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Strategy to Estimate Size

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by

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Introduction

Have you ever wondered exactly how much land and space is included in a national park? In this paper, we will explore the vastness of a fictitious Park and calculate its total area.

We will:

1. Create a fictitious Park, determine a scale that can be used to perform calculations using a practical unit of measurement.
2. Develop a strategy that students can use to find the area of the Park.
3. Demonstrate the use of the strategy by solving for the area of the fictitious park.

We start with a real National Park

We investigated several National Parks in the United States (Acadia National Park, Yellowstone, Grand Canyon, Yosemite, Glacier, Denali, and Big Bend). Five out of the seven parks had irregular shapes that roughly resemble a circle. Using the scale provided on the image, we can estimate the size of the national park.

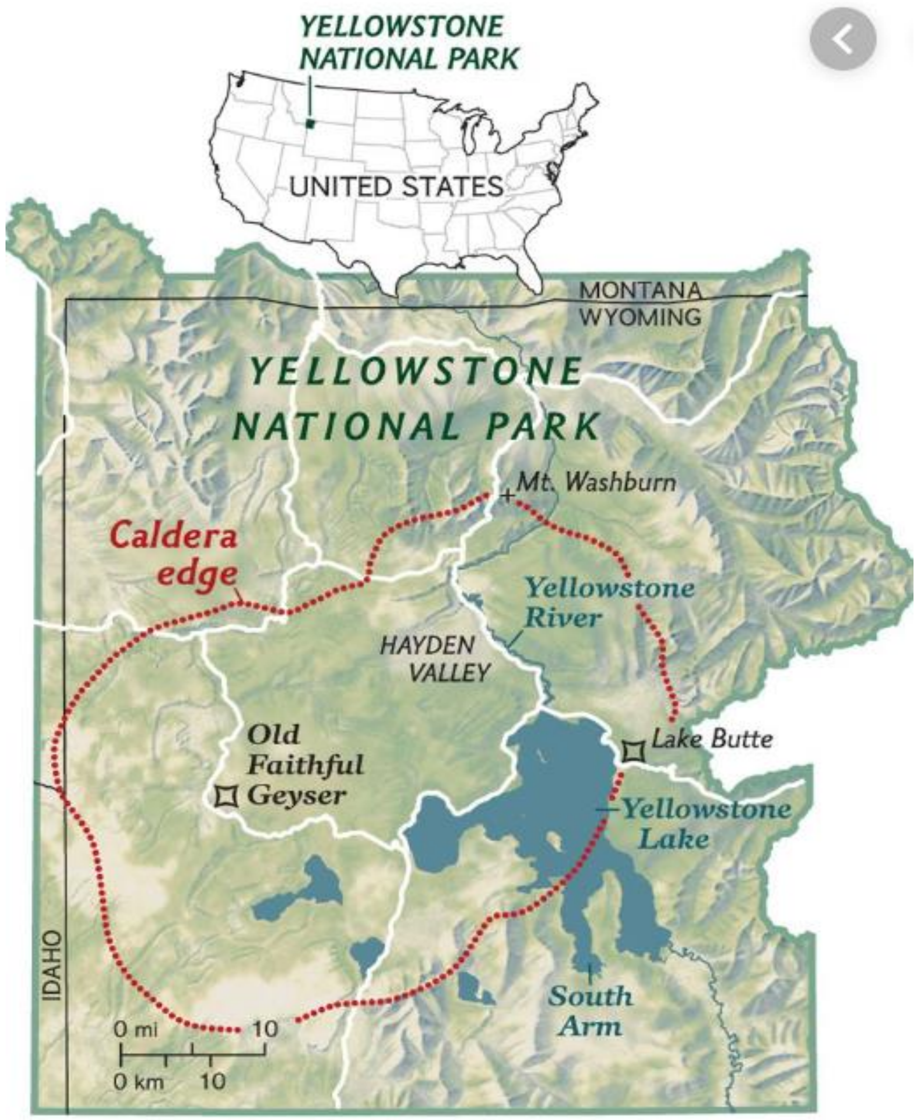

Figure 1. Map of Yellowstone National Park.

Referencing the scale of the map, a length slightly greater than 0.5 in. represents 10 miles.

Placing the map on a grid allows us to see how many square-units occupy the region enclosed by the boundary.

