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Using GeoGebra, Photography, and Vocabulary to Teach Mathematics while Aiding our ESOL Populations

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Abstract

This paper shares many important examples of using real world photography and GeoGebra software in teaching math to all students including the ESOL population. ESOL learners can benefit immensely from the use of real-world photography imported into GeoGebra software because new vocabulary and math concepts are concretized. Students are often excited and motivated by photography and technology. Thus, this paper will demonstrate how mathematics teachers can insert photos into GeoGebra Software and then explore math concepts relevant in the photographs, i.e. shapes, symmetry, measurement, fractions, parabolas, etc. By using software and a visual approach to teaching math with photographs, many of the Common Core and state math standards are met while also helping all students including the ESOL learners. GeoGebra software is available as a free download and many school districts are adopting this software in their classrooms as it is a powerful tool for teaching and connecting geometry, measurement, and algebra. The authors advocate teaching mathematics using authentic and relevant materials such as photography and technology in meeting the needs of all students, particularly the ESOL learners who have limited English language skills. Undoubtedly, using photographs is one way to motivate ESOL learners to learn mathematics and English simultaneously.

Keywords: Teaching Mathematics, GeoGebra, Photography, ESOL, Visuals, Vocabulary

Introduction

"Do not worry about your difficulties in Mathematics. I can assure you mine are still greater." And "…that is why I think a photograph can be kind."

-Albert Einstein

"Every child is an artist. The problem is how to remain an artist once he grows up."

-Pablo Picasso

In this high-tech and globally competitive STEM driven world it is becoming more and more important that all citizens be confident in their ability to

do mathematics. Knowledge of mathematics is an important skill necessary to succeed in today's world. All students deserve equal access to learning math and teachers must make the effort to ensure this. The National Council of Teachers of Mathematics (NCTM, 2000) in their revised and updated standards document specifically identified "Equity" as their first principle for school mathematics. "Excellence in mathematics education requires-high expectations and strong support for all students" (p. 11). NCTM believes that "Equity requires accommodating differences to help everyone learn mathematics" (p. 13). The NCTM has taken a prominent stand that as educators we must take an "equity for all students" approach to teaching mathematics. All students have the right to learn math and feel confident in their ability to do math. Teachers shoulder the responsibility of ensuring that "mathematics can and will be learned by all students" (NCTM, 2000, p. 13).

This paper includes strategies for teaching mathematics to reach all students using photography in GeoGebra along with some English as a Second Language (ESOL) best practices. These teaching strategies may be effective with Exceptional Education (ESE), English Language Learners (ELL), and/or mainstream students. With ESE learners, the use of the multi-modal approach that incorporates the multiple intelligence caters to students' short attention span, as they are not expected to only sit still to learn the materials. English Language Learners (ELL) often need specially designed instruction in English such as the Sheltered Instruction Observation Protocol (SIOP). The strategies and approaches in this paper could help mainstream teachers support ELLs' language acquisition while teaching them the content areas such as Mathematics by emphasizing vocabulary through the use of visuals (Furner, Yahya, & Duffy, 2005). According to Heera Kang (2016), providing ELLs access to the same level of math rigor as other students is an essential aspect of supporting their overall academic success. When considering instructional solutions inclusive of ELLs–particularly technology-based solutions–here are some features to look for:

- Presents math problems and concepts visually
- Allows interactive, self-directed exploration
- Delivers scaffolded, mastery-based learning
- Introduces mathematical language in strategic intervals
- Provides data and real-time, informative feedback so players can monitor progress and adjust their solutions.

In reality, these strategies really are just best practice for the teaching of mathematics in general.

Using Photographs in GeoGebra Math Tools to Teach Vocabulary

English language learners, especially the beginners, rely on materials with less print and more visuals such as pictures to facilitate their comprehension of the concepts they are learning. Mathematical difficulties of ELLs appear to be related to the language demands of mathematics tasks because of linguistic as well as nonlinguistic processing constraints. (Alt, Arizmendi & Beal, 2014) Most times, ELLs already have understood the math concepts in their L1; all they need to do is to re-label these concepts in English. Another approach in teaching vocabulary to ELLs is through the use of Semiotics. Research has shown that semiotics plays a very vital role in the field of language learning and teaching. According to Senel (2007), semiotics provides a practical teaching/learning process by using body language, pictures, visuals, film-strips, video, photography, etc. With the help of semiotics, language learning becomes more productive and exciting. Qadha and Mahdi (2019) investigated the effect of semiotics on learning abstract words. Fifty-five Arab learners of English as a foreign language (EFL)participated in their study and participants were assigned into three groups. The first group was taught abstract words using semiotics. The second group was taught concrete words using semiotics. The third group was taught the same words using a traditional way, i.e., without semiotics. Results of the post-test indicated that participants in semiotics groups (either concrete or abstract) outscored the participants who did not use semiotics to learn new words. The study concluded that semiotics is a useful tool to enhance learning new words. Also, semiotics can be more helpful in learning concrete words than abstract words. Other semiotics studies have also examined digital computer games for second language acquisition especially vocabulary. Aghlara and Tamjid (2011) examined the effect of using a digital computer game on preschoolers' vocabulary gain; they noticed that children in the experimental group did better than in the control group. Their study discussed the positive effects of using digital computer games on vocabulary gain at the preschool level. Likewise, Segers and Forhoeven (2003) conducted a study in which they studied the impact of using a computer on preschoolers' vocabulary learning. The study concluded that vocabulary training through computer had positive effects on preschool children's vocabulary learning. In addition, Silverman and Hines (2009) conducted a study in which they tried to make a comparison between the classical and multimedia-enhanced read-aloud vocabulary instructions concerning their effects on vocabulary gain of English language learners (ELLs) and non-English language learners (non-ELLs). The study came up with results showing that there was a positive effect for ELLs on a researcher- designed measure and a measure of general vocabulary

