourselves to the scenario with only one vector resonance multiplet studied here the vector form-factor relations turn into [30]

$$M_V\left(2\kappa_{12}^V + \kappa_{16}^V - 2\kappa_{17}^V\right) = c_1 - c_2 + c_5 = 0, \tag{23}$$

$$-M_V\kappa_{17}^V = c_5 - c_6 = -\frac{F_V}{\sqrt{2}M_V}d_3, \tag{24}$$

with $d_3 = \kappa_3^{VV}$ in $\tilde{\mathcal{L}}_{R\chi T}^{\text{odd}}$ notation.

3.5. Compatibility between constraints

In a first step, we find that the three $\tau^- \to (KK\pi)^-\nu_\tau$ relations (16)–(18) are compatible with the $\langle VVP \rangle$ relations in Eqs. (11)–(15) provided

$$1 + \frac{32\sqrt{2}F_V d_m \kappa_3^{PV}}{F^2} = 0, \tag{25}$$

$$F_V^2 = 3F^2. \tag{26}$$

The first relation, Eq. (25), was previously obtained in Ref. [17] (Eq. (22)) after requiring the right short-distance behaviour for both the $\langle VVP \rangle$ Green function and the $\pi^0 \to \gamma\gamma^*$ TFF. This condition obviously requires $\kappa_3^{PV} \neq 0$, i.e., the presence of a pseudoscalar resonance contribution.

The second relation, Eq. (26), was also found in the study of the radiative $\tau^- \to \varphi^-\gamma\nu_\tau$ processes in Ref. [28]. Ref. [26] pointed out that, while the $\langle VVP \rangle$ constraints (6)–(10) without pseudoscalar resonances yielded a wrong high-energy structure for $\pi^0 \to \gamma\gamma^*$, the $\tau^- \to (KK\pi)^-\nu_\tau$ conditions (16)–(18) ensured the proper Brodsky–Lepage asymptotic behaviour for the $\pi^0 \to \gamma\gamma^*$ TFF, provided that the constraint (26) is fulfilled.

It is noteworthy that the conditions (25) and (26) exactly agree with the high-energy constraints for the $\tau^- \to \varphi^-\gamma\nu_\tau$ vector form-factor (Eqs. (19) and (20)), $\pi^0 \to \gamma^*\gamma^*$ TFF (Eqs. (19)–(21)) and the $\tau^- \to (\varphi V)^-\nu_\tau$ vector form-factor (Eqs. (23) and (24)).

4. Discussion and comparison with the phenomenology

In summary, we provide in the present letter the unique set of consistent high-energy constraints in the odd intrinsic-parity sector

$$M_V\left(2\kappa_{12}^V + 4\kappa_{14}^V + \kappa_{16}^V - \kappa_{17}^V\right) = 4c_3 + c_1 = 0,$$
$$M_V\left(2\kappa_{12}^V + \kappa_{16}^V - 2\kappa_{17}^V\right) = c_1 - c_2 + c_5 = 0,$$
$$-M_V\kappa_{17}^V = c_5 - c_6 = \frac{N_C M_V}{64\sqrt{2}\pi^2 F_V},$$
$$8\kappa_2^{VV} = d_1 + 8d_2 = \frac{F^2}{8F_V^2} - \frac{N_C M_V^2}{64\pi^2 F_V^2},$$
$$\kappa_3^{VV} = d_3 = -\frac{N_C}{64\pi^2}\frac{M_V^2}{F_V^2},$$
$$1 + \frac{32\sqrt{2}F_V d_m \kappa_3^{PV}}{F^2} = 0,$$
$$F_V^2 = 3F^2, \tag{27}$$

compatible for the $\langle VVP \rangle$ Green function [16,17] and a series of related odd intrinsic-parity amplitudes: the $\tau^- \to X^-\nu_\tau$ vector form-factors ($X^- = (KK\pi)^-$ [26], $\varphi^-\gamma$ [28], $(\varphi V)^-$ [30])

and the $\pi^0 \to \gamma^*\gamma^*$ TFF [17]. The consistent set of high-energy relations (27) for these anomalous QCD amplitudes constitutes the central outcome of this letter. Notice however that only the pseudo-Goldstones and the lightest vector and pseudoscalar resonance multiplets have been considered here, so these relations would change if we varied the resonance content of the theory.

