Singlet-Triplet Conversion and the Long-Range Proximity Effect in Superconductor-Ferromagnet Structures with Generic Spin Dependent Fields

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The long-range proximity effect in superconductor-ferromagnet (S/F) hybrid nanostructures is observed if singlet Cooper pairs from the superconductor are converted into triplet pairs which can diffuse into the ferromagnet over large distances. It is commonly believed that this happens only in the presence of magnetic inhomogeneities. We show that there are other sources of the long-range triplet component (LRTC) of the condensate and establish general conditions for their occurrence. As a prototypical example, we consider first a system where the exchange field and spin-orbit coupling can be treated as time and space components of an effective SU(2) potential. We derive a SU(2) covariant diffusive equation for the condensate and demonstrate that an effective SU(2) electric field is responsible for the long-range proximity effect. Finally, we extend our analysis to a generic ferromagnet and establish a universal condition for the LRTC. Our results open a new avenue in the search for such correlations in S/F structures and make a hitherto unknown connection between the LRTC and Yang-Mills electrostatics.

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The odd-triplet superconductivity in superconductor-ferromagnet (S/F) structures has been intensively studied, both theoretically and experimentally, since its prediction in 2001 [1,2]. In that context, it is of crucial interest to understand the process of converting the singlet Cooper pairs from the superconductor into triplet pairs of electrons with equal spins in the ferromagnet. Apart from its interest for fundamental research, the study of triplet superconducting correlations might find useful applications for spintronics [3].

It is well established that triplet pairs, once created, diffuse into the ferromagnetic materials over distances much larger than the singlet ones. This leads to the long-range proximity effect which explains the observation of Josephson currents through $S/F/S$ junctions over large distances [4–10] and the long-range propagation in S/F structures of superconducting correlations [11]. The usual theoretical interpretation of these experiments assumes that the singlet-triplet conversion is mediated by a magnetic inhomogeneity in the vicinity of the S/F interface [12]. This can be caused by a domain wall [2], a spin-active S/F interface [13], or by a multilayered ferromagnetic structure with different magnetic orientations [14].

Formally, the existence of the long-range triplet component (LRTC) can be inferred by inspecting the spin structure of the quasiclassical condensate (anomalous) Green's function (GF) $\hat{f} = f_s + \mathbf{f}_t\hat{\sigma}$. Here, f_s is the amplitude of the singlet component, and the vector $\mathbf{f}_t$ describes the triplet correlations ($\hat{\sigma}$ is the vector of Pauli matrices). The component of the vector $\mathbf{f}_t$ parallel to the magnetization describes the triplet state with zero spin projection (i.e., $|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle$) [15,16]. The LRTC corresponding to the pairs with spin projections ± 1 only exists if $\mathbf{f}_t$ is noncollinear with the magnetization direction, which is the case for certain magnetic inhomogeneities [15–17]. Indeed, it is commonly believed that the only way to generate the LRTC in S/F hybrids is by means of creating a nonhomogeneous magnetic configuration.

In this Letter, we study the LRTC in S/F structures from a more general perspective and demonstrate that, besides an inhomogeneous magnetization, there are other sources of long-range triplet correlations. In particular, the momentum dependence of an effective exchange field, which can be attributed to the spin-orbit (SO) coupling, naturally generates LRTC, provided that certain conditions are fulfilled. We also show that the physical mechanism of the singlet-triplet conversion can be linked to the local SU(2) invariance of magnetized systems with SO interaction [19,20]. To reveal this physics, we first analyze a prototypical example of a spin dependent field consisting of a momentum independent Zeeman term and a SO coupling that is linear in momentum. These two contributions act as the time and space components of the SU(2) gauge potential, which ensures that that the Zeeman and SO fields enter physical quantities only in gauge covariant combinations [21]. We derive a SU(2) covariant Usadel equation and identify the SU(2) electric field as a key object responsible for the long-range triplet proximity effect and for the existence of the LRTC in S/F structures. By solving the Usadel equation for a lateral S/F junction, we demonstrate that, even in the case of a uniform exchange field, the LRTC is present in the system. In the second part, we generalize our results to systems with possibly anisotropic Fermi surfaces and generic spin dependent fields with an arbitrary momentum dependence. We derive the general quasiclassical equations for the anomalous GF, which allows us to establish universal conditions for the creation of triplet long-range superconductivity in diffusive S/F hybrids.