knowledge. With many studies supporting the notion that teaching vocabulary to ELLs is enhanced with the use of technology, teachers can facilitate ELLs' learning of vocabulary by using technology in teaching math as well. For instance, teachers can point out all the math concepts and vocabulary through the use of GeoGebra math tools. Real objects embedded in the photos can be placed in GeoGebra math tools. This can reinforce the math ideas and concepts visually and enable ELLs to simultaneously learn general English and mathematics vocabulary. It is important to utilize concrete objects that students can use in hands-on activities; this not only makes the comprehension of abstract math concepts more comprehensible, but also fun while connections to the representational models are made in the photos that students can import into the GeoGebra software [See the Figures 1-11 below].

Using Photographs in GeoGebra to Teach Mathematics to the Culturally Diverse Students

Especially for the teaching of students from diverse background, teachers can research the math strategies used by their students from different cultures. For instance, Chinese students may be familiar with the use of abacus to do their calculations. Teachers can perhaps ask these students to show the others in class how abacus is used. Honoring and recognizing students' individual ways of solving math problems will boost their self-esteem because students would feel that they too have something to contribute to the learning process despite their English language constraints. Even students with learning disabilities in the area of math can be powerful teachers to other students when they explain how they learned to accomplish specific skills or concepts (Mastropieri, et al, 2001, p.23). In addition, teachers can prompt students to talk about their experience in learning some of the math concepts in their country. By importing photographs into the GeoGebra, ELL can share photos of their cultural background to explore the math concepts and in doing so learn the English vocabulary that is depicted within these photos.

By capitalizing on students' prior knowledge and using photos to teach math, teachers who are empathetic to their students' needs and backgrounds, bridge the new knowledge to the old, making learning new math concepts more manageable for these students. In some cases, by understanding how students solve problems, teachers can troubleshoot or fine tune the individual student's process and make them more efficient learners, and by connecting the math concepts they learn to the world around them through photos makes the process easier for the ELLs.

Using Photography in Mathematics to Apply Problems to Daily Life Situations

Creative teachers can use a variety of ways to coach students when applying problems to daily life situations especially when they make it meaningful by incorporating photos within the math lesson. For example, teachers can use realia such as restaurant take-out menus to teach students multiplication and division. Not only do such activities involve students with real life situations they also create a fun learning environment for students. Such environment will also promote English language acquisition for ELLs. Krashen's (1985) metaphoric use of affective filter in his affective filter hypothesis reinforces the idea that teachers can lower students' "affective filter" by fostering a spirit of mutual respect, high expectations, and cooperative learning. A group project that students can do now in this pandemic is to go on virtual tours of Museums or Monuments. Students can be given some mathematical problems to solve based on what they have seen on this virtual tour. By using real world practice activities, the goal of generalizing skills learned in class to their lives becomes more attainable. When children see and relate things to real life it makes the learning of math much more meaningful [Refer to the model figures of photography in GeoGebra with vocabulary, below].

Using Drawings and Visualizations to teach Mathematical Concepts

The Natural Approach (Krashen, 1985; Terrell, 1981) is used extensively when teaching ELLs. One of the four principles of this approach is that the teacher understands that the new ELLs go through a silent period before they begin to speak English. One of the subsequent strategies of the Natural Approach is to allow students who are at the early developmental stage of their language acquisition process to use drawings and symbols in solving some of the math problems. The same techniques are employed with students with learning disabilities to allow them to process auditory information before making a verbal response. In fact, as a comprehension strategy, teachers can use students' drawings and verbal rehearsals as testimony of their understanding of math concepts. This approach can alleviate frustrations for both teachers and students. Students can use the GeoGebra software and by inserting photos into the GeoGebra they can then draw on the photo and employ several math functions such as measuring, plotting points, identifying vertices and at the same time learning general English vocabulary and technical Math terminologies. Additionally, students can be encouraged to explore and create drawings and designs using the geometry tools within GeoGebra to discover their artistic talents as they learn mathematics.

