

Holographic kinetic k-essence model

Norman Cruz,^{1,*} Pedro F. González-Díaz,^{2,†} Alberto Rozas-Fernández,^{2,†} and Guillermo Sánchez^{3,‡}

¹*Departamento de Física, Facultad de Ciencia, Universidad de Santiago de Chile, Casilla 307, Santiago, Chile.*

²*Colina de los Chopos, Instituto de Física Fundamental,*

Consejo Superior de Investigaciones Científicas, Serrano 121, 28006, Madrid, Spain

³*Departamento de Matemática y Ciencia de la Computación, Facultad de Ciencia, Universidad de Santiago de Chile, Casilla 307, Santiago, Chile.*

(Dated: January 3, 2014)

We consider a connection between the holographic dark energy density and the kinetic k-essence energy density in a flat FRW universe. With the choice $c \geq 1$, the holographic dark energy can be described by a kinetic k-essence scalar field in a certain way. In this paper we show this kinetic k-essential description of the holographic dark energy with $c \geq 1$ and reconstruct the kinetic k-essence function $F(X)$.

There is a strong belief that the universe is undergoing an epoch of accelerated expansion. Recent cosmological observations from Type Ia supernovae (SN Ia) [1], Cosmic Microwave Background (CMB) anisotropies measured with the WMAP satellite [2], Large Scale Structure [3], weak lensing [4] and the integrated Sachs-Wolfe effect [5] provide an overwhelming evidence in favour of a present accelerating universe. Within the framework of the standard Friedman-Robertson-Walker cosmology, this present acceleration requires the existence of a negative pressure fluid, dubbed dark energy, whose pressure p_Λ and density ρ_Λ satisfy $\omega_\Lambda = p_\Lambda/\rho_\Lambda < -1/3$. Unsurprisingly, the unknown nature and origin of dark energy has become a fundamental problem in theoretical physics and observational cosmology. The cosmological constant (or vacuum energy) is the most obvious candidate to address this issue as it complies well with the cosmological tests at our disposal. However, the well known problem of the cosmological constant and the coincidence problem [6] are enough reasons to look for alternatives. Interesting proposals are the quantum cosmic model [7] and $f(R)$ theories (see [8] for recent reviews and references therein). On the other hand, we have a plethora of dynamical dark energy models such as quintessence [9], tachyon [10], phantom [11], quintom [12], etc. But these scalar field dark energy models are only seen as an effective description of the underlying theory of dark energy.

In search of a more profound approach, holographic dark energy models [13, 14, 15] have been recently advanced which rely on the holographic principle [16], which is believed to be a fundamental principle in the quantum theory of gravity. Therefore these models incorporate significant features of the underlying theory of dark energy. The holographic principle is a conjecture stating that all the information stored within some volume can be described by the physics at the boundary of the volume and, in the cosmological context, this principle will

set an upper bound on the entropy of the universe. With the Bekenstein bound in mind, it seems to make sense to require that for an effective quantum field theory in a box of size L with a short distance cutoff (UV cutoff: Λ), the total entropy should satisfy the relation

$$L^3 \Lambda^3 \leq S_{BH} = \pi L^2 M_p^2, \quad (1)$$

where M_p is the reduced Planck mass and S_{BH} is the entropy of a black hole of radius L which acts as a long distance cutoff (IR cutoff: L). However, based on the validity of effective quantum field theory Cohen et al [13] suggested a more stringent bound, requiring that the total energy in a region of size L should not exceed the mass of a black hole of the same size.

Therefore, this $UV - IR$ relationship gives an upper bound on the zero point energy density

$$\rho_\Lambda \leq L^{-2} M_p^2 \quad (2)$$

which means that the maximum entropy is

$$S_{max} \approx S_{BH}^{3/4}. \quad (3)$$

The largest L is chosen by saturating the bound in Eq.(2) so that we obtain the holographic dark energy density

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2} \quad (4)$$

where c is a free dimensionless $O(1)$ parameter and the coefficient 3 is chosen for convenience. Interestingly, this ρ_Λ is comparable to the observed dark energy density $\sim 10^{-10} eV^4$ for $H = H_0 \sim 10^{-33} eV$, the Hubble parameter at the present epoch. The fact that quantum field theory over-counts the independent physical degrees of freedom inside the volume explains the success of this estimation over the naive estimate $\rho_\Lambda = O(M_p^4)$. Therefore, holographic dark energy models have the advantage over other models of dark energy in that they do not need an *ad hoc* mechanism to cancel the $O(M_p^4)$ zero point energy of the vacuum.

