Discharge coefficient analysis for triangular sharp-crested weirs using a low-speed photographic technique

by

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Abstract

Triangular weirs are commonly used for the measurement of discharge in open channel flow, representing an inexpensive, reliable methodology for the monitoring of water allocation. In this work, a low-speed photographic technique was used to characterize the upper and lower nappe profiles of flow over fully aerated triangular weirs. A total of 112 experiments were performed covering a range of weir vertex angles (from 30° to 90°), crest elevations (8 or 10 cm) and discharges (0.01 – 7.82 l s^{-1}). The experimental nappe profiles were mathematically modeled and combined with elements of free-vortex theory to derive a predictive equation for the weir discharge coefficient. Comparisons were established between measured C_d, the proposed discharge coefficient equation and discharge coefficient equations identified in the literature. The proposed equation can predict C_d with a Mean Estimation Error (MEE) of 0.001, a Root

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Mean Square Error (RMSE) of 0.004, and an Index of Agreement (IA) of 0.984. In the experimental conditions of this study, this performance slightly improves that of the equation proposed by Greve in 1932, showing the same absolute value of MEE, but lower values of RMSE and IA.

Keywords: weir vertex angle, flow measurement, hydrometry, free-vortex theory

Introduction

Weirs are elevated barriers located perpendicular to the main direction of water movement to cause the fluid to rise above the obstruction in order to flow through an opening of regular shape. For a properly designed and operated weir of a given geometry there is a unique discharge corresponding to each measurement of flow depth (El-Hady 2011). The geometrical parameters involved in the hydraulic operation of weirs are the length of the weir crest and the shape of the flow control section (Emiroglu et al. 2010; USBR 2001). In sharp-crested or thin-plate weirs the upstream head (h) to length of crest in the direction of flow (L) ratio is greater than 15 (Fig. 1). Specific assumptions are adopted to estimate the relation between discharge and upstream head (Bagheri and Heidarpour 2010; Sotelo 2009; El-Alfy 2005; Bos 1989). These structures have been extensively studied using classical physics and experimental analyses to understand the characteristics of flow as well as to determine the coefficient of discharge (C_d). This coefficient represents the effects not taken into consideration in the derivation of the equations used to estimate discharge from flow depth. Such effects include viscosity, capillarity, surface tension, velocity distribution in the approach section and streamline curvature due to weir contraction (Aydin et al. 2011; El-Hady 2011).

In the particular case of triangular sharp-crested weirs, Shen (1981) described experimental procedures used by different authors to determine C_d. El-Alfy (2005) experimentally evaluated the effect of vertical flow curvature on the discharge coefficient, and reported that C_d is inversely proportional to the V-notch angle (θ) and directly proportional to the relative head (h/P). Recently, Bagheri and Heidarpour

(2010) obtained a discharge coefficient equation for rectangular sharp-crested weirs based on the upper and lower nappe profiles and free-vortex theory.

Photography has been used for the characterization of flow over hydraulic structures, particularly weirs. For instance, Del Giudice et al., (1999) used photographs to illustrate complex flow patters near a sewer sideweir. Novak et al. (2013) photographed the planes displayed by a laser on the flow near a side weir, and used these images to determine flow depth profiles and flow velocity (from the movement of hydrogen bubles). Photography was recently applied to a different hydraulic problem: the characterization of sprinkler irrigation drops moving in the air. Salvador et al. (2009) and Bautista et al. (2009) performed out-door and in-door experiments to evaluate drop geometrical and kinematic characteristics using a low-speed photographic technique.

The objective of this study was to determine a discharge coefficient equation for triangular sharp-crested weirs based on: 1) the free vortex theory as described by Bagheri and Heidarpour (2010); and 2) measurements of the upper and lower nappe profiles using an adaptation of the low-speed photographic technique proposed by Salvador et al. (2009).