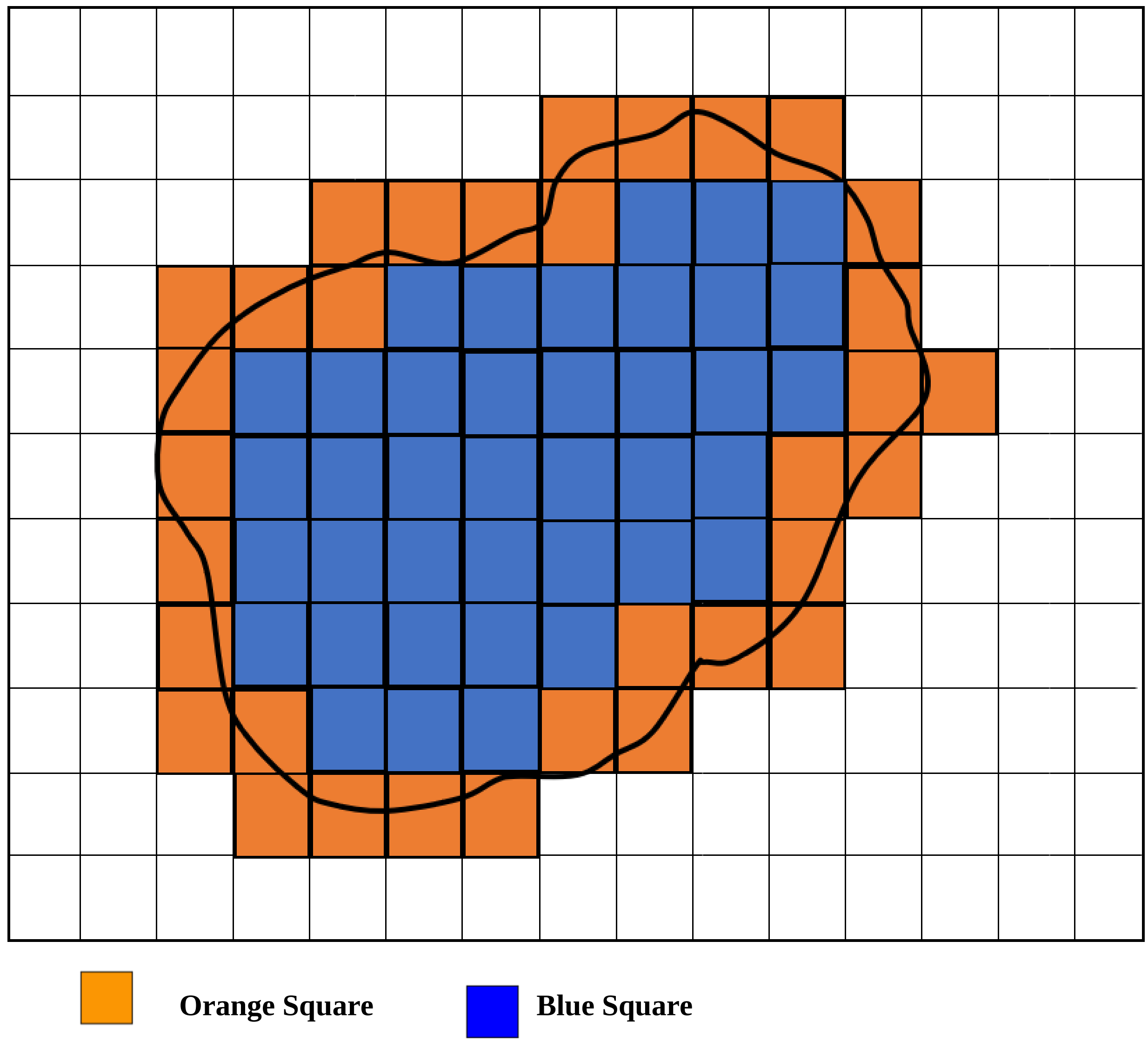

Figure 2. Outline of Yellowstone National Park (Depiction of Inner and Outer Area).