If we take L as the Hubble scale H^{-1} , then the dark energy density will be close to the observational result.

*ncruz@lauca.usach.cl

†a.rozas@cfmac.csic.es

‡gsanchez@usach.cl

However, Hsu [14] pointed out that this yields a wrong equation of state for dark energy. This led Li [15] to propose that the IR cut-off L should be taken as the size of the future event horizon of the universe

$$R_{\text{eh}}(a) = a \int_t^\infty \frac{dt'}{a(t')} = a \int_a^\infty \frac{da'}{Ha'^2}. \quad (5)$$

This allows to construct a satisfactory holographic dark energy model that may provide natural solutions to both dark energy problems as showed in [15]. It is worth remarking, too, that the holographic dark energy model has been tested and constrained by several astronomical observations [17]. If we then assume the holographic vacuum energy scenario as the underlying theory of dark energy, we want to see how the scalar field model can be used to effectively describe it. Some work has been done in this direction. Holographic quintessence and holographic quintom models have been discussed in [18] and [19] respectively and the holographic tachyon model in [20]. Other relevant works can be found in [21]. Our present work aims at constructing the holographic kinetic k-essence model of dark energy, relating the kinetic k-essence scalar-field with the holographic dark energy.

In order to build our holographic model, we impose the holographic nature to the kinetic k-essence, i.e., we identify ρ_ϕ with ρ_Λ .

We consider a universe filled with a matter component ρ_m (including both baryons and cold dark matter) and an holographic kinetic k-essence component ρ_ϕ , the Friedman equation reads

$$3M_P^2 H^2 = \rho_m + \rho_\phi, \quad (6)$$

or equivalently

$$H(z) = H_0 \left(\frac{\Omega_{m0}(1+z)^3}{1-\Omega_\phi} \right)^{1/2} \quad (7)$$

where $z = (1/a) - 1$ is the redshift of the universe. From the definition of the holographic dark energy and the definition of the future event horizon, we find

$$\int_a^\infty \frac{da'}{Ha'^2} = \int_x^\infty \frac{dx}{Ha} = \frac{C}{\sqrt{\Omega_\phi} Ha} \quad (8)$$

The Friedman equation (7) implies

$$\frac{1}{Ha} = \sqrt{a(1-\Omega_\phi)} \frac{1}{H_0 \sqrt{\Omega_{m0}}} \quad (9)$$

Substituting (9) into (8), we obtain the following equation

$$\int_x^\infty e^{x'/2} \sqrt{1-\Omega_\phi} dx' = C e^{x/2} \sqrt{\frac{1}{\Omega_\phi} - 1}, \quad (10)$$

where $x = \ln a$. The differential equation for the fractional density of dark energy is obtained by taking the

derivative with respect to x in both sides of equation (10), yielding

$$\Omega'_\phi = -(1+z)^{-1} \Omega_\phi (1-\Omega_\phi) \left(1 + \frac{2}{c} \sqrt{\Omega_\phi} \right), \quad (11)$$

where the prime denotes the derivative with respect to the redshift z . This equation has an exact solution [15] and describes the evolution of the holographic dark energy as a function of the redshift. Since Ω'_ϕ is always positive, the fraction of dark energy increases with time. From the energy conservation equation of dark energy, the equation of state of dark energy can be given [15]

$$\omega_\phi = -1 - \frac{1}{3} \frac{d \ln \rho_\phi}{d \ln a} = -\frac{1}{3} \left(1 + \frac{2}{c} \sqrt{\Omega_\phi} \right). \quad (12)$$

Note that the formula $\rho_\phi = \frac{\Omega_\phi}{1-\Omega_\phi} \rho_m^0 a^{-3}$ and the differential equation of Ω_ϕ , Eq.(11), are used in the second equal sign.