The relations (27) also agree with the short-distance constraints obtained in the analysis of other anomalous processes in the resonance region: the $\tau^- \to \eta\pi^-\pi^0\nu_\tau$ decay [31]; $V \to \varphi\gamma^{(\star)}$ and $\varphi \to \gamma\gamma^{(\star)}$ decays [32]; holographic studies of three-point Green functions and associated form-factors [33]; $e^+e^- \to \varphi\pi^+\pi^-$ ($\varphi = \pi^0, \eta$) [34]; and the $\tau^- \to \pi^-\nu_\tau\ell^+\ell^-$ decay [35].

Interestingly, an analogous set of consistent relations was extracted in the even intrinsic-parity sector in Ref. [15] (Eqs. (4.1), (4.2), (5.7) and (5.12)) for the $\langle VAP \rangle$ Green function [22] and related form-factors [36,37], in combination with the $\pi\pi$ vector form-factor constraint and the two $VV - AA$ WSRs [9].

The last constraint in (27), $F_V^2 = 3F^2$, is particularly interesting, as it was also previously derived in the even intrinsic-parity sector in an independent way. It was found in the high-energy analysis of the $\pi\pi$ vector form-factor at NLO in $1/N_C$ [39] if the scalar resonance effects are disregarded. Likewise, the combination of the large-N_C constraints for $\pi\pi$ vector form-factor ($F_V G_V = F^2$) [9] and $\pi\pi$-scattering ($3G_V^2 = F^2$ if the scalar resonance contributions are neglected) [40] also reproduces the condition $F_V^2 = 3F^2$. These two relations from $\pi\pi$ VFF and scattering also show up in the context of holographic models of QCD [33,41,42] when the derived sum-rules are restricted to the single vector resonance approximation. Indeed, recent studies in that field [33] are pointing out an interconnection between the even intrinsic-parity and anomalous sectors of QCD [33,42,43]. It is also worth to note that this value $F_V = \sqrt{3}F = 3G_V$ was found to be a low-energy fixed point of the renormalized couplings $F_V(\mu)$ and $G_V(\mu)$ within the single vector resonance approximation, being NLO corrections in $1/N_C$ of the order of $\Gamma_\rho/M_\rho \sim 20\%$ [54].

The combined study of the $\pi\pi$ VFF and the $\pi \to \gamma\ell\nu$ axial-vector form-factor produces $F_V = \sqrt{2}F = 2G_V$ if operators with two or more resonance fields are disregarded [9], although this is no longer so when they are taken into full consideration [22]. The large-N_C study of the $\pi\pi$ partial-wave scattering amplitudes $T_J^I(s)$ at high energies[4] yields the generalized version of the KSRF relation [44] including the effect of scalar resonances [40],

$$2c_d^2 + 3G_V^2 = F^2. \tag{28}$$

Nonetheless, Ref. [45] observed that this relation left a residual logarithmically divergent behaviour in the $T_J^I(s)$ at high energies. Moreover, Eq. (28) is incompatible with the $\pi\pi$ VFF constraint $F_V G_V = F^2$ [9] and the odd-sector relation $F_V^2 = 3F^2$ in Eq. (27) if the scalar is taken into account and only the lightest multiplets are accounted for. On the other hand, phenomenologically, it is found to be reasonably well fulfilled and violations are mild: the resonance couplings extracted from pion and kaon scattering phase-shift fits to data were found to fulfill Eq. (28) within $15 \leftrightarrow 20\%$ violations [46], regardless of taking or not the $c_d \to 0$ limit. This slight tension can be relaxed if one discards the $\pi\pi$ partial-wave constraint in Eq. (28) when only the lowest resonance multiplets are included. Alternatively, Ref. [45] advocated that the forward $\pi\pi$ scattering amplitude could be well described at large N_C with just the lightest vector and scalar, obtaining the