The edge length of each square is 0.44 inches which results in an area of

$(0.44\ in)^2 = 0.194\ in^2$. for each square. Applying dimensional analysis (conversion factors) will allow us to find the area of the park in units of square miles. Before doing so, we need to calculate the amount of square units occupied by the figure. The inner area of the figure consists of the total number of squares completely contained within the contour, i.e., the blue squares…

$$A_{inner} = 39\ unit^2.$$

The outer area consists of the total number of orange *and* blue squares:

$$A_{outer} = 39 + 72 = 111\, unit^2.$$

Finally, the mean (average) of these values is an approximation of the area (Pirnot, 2017):

$$A \approx (A_{inner} + A_{outer})/2$$

$$A \approx 55.5\, unit^2.$$

This process will be explained in depth in the following sections. In the meantime, let us proceed with the estimation. The area of each square unit can be applied as a conversion factor, along with the scale provided on the map. The area of Yellowstone National Park is given by…

$$55.5\, unit^2 \times \frac{0.194\, in^2}{1\, unit^2} \times \frac{100\, mi^2}{0.25\, in^2} = 4{,}307\, mi^2.$$

Note that the actual area of the national park is 3471 mi^2.Using the equation for percent error allows us to compare the estimated value with the actual size of Yellowstone:

$$\%\, Error = \frac{|estimated - actual|}{actual} \times 100\%$$

$$= \frac{|4307 - 3471|}{3471} \times 100\%$$

$$= 24.1\, \%\, .$$

Note how limitations in the measuring device will skew the results. For instance, suppose the ruler had a greater capacity to display significant figures. This would result in more certainty up to a greater number of decimal places. Unfortunately, however, 0.53 inches may be more accurate for the length of the scale (there could be any number following the digit "5"). If we report our measurements with a greater number of significant figures, perhaps we will obtain a more accurate estimation. Suppose the length of the line is 0.53 inches.

Let the scale of the map read 0.53 inches in the image to every 10 miles in the actual park. Therefore,

$$(0.53\ in)^2 \ = \ (10\ mi)^2$$

$$0.2809\ in^2 \ = \ 100\ mi^2\ .$$

Combining the above relationship into a fraction allows us to utilize it as a conversion factor:

$$55.5\ unit^2 \ \times \frac{0.194\ in^2}{1\ unit^2} \times \frac{100\ mi^2}{0.2809\ in^2} = 3{,}833\ mi^2.$$

Notice how this numerical value is much closer to that of the actual area. Using the scale of the map and conversion factors is a great problem-solving strategy.

Diagram (Map) of Our Park

Suppose a park takes the (irregular) shape of an arbitrary closed curve. Since the shape is not a polygon or a recognizable figure, there is no formula for perimeter or area that quantifies its size. The goal is to devise a method, allowing one to estimate the size of this park.

Method 1. Place the Figure on a Grid

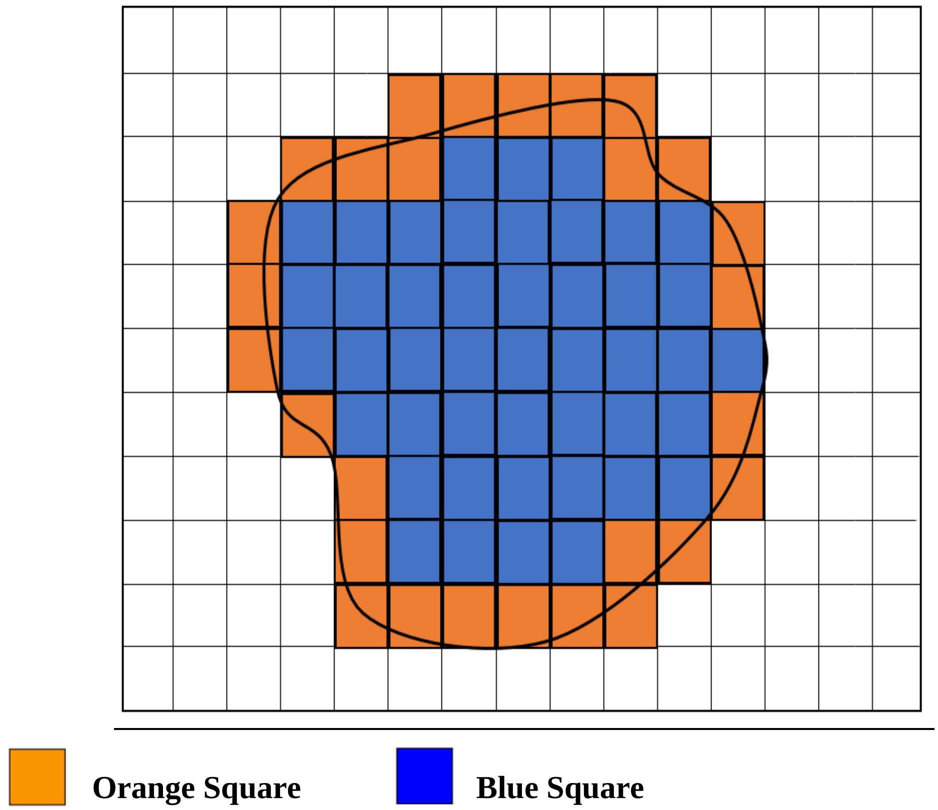

Figure 3. Outer and inner area of our national park.