Usually k-essence is defined as a scalar field ϕ with a non-canonical kinetic energy associated with a lagrangian $\mathcal{L} = -V(\phi)F(X)$. In the subsequent calculations, we shall restrict ourselves to the simple k-essence models for which the potential $V = V_0 = \text{constant}$. We also assume that $V_0 = 1$ without any loss of generality. One reason for studying k-essence it that it is possible to construct a particularly interesting class of such models in which the k-essence energy density tracks the radiation energy density during the radiation-dominated era, but then evolves toward a constant-density dark energy component during the matter-dominated era. Such behavior can to a certain degree solve the coincidence problem [22].

We investigate a dark energy model described by an effective minimally coupled scalar field with a non-canonical term. If for a moment we neglect the part of the Lagrangian containing ordinary matter, the general action for a k-essence field ϕ minimally coupled to gravity is

$$S = S_G + S_\phi = - \int d^4x \sqrt{-g} \left(\frac{R}{2} + F(\phi, X) \right), \quad (13)$$

where $F_k(\phi, X)$ is an arbitrary function of ϕ that represents the k-essence action and $X = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi$ is the kinetic term.

We now restrict ourselves to the subclass of kinetic k-essence, with an action independent of ϕ

$$S = - \int d^4x \sqrt{-g} F(X). \quad (14)$$

We assume a Friedman-Robertson-Walker metric $ds^2 = dt^2 - a^2(t) d\vec{x}^2$ (where $a(t)$ is the scale factor) and work in units $c = \hbar = 1$. Unless stated otherwise, we consider ϕ to be smooth on scales of interest so that $X = \frac{1}{2} \dot{\phi}^2 \geq 0$. The energy-momentum tensor of the k-essence is obtained by varying the action (14) with respect to the metric, yielding

$$T_{\mu\nu} = F_X \partial_\mu \phi \partial^\mu \phi - g_{\mu\nu} F, \quad (15)$$

where the subscript X denotes differentiation with respect to X . Identifying (15) with the energy-momentum tensor of a perfect fluid we have the k-essence energy density ρ_ϕ and pressure p_ϕ

$$\rho_\phi = F - 2XF_X \quad (16)$$

and

$$p_\phi = -F. \quad (17)$$

Throughout this paper, we will assume that the energy density is positive so that $F - 2XF_X > 0$. The equation of state for the k-essence fluid can be written as $p_\phi = w_\phi \rho_\phi$ with $F > 0$,

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{F}{2XF_X - F}. \quad (18)$$

For a flat FRW metric, applying the Euler-Lagrange equation for the field to the action (14) we find the equation of motion for k-essence field

$$(F_X + 2XF_{XX})\ddot{\phi} + 3HF_X\dot{\phi} = 0, \quad (19)$$

which can be rewritten in terms of X as

$$(F_X + 2XF_{XX})\dot{X} + 6HF_X X = 0, \quad (20)$$

where the dot denotes differentiation with respect to the cosmic time and $H = \dot{a}/a$ is the Hubble parameter. If we now change the independent variable from time t to the scale factor a , we obtain

$$(F_X + 2XF_{XX})a \frac{dX}{da} + 6F_X X = 0. \quad (21)$$

This equation can be integrated exactly, for arbitrary F , yielding

$$XF_X^2 = ka^{-6}, \quad (22)$$

where k is a constant of integration [23]. Given a function $F(X)$, Eq.(22) allows us to find solutions $X(a)$ and then the other parameters of the k-essence fluid like ρ_ϕ , p_ϕ and ω_ϕ as a function of the scale factor, a .