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[4] A generalization of the KSRF relation was derived in Ref. [40] by demanding that the $T_J^I(s)$ obeyed once-subtracted dispersion relations at large N_C. The contribution from the lightest vector and scalar resonance exchanges in the s, t and u channels were computed therein, leading to the large-N_C constraint given in Eq. (28).

relation $2c_d^2 + G_V^2 = F^2$. This stems from assuming that the expected power behaviour from Regge theory at high energies (or a less divergent one). However, the latter constraint in combination with the $\pi\pi$ VFF ($F_V G_V = F^2$) and odd-sector one ($F_V^2 = 3F^2$) leads to a scalar coupling $c_d = F/\sqrt{3} \approx 50$ MeV, which is far too large in comparison with previous phenomenological determinations where $c_d \lesssim 30$ MeV (see Refs. [46,47] and references therein).

A good probe of the $F_V^2 = 3F^2$ relation are the $\tau^- \to (\pi\pi\pi)^- \nu_\tau$ decays, even though its check is not free of ambiguities related to the treatment of the $\rho(1450)$ resonance or the $\pi\pi$ rescattering effects producing the σ resonance. In Ref. [37] only the first contribution was included (phenomenologically) and fitting the differential decay widths as a function of the three-pion invariant masses yielded a deviation of $\sim 13\%$ with respect to this prediction. The updated RχT TAUOLA currents [48] added also a modelization of the σ effect and benefited from the two-pion invariant mass distributions measured by BaBar [49] to reach good agreement with data [51]. The fitted value of F_V is consistent with the previous prediction at one sigma, which is remarkable since the quoted error is slightly below 5%.

The consistent set of relations (27) can also be tested through the lightest meson ($\pi^0/\eta/\eta'$) TFF, which are an essential ingredient for predicting the related pseudoscalar-exchange hadronic light-by-light contribution to the muon anomalous magnetic moment. Refs. [17,52] obtain their best agreement with the π TFF data violating only the prediction for κ_3^{PV} in Eqs. (27) by $4 \leftrightarrow 6\%$. The current understanding of the $\eta - \eta'$ mixing in the double-angle mixing scheme allows the prediction of the related $\eta - \eta'$ TFF [52], which are found in good agreement with data as well.

We find, therefore, that the minimal hadronical ansatz [38] – consisting on including as many resonance multiplets as needed to achieve a consistent set of short-distance constraints – reduces to the single (vector and pseudoscalar) resonance approximation for the $\langle VVP \rangle$ Green function and related anomalous form-factors. One must be aware that the OPE constraints from the $\langle VVP \rangle$ Green function in Section 3.1, where the three four-momenta p, q, r are taken to infinity at the same rate, depend crucially on the inclusion of both the lightest pseudoscalar and vector resonances. For instance, the total absence of P resonances leads to roughly a factor 2 difference with respect to alternative determinations of d_3 [50], far more important than the impact of considering $F_V = \sqrt{2}F$ or $F_V = \sqrt{3}F$. Obviously, the unique consistent set of short-distance constraints (27) would be modified if the spectrum of the theory is enlarged.

Despite we deal with the odd-intrinsic parity sector, the constraint $F_V = \sqrt{3}F$ belongs also to the normal parity sector, where one can test the impact of heavier states. Therein, the study of two-meson vector form factors in hadronic tau decays has been sensitive to the effect of excited resonances thanks to the very good quality data taken at B-factories. In the discussion at the beginning of Section 4 in Ref. [35] (see also references therein) it is seen that the modifications induced by the excited resonances in the short-distance relations obtained within the single resonance approximation are – at most – of 5%.

This observation cannot be *a priori* generalized to the asymptotic relations involving couplings which describe interactions between more than one resonance field, where current data are not precise enough to settle this issue, although large deviations are not hinted by the time being.