Algebraic Expression to Estimate Size

All squares completely contained inside the shaded region represent the **inner area**. The orange squares are not fully enclosed by the boundary; rather, the contour crosses *through* these squares (this denotes the **outer area** of the figure) (Pirnot, 2017).

The area of an individual square is given by l^2, where l denotes the edge length of the square. The inner area is the sum of individual areas enclosed within the boundary. Since multiplication is repeated addition…

$$A_{inner} = n_{blue}\, l^2 \,.$$

The outer area is the sum of the orange squares *and* blue squares, collectively. It is very important to emphasize this to students. A common misconception is that *only* the orange squares constitute the outer area. The name is somewhat unfortunate because, as we mentioned, the outer area is not simply the orange squares; rather, it is the total area (squares) of the figure.

$$A_{outer} = n_{blue}\, l^2 + n_{orange}\, l^2.$$

The inner area is too small to represent the size of the actual park, whereas the outer area overcompensates. This suggests that the actual area must be some value in between. In order to approximate the area of the entire park, we will take the average of these two values (Pirnot, 2017):

$$A \approx \frac{1}{2}\,(A_{inner} + A_{outer})$$

Substituting in the expressions for the outer and inner areas:

$$A \approx \frac{1}{2}\,(\,n_b\, l^2 + n_b\, l^2 + n_o\, l^2)$$

Note that l^2 appears in all terms; therefore, we can distribute this factor outside the parentheses…

$$A \approx \frac{1}{2}\, l^2\,(n_b + n_b + n_o)$$

Combining like-terms gives…

$$A \approx \frac{1}{2}\, l^2\,(2n_b + n_o)$$

An equation has been derived which yields the approximate area of the park. Suppose one unit on the grid represents one decameter (1 Dm). Substituting this length into the equation allows us to find a numerical value for area:

$$A \approx \frac{1}{2}\, l^2\, (2n_b\; + n_o)$$

$$= (1/2)(1)^2\, [\,(2)(45)\; +\; 28]$$

$$= (1/2)(118)$$

$$= 59.$$

Therefore, the area of the park is approximately equal to $59\ Dm^2$.

Notice how it may be an inconvenience to individually count the number of squares. We can generalize our method to regions consisting of any number of square units, and those units can be calculated via $A = lw$.Suppose the presented problem asked you to find the area of an irregularly shaped figure. Again, if the area consisted of a finer grid (more squares), you would encounter yet *another* problem: what is the most efficient method to find the inner and outer areas? How can I find the number of blue squares (inner area), for example, without having to manually count them one-by-one? The following example illustrates how to solve this problem. Instead of using more squares, however, we will keep the example simple and use our current grid from *Figure 3.*

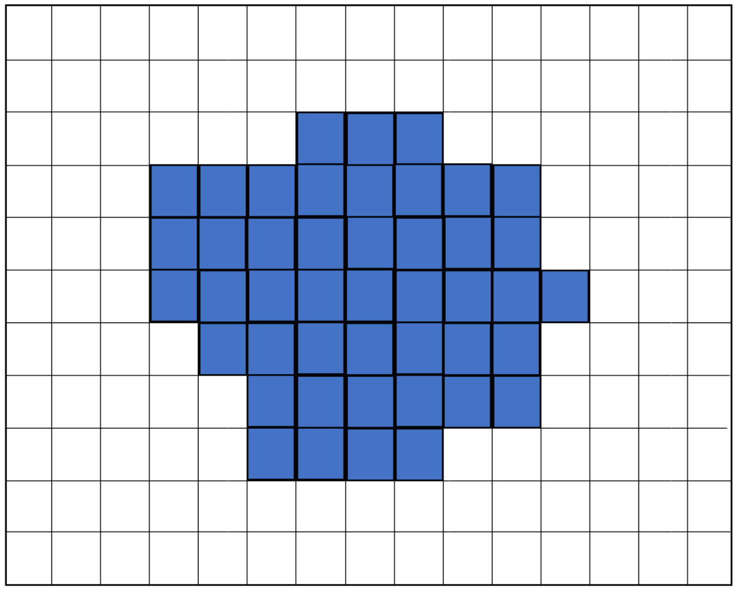

Figure 4. Depiction of the inner area of the park.