Using Technology and Mathematics Software such as GeoGebra to make Math Representational

Today the Internet and computer software such as GeoGebra are now being used as an instructional tool to explore, investigate, problem-solve, interact, reflect, reason, communicate, and learn many concepts that are in U.S. School curricula. Students are able to take virtual tours of places like the Bronx Zoo, the White House, the Louvre Museum, as well as access information from NASA, the United Nations, etc. The number of websites and educational software designed for teachers and students with the intent to teach concepts is becoming infinite. GeoGebra can be used right on the computer or downloaded free and used even off line. There are many websites today like funschool.com or www.funbrain.com which are ideal for both teachers and students to teach/learn a multitude of math and reading/language concepts of K-12 and software like *Mathblaster* or *Tesselmania* can really make the learning of mathematics dynamic. GeoGebra software is one of the most diverse math tools out there now and it is free where students can explore and create tessellating pieces of art and so much more.

Ameis and Ebenezer (2000) wrote a book called *Mathematics on the Internet* where they provide resources and suggestions for teaching mathematics via the Internet. The book connects many math K-12 concepts to many websites that can be used to help teach these concepts. Parents of home-schoolers too can greatly benefit from the use of the Internet as a means to learn via Internet Field Trips. The Internet has a definite role to play in reforming traditional teaching. By using educational software like GeoGebra and the Internet (technology) and photography, all students, especially ELLs, can now learn in ways that are challenging yet more exciting. The Internet too provides students with the tools to use the computer to access information and become independent life-long learners in an age that is increasingly dependent on technology to survive a complex multicultural world.

Educators today should advocate the use of emerging technologies such as the GeoGebra which demonstrates how photos can be imported and used to teach the math concepts[See Figure 1 below on how to insert photos in GeoGebra]. It is critical in the 21st Century that students have an appreciation for math around them and in their everyday life (Gorriz & Vilches 2019). When using visuals, students are often highly motivated to use photos that appeal to their senses while exploring the mathematics within them. Antje, Hannula, & Toivanen (2018) found that using outdoor photography when teaching math has a positive impression on students and their learning of mathematics.

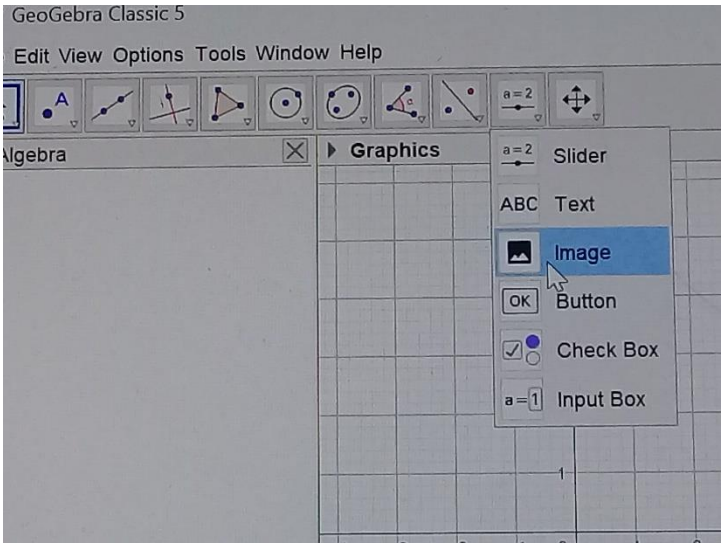

Figure 1. Inserting Photos into the GeoGebra Software

GeoGebra is a multi-platform dynamic mathematics software for all levels of education from elementary through university that joins dynamically geometry, algebra, tables, graphing, spreadsheets, statistics and calculus in one easy-to-use package (Hewson, 2009; Hohenwarter, Hohenwarter, & Lavicza, 2009). This open-source dynamic mathematics software can be downloaded for free and accessed at: http://www.geogebra.org/cms/en/info. Since there are no licensing issues, students and teachers have the freedom to use it both within the classroom and at home. GeoGebra has a large international user and developer community with users from 190 countries is currently translated into 55 languages. This paper advocates the use of GeoGebra, and also how the insertion of photographs and the use GeoGebra tools help teachers to teach important math concepts in grades K-12.

Connections need to be made when we teach mathematics (Zengin, 2019). Munakata and Vaidya (2012) based on their research found that students do not consider mathematics and science to be creative endeavors, although the traditional artistic disciplines rank high in this regard. To address this problem in perception, the authors used photography as a means to encourage students to find the deep-rooted connections between science and mathematics and the arts. The photography project was used in a formal classroom setting as well as an outside activity, i.e. in a more informal setting. Students' interest and motivation were peaked when photography was part of the instructional strategies to teach new material while making meaningful connections to math through photography. Jones (2012) also in her book, *Visualizing Mathematics*, discusses how teachers need to help students visualize and create representations of their math understanding in order to make them excited about the subject.

By using technology and photographs, our young learners are excited to construct and investigate math ideas and learn some history as well with GeoGebra. Students like and enjoy math more by way of photography and the software GeoGebra. Students will see and appreciate the world around them better and this may get them excited about the STEM fields that are so critical today while also instilling a passion for photography. Abraham (2019) contends that when someone loves what they do or have a passion for something such as photography, they can influence others to develop that passion as well.