From Eqs.(16),(18) and (6), we can obtain the expression for F as a function of the redshift z

$$F(z) = -\rho_\phi \omega_\phi = -3M_p^2 H^2(z) \Omega_\phi(z) \omega_\phi(z). \quad (23)$$

Note that, since $\omega_\phi(z) < 0$, the above expression indicates that F is positive in this approach. If we demand that the energy density be positive, Eq.(16) implies that $F_X < F/2X$. Therefore, for kinetic k-essence, $F > 0$ and $F_X < 0$ imply that $w > -1$ (cf.[24] noticing the difference in the sign convention for the energy density and the pressure). Now we focus on the reconstruction of $F(X)$ in the redshift range between $z = 0$ and $z = 1.8$ which is the current range for the supernova data. We shall do so in the light of the holographic dark energy

with $c \geq 1$ as the future event horizon is only well defined when $w \geq -1$ (see [15]). As an example, we plot in Fig.1 some evolutions of the equation of state of the holographic dark energy. We show in the plot the cases $c = 1, 1.1, 1.2$ and 1.3 . It is clear that for these cases $c \geq 1$, they always evolve in the region of $w \geq -1$.

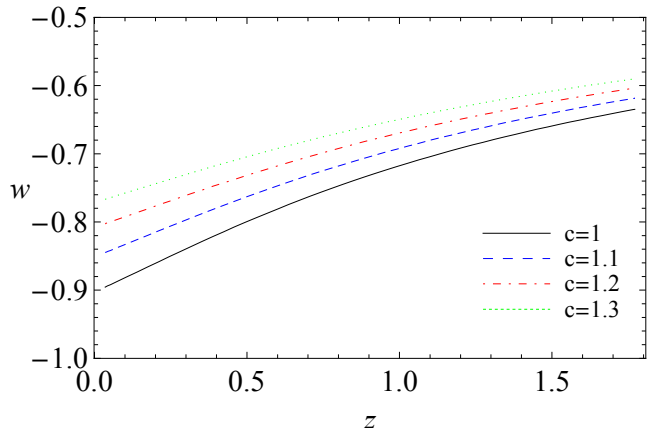


FIG. 1: The evolutions of the equation of state of holographic dark energy. Here we take $\Omega_{m0} = 0.27$, and show the cases for $c = 1, 1.1, 1.2$ and 1.3 .

In order to carry out the numerical evaluation which allows to find F as a function of X , we use the dimensionless variable $\mathcal{F} = F/(M_p^2 H_0^2)$. Rewriting Eq.(22) yields

$$X \left(\frac{d\mathcal{F}}{dz} \frac{dz}{dX} \right)^2 = \frac{k}{(M_p^2 H_0^2)^2} (1+z)^6, \quad (24)$$

where the usual relation $1+z = 1/a$ has been used. Defining the function $g(z)$ by

$$g(z) \equiv \left(\frac{d\mathcal{F}}{dz} \right)^2, \quad (25)$$

we can obtain the following expression which allows us to determine X as a function of z

$$\int_{X_0}^X \left(\frac{1}{M_p^2 H_0^2} \right) \sqrt{\frac{k}{X'}} dX' = \int_0^z \frac{\sqrt{g(z)}}{(1+z)^3} dz. \quad (26)$$

We assume that $k/X > 0$ in order to have real solutions for X . Integrating the above equation yields

$$\frac{X}{X_0}(z) = \left(\frac{1}{2} \left(\frac{M_p^2 H_0^2}{\sqrt{kX_0}} \right) \int_0^z \frac{\sqrt{g(z)}}{(1+z)^3} dz + 1 \right)^2, \quad (27)$$

which admits the following analytical solution

$$\frac{X}{X_0} = \left(\frac{\Omega_{m0} \Omega_\phi (c - \sqrt{\Omega_\phi})}{c(1 - \Omega_\phi) \left(\frac{3}{2} (\Omega_{m0} - 1) + \Omega_{\phi 0} \left(\frac{1}{2} + \frac{\sqrt{\Omega_{\phi 0}}}{c} \right) \right)} \right)^2 \quad (28)$$

where X_0 and Ω_{m0} are the current values for X and Ω_m .

From Eqs. (23) and (28) we can obtain the function $\mathcal{F} = \mathcal{F}(X/X_0)$.

As we mention before, from Eq. (23), F must be necessarily positive and a monotonically increasing function with z within the relevant redshift range, for an accelerating universe with holographic dark energy. This behaviour is shown in Fig.2.