Materials and methods

Governing equations

For a sharp-crested weir of any geometrical section with the crest elevation (P) being high enough to neglect the velocity head (Figure 1), discharge equations are usually obtained from the mathematical integration of an elemental flow strip over the nappe (Singh et al. 2010). The total discharge flowing between elevations 0 and *h* can be obtained solving the following expression:

$$Q = 2\sqrt{2g}\, C_d \int_0^h x(h-y)^{\frac{1}{2}} dy \qquad [1]$$

where *Q* is the discharge over the weir ($m^3\ s^{-1}$); *g* is the gravitational acceleration ($m\ s^{-2}$); C_d is the discharge coefficient (dimensionless); *h* is the water head (m); *x* is the flow width, with $x = f(y)$ depending of the weir geometry; and *dy* is the vertical thickness of elemental flow strip. A sharp-crested weir with symmetrical triangular section and vertex angle (θ) entails that $x = y\tan\left(\frac{\theta}{2}\right)$, as shown in Figure 1b. The resulting discharge equation is:

$$Q = \frac{8}{15} C_d \sqrt{2g} \tan\left(\frac{\theta}{2}\right) h^{\frac{5}{2}} \qquad [2]$$

Considering free-vortex motion theory, Bagheri and Heidarpour (2010) proposed an expression to derive the discharge coefficient of flow passing over a rectangular sharp-crested weir. Following the reasoning of these authors, a similar expression could be obtained for a triangular sharp-crested weir (Equation 3):

$$Q = 2V_b R_b \tan\left(\frac{\theta}{2}\right)\left[kY + R_b \ln\left(\frac{R_b}{kY + R_b}\right)\right] \qquad [3]$$

where V_b is the lower nappe velocity, obtained at the section of maximum elevation of the lower nappe (m s^{-1}); R_b is the radius of streamline curvature at lower nappe of the profile in segment OB (m); k is the non-concentricity coefficient; and Y is the flow depth at the section of maximum elevation of lower nappe (m) (Figure 1).

Experimental setup and measuring techniques

Experiments were performed in a horizontal rectangular recirculating plexiglass laboratory channel 7.2 m long, 0.3 m wide, and 0.3 m high. Canal cross section was designed for a maximum discharge of 10 l s^{-1}, having in mind a common application of this type of weirs: the analysis of furrow irrigation inflow and outflow. Mild steel plates (galvanized sheet metal) with a thickness of 1.5 mm were used to manufacture weirs. Vertex angles were 30º, 45º, 60º and 90º, each of them with 8 and 10 cm of crest height. Water was supplied to the channel through an overhead tank provided with an overflow arrangement to maintain constant head. A grid wall was installed into the channel to dissipate flow velocity. To avoid the area of water surface draw-down, head over the weir was measured 1.0 m upstream of the vertical weir plane using a point gage with accuracy of ± 0.1 mm. Discharge over the weirs was volumetrically measured, using a prismatic steel measuring tank with base dimensions of 0.75 m x 0.75 m. Weirs were installed at the end of the channel to provide an unrestricted supply of air under the nappe. Consequently, all data for this study correspond to the conditions of fully aerated flow. Equations of flow nappe profiles and discharge coefficients for triangular sharp-crested weirs were obtained for four different models. Table 1 presents a summary of the weir characteristics and test conditions. Weir models were tested using 14 flow rates. A total of 112 experiments were conducted (4 vertex angles x 2 values of P x 14 flow rates). Additionally, each discharge was measured five times. The average of these replications was used to obtain the discharge coefficient.

An adaptation of the low-speed photographic technique proposed by Salvador et al. (2009) was implemented in order to identify a set of points (z, y) along the upper and lower nappes to characterize the profiles. Coordinate z corresponds to the horizontal distance downstream from the weir. All coordinate values were initially registered in pixels and then transformed to millimeters using the pixel per millimeter ratio obtained from image analysis (all images included a reference ruler). In order to assess the differences between measured-estimated values and different estimation equations proposed by other authors, the following statistic parameters were used: mean estimation error (MEE), root mean square error (RMSE), and index of agreement (IA) (Willmott, 1982).