We consider a hybrid structure consisting of a conventional s-wave superconductor (S) with the order parameter Δ in contact with a ferromagnet (F) with a SO coupling. Let us first assume that SO effects can be modeled by a generic form $H_{SO} = \frac{1}{2m}\{p_j, \hat{\mathcal{A}}_j\}$ that is linear in momentum, where $\hat{\mathcal{A}}_j = \mathcal{A}_j^a \sigma^a$ are the components of a 2×2 matrix valued vector that parametrizes the SO coupling. In this case, the Hamiltonian in the F region can be represented in the form

$$H = \frac{1}{2m}(p_j - \hat{\mathcal{A}}_j)^2 - \hat{\mathcal{A}}_0 + V_{\text{imp}}, \tag{1}$$

where $\hat{\mathcal{A}}_0 = \mathcal{A}_0^a \sigma^a \equiv h^a \sigma^a$ is the exchange field and V_{imp} is the spin independent impurity scattering term [22]. The SO coupling and the Zeeman term enter the problem as the space and time components of the SU(2) gauge potential, which implies the SU(2) gauge invariance. The Hamiltonian remains unchanged under any local SU(2) rotation with a matrix $\hat{U}(\mathbf{r})$ supplemented with the gauge transformation of the potentials $\hat{\mathcal{A}}_j \mapsto \hat{U}\hat{\mathcal{A}}_j\hat{U}^{-1} - i(\partial_j \hat{U})\hat{U}^{-1}$ and $\hat{\mathcal{A}}_0 \mapsto \hat{U}\hat{\mathcal{A}}_0\hat{U}^{-1}$. Since we are interested in equilibrium quantities, we work with the Matsubara 4×4 matrix (in the Nambu $\times$ spin space) GF $\check{\mathcal{G}}_\omega(\mathbf{r}_1, \mathbf{r}_2)$ at the discrete frequencies $\omega = \pi T(2n+1)$. To keep track of exact SU(2) gauge symmetry, we employ a technique developed in the context of quark-gluon kinetics [24] and used recently to describe spin dynamics in semiconductors [25]. Namely, we introduce the covariant GF as follows: $\tilde{\check{\mathcal{G}}}_\omega(\mathbf{r}_1, \mathbf{r}_2) = \hat{W}(\mathbf{R}, \mathbf{r}_1)\check{\mathcal{G}}_\omega(\mathbf{r}_1, \mathbf{r}_2)\hat{W}(\mathbf{r}_2, \mathbf{R})$, where $\hat{W}(\mathbf{R}, \mathbf{r}_1)$ and $\hat{W}(\mathbf{r}_2, \mathbf{R})$ are the Wilson link operators which "covariantly connect" the arguments of the GF to the "center-of-mass" coordinate $\mathbf{R} = \frac{\mathbf{r}_1+\mathbf{r}_2}{2}$ [26]. The advantage of the covariant GF is that its Wigner transform, and thus the corresponding quasiclassical GF $\tilde{\check{g}}(\mathbf{n}, \mathbf{R})$, transform locally covariantly under a nonuniform SU(2) rotation, i.e., $\tilde{\check{g}} \mapsto \hat{U}\,\tilde{\check{g}}\,\hat{U}^{-1}$. By using the method of Ref. [24], we can derive the equation of motion for $\tilde{\check{\mathcal{G}}}_\omega$ and then proceed further to the quasiclassical limit and eventually to the diffusive Usadel equation [27]. Here, we only show the final linearized Usadel equation in the ferromagnet for the covariant anomalous function $\tilde{\hat{f}}$

$$D\tilde{\nabla}_k(\tilde{\nabla}_k\tilde{\hat{f}}) - 2|\omega|\tilde{\hat{f}} - i\,\text{sgn}\omega\{\hat{\mathcal{A}}_0, \tilde{\hat{f}}\} = 0. \tag{2}$$

