

Renormalization group approach to chiral symmetry breaking in graphene

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We investigate the development of a gapped phase in the field theory of Dirac fermions in graphene with long-range Coulomb interaction. In the large- N approximation, we show that the chiral symmetry is only broken below a critical number of two-component Dirac fermions $N_c = 32/\pi^2$, that is exactly half the value found in quantum electrodynamics in 2+1 dimensions. Adopting otherwise a ladder approximation, we give evidence of the existence of a critical coupling at which the anomalous dimension of the order parameter of the transition diverges. This result is consistent with the observation that chiral symmetry breaking may be driven by the long-range Coulomb interaction in the Dirac field theory, despite the divergent scaling of the Fermi velocity in the low-energy limit.

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I. INTRODUCTION

The fabrication of single layers of carbon with atomic thickness has provided us with a laboratory to explore new physics,^{1,2} as the electrons in this so-called graphene behave at low energies as massless Dirac fermions, displaying conical valence and conduction bands.³ Apart from its quite interesting properties from the applied point of view, the new material offers the possibility of studying an electron system that is a variant of quantum electrodynamics (QED) in the strong-coupling regime with unusual features as shown, for instance, in Refs. 4–8.

A remarkable feature of this field theory of electrons in graphene is its scale-invariant character.⁹ This means for practical purposes that, while many-body corrections give rise, in general, to dependences on the high-energy cutoff, these are susceptible of being reabsorbed into the definition of physical quantities. Consequently, some of the parameters of the theory may have a nontrivial scaling in the low-energy limit. The quasiparticle weight is, for instance, renormalized, and it would be driven to zero if its flow were not arrested by the divergence of the Fermi velocity in the infrared.¹⁰ This marginal behavior leaves anyhow an imprint in the quasiparticle decay rate¹¹ with an unconventional dependence on energy which has been observed experimentally.¹²

An important phenomenon that may take place in a system of massless Dirac fermions is the opening of a gap in the regime of strong interaction. In this respect, the case of QED in 2+1 dimensions can serve as a good example, in which the original $U(N)$ chiral symmetry of the theory with N massless two-component Dirac fermions is spontaneously broken below a critical number of flavors N_c .¹³ This chiral symmetry breaking (CSB) has been also studied in graphene by a number of analytical^{14–19} as well as numerical methods.^{20–23} The conclusion to be drawn from different approaches is that a gap can open up in the Dirac spectrum, though the effect may only appear below some critical value of N and above some critical interaction strength. In this picture, there remain, however, important questions to be addressed, related to the effect of the above-mentioned scaling of the parameters in the model. We point out, in particular, that the strength of any four-fermion interaction in the Dirac

field theory has to be measured relative to the weight of the kinetic energy that scales with the Fermi velocity. Then, it is crucial to clarify whether the divergence of this parameter in the infrared may prevent the CSB even for a small number of Dirac fermions.

In this paper we apply renormalization group methods to study the CSB in the field theory of Dirac fermions in graphene. We consider that this electron system is governed at low energies by the Hamiltonian

$$H = iv_F \int d^2r \bar{\psi}_i(\mathbf{r}) \boldsymbol{\gamma} \cdot \nabla \psi_i(\mathbf{r}) + \frac{e^2}{8\pi} \int d^2r_1 \int d^2r_2 \rho(\mathbf{r}_1) \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} \rho(\mathbf{r}_2), \quad (1)$$

where $\{\psi_i\}$ is a collection of $N/2$ four-component Dirac spinors, $\bar{\psi}_i = \psi_i^\dagger \gamma_0$, and $\rho(\mathbf{r}) = \bar{\psi}_i(\mathbf{r}) \gamma_0 \psi_i(\mathbf{r})$. The matrices γ_σ satisfy $\{\gamma_\mu, \gamma_\nu\} = 2 \text{diag}(1, -1, -1)$ and can be conveniently represented in terms of Pauli matrices as $\gamma_{0,1,2} = (\sigma_3, \sigma_3 \sigma_1, \sigma_3 \sigma_2) \otimes \sigma_3$, where the first factor acts on the two sublattice components of the graphene lattice. Our aim is to elucidate whether a term of the type