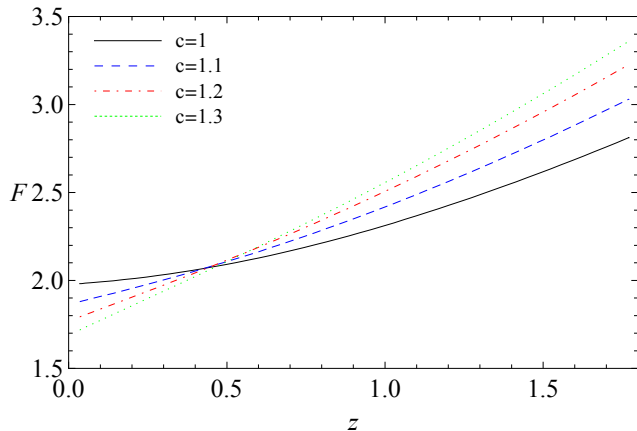


FIG. 2: Variation of $F(z)$ in units of $M_P^2 H_0^2$. Here we take $\Omega_{m0} = 0.27$, and show the cases for $c = 1, 1.1, 1.2$ and 1.3 .

Likewise, the behaviour of X/X_0 as a function of the redshift z is showed in Fig.3.

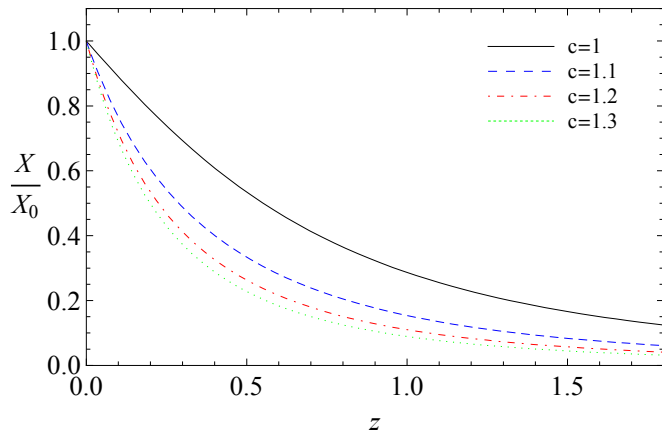


FIG. 3: Variation of $\frac{X}{X_0}(z)$. Here we take $\Omega_{m0} = 0.27$, and show the cases for $c = 1, 1.1, 1.2$ and 1.3 .

The holographic kinetic k-essence, represented by the function \mathcal{F} is plotted in Fig.4 as a function of X/X_0 . From Figs. 3 and 4 we can see the dynamic of the k-essence field explicitly. F is a monotonically decreasing function of X in the relevant redshift range. This is because for $X > 0$, the sign of $\frac{F_X}{F}$ is related to the value of w_ϕ . We should emphasise that the reconstruction of $F(X)$ only involves the portion of it over which the field evolves to give the requires $H(z)$. Incidentally, Figs. 2, 3

and 4 are very similar to the ones shown in [25] for the transient case although the author was dealing there with a non-holographic model in which the ansatz for the Hubble parameter $H(z)$ was obtained by modelling the dark energy as a Generalised Chaplygin gas. We see that the reconstructed $\mathcal{F} = \mathcal{F}(X/X_0)$ is a well-behaved, single valued function.