In the anomaly sector, it is more difficult to quantify possible deviations and the impact from heavier states, due to both the higher complexity of the asymptotic constraints and the larger uncertainty of the measurements. The addition of the first excited vector meson multiplet does not change the analysis of asymptotic constraints derived from the $V\varphi$ form factors [52]. How-

ever, it does modify the high-energy restrictions from the radiative pion form factor (see Eqs. (A.9) and (A.10) in Ref. [53]). According to [53], the first two relations in our Eq. (27) do not get modified by the inclusion of the first excited multiplet.

Nonetheless, the next three relations in (27) do indeed change. For instance, the $(c_5 - c_6)$ constraint becomes [53]

$$(c_5 - c_6) + \frac{F_V' M_V}{F_V M_V'}\left(c_5' - c_6'\right) = \frac{N_C M_V}{64\sqrt{2}\pi^2 F_V}, \tag{29}$$

where the primed parameters refer to the excited vector V' and are defined in analogy to the respective V couplings.

The second term on the left-hand side of Eq. (29) modifies our original equation. Apart from dynamical reasons which might suppress $c_5' - c_6'$ with respect to $c_5 - c_6$, the factor $\frac{F_V' M_V}{F_V M_V'}$ reduces the effect of the former coupling combination by ~ 0.3 [30]. More complicated equations are found in [53] involving d_3, $d_1 + 8d_2$ and analogous and new excited resonances couplings. Likewise, there is no significant tension between the phenomenological analysis of the $\tau^- \to \eta^{(\prime)}\pi^-\pi^0 \nu_\tau$ data [31] and the short-distance constraints (27), which allows us to extract a conservative estimate of the impact of excited multiplets as less than 20% (see Ref. [31] for details).

New, more precise measurements of hadronic (radiative) tau decays, e^+e^- hadroproduction, vector meson decays, meson TFF and meson–meson scattering phaseshifts would be extremely helpful tools in increasing our knowledge of the hadronization of QCD currents in its non-perturbative regime and, in particular, in ascertaining the role of excited resonance multiplets in the corresponding dynamics and its effect in the short-distance relations obtained within the single resonance approximation. These are expected at Belle-II and forthcoming facilities.

Finally, we want to call the attention of the reader to the relations in Eq. (4), which provide a dictionary between the two RχT bases $\mathcal{L}_{R\chi T}^{\text{odd}}$ [16] and $\tilde{\mathcal{L}}_{R\chi T}^{\text{odd}}$ [17] that can be useful in future comparisons.

Acknowledgements

References

[1] S. Weinberg, Physica A 96 (1979) 327.
[2] J. Gasser, H. Leutwyler, Ann. Phys. 158 (1984) 142.
[3] J. Gasser, H. Leutwyler, Nucl. Phys. B 250 (1985) 465.
[4] J. Bijnens, G. Colangelo, G. Ecker, J. High Energy Phys. 9902 (1999) 020, arXiv:hep-ph/9902437.
[5] J. Bijnens, L. Girlanda, P. Talavera, Eur. Phys. J. C 23 (2002) 539, arXiv:hep-ph/0110400.
[6] J. Wess, B. Zumino, Phys. Lett. B 37 (1971) 95.
[7] E. Witten, Nucl. Phys. B 223 (1983) 422.
[8] G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B 321 (1989) 311.
[9] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, E. de Rafael, Phys. Lett. B 223 (1989) 425.
[10] G. 't Hooft, Nucl. Phys. B 72 (1974) 461.
[11] G. 't Hooft, Nucl. Phys. B 75 (1974) 461.
[12] E. Witten, Nucl. Phys. B 160 (1979) 57.
[13] J. Bijnens, E. Gamiz, E. Lipartia, J. Prades, J. High Energy Phys. 0304 (2003) 055, arXiv:hep-ph/0304222.