Here, $\tilde{\nabla}_k$ is the covariant gradient operator defined by $\tilde{\nabla}_k\Psi = \partial_k\Psi - i[\hat{\mathcal{A}}_k, \Psi]$. At the S/F boundary, we use the Kupriyanov-Lukichev boundary conditions [28] which take the form

$$N_k\tilde{\nabla}_k\tilde{\hat{f}}|_I = -\gamma f_\Delta, \tag{3}$$

where $f_\Delta = \Delta/\sqrt{\omega^2 + \Delta^2}$ is the anomalous GF in the S region and N_k is the k component of the vector normal to the S/F interface. Hence, in the covariant formalism, the usual gradients are replaces by the covariant ones, and this is the only place where the SO coupling enters the theory. Equations (2) and (3) are manifestly gauge covariant, and their structure is very physically appealing. In fact, they can be written immediately by using only the gauge symmetry arguments [29].

Next, we write $\tilde{\hat{f}}$ as the sum of the singlet f_s and triplet $\tilde{\hat{f}}_t$ contributions, splitting out of $\tilde{\hat{f}}_t$ the part parallel to the exchange field $\hat{\mathcal{A}}_0$:

$$\tilde{\hat{f}} = f_s + \tilde{\hat{f}}_t = f_s + \hat{\mathcal{A}}_0 f_t^{\parallel} + \tilde{\hat{f}}_t^{\perp}. \tag{4}$$

For any matrix $\hat{M}$, we have defined $\hat{M}^{\perp} = \frac{1}{4}[\hat{\mathcal{A}}_0, [\hat{\mathcal{A}}_0, \hat{M}]]/|\hat{\mathcal{A}}_0|^2$, where $|\hat{\mathcal{A}}_0| = \sqrt{\mathcal{A}^a\mathcal{A}^a}$ is the amplitude of the exchange field.

A trace of Eqs. (2) and (3) gives the equations for the singlet amplitude f_s coupled to the parallel triplet amplitude $f_t^{\parallel}$:

$$\begin{aligned} \nabla^2 f_s - \kappa_\omega^2 f_s - 2i\frac{\text{sgn}\omega}{D}|\hat{\mathcal{A}}_0|^2 f_t^{\parallel} = 0, \\ N_k\partial_k f_s|_I = -\gamma f_\Delta, \end{aligned} \tag{5}$$

where ∇^2 is the usual Laplace operator and $\kappa_\omega^2 = 2|\omega|/D$. The traceless part of Eqs. (2) and (3), can be rearranged as follows:

$$\begin{aligned} &\hat{\mathcal{A}}_0\left[\nabla^2 f_t^{\parallel} - \kappa_\omega^2 f_t^{\parallel} - i2\frac{\text{sgn}\omega}{D} f_s\right] \\ &\quad + [\tilde{\nabla}_k(\tilde{\nabla}_k\tilde{\hat{f}}_t^{\perp}) - \kappa_\omega^2\tilde{\hat{f}}_t^{\perp} + 2\hat{\mathcal{F}}_{k0}\partial_k f_t^{\parallel} + f_t^{\parallel}\tilde{\nabla}_k\hat{\mathcal{F}}_{k0}] = 0, \end{aligned} \tag{6}$$

$$N_k(\hat{\mathcal{A}}_0\partial_k f_t^{\parallel} + \tilde{\nabla}_k\tilde{\hat{f}}_t^{\perp} + \hat{\mathcal{F}}_{k0} f_t^{\parallel})|_I = 0. \tag{7}$$

Here, $\hat{\mathcal{F}}_{k0}$ is the SU(2) electric field, defined as

$$\hat{\mathcal{F}}_{k0} = \partial_k\hat{\mathcal{A}}_0 - i[\hat{\mathcal{A}}_k, \hat{\mathcal{A}}_0]. \tag{8}$$

An inspection of Eqs. (6) and (7) shows that $\hat{\mathcal{F}}_{k0}$ is the key object for the long-range proximity effect: If $\hat{\mathcal{F}}_{k0} = 0$, the homogeneity of the boundary condition (7) implies that $\hat{\tilde{f}}_t^{\perp} = 0$. In other words, no transverse triplet is generated in the absence of the SU(2) electric field. $\hat{\mathcal{F}}_{k0}$ vanishes in the case of a uniform magnetization and if the SO and Zeeman terms commute.