GeoGebra

While the study of geometry has existed for many millennia, it is within the past twenty years that a shift has occurred in how geometry may be learned through computer-based interactive geometric software. Software programs like GeoGebra allow users to construct interactive representations of points, lines, shapes, and circles. These geometric objects are interactive in that they may be resized and shifted around onscreen through clicking and dragging actions. Furthermore, interactive geometric software like GeoGebra in the K-12 mathematics curriculum has been used at the elementary level, middle school level, and the high school level (Yu & Tawfeeq, 2011) and has proven to be a very effective means for teaching and learning mathematics.

Fahlberg-Stojanovska, & Stojanovski (2009) discovered that using GeoGebra software motivates and helps young people learn at a higher level while exploring and conjecturing as they draw and measure onscreen. Rosen & Hoffman (2009) found that it is very important to integrate both concrete and virtual manipulatives into the primary-age math classroom. Furner & Marinas (2015) found that children easily transition to the abstract when using geometry sketching software by first using geoboards and the software like *Paint* before going directly to the sketching software GeoGebra.

GeoGebra is a great resource and technological tool that when used in the math classroom provides a focus to:

- promote technology as an essential tool for learning mathematics in the 21st century
- integrate the principles and process standards with teaching the content standards
- provide access to all five mathematics content standards for all students
- support learner-centered strategies that address the diverse needs of all learners of mathematics

GeoGebra allows students to actively construct their own understanding of geometry, measurement, and algebra using this technology. Using GeoGebra educators can meet the Common Core Math Standards, students can master many math concepts such as:

- use the polygon and circle tools to draw shapes
- measure angles and distance
- use GeoGebra sliders to adjust values of different problems
- insert images into a file to demonstrate mathematical problem solving
- recognize perimeter as an attribute of plane figures and distinguish between linear and area measures
- reason with shapes and their attributes

Effective mathematics teachers can maximize the potential of GeoGebra to:

- develop students' understanding
- stimulate their interest
- increase their proficiency in mathematics

GeoGebra is a free software, a multi-platform dynamic mathematics software for all levels of education that joins geometry, algebra, tables, graphing, statistics and calculus in one easy-to-use package (Hohenwarter, Hohenwarter, & Lavicza, 2009). GeoGebra has a large international user and developer community with users from 190 countries. The software is currently translated into 55 languages and attracts close to 300,000 downloads per month. It can be downloaded for free and accessed at: http://www.geogebra.org/cms/en/info.

Making Connections when Teaching Mathematics is Important for all Students

Today it is important that educators make important connections between the school curriculum and the real-world to better reach learners. Connections need to be made when we teach mathematics according to Zengin (2019). Connections between the Common Core State Standards and the National Council of Teachers of Mathematics (NCTM) Standards need to meet the challenges of differentiating mathematics instruction in the K-12 classroom. Small (2012) explains two powerful and universal strategies that teachers can use across all math content: Open Questions and Parallel Tasks. Showing teachers how to get started and become experts with these connection strategies. Small also demonstrates more inclusive learning conversations that promote broader student participation and mathematical thinking. Guidance for creating a more inclusive classroom learning community with mathematical talk that engages participants from all levels is important (Small, 2012). While using the technology and covering all the required math standards by making all necessary connections for sound learning, students can work together and use the software like GeoGebra to engage in higher level mathematical understanding.

Andresen & Misfeldt (2010) found that with the Common Core Standards (National Governors Association Center for Best Practices - NGA Center, 2010) in teaching mathematics, teachers need to be trained and learn new mathematics

content and technology like GeoGebra in order to be effective in teaching and reaching their students. Knowledge of technology cannot be isolated from the content and good mathematics teaching requires an understanding on how technology is related to the pedagogy and mathematics (Hughes, 2005).

Today, most schools and states are adhering to the Common Core Math Standards (National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO), 2010) found at: http://www.corestandards.org/. A sampling of objectives from the Common Core Standards that can be addressed using the GeoGebra software are presented in many of the photos and examples below [See Figures 2-11]:

Teaching the Common Core Math Standards: Using Photography and GeoGebra

CCSS.MATH.CONTENT.K.G.A.2

Students can correctly name shapes regardless of their orientations or overall size.

CCSS.MATH.CONTENT.2.G.A.1

Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.1 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.

CCSS.MATH.CONTENT.4.G.A.1

Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

CCSS.MATH.CONTENT.6.G.A.3

Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

CCSS.MATH.CONTENT.8.G.B.7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

CCSS.MATH.CONTENT.HSG.CO.A.1

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Using Photography while Emphasizing Vocabulary and Math Ideas

Math teachers may want to do a two-minute math starter problems like the following using photos to start and motivate a class lesson (Furner & Marinas, 2020) by using the photos and questions below:

Can you identify where each one of these photos was taken [See Figure 2]?

Can you see some mathematical idea or math vocabulary within the photo?