Results

The points obtained from the photographs were plotted as shown in Figure 2, where *y* is the vertical depth of flow at *z* distance downstream from the weir. Plotted information corresponds to all measured upper and lower flow nappe profiles for the different values of vertex angle. Figure 2 shows pairs (*z*, *y*) relative to head (h) as well as the polynomials that best fit each case. The upper and lower nappe profiles could be successfully adjusted to quadratic equations. Polynomials were used to determine distances OA, OB, AC, and AE (Figure 1) for each weir model using the general regression equations in Figure 2. The same procedure was used to determine the mean radius of curvature of the streamline along the distance of OB at lower nappe profiles (R_b), the flow depth at the section of maximum elevation of the lower nappe (Y), and the correction coefficient of non-concentricity streamline (k) (Bagheri and Heidarpour, 2010). The analysis of ratios R_b/h and Y/h against weir vertex angle expressed as tan (θ/2) shows potential relations in both cases. Regarding the non-concentricity coefficient, the best relation between k and h tan (θ/2) is represented by a potential equation. Substituting R_b/h, Y/h, and k expressions into Equation 3 results in Equation 4:

$$Q = 8.859 h^{\frac{5}{2}} \tan\left(\frac{\theta}{2}\right) Z_1 \left[k Z_0 + Z_1 \ln\left(\frac{Z_1}{k Z_0 + Z_1} \right) \right] \qquad [4]$$

where

$$Z_0 = 0.682 \left[\tan\left(\frac{\theta}{2}\right) \right]^{0.044} \qquad [5]$$

and

$$Z_1 = 0.445\left[\tan\left(\frac{\theta}{2}\right)\right]^{-0.098} \quad [6]$$

Combining Equations 2 and 4, the discharge coefficient can be expressed as Equation 7:

$$C_d = 3.750 Z_1\left[kZ_0 + Z_1 \ln\left(\frac{Z_1}{kZ_0 + Z_1}\right)\right] \quad [7]$$

Estimated discharge coefficients (for head over the weir ranging from 1.5 cm to 15 cm) ranged between 0.669-0.607, 0.674-0.614, 0.677-0.618, and 0.680-0.624 for weir angles of 30°, 45°, 60°, and 90° respectively. Measured discharge coefficients (for heads over the weir of 1.5-15 cm for weir angles of 30°, 45°, and 60°; and for heads over the weir of 1.5-12 cm for a weir angle of 90°) ranged between 0.665-0.614, 0.668-0.616, 0.672-0.620, and 0.677-0.624 for the same weir vertex angles. Figure 3 presents a comparison of the experimental data, the proposed discharge coefficient (Equation 7) and the estimates obtained using some references discussed by Shen (1981). The proposed equation can predict C_d for the range or 30°-90° weir vertex angles with MME = 0.001, RMSE = 0.004, and IA = 0.984. In the experimental conditions of this study, this performance can only be compared to that of the equation proposed by Greve (1932), which showed the same absolute value of MEE but lower values of RMSE and IA.

Conclusions

An experimental analysis was performed to estimate the discharge coefficient for four triangular sharp-crested weir models. Regression equations of the upper and lower nappe profiles developed from experimental data and free-vortex theory were used to derive a discharge coefficient equation as a function of head over the weir (h) and weir vertex angle expressed as tan (θ/2). Experimental data showed that both nappe profiles can be successfully represented by second-degree polynomials. Results also indicated that the non-dimensional mean radius of curvature of the streamline along the distance OB at lower nappe profiles (R_b/h) and the non-dimensional flow depth at the section of maximum elevation of the lower nappe (Y/h) show potential relations with the weir vertex angle expressed as tan (θ/2). To take into account the non-concentricity of the streamlines, a correction coefficient was proposed as a function of h and θ. Comparisons between measured C_d, the proposed discharge coefficient equation and discharge coefficient equations proposed by a number of authors were established. In the experimental conditions, the proposed equation represents an improvement in the estimation of discharge from triangular weirs, and confirms the validity of a predictive equation proposed by Greve in 1932.