$$\rho_m(\mathbf{r}) = \bar{\psi}(\mathbf{r}) \psi(\mathbf{r}) \quad (2)$$

is generated spontaneously in the Hamiltonian of the electron system. A convenient way to address this question is to look at the susceptibility built from that operator, that is, at the correlator

$$\Pi(\mathbf{q}, \omega) = i \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle T \rho_m(\mathbf{q}, t) \rho_m(-\mathbf{q}, 0) \rangle. \quad (3)$$

$\Pi(\mathbf{q}, \omega)$ is actually a response function measuring the reaction of the system under a slight difference of scalar potential in the two sublattices of the graphene lattice. A divergence of $\Pi(\mathbf{0}, 0)$ at some particular value of the coupling constant can be interpreted as the signal that ρ_m is getting a nonvanishing expectation value, which is, in turn, the signature of the opening of a gap in the Dirac spectrum.

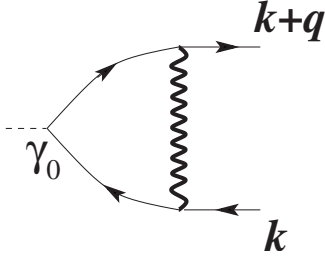


FIG. 1. Quantum corrections to the vertex built from $\bar{\psi}\psi$ in the large- N approximation, where the interaction between electrons is taken as the RPA dressed Coulomb potential (thick wavy line).

We will take advantage of the power of the renormalization group to characterize the possible singular behavior of Π as a function of e^2/v_F . For this purpose, we concentrate on the corrections to the vertex built from $\bar{\psi}\psi$, as shown in Figs. 1 and 3. In the process of renormalization, ρ_m may get, in general, an anomalous dimension γ_{ψ^2} , modifying the naive scaling of the susceptibility,

$$\Pi(\mathbf{q}, 0) \sim |\mathbf{q}|^{1-2\gamma_{\psi^2}}. \quad (4)$$

In what follows, we apply different approaches for the determination of γ_{ψ^2} , in order to establish the existence of a singular behavior in the long-distance scaling of the susceptibility Π .

II. LARGE- N APPROXIMATION

We can go beyond the usual perturbative approach in the coupling e^2/v_F by taking formally a large number N of fermion flavors to perform then the sum of all the diagrams that arise to leading order in a $1/N$ expansion. If we think of all possible contributions to the expectation value $\langle \rho_m(\mathbf{q})\psi(\mathbf{k} + \mathbf{q})\psi^\dagger(\mathbf{k}) \rangle$, it is clear that the leading corrections in $1/N$ are given by the iteration of the exchange of electron-hole bubbles in the interaction between the ψ and ψ^\dagger fields. This amounts to adopt the random-phase approximation (RPA) for the dressed Coulomb interaction represented in Fig. 1. Introducing the polarization $\chi(\mathbf{q}, \omega_q)$, we get for the corresponding vertex function

$$\Gamma(\mathbf{q}; \mathbf{k}) = \gamma_0 + i \sum_{n=0}^{\infty} \int \frac{d^2p}{(2\pi)^2} \frac{d\omega_p}{2\pi} G_0(\mathbf{p}, \omega_p) \gamma_0 G_0(\mathbf{p} + \mathbf{q}, \omega_p) \times \frac{e^2}{2|\mathbf{p} - \mathbf{k}|} \left(\frac{e^2 \chi(\mathbf{p} - \mathbf{k}, \omega_p - \omega_k)}{2|\mathbf{p} - \mathbf{k}|} \right)^n, \quad (5)$$

where G_0 stands for the free Dirac propagator. We recall that, in the case of N two-component Dirac fermions, $\chi(\mathbf{q}, \omega_q) = -(N/16)\mathbf{q}^2 / \sqrt{v_F^2 \mathbf{q}^2 - \omega_q^2}$.