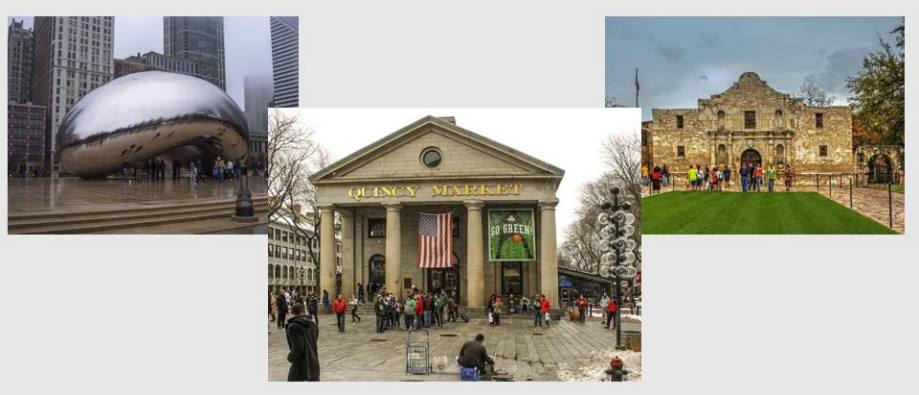

Figure 2. Identify the Photo Locations and Mathematics in them (Photos by Dr. Carol Marinas)

In this example in Figure 2, students may know these famous locations: Chicago, Boston, and San Antonio and they should be able to identify some important vocabulary: **parallel lines, circles, rectangles, symmetry, 3-D solids, angles,** and **repeating patterns**.

Munakata and Vaidya (2012) grounded on their research discovered that students do not consider mathematics and science to be creative endeavors, though the traditional artistic disciplines rank high in this regard. To address this problem in perception, the authors used photography as a means to encourage students to find the deep-rooted connections between science and mathematics and the arts. The photography project was implemented in a formal classroom setting as well as an outside activity, i.e. in a more informal setting. Through the project, it was observed that students' interest and motivation were peaked when photography was part of the instructional approaches to teach new material while making meaningful connections to the math concepts. Bragg & Nicol (2011) contend that students can have success with learning math when they use photos and problem solving together to do the math based on the photos, relating and understanding real-world problems through the use of interactive technology like GeoGebra and connecting them with photography to make important connections in mathematics for students.

Rizzo, del Río, Manceñido, Lavicza, & Houghton (2019) through their research found that by using photograph and GeoGebra and students collaborating to learn math, students were found to be much more motivated to learn the math concepts, and the photography helped students to connect math to the real-world. Through their research and analysis, they found using a combination of three strategies helped to engage learners: relating photography/art, students' surroundings/real-world, and the mathematics content. These three strategies proved to enhance instruction and success in delivering the subject. Portaankorva-Koivisto & Havinga (2019) found that when educators support learning through

the use of photography and the visual arts in teaching math, it enhances students' learning and success of understanding mathematics and its values in everyday life. In addition, students can see shared values and ideas in both math and photographs, such as ratio, shapes, and other mathematical and shared ideas.

It is important that we develop a passion for photography in our students for those who are interested. Abraham (2019) contends that when someone loves what they do or have a passion for something like photography, they can influence others to develop that passion as well. Antje, Hannula, & Toivanen (2018) found that when using outdoor photography when teaching math that it had a positive impression on students and their learning of mathematics. It is important we encourage our learners to recognize that geometry and shapes/mathematics that surround us! Students can see this in photos and so much more math can be learned using photographs to teach mathematics. Spring(2020) contends that educators can teach math, English, physics and other subjects using photography. Oswald (2008) reported in her article that teachers can use photography to teach and learn science, math, and writing. Jaqua (2017) contends using photos and even selfies have benefits for students in learning mathematics. Math is already a part of students' lives like selfies are and encouraging students to see the math values when they take photos, valuing the world around them and the math that exists as well, math can then be more real for learners. Encouraging students to take photos may help them develop a passion for photography and help develop a long-term creative outlet and passion for live. Mathematics lends itself well to incorporating photography into its discipline.

Math teachers can ask their students what math concepts can they see in the photo below [See Figure 3]?

Figure 3. Identify Math Concepts in Photo in Orlando taken by Dr. Joseph Furner

Figure 3 offers some vocabulary words in the photo, words like: **points, lines, circles, parallel lines,** to name a few. Oswald (2008) reported on a school-based project where the head teacher, Liz Becks created a project to learn science, math, and writing using photography, Beck is cited in her report saying "There's something in nearly every subject that relates to photography, and it's a topic that appeals to kids" (p. 1). By using photography within instruction educators are appealing to learners and also creating a passion for photography in many of them. Please see Figure 3 for example of parallel lines within a photo of the real-world imported into GeoGebra and the parallel lines drawn on the photo using the GeoGebra tools.