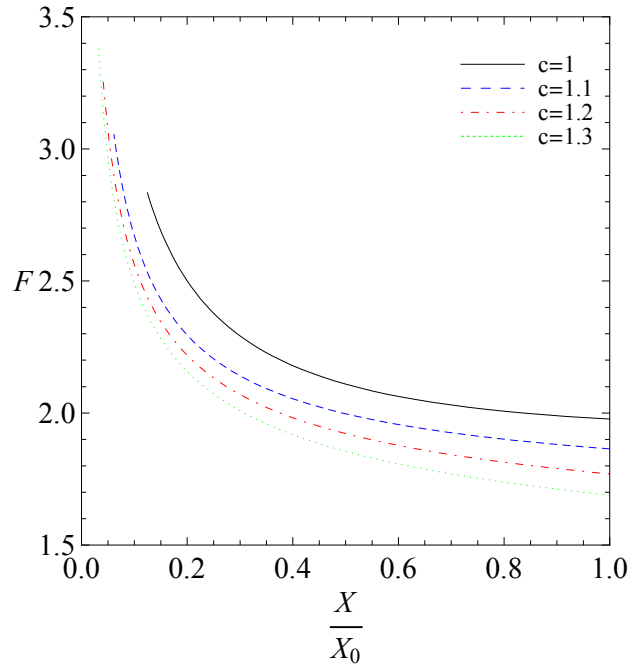


FIG. 4: Reconstructed $F(X/X_0)$ in units of $M_P^2 H_0^2$. Here we take $\Omega_{m0} = 0.27$, and show the cases for $c = 1, 1.1, 1.2$ and 1.3 .

The holographic dark energy models depend mainly on the parameter c . From Eq.(12), we see that the equation of state satisfies $-(1 + 2/c)/3 \leq w \leq -1/3$ due to $0 \leq \Omega_\phi \leq 1$, showing that the parameter c plays a key role in the holographic evolution of the universe.

When $c \geq 1$, the case we are studying, the equation of state will evolve in the region of $-1 \leq w \leq -1/3$. The value of c should be determined by cosmological observations in the holographic scenario. The case $c \geq 1$ is worth investigating as current observational data cannot determine the value of c accurately. In recent fit studies, different groups gave different values for c . An analysis of some of the latest observational data, including the gold sample of 182 SNIa, the CMB shift parameter given by the 3-year WMAP observations, and the BAO measurement from the SDSS, showed that the possibilities of $c > 1$ and $c < 1$ both exist and their likelihoods are almost equal within 3 sigma error range [26].

K-essence models with different $F(X)$ have been discussed in the literature. For the holographic kinetic k-essence model constructed in this paper, the reconstructed $F(X)$ can be determined from Eqs.(23) and (28). If we take $c = 1$, the behaviour is similar to the cosmological constant.

If $c > 1$, the equation of state of dark energy will be always larger than -1 and therefore the universe does not enter the de Sitter phase and avoids the occurrence of a Big Rip. Thus, we see explicitly that the value of c is paramount for the holographic dark energy model as it determines the feature of the holographic dark energy as well as the ultimate fate of the universe.

As has been analysed above, the holographic dark energy scenario reveals the dynamical nature of the vacuum energy. On the other hand, as has already been mentioned, the scalar field dark energy models are often viewed as effective description of the underlying theory of dark energy. However, the latter theory cannot be achieved before a complete theory of quantum gravity is established. In spite of this, we can speculate about the underlying theory of dark energy by taking some principles of quantum gravity into account. The holographic dark energy model is no doubt a tentative in this way.

To sum up, we have shown that a holographic dark energy with $c \geq 1$ can be totally described by kinetic k-essence in a certain way. A correspondence between

holographic dark energy and kinetic k-essence has been established, and the holographic kinetic k-essence function $F(X)$ has been reconstructed for the redshift range between $z = 0$ and $z = 1.8$.

Acknowledgements

NC acknowledges the warm hospitality of Prof. Pedro F. González-Díaz during the visit to the Instituto de Física Fundamental, Consejo Superior de Investigaciones Científicas, where part of this investigation was carried out. This work was supported by CONICYT through grant N^o 1040229 (NC), DICYT N^o 120618, of Dirección de Investigación y Desarrollo, Universidad de Santiago de Chile (GS), as well as by DGICYT under Research Project No. FIS2005-01180 (P. F. G. D and A. R. F.). A. R. F. acknowledges MEC for a FPU grant. A. R. F. benefited from discussions with David Brizuela and Iñaki Garay.