If $\hat{\mathcal{F}}_{k0}$ is small, Eqs. (6) and (7) can be treated perturbatively. The leading contribution is given by the terms proportional to $\hat{\mathcal{A}}_0$, i.e., by the first line in Eq. (6) and by the first term in Eq. (7). This yields the well known equations for the short-range triplet component $f_t^{\parallel}$ coupled to the singlet one f_s,

$$\nabla^2 f_t^{\parallel} - \kappa_\omega^2 f_t^{\parallel} - 2i\frac{\mathrm{sgn}\omega}{D} f_s = 0, \qquad N_k \partial_k f_t^{\parallel}|_I = 0. \tag{9}$$

Equations for the lowest in $\hat{\mathcal{F}}_{k0}$ correction to $\hat{\tilde{f}}_t$ come from the second line in Eq. (6) and the last two terms in Eq. (7):

$$\nabla^2 \hat{\tilde{f}}_t^{\perp} - \kappa_\omega^2 \hat{\tilde{f}}_t^{\perp} = -[2\hat{\mathcal{F}}_{k0}\partial_k f_t^{\parallel} + f_t^{\parallel}\tilde{\nabla}_k \hat{\mathcal{F}}_{k0}]^{\perp}, \tag{10}$$

$$N_k(\partial_k \hat{\tilde{f}}_t^{\perp} + \hat{\mathcal{F}}_{k0}^{\perp} f_t^{\parallel})|_I = 0. \tag{11}$$

In the first terms in the left-hand sides, we replaced $\tilde{\nabla}_k$ with ∂_k as the difference of these operators gives higher order corrections compared to those determined by the terms $\sim f_t^{\parallel}$.

Equations (5) and (9)–(11) provide a complete description of S/F structures, which clearly demonstrates a common physical role of the SO coupling and inhomogeneous magnetization in the problem of the singlet-triplet conversion. As is expected on general grounds, they appear in the theory in the form of a single gauge covariant object—the SU(2) electric field $\hat{\mathcal{F}}_{k0}$ Eq. (8) entering the "source part" of Eq. (10) for $\hat{\tilde{f}}_t^{\perp}$.

It is easy to see that the well known generation of LRTC by magnetic inhomogeneities follows naturally from our covariant formulation. Consider, for example, a transversal multilayer S/F structure shown in Fig 1(a) in the absence of SO coupling. In this case, a nonzero $\hat{\mathcal{F}}_{k0}$ is solely due to inhomogeneity of the exchange field $\hat{\mathcal{A}}_0$. Assume that $\hat{\mathcal{A}}_0$ has only an in-plane component (eventually rotating). Then, the first term in the right-hand side of Eq. (8) generates the LRTC for a Bloch domain wall parallel to the interface [2], while the second term $\sim\tilde{\nabla}_k\hat{\mathcal{F}}_{k0}$ is responsible for the LRTC in the presence of a finite Néel wall perpendicular to the interface plane [30]. It is interesting to note that the covariant derivative $\tilde{\nabla}_k\hat{\mathcal{F}}_{k0}$ of the non-Abelian electric field is exactly the right-hand side of the Yang-Mills electrostatic equation. The general gauge symmetry arguments of Ref. [21] (used there to uncover the nature of the equilibrium spin currents) show that $\tilde{\nabla}_k\hat{\mathcal{F}}_{k0}$ is proportional to the magnetization induced in the F region by a nonuniform exchange field and/or SO coupling. This reveals the nature of the second term in Eq. (10) and provides an interesting connection between the generation of LRTC at the edges of Néel domain walls and the Yang-Mills electrostatics.