[14] M. Golterman, S. Peris, Phys. Rev. D 74 (2006) 096002, arXiv:hep-ph/0607152;
P. Masjuan, S. Peris, J. High Energy Phys. 0705 (2007) 040, arXiv:0704.1247 [hep-ph].

[15] V. Cirigliano, G. Ecker, M. Eidemüller, R. Kaiser, A. Pich, J. Portolés, Nucl. Phys. B 753 (2006) 139, arXiv:hep-ph/0603205.

[16] P.D. Ruiz-Femenía, A. Pich, J. Portolés, J. High Energy Phys. 0307 (2003) 003, arXiv:hep-ph/0306157.

[17] K. Kampf, J. Novotny, Phys. Rev. D 84 (2011) 014036, arXiv:1104.3137 [hep-ph].

[18] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 385;
M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B 147 (1979) 448.

[19] S. Weinberg, Phys. Rev. Lett. 18 (1967) 507.

[20] M. Knecht, A. Nyffeler, Eur. Phys. J. C 21 (2001) 659, arXiv:hep-ph/0106034.

[21] B. Moussallam, Phys. Rev. D 51 (1995) 4939, arXiv:hep-ph/9407402;
B. Moussallam, Nucl. Phys. B 504 (1997) 381, arXiv:hep-ph/9701400.

[22] V. Cirigliano, G. Ecker, M. Eidemüller, A. Pich, J. Portolés, Phys. Lett. B 596 (2004) 96, arXiv:hep-ph/0404004.

[23] V. Cirigliano, G. Ecker, M. Eidemüller, R. Kaiser, A. Pich, J. Portolés, J. High Energy Phys. 0504 (2005) 006, arXiv:hep-ph/0503108.

[24] M. Jamin, V. Mateu, J. High Energy Phys. 0804 (2008) 040, arXiv:0802.2669 [hep-ph].

[25] S.J. Brodsky, G.R. Farrar, Phys. Rev. Lett. 31 (1973) 1153;
G.P. Lepage, S.J. Brodsky, Phys. Rev. D 22 (1980) 2157.

[26] D. Gómez Dumm, P. Roig, A. Pich, J. Portolés, Phys. Rev. D 81 (2010) 034031, arXiv:0911.2640 [hep-ph].

[27] E.G. Floratos, S. Narison, E. de Rafael, Nucl. Phys. B 155 (1979) 115.

[28] Z.-H. Guo, P. Roig, Phys. Rev. D 82 (2010) 113016, arXiv:1009.2542 [hep-ph].

[29] F. Jegerlehner, A. Nyffeler, Phys. Rep. 477 (2009) 1, arXiv:0902.3360 [hep-ph];
E.V. Shuryak, A.I. Vainshtein, Nucl. Phys. B 199 (1982) 451.

[30] Z.-H. Guo, Phys. Rev. D 78 (2008) 033004, arXiv:0806.4322 [hep-ph].

[31] D. Gómez Dumm, P. Roig, Phys. Rev. D 86 (2012) 076009, arXiv:1208.1212 [hep-ph].

[32] Y.-H. Chen, Z.-H. Guo, H.-Q. Zheng, Phys. Rev. D 85 (2012) 054018, arXiv:1201.2135 [hep-ph].

[33] P. Colangelo, J.J. Sanz-Cillero, F. Zuo, J. High Energy Phys. 1211 (2012) 012, arXiv:1207.5744 [hep-ph];
P. Colangelo, J.J. Sanz-Cillero, F. Zuo, J. High Energy Phys. 1306 (2013) 020, arXiv:1304.3618 [hep-ph].

[34] L.Y. Dai, J. Portolés, O. Shekhovtsova, Phys. Rev. D 88 (2013) 056001, arXiv:1305.5751 [hep-ph].

[35] P. Roig, A. Guevara, G. López Castro, Phys. Rev. D 88 (2013) 033007, arXiv:1306.1732 [hep-ph].

[36] D. Gómez Dumm, A. Pich, J. Portolés, Phys. Rev. D 69 (2004) 073002, arXiv:hep-ph/0312183.