-
- [1] A. G. Riess et al. [Supernova Search Team Collaboration], *Astron. J.* **116** (1998) 1009 [arXiv:astro-ph/9805201]; S. Perlmutter et al. [Supernova Cosmology Project Collaboration], *Astrophys. J.* **517** (1999) 565, [arXiv:astro-ph/9812133]; W. J. Percival et al. [The 2dFGRS Collaboration], *Mon. Not. Roy. Astron. Soc.* **327** (2001) 1297, [arXiv:astro-ph/0105252]; P. Astier et al., *Astron. Astrophys.* **447**, 31 (2006); A. G. Riess et al. [Supernova Search Team Collaboration], *Astrophys. J.* **607**, 665 (2004) [arXiv:astro-ph/0402512], P. de Bernardis et al., *Nature (London)* **404**, 955 (2000); R.A. Knop et al., *Astrophys. J.* **598**, 102 (2003); J.L. Tonry et al., *Astrophys. J.* **594**, 1 (2003); M.V. John, *Astrophys. J.* **614**, 1 (2004).
- [2] D.N. Spergel et al., *Astrophys. J. Suppl.* **170**, 377 (2007); G. Hinshaw et al., *Astrophys. J. Suppl.* **170**, 288 (2007).
- [3] M. Colless et al., *Mon. Not. R. Astron. Soc.* **328**, 1039 (2001); M. Tegmark et al., *Phys. Rev. D* **69**, 103501 (2004); V. Springel, C.S. Frenk, and S.M.D. White, *Nature (London)* **440**, 1137 (2006).
- [4] C.R. Contaldi, H. Hoekstra, and A. Lewis, *Phys. Rev. Lett.* **90**, 221303 (2003).
- [5] S. Cole et al., *Mon. Not. R. Astron. Soc.* **362**, 505 (2005); S.P. Boughn, and R.G. Crittenden, *Nature*, **427**, 45 (2004); S.P. Boughn, and R.G. Crittenden, *Mon. Not. R. Astr. Soc.* **360** 1013, (2005); S.P. Boughn, and R.G. Crittenden, *New Astron. Rev.* **49**, 75 (2005).
- [6] P.J. Steinhardt, in *Critical Problems in Physics*, edited by V.L. Fitch and D.R. Marlow (Princeton University, Princeton, NJ, 1997).
- [7] P. F. Gonzalez-Diaz, *AIP Conf. Proc.* **878** (2006) 227 [arXiv:hep-th/0608204]; P. F. Gonzalez-Diaz and A. Rozas-Fernandez, *Phys. Lett. B* **641** (2006) 134 [arXiv:astro-ph/0609263]; P. F. Gonzalez-Diaz and A. Rozas-Fernandez, *Class. Quant. Grav.* **25** (2008) 175023 [arXiv:0807.2055 [gr-qc]].
- [8] F. S. N. Lobo, arXiv:0807.1640 [gr-qc]; S. Nojiri and S. D. Odintsov, arXiv:0807.0685 [hep-th].
- [9] P. J. E. Peebles and B. Ratra, *Astrophys. J.* **325** L17 (1988); B. Ratra and P. J. E. Peebles, *Phys. Rev. D* **37** 3406 (1988); C. Wetterich, *Nucl. Phys. B* **302** 668 (1988); J. A. Frieman, C. T. Hill, A. Stebbins and I. Waga, *Phys. Rev. Lett.* **75**, 2077 (1995) [astro-ph/9505060]; M. S. Turner and M. J. White, *Phys. Rev. D* **56**, 4439 (1997) [astro-ph/9701138]; R. R. Caldwell, R. Dave and P. J. Steinhardt, *Phys. Rev. Lett.* **80**, 1582 (1998) [astro-ph/9708069]; A. R. Liddle and R. J. Scherrer, *Phys. Rev. D* **59**, 023509 (1999) [astro-ph/9809272]; I. Zlatev, L. M. Wang and P. J. Steinhardt, *Phys. Rev. Lett.* **82**, 896 (1999) [astro-ph/9807002]; P. J. Steinhardt, L. M. Wang and I. Zlatev, *Phys. Rev. D* **59**, 123504 (1999) [astro-ph/9812313].
- [10] A. Sen, *JHEP* **0207**, 065 (2002) [hep-th/0203265]; A. Sen, *JHEP* **0204**, 048 (2002) [hep-th/0203211].
- [11] R. R. Caldwell, *Phys. Lett. B* **545**, 23 (2002) [astro-ph/9908168]; R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, *Phys. Rev. Lett.* **91**, 071301 (2003) [astro-ph/0302506].
- [12] B. Feng, X. L. Wang and X. M. Zhang, *Phys. Lett. B* **607**, 35 (2005) [astro-ph/0404224]; Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, *Phys. Lett. B* **608**, 177 (2005) [astro-ph/0410654]; X. Zhang, *Commun. Theor. Phys.* **44**, 762 (2005); H. Wei, R. G. Cai and D. F. Zeng, *Class. Quant. Grav.* **22**, 3189 (2005) [hep-th/0501160]. H. S. Zhang and Z. H. Zhu, *Phys. Rev. D* **75**, 023510 (2007) [astro-ph/0611834].
- [13] A. G. Cohen, D. B. Kaplan and A. E. Nelson, *Phys. Rev. Lett.* **82**, 4971 (1999) [hep-th/9803132].
- [14] S. D. H. Hsu, *Phys. Lett. B* **594**, 13 (2004) [hep-th/0403052].
- [15] M. Li, *Phys. Lett. B* **603**, 1 (2004) [hep-th/0403127].
- [16] P.F. González-Díaz, *Phys. Rev. D* **27** (1983) 3042; G. 't Hooft, gr-qc/9310026; L. Susskind, *J. Math. Phys.* **36**, 6377 (1995) [hep-th/9409089].