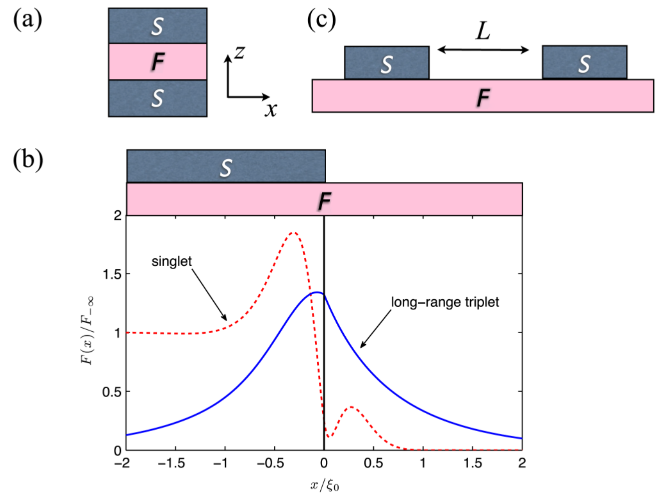

FIG. 1 (color online). Different geometers considered in the text. (a) The transversal $S/F/S$ junction. (b) The lateral S/F structure and space dependence of $F = \sum_\omega |f|^2$ for the singlet and LRTC normalized with the asymptotic value at $x = -\infty$. Here, $\xi_0 = \sqrt{D/\Delta}$, $h = 10\Delta$, $T = 0.05\Delta$, and Δ is the order parameter in the superconductor. (c) The lateral Josephson junction.

We now analyze a SO-generated LRTC in the case of a uniform magnetization. A special type of SO coupling in Eq. (1) should naturally occur in the vicinity of hetero-interfaces where inversion asymmetry exists [31–33]. Hence, we concentrate on the situation when the SU(2) vector potential is localized around the S/F interface and has in-plane components $\hat{\mathcal{A}}_x$ and $\hat{\mathcal{A}}_y$. This implies that only $\hat{\mathcal{F}}_{x0}$ and $\hat{\mathcal{F}}_{y0}$ are nonzero.

As a first example, we consider a transversal S/F structure [Fig. 1(a)] assuming for definiteness a Rashba-Dresselhaus SO term with $\hat{\mathcal{A}}_x = \beta\hat{\sigma}_x - \alpha\hat{\sigma}_y$ and $\hat{\mathcal{A}}_y = (\alpha\hat{\sigma}_x - \beta\hat{\sigma}_y)$, where α and β are the Rashba and Dresselhaus constants. We assume a contstant in-plane magnetization. Thus, only the second term in the right-hand side of Eq. (10) serves as source for the LRTC. One can easily show that this term is nonzero only if $\alpha\beta \neq 0$ and $\mathcal{A}_0^x \neq \mathcal{A}_0^y$. This in particular means that a pure Rashba or Dresselhaus SO coupling does not induce the LRTC in a transversal geometry with an in-plane magnetization. We emphasize that the LRTC discussed here has s-wave symmetry, in contrast with the odd in momentum triplet component predicted in Refs. [32,33] for pure ballistic S/F and S/N systems in the presence of an interface SO coupling.

Lateral S/F structures are more favorable for the existence of LRTC. Consider the structure shown in Fig. 1(b).

Assuming for simplicity, but without loss of generality, that the F film is thin enough, we integrate Eqs. (5) and (9)–(11) over the z direction and obtain the following set of 1D equations

$$\partial_x^2 f_s - \kappa_\omega^2 f_s - 2i\frac{\mathrm{sgn}\omega}{D}|\hat{\mathcal{A}}_0|^2 f_t^{\parallel} = -\theta(-x)\bar{\gamma} f_\Delta, \quad (12)$$

$$\partial_x^2 f_t^{\parallel} - \kappa_\omega^2 f_t^{\parallel} - \frac{2i\mathrm{sgn}\omega}{D} f_s = 0, \quad (13)$$

$$\partial_x^2 \hat{\vec{f}}_t^{\perp} - \kappa_\omega^2 \hat{\vec{f}}_t^{\perp} = -2\theta(-x)\hat{\bar{\mathcal{F}}}_{x0}\partial_x f_t^{\parallel}, \quad (14)$$