Acknowledgements

This research was funded by the Secretaría de Agricultura, Ganadería, Desarrollo Rural, Pesca y Alimentación of the Mexican Government (SAGARPA, Mexico) and the Secretaría del Campo of Zacatecas State Government (SECAMPO, Zacatecas). Thanks are also due to the Universidad Autónoma of Zacatecas, Mexico. Cruz Octavio Robles Rovelo received a scholarship from the Mexican Consejo Nacional de Ciencia y Tecnología (CONACYT).

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Table 1. *Triangular sharp-crested weir characteristics and test conditions.*

Weir model	Vertex angle (θ, °)	P (cm)	Q (l s^{-1})	h (cm)
1	30	8, 10	0.01-3.56	1.5-15.0
2	45	8, 10	0.02-5.52	1.5-15.0
3	60	8, 10	0.03-7.74	1.5-15.0
4	90	8, 10	0.04-7.82	1.5-12.0

Figure 1. *Experimental parameters: a) direction of flow view, b) frontal view.*

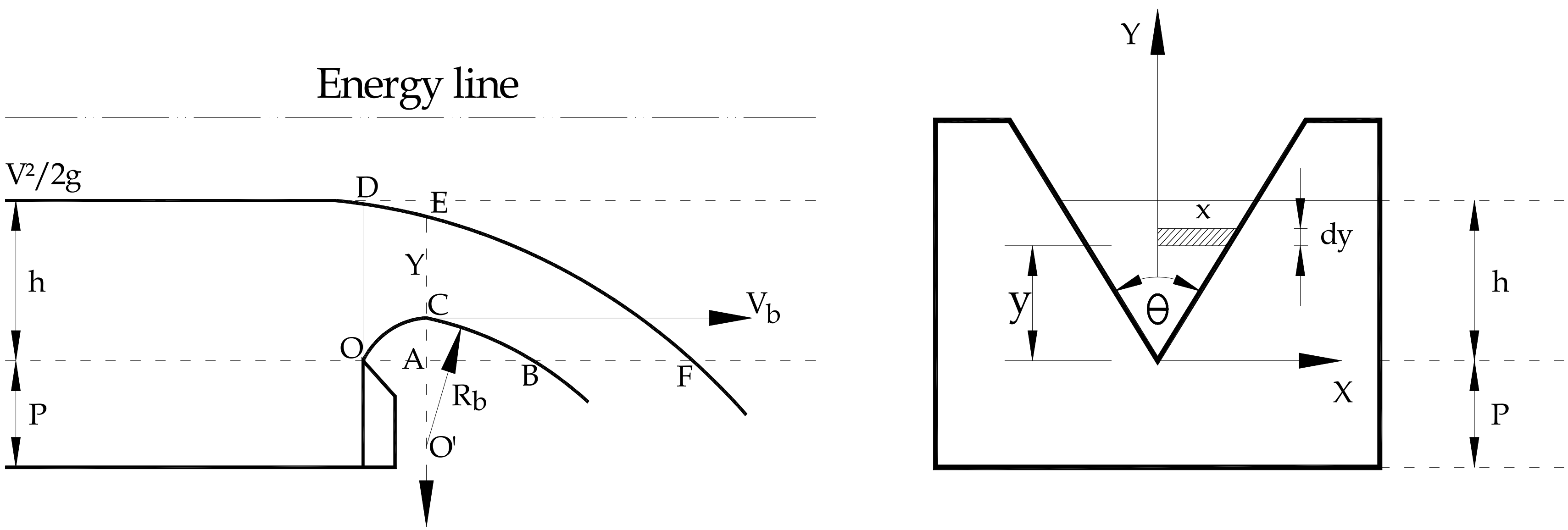

Figure 2. *Upper and lower nappe profiles. Weir vertex angles: a) 30°, b) 45°, c) 60°, and d) 90°.*