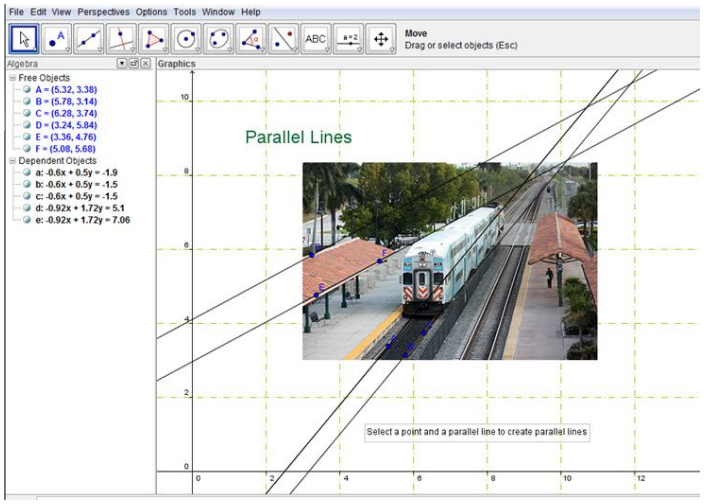

Figure 4. Examples of Parallel Lines in a Photo

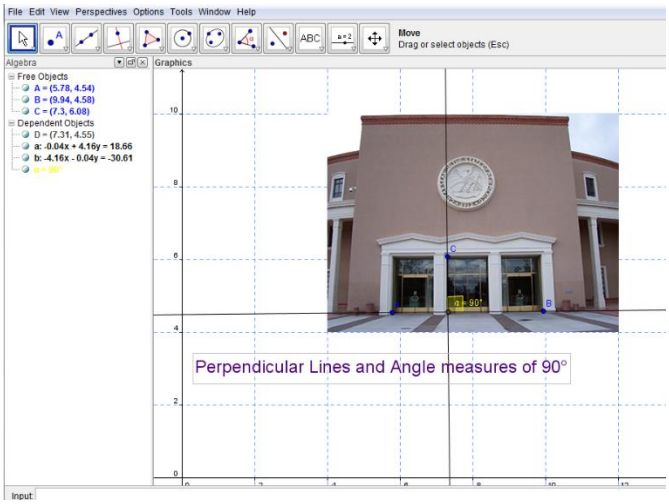

Figure 4. GeoGebra file showing Photo of Perpendicular Lines

Perpendicular lines create **right angles**, **90 degree angles**, and like in the GeoGebra file photo above [See Figure 5] right angles and perpendicular lines are drawn on the photo with the GeoGebra software all allowing students to identify vocabulary and math ideas.

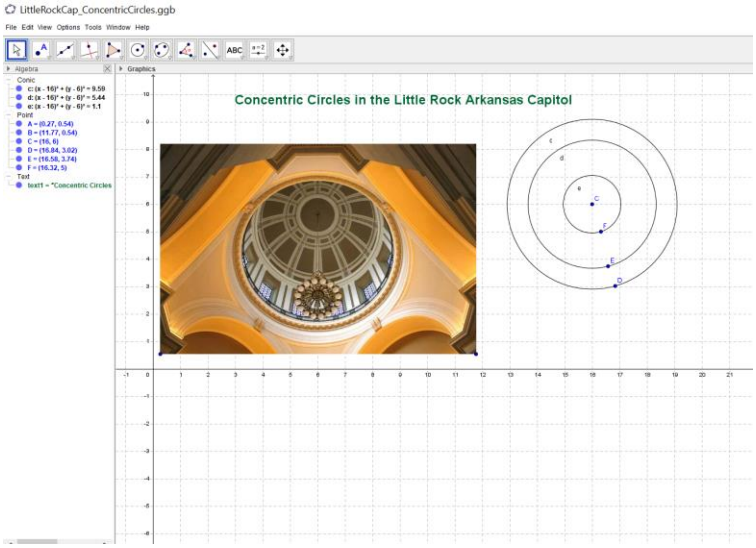

Figure 6. Circles and Concentric Circles in GeoGebra

The photo in Figure 6 was imported into GeoGebra and then students were asked to draw **circles** and **concentric circles** seen in the photo of the Little Rock Capitol building.