where $\bar{\gamma}$ and $\bar{\mathcal{F}}_{x0}$ are effective values averaged over the thickness. The boundary conditions at $x = 0$ are the continuity of all functions and the continuity of $\partial_x f_s$, $\partial_x f_t^{\parallel}$, and $\partial_x \hat{\vec{f}}_t^{\perp} + \hat{\mathcal{F}}_{x0} f_t^{\parallel}$. This boundary problem can be solved straightforwardly. In the interesting limiting case of the exchange field $|\hat{\mathcal{A}}_0|$ much larger than the superconducting gap Δ, the condensate function at $x > 0$ [cf. Fig. 1(b)] is given by the expression

$$\hat{\vec{f}}(x) = \sum_\alpha C_\alpha e^{-\kappa_\alpha x}[1 + (-1)^\alpha \hat{\mathcal{A}}_0/|\hat{\mathcal{A}}_0|] + \hat{\vec{\mathcal{F}}}_{x0}^{\perp} C e^{-\kappa_\omega x}, \quad (15)$$

where $\alpha = 1, 2$; $C_{1,2} \approx -\bar{\gamma} f_\Delta D^2 \kappa_{2,1}^2/(4|\mathcal{A}_0|^2)$; and $\kappa_{1,2}^2 = \pm 2i\mathrm{sgn}\omega|\mathcal{A}_0|/D$, with $\mathrm{Re}\kappa_{1,2} > 0$ and $C \approx -(3i/2)\bar{\gamma} f_\Delta D\mathrm{sgn}\omega/(|\mathcal{A}_0|^2\kappa_\omega)$. All components decay away from the edge plane $x = 0$. The singlet and parallel to $\hat{\mathcal{A}}_0$ triplet components [the first term in Eq. (15)] decay over the short magnetic distance $\sqrt{D/2h}$, while the triplet component perpendicular to $\hat{\mathcal{A}}_0$ [the second term in Eq. (15)] decays over a larger length of the order of $\sqrt{D/2T}$, confirming its long-range character. Figure 1(b) shows that the LRTC decays in both directions from the inhomogeneity at $x = 0$, which looks very similar to the LRTC generated at the edge of a Néel domain wall [30]. This similarity is not accidental. In fact, in the particular case of $\hat{\mathcal{A}}_y = 0$, our system is gauge equivalent to the Néel wall with an edge at $x = 0$.

We now consider a lateral structure with two S electrodes separated by a distance L [see Fig. 1(c)]. If $L \gg \sqrt{D/2h}$, the Josephson coupling is only mediated by the LRTC, and the critical current is given by

$$I_c = \left(\frac{\mathcal{S}\sigma_F}{e}\right)\mathrm{tr}(\hat{\vec{\mathcal{F}}}_{x0}^{\perp})^2 T\sum_{\omega_n}\kappa_\omega C^2(\omega_n)e^{-\kappa_\omega L}, \quad (16)$$

where $\mathcal{S}$ and σ_F are the cross section and conductivity of the F region. From this equation, one concludes that a finite SU(2) electric field with a component perpendicular to the magnetization is the source of the long-range Josephson effect. The lateral geometry shown in Fig. 1(c) is equivalent to the one explored in the experiments of Refs. [4,8]. Thus, our theory gives a plausible explanation for the long-range effects observed in these experiments. One argues in that case that the long-range Josephson current is either due to a SO coupling at the S/F interfaces [4] or to a Rashba-type SO coupling in the quasi-1D geometry of Ref. [8]. A triplet component can also be induced in a superconductor-normal metal-superconductor lateral structure with Rashba SO coupling in an external Zeeman field [34].

We finally generalize our results to ferromagnets with a generic momentum dependent effective exchange field. Our starting point is the following Hamiltonian in the F region

$$H = \xi_{\mathbf{p}} - [b^a(\mathbf{p}) + h^a(\mathbf{p})]\sigma^a. \quad (17)$$

Here, $\xi(\mathbf{p})$ is the spin independent part of the quasiparticle energy. The spin dependent contribution is written as the sum of an even $\hat{h}(-\mathbf{p}) = \hat{h}(\mathbf{p})$ and an odd $\hat{b}(-\mathbf{p}) = -\hat{b}(\mathbf{p})$ in momentum parts. $\hat{h}(\mathbf{p})$ describes the Zeeman-type exchange term, which is, in general, momentum dependent in realistic systems. The odd part $\hat{b}(\mathbf{p})$ corresponds to a generic SO interaction which preserves the time reversal symmetry [35].