[37] D. Gómez Dumm, P. Roig, A. Pich, J. Portolés, Phys. Lett. B 685 (2010) 158, arXiv:0911.4436 [hep-ph].

[38] M. Knecht, S. Peris, E. de Rafael, Phys. Lett. B 443 (1998) 255, arXiv:hep-ph/9809594;
M. Knecht, S. Peris, E. de Rafael, Phys. Lett. B 457 (1999) 227, arXiv:hep-ph/9812471;
S. Peris, M. Perrottet, E. de Rafael, J. High Energy Phys. 9805 (1998) 011, arXiv:hep-ph/9805442;
M.F.L. Golterman, S. Peris, Phys. Rev. D 61 (2000) 034018;
M. Knecht, S. Peris, M. Perrottet, E. de Rafael, Phys. Rev. Lett. 83 (1999) 5230, arXiv:hep-ph/9908283.

[39] A. Pich, I. Rosell, J.J. Sanz-Cillero, J. High Energy Phys. 1102 (2011) 109, arXiv:1011.5771 [hep-ph].

[40] Z.H. Guo, J.J. Sanz Cillero, H.Q. Zheng, J. High Energy Phys. 0706 (2007) 030, arXiv:hep-ph/0701232.

[41] J. Hirn, V. Sanz, J. High Energy Phys. 12 (2005) 030, arXiv:hep-ph/0507049;
R.S. Chivukula, M. Kurachi, M. Tanabashi, J. High Energy Phys. 06 (2004) 004, arXiv:hep-ph/0403113.

[42] T. Sakai, S. Sugimoto, Prog. Theor. Phys. 114 (2005) 1083, arXiv:hep-th/0507073.

[43] D.T. Son, N. Yamamoto, arXiv:1010.0718 [hep-ph].

[44] K. Kawarabayashi, M. Suzuki, Phys. Rev. Lett. 16 (1966) 255;
Riazuddin, Fayazuddin, Phys. Rev. 147 (1966) 1071.

[45] J. Nieves, A. Pich, E. Ruiz Arriola, Phys. Rev. D 84 (2011) 096002, arXiv:1107.3247 [hep-ph].

[46] Z.-H. Guo, J.A. Oller, Phys. Rev. D 84 (2011) 034005, arXiv:1104.2849 [hep-ph];
Z.-H. Guo, J.A. Oller, J. Ruiz de Elvira, Phys. Rev. D 86 (2012) 054006, arXiv:1206.4163 [hep-ph].

[47] R. Escribano, P. Masjuan, J.J. Sanz-Cillero, J. High Energy Phys. 1105 (2011) 094, arXiv:1011.5884 [hep-ph].

[48] O. Shekhovtsova, T. Przedzinski, P. Roig, Z. Was, Phys. Rev. D 86 (2012) 113008, arXiv:1203.3955 [hep-ph].

[49] I.M. Nugent, in: Proceedings of the 12th International Workshop on Tau Lepton Physics (TAU 2012), Nucl. Phys. B Proc. Suppl., in press, arXiv:1301.7105 [hep-ex].

[50] B. Ananthanarayan, B. Moussallam, J. High Energy Phys. 0406 (2004) 047, arXiv:hep-ph/0405206.

[51] I.M. Nugent, T. Przedzinski, P. Roig, O. Shekhovtsova, Z. Was, Phys. Rev. D 88 (2013) 093012, arXiv:1310.1053 [hep-ph].

[52] P. Roig, A. Guevara, G. López Castro, Phys. Rev. D 89 (2014) 073016, http://dx.doi.org/10.1103/PhysRevD.89.073016, arXiv:1401.4099 [hep-ph].

[53] V. Mateu, J. Portolés, Eur. Phys. J. C 52 (2007) 325, arXiv:0706.1039 [hep-ph].

[54] J.J. Sanz-Cillero, Phys. Lett. B 681 (2009) 100, arXiv:0905.3676 [hep-ph].