Placing additional square units in the unoccupied cells allows us to create a rectangle. As shown in the figure below, this allows us to calculate the area using its length and its width. Turning this problem into a previously encountered situation (finding the area of a rectangle) is a wonderful problem-solving strategy. These are strategies that should be highlighted as the instructor transitions from step-to-step sequentially, rather than being emphasized in an isolated context (when a problem is not being solved real-time). Moreover, emphasizing such methods is a great addition to a mathematics curriculum: making diagrams, organizing information, reading the problem multiple times, looking for patterns, and so on (Lawson, 1990). Developing such strong habits is essential for a student's problem-solving skills, as they are universally applicable to all problems.

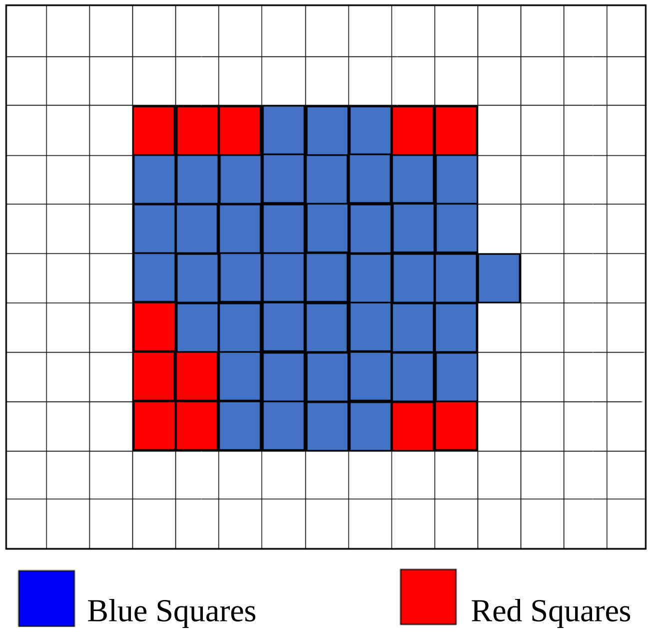

Figure 5. Inner area with additional areas $A_1, A_2, A_3, and\ A_4$.

Notice how another dilemma is encountered. There is a blue square occupying a column of the grid by itself. It is located four units left and six units up from the bottom right cell. We could occupy that column with red squares; however, there are more efficient ways to take this into account. In this case, let's find the area of this rectangle (as if the blue square was not there) and simply add the area of the blue square *after*.

$$A_{rectangle} = lw$$

$$= (8)(7) + 1$$

$$= 57\ units^2\ .$$

Finally, subtracting away the additional (red) areas will leave the actual inner area (the total number of blue squares). Letting A_1denote the area of the (red) upper left corner, A_2denote the area of the upper right corner, and so on...

$$A_{inner} = A_{rectangle} - (A_1 + A_2 + A_3 + A_4)$$

$$A_{inner} = 57 - (3 + 2 + 5 + 2)$$

$$A_{inner} = 45\, units^2\,.$$

Method 2. Approximating the Area with a Circle

Although the shape is highly irregular, we can estimate the size of the park using a circle. Using this alternative method ensures the previous approximation was valid, if we obtain a consistent numerical value. Utilizing a shape that is more familiar to us is a wonderful problem-solving strategy, because it simplifies the given task. Employing Polya's Four-Step method, it is clear that we are identifying the main problem: how can we devise a way to quantify the size of an irregularly shaped closed curve? Is there a shape that this figure most closely resembles? If so, what formula can we apply?