An interesting feature of the sum in Eq. (5) is that, while a high-energy cutoff Λ has to be imposed to make the integrals finite, all the terms show the same degree of logarithmic dependence on the cutoff. By passing to imaginary frequency $i\bar{\omega}_p = \omega_p$, we can compute the divergent contribution to the vertex as

$$\begin{aligned} \Gamma &\approx \gamma_0 + \gamma_0 \frac{e^2}{2} \sum_{n=0}^{\infty} (-1)^n \left(\frac{Ne^2}{32} \right)^n \\ &\times \int \frac{d^2p}{(2\pi)^2} \frac{d\bar{\omega}_p}{2\pi} \frac{|\mathbf{p}|^{n-1}}{(\bar{\omega}_p^2 + v_F^2 \mathbf{p}^2)^{n/2+1}} \\ &\approx \gamma_0 + \gamma_0 \frac{4}{\pi^{3/2} N} \sum_{n=0}^{\infty} (-1)^n g^{n+1} \frac{\Gamma\left(\frac{1}{2} + \frac{n}{2}\right)}{\Gamma\left(1 + \frac{n}{2}\right)} \int^\Lambda \frac{d(v_F |\mathbf{p}|)}{v_F |\mathbf{p}|} \\ &\approx \gamma_0 + \gamma_0 \frac{8}{\pi^2 N} g \frac{\arccos(g)}{\sqrt{1-g^2}} \log \Lambda, \end{aligned} \quad (6)$$

where $g = (N/32)e^2/v_F$. We note that the singularity at $g=1$ is only apparent, as the function in Eq. (6) can be continued analytically to $g > 1$ by taking $\arccos(g) = i \log(g + \sqrt{g^2 - 1})$. In this way, we end up with an expression of the vertex that becomes sensible for arbitrarily large values of the effective coupling.

The divergence of the vertex Γ at large Λ has to be removed by absorbing the dependence on the cutoff into the scale Z_{ψ^2} of the composite field $\bar{\psi}\psi$.²⁴ However, this is not the only field redefinition to be accomplished, as the finiteness of the full Dirac propagator demands the introduction of a cutoff-dependent scale for the Dirac field, such that $\psi(\Lambda) = Z_\psi^{1/2}(\Lambda) \psi_{\text{ren}}$. The electron self-energy can be actually found in Ref. 10 to dominant order in the $1/N$ approximation, providing the result

$$Z_\psi \approx 1 + \frac{8}{\pi^2 N} \left(2 + \frac{2-g^2 \arccos g}{g \sqrt{1-g^2}} - \frac{\pi}{g} \right) \log \Lambda. \quad (7)$$

The cutoff independence of the vertex Γ must be guaranteed after multiplication by Z_ψ and the scale Z_{ψ^2} of the composite field. We define the renormalized vertex as $\Gamma_{\text{ren}} = Z_{\psi^2} Z_\psi \Gamma$. By imposing the finiteness of Γ_{ren} , we obtain to leading order in the $1/N$ expansion

$$Z_{\psi^2} \approx 1 - \frac{8}{\pi^2 N} \left(2 + \frac{2 \arccos g}{g \sqrt{1-g^2}} - \frac{\pi}{g} \right) \log \Lambda. \quad (8)$$

The knowledge of Z_{ψ^2} can now be used to determine the anomalous scaling of the susceptibility $\Pi(\mathbf{q}, \omega)$. This correlator involves two composite operators $\bar{\psi}\psi$, and it can be made cutoff independent by multiplying each of them by their renormalization factor. The finite susceptibility is then $\Pi_{\text{ren}}(\mathbf{q}, \omega) = Z_{\psi^2}^2 \Pi(\mathbf{q}, \omega)$. A renormalization group equation can be obtained for Π , relying on the independence of the susceptibility on Λ in the renormalized theory.²⁴ We obtain from the invariance of Π_{ren}

$$\left[\Lambda \frac{\partial}{\partial \Lambda} + \beta(g) \frac{\partial}{\partial g} - 2\gamma_{\psi^2} \right] \Pi(\mathbf{q}, \omega) = 0 \quad (9)$$