Following the standard procedure (see, for example, Ref. [36]), one derives the Eilenberger equation [37] for a generic spin dependent Hamiltonian

$$v_k^F(\mathbf{n})\partial_k\check{g} + [\omega\tau_3, \check{g}] - i[\hat{h}(\mathbf{n})\tau_3 + \hat{b}(\mathbf{n}), \check{g}] = -\frac{1}{2\tau}[\langle\check{g}\rangle, \check{g}], \quad (18)$$

where $\mathbf{n}$ is a unit vector pointing in the direction of momentum, $v_k^F(\mathbf{n})$ are components of the Fermi velocity, τ is the impurity scattering time, and the angled brackets denote averaging over $\mathbf{n}$. This equation allows for a general anisotropy with different velocities and spin splittings at different points on the Fermi surface. In the particular case of an isotropic h, Rashba SO coupling, and a pure ballistic system, we recover the equation used in Ref. [38]. We focus here on the diffusive limit, in which τ^{-1} determines the largest energy scale in Eq. (18). In this case, Eq. (18) reduces to the Usadel equation for the angle-averaged GF [27,39]. For a general anisotropic ferromagnet, the linearized Usadel equation takes the form

$$D_{kj}\partial_k\partial_j\hat{f} - 2\omega\hat{f} - i\mathrm{sgn}\omega\{\langle\hat{h}\rangle, \hat{f}\} - 2i\tau[\langle v_k^F\hat{b}\rangle, \partial_k\hat{f}] - i\tau[\partial_k\langle v_k^F\hat{b}\rangle, \hat{f}] - \tau\langle[\hat{b}, [\hat{b}, \hat{f}]]\rangle - \tau\langle\{\delta\hat{h}, \{\delta\hat{h}, \hat{f}\}\}\rangle = 0, \quad (19)$$

where $D_{kj} = \tau\langle v_k^F v_j^F\rangle$ is the tensor of diffusion coefficients and $\delta\hat{h}(\mathbf{n}) = \hat{h}(\mathbf{n}) - \langle\hat{h}\rangle$ is the variation of the Zeeman field at the Fermi surface. With the exception of the last term containing $\delta\hat{h}$, Eqs. (19) and the gauge covariant Usadel equation, Eq. (2), are structurally equivalent. The first three terms in Eq. (19) correspond to Eq. (2) without the commutators coming from the covariant gradients. These terms in Eq. (2) correspond to the commutators in Eq. (19) involving the SO field $\hat{b}$. Because of this similarity, our analysis of Eq. (2) is directly applicable to Eq. (19). Hence, we conclude that the last four terms in Eq. (19) serve as a the source for the LRTC. More precisely, the LRTC is generated if any of these terms has a finite component perpendicular to the exchange field $\langle \hat{h} \rangle$ averaged over the Fermi surface. We can draw a remarkable conclusion from this result: From the knowledge of the electronic properties at the Fermi level of S/F systems, namely, from $\xi(\mathbf{p})$, $\hat{h}(\mathbf{p})$, and $\hat{b}(\mathbf{p})$ in Eq. (17), one can easily infer whether or not the LRTC would exist in the hybrid structure. Moreover, Eq. (19) is quite general and can be used in a broad context of problems involving superconductivity and spin fields.

In conclusion, we presented a general description of the long-range triplet superconductivity in S/F structures. Starting from the linear in momentum SO coupling we developed the SU(2) covariant theory describing the diffusion of the condensate and identified the SU(2) electric field as a physical source for the LRTC. We also considered the case of an arbitrary momentum dependence of the spin fields and derived a useful equation from which, by knowledge of the electronic structure of the ferromagnet and the interfaces, one can directly predict whether the LRTC is generated or not. Our results not only unify in an elegant way all models describing the long-range proximity effect in S/F structures but also predict new sources for the singlet-triplet conversion and provide a useful tool in the search for triplet superconducting correlations.

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