- [17] X. Zhang and F. Q. Wu, Phys. Rev. D **72**, 043524 (2005) [astro-ph/0506310]; Z. Chang, F. Q. Wu and X. Zhang, Phys. Lett. B **633**, 14 (2006) [astro-ph/0509531]; Q. G. Huang and Y. G. Gong, JCAP **0408**, 006 (2004) [astro-ph/0403590]; K. Enqvist, S. Hannestad and M. S. Sloth, JCAP **0502** 004 (2005) [astro-ph/0409275]; J. Shen, B. Wang, E. Abdalla and R. K. Su, Phys. Lett. B **609** 200 (2005) [hep-th/0412227]; H. C. Kao, W. L. Lee and F. L. Lin, Phys. Rev. D **71** 123518 (2005) [astro-ph/0501487].
- [18] X. Zhang, Phys. Lett. B **648** (2007) 1 [arXiv:astro-ph/0604484].
- [19] X. Zhang, Phys. Rev. D **74** (2006) 103505 [arXiv:astro-ph/0609699].
- [20] M. R. Setare, Phys. Lett. B **653** (2007) 116 [arXiv:0705.3517 [hep-th]]; J. Zhang, X. Zhang and H. Liu, Phys. Lett. B **651** (2007) 84 [arXiv:0706.1185 [astro-ph]].
- [21] S. Nojiri and S. D. Odintsov, Gen. Rel. Grav. **38** (2006) 1285 [arXiv:hep-th/0506212]; E. Elizalde, S. Nojiri, S. D. Odintsov and P. Wang, Phys. Rev. D **71** (2005) 103504 [arXiv:hep-th/0502082]; M. R. Setare, Phys. Lett. B **648** (2007) 329 [arXiv:0704.3679 [hep-th]].
- [22] C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. **85** (2000) 4438 [arXiv:astro-ph/0004134]; C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. D **63** (2001) 103510 [arXiv:astro-ph/0006373]; T. Chiba, Phys. Rev. D **66** (2002) 063514 [arXiv:astro-ph/0206298].
- [23] R. J. Scherrer, Phys. Rev. Lett. **93** (2004) 011301 [arXiv:astro-ph/0402316].
- [24] R. de Putter and E. V. Linder, Astropart. Phys. **28** (2007) 263 [arXiv:0705.0400 [astro-ph]].
- [25] A. A. Sen, JCAP **0603** (2006) 010 [arXiv:astro-ph/0512406].
- [26] X. Zhang and F. Q. Wu, Phys. Rev. D **76** (2007) 023502 [arXiv:astro-ph/0701405].