with the anomalous dimension

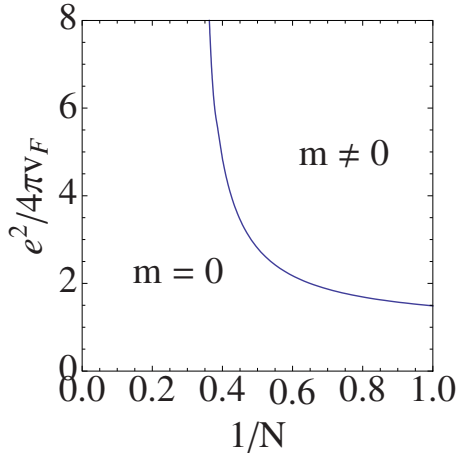


FIG. 2. (Color online) Phase diagram obtained to leading order in the $1/N$ approximation, showing the regime with massless Dirac fermions ($m=0$) and the phase with CSB ($m \neq 0$).

$$\gamma_{\psi^2} = -\Lambda \frac{\partial \log Z_{\psi^2}}{\partial \Lambda} \quad (10)$$

and $\beta(g) = \partial g / \partial \log \Lambda$. In the bare theory with a cutoff Λ , it follows from dimensional analysis that the susceptibility Π can be written in terms of a dimensionless function $\Phi(x)$ as $\Pi(\mathbf{q}, 0) = (|\mathbf{q}|/v_F) \Phi(v_F |\mathbf{q}|/\Lambda)$. Then, neglecting in a first approximation the scaling of the effective coupling, the solution of Eq. (9) implies that $\Pi(\mathbf{q}, 0) \sim (|\mathbf{q}|/v_F)(v_F |\mathbf{q}|/\Lambda)^{-2\gamma_{\psi^2}}$ with the behavior anticipated in Eq. (4).

The anomalous dimension obtained from Eq. (8) is

$$\gamma_{\psi^2} = \frac{8}{\pi^2 N} \left(2 + \frac{2 \arccos g}{g \sqrt{1-g^2}} - \frac{\pi}{g} \right) \quad (11)$$

and it turns out to be a monotonous, increasing function of g . This means that, provided that it gets sufficiently large, there may exist a critical value g_c at which $\Pi(\mathbf{q}, 0)$ becomes singular in the limit $\mathbf{q} \rightarrow 0$. The divergence of this susceptibility implies a long-wavelength instability, which can be interpreted as the development of a nonvanishing expectation value of $\bar{\psi}\psi$. On the other hand, the value of g_c depends, in general, on the number of flavors N . We can draw then a boundary marking the onset of CSB in (N, g) space. This line of transition, characterized by the condition $1 - 2\gamma_{\psi^2} = 0$, is shown in Fig. 2.

The expression in Eq. (11) leads to the existence of a critical number of flavors N_c , above which CSB cannot take place. If we let $g \rightarrow \infty$ in that equation, we approach the maximum value of γ_{ψ^2} from which we find $N_c = 32/\pi^2$. It is very suggestive that this critical N is precisely half the value obtained in QED in 2+1 dimensions.¹³ Technically, the methods used to derive N_c in each model cannot be easily compared, as QED is not a scale-invariant field theory in that number of dimensions. On intuitive grounds, however, one can understand the relation between the two values of N_c , as the photon propagating in QED has two different degrees of freedom. This may explain that twice the number of flavors

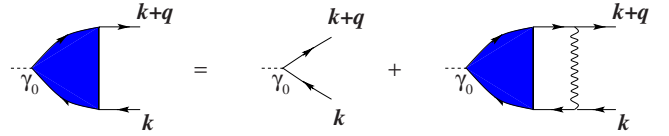


FIG. 3. (Color online) Self-consistent diagrammatic equation for the vertex $\Gamma(\mathbf{q}; \mathbf{k})$, equivalent to the sum of ladder diagrams built from the iteration of the bare Coulomb interaction (thin wavy line).

are needed there to equally screen the interaction, in comparison to our model with just the scalar Coulomb potential.