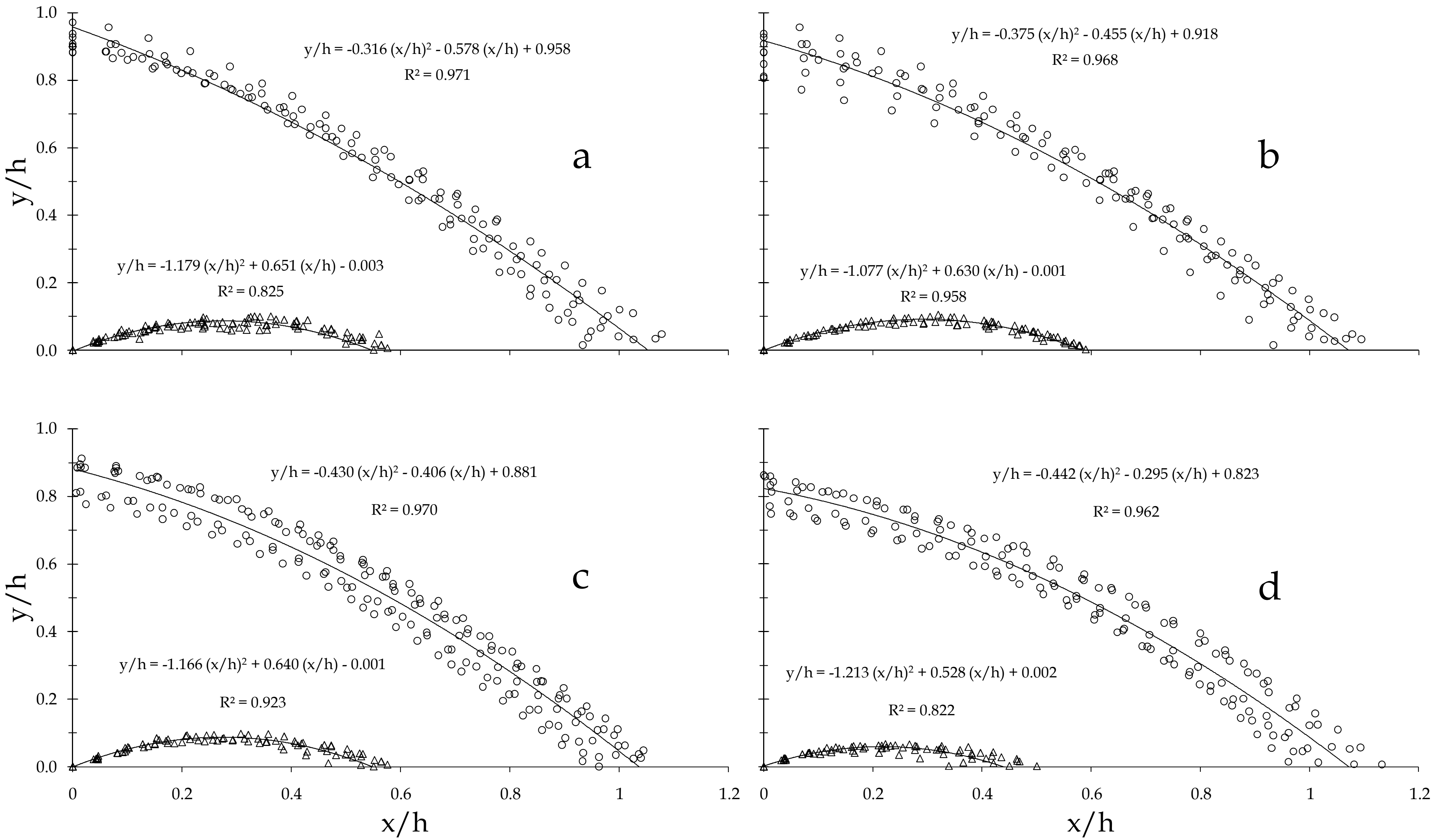

Figure 3. *Discharge coefficient* (C_d) vs. *head over triangular sharp-crested weir. Weir vertex angles: a) 30°, b) 45°, c) 60°, and d) 90°.*