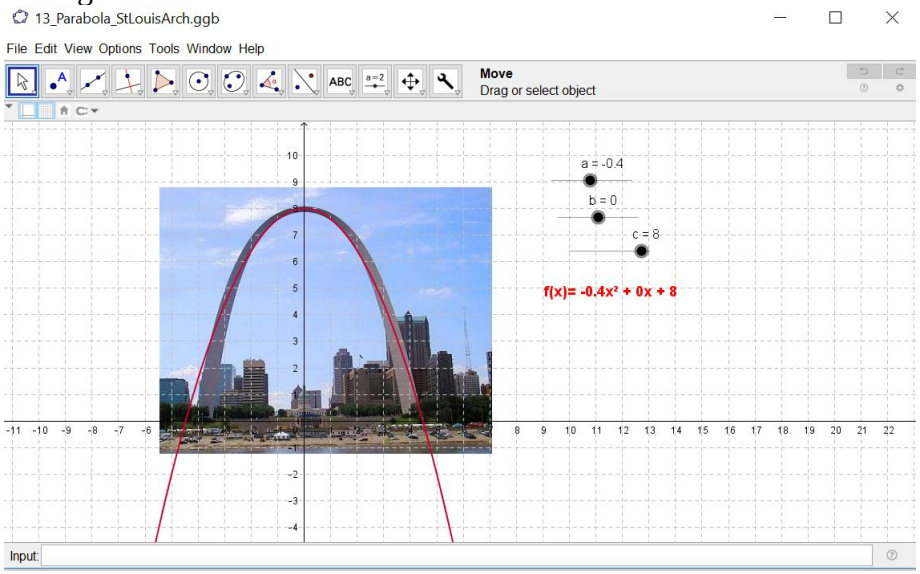

Figure 7. Parabola Examples in the Real World

Parabolas often act as a frame around the central object and can be seen in Figure 7 above with the example of a photo of the St. Louis Arch is like a

parabola and students can import the photo into GeoGebra and then try to draw a parabola on it to find its formula.

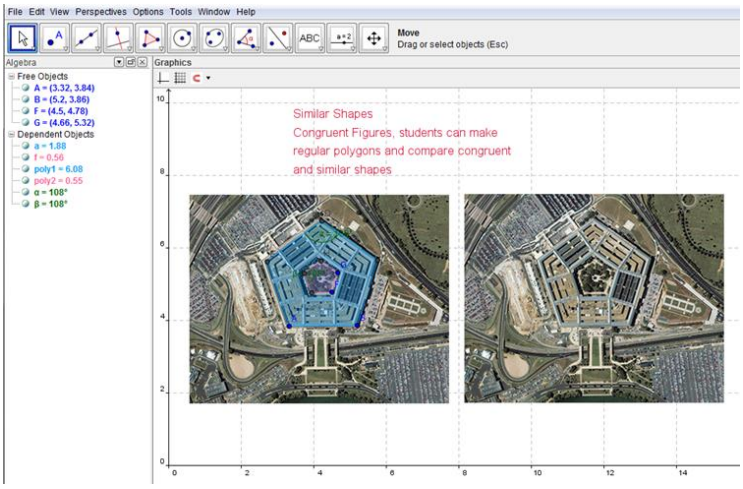

Figure 8. Similar Shapes

Figure 8 depicts an aerial view of the Pentagon photo inserted into GeoGebra and then the students can use the GeoGebra tools to compare **angles** and **pentagon shapes** to see the **similar shapes** of the **interior** and **exterio**r shapes of the Pentagon.

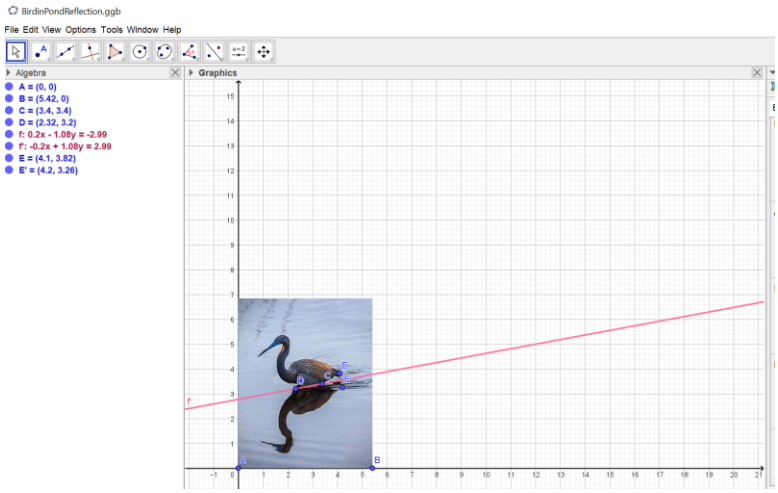

Figure 9. GeoGebra File Showing Photo and Line of Reflection

Reflections can often show up when taking photos of water, glass, or any other type of reflective surface. The photo above in Figure 9 shows a photo of a **reflection** of a bird in water with a **line of reflection** drawn in GeoGebra.

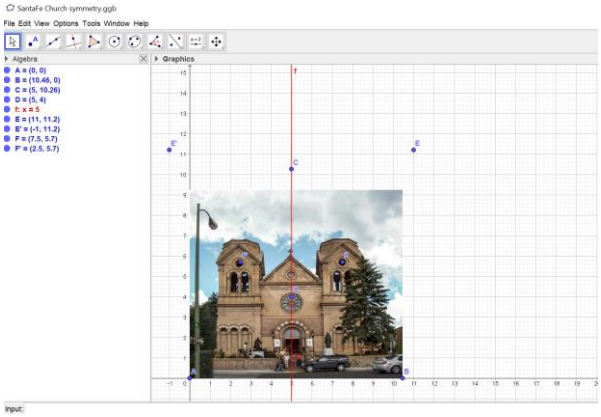

Figure 10. GeoGebra File of Symmetry of Photo of a Building with Symmetry

Figure 10 shows a photo inserted into the GeoGebra software, a line was drawn through the center of the photo and then a **point** was selected and **reflected along the line** to show them as **symmetrical** to each other.