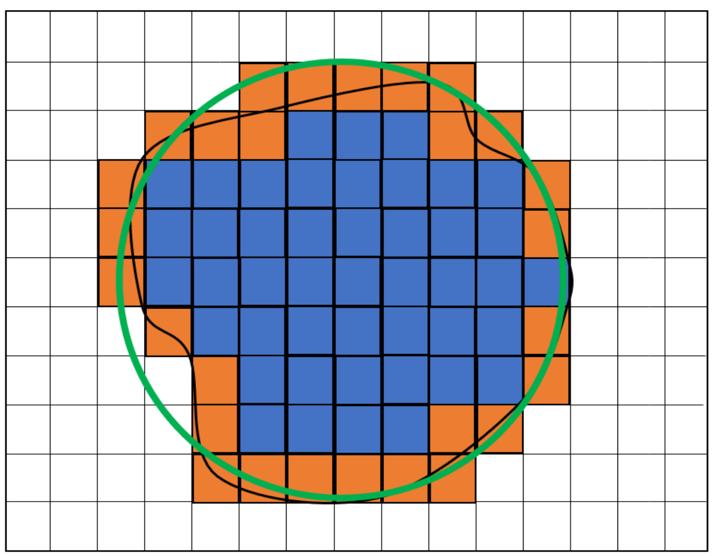

Figure 6 . Estimation of the area using a circle of radius 4 units.

Applying step two of Polya's Four-Step method, we need to devise a plan. Using *Figure 1*, we constructed a circle with a radius of roughly four units (*Figure 6*). The area of the circle is given by $A = \pi R^2$. Plugging in the value for radius *R* allows us to determine a numerical value for the area. Employing step three of Polya's problem-solving method…

$$A = (3.14)(4)^2 = 50.24\, Dm^2.$$

The units of area should be units of length squared. The radius is measured in decameters, so it logically follows that our area will be units of decameters squared.

There is one last way to quantify the size of this national park. Although the area is a great representation of its size, the perimeter of the park has not been calculated. Geometrically, the perimeter of a circle is its circumference…

$$C = 2\pi R$$

$$25.12\ Dm.$$

Problem-Solving Model Utilized: Polya's Four-Step Method

<u>Step 1: Understand the problem</u>

Identify the problem. What are we asked to find? What is the fundamental principle? Is there a particular method or equation that applies to this specific problem?

<u>Step 2: Devise a plan</u> Assign variables to given quantities, set up algebraic equations, construct diagrams, and consider formulas which may apply to the given situation. Organizing your data is a great starting point when approaching a challenging problem.

<u>Step 3: Execute the plan</u>

Isolate any desired quantities and make inferences from any diagrams. Although some diagrams are provided by the author of this particular problem, it may be necessary to expand on the given schematic. Check that the final numerical result has the appropriate units and sign. For instance, suppose you are computing the volume of a cylinder. A negative volume is not possible; furthermore, the volume should be expressed in units of length cubed.

<u>Step 4: Looking Back</u>

<u>Students have done this when they used the circle in addition to using the grid shapes to find the area. Answers the question: Is there another way to find the solution? Is there a more elegant solution? It is more than just verifying calculations are correct. (p. 14 -15 of Polya's *How to Solve it.*</u>

Conclusion:

Using Polya's Four Step Method, this problem was approached in a similar way that most students would attempt to solve this problem. Relating new information to previous information or applying previous skills to new types of problems are the true measure of the problem-solving skills teachers strive for their students to possess. The formulas that were used to approximate the area of our National Park started relatively simple with the area of a rectangle and a circle. This type of artifact can have classroom applications from middle school to post-secondary education because it depends on simple formulas that students are introduced to very early. All of the results of our areas were relatively consistent which supports our approximations being correct. All calculations have been included and this could easily be recreated in any classroom and serve as a guide; as well as being added to because even though we included several ways to solve the same problem, the beauty of problem solving is that there can always be another way.

Use in the Classroom

The main goal of this project is for students to use whatever knowledge they have to estimate the size of a National Park. In a fifth-grade classroom this project could be modified to test mastery of multiple common core standards. The standards are CCSS.MATH.CONTENT.5.G.A.1 and CCSS.MATH.CONTENT.5.MD.A.1. These standards address student ability to plot points on a coordinate plane and convert measurements within the same measurement system. By adding an aspect to the problem where students have to not only estimate the size of a park, but also define the ecosystem in the park, allows students to show mastery of the Next Generation Science standard 5-LS2-1. This activity blends together the benefits of project based learning and interdisciplinary instruction. The goal of interdisciplinary instruction as well as Project Based Learning are used to prepare students for real world problem solving, by working with a group of students to discover something.

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