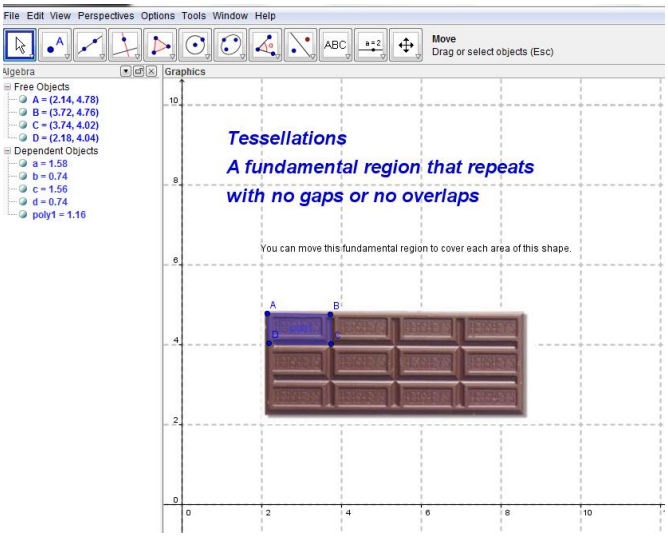

Figure 11. Photos of Tessellations-Using a Photo of a Chocolate Bar

Tessellating patterns are patterns that repeat with the same **fundamental region** covering a space, with no **gaps** and no **overlaps** like seen in the chocolate bar rectangular pieces seen in Figure 11. Students can use the GeoGebra and move the pieces to test their hypothesis of the same shape through repetition. By using GeoGebra and inserting photos into the software, students can then use the tools in GeoGebra to do the math, learn the vocabulary, and understand mathematical concepts better.

In Conclusion

Today young people intrigued by technology will construct and investigate geometric shapes and many math ideas with GeoGebra and will start enjoying math and even see some connections in mathematics and maybe even develop a passion for photography when teachers have them import photos into the GeoGebra software! It is important to make such connections when teaching math using GeoGebra and Photography because they:

- Show a purpose for math
- Better reach all types of learners even ESE and ESOL
- Make connections and relationships between math and photography while covering math concepts
- Employ emerging technologies in math with the real world
- Show practical applications to math in everyday life
- Employ innovative teaching in the classroom
- Stimulate excitement through Photography/Modeling
- May help students develop a passion for photography in the process

In a globally competitive world where it is more important to prepare our students for science, technology, engineering, and mathematics (STEM) fields, using GeoGebra software covers all the STEM areas. It applies math, science, and engineering ideas while using and applying the technology of GeoGebra along with digital photography to teach math and motivate and help develop a passion toward photography in our young people. What better way to learn math than to use pictures and photos and to see the math in them, applying the GeoGebra math tools to understand and apply the mathematics. Pictures still speak the most universal language, a picture is worth a thousand words as the saying goes, photographs can be so inspiring and kind! Our young people can become artists with their photography and also use the technology to understand the mathematics they have to learn in school! When our young people grow up they can still remain artists with a passion for photography! The GeoGebra technology, photography, and emphasizing math vocabulary and concepts when teaching can help teachers reach all types of students, especially our ESOL populations.

The suggested means to teach math to all students are by no means exhaustive. With the ESE learners, the use of the multi-modal approach that incorporates the multiple intelligence caters to students' short attention span as they are not expected to only sit still to learn the materials, going out and taking photos and then uploading and inserting them into GeoGebra and exploring all the math on them can be more exciting for young learners. With the English Language Learners (ELL), we have included the SDAIE (Specially Designed Academic Instruction in English) approach which supports the teaching of language acquisition while teaching the content area such as Math. Language objectives are incorporated in teaching Math concepts, therefore, ELL are given the opportunity to simultaneously develop their English language skills as they are learning Math. Math teachers today must work hard to eliminate and prevent any math anxiety their students may develop or carry with them (Furner & Duffy, 2002). Our children today are not only competing for jobs with others in the U.S.A., but with others from around the globe--being confident in their ability to do mathematics in this competitively global society. With these approaches and strategies mentioned in this article, content area teachers can also play a crucial role in the language learning process of their students and the primary teachers no longer have to be the sole responsible party in teaching ELL the English language; rather, both the content area teachers and the primary teachers can work as a team to help the ELL in their academic work. Each day the diversity of students grows within the confinement of our classrooms, therefore, teachers have to tirelessly keep abreast with their research of diverse teaching strategies to reach all students to learn. Teachers must see to it that they reach all students equally. Equity in mathematics instruction requires teachers to provide accommodations so everyone in the class can learn mathematics. The 'best practices" mentioned

here can better assist any teacher in reaching all students mathematically while using technology like GeoGebra and photography which can really appeal to the learners and make math not only appealing but getting young people ready for a STEM world.

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