III. LADDER APPROXIMATION

We resort now to an approach that can better capture the behavior of the system to the right of the phase diagram of Fig. 2. For this purpose, it is pertinent to adopt a self-consistent approximation in the calculation of the vertex Γ , equivalent to the sum of ladder diagrams, by which the most divergent logarithmic dependences are taken into account at each perturbative level.²⁵ The approach is encoded in the self-consistent equation shown in Fig. 3. The perturbative solution leads to a power series in the effective coupling $\lambda \equiv e^2/8\pi v_F$, where the term of order λ^n diverges, in general, with the high-energy cutoff as $\log^n(\Lambda)$. The important point is that the set of diagrams considered in this way allows to implement a consistent renormalization of the theory, where Z_{ψ^2} is free of nonlocal divergences, making possible a precise computation of the anomalous dimension γ_{ψ^2} .

A solution of the equation in Fig. 3 has been given in Ref. 26 regularizing the momentum integrals with an infrared and a high-energy cutoff. Here, in order to facilitate the calculation of the divergences of the vertex Γ , we define instead the field theory by analytic continuation to spatial dimension $d = 2 - \epsilon$. After integration in the frequency variable, the self-consistent equation for the vertex takes the form

$$\Gamma(\mathbf{0}; \mathbf{k}) = \gamma_0 + 2\pi\lambda_0 \int \frac{d^d p}{(2\pi)^d} \Gamma(\mathbf{0}; \mathbf{p}) \frac{1}{|\mathbf{p}|} \frac{1}{|\mathbf{p} - \mathbf{k}|}, \quad (12)$$

where the dimensionful coupling λ_0 is given in terms of an auxiliary momentum scale ρ by $\lambda_0 = \lambda \rho^\epsilon$ (see below). Thus, powers of $\log \Lambda$ are traded by poles at $\epsilon=0$ in the different perturbative contributions, which are easier to compute. In principle, Eq. (12) could also afford a nonperturbative resolution, but the computation of the anomalous dimension would be complicated then as this is obtained from the residue of the $1/\epsilon$ pole. Unfortunately, a closed equation for that quantity cannot be written from Eq. (12), which couples the equations for the coefficients of the different powers of ϵ . This is otherwise a natural consequence of the regularization of the diagrams, since the interdependence of the different poles is a key property of a renormalizable theory, as we illustrate below.

We resort then to an iterative resolution of Eq. (12), by which we can obtain a recursion between consecutive orders in the power series for the vertex

$$\Gamma(\mathbf{0}; \mathbf{k}) = \gamma_0 \left[1 + \sum_{n=1}^{\infty} \lambda_0^n \Gamma_n(\mathbf{k}) \right]. \quad (13)$$

It can be easily seen that the momentum dependence of the different orders takes, in general, the form

$$\Gamma_n(\mathbf{k}) = \frac{a_n}{|\mathbf{k}|^{n\epsilon}}. \quad (14)$$

Inserting the n th term of the series in the right-hand side of Eq. (12), we get

$$\begin{aligned} \frac{a_{n+1}}{|\mathbf{k}|^{(n+1)\epsilon}} &= 2\pi \int \frac{d^d p}{(2\pi)^d} \frac{a_n}{|\mathbf{p}|^{n\epsilon}} \frac{1}{|\mathbf{p}|} \frac{1}{|\mathbf{p} - \mathbf{k}|} \\ &= 2\pi a_n \int \frac{d^d p}{(2\pi)^d} \frac{\Gamma\left(1 + \frac{n\epsilon}{2}\right)}{\sqrt{\pi} \Gamma\left(\frac{1+n\epsilon}{2}\right)} \\ &\quad \times \int_0^1 dx \frac{x^{-1/2}(1-x)^{-(1+n\epsilon)/2}}{[(\mathbf{p} - \mathbf{k})^2 x + \mathbf{p}^2(1-x)]^{1+n\epsilon/2}} \\ &= \frac{1}{2\sqrt{\pi}} (4\pi)^{\epsilon/2} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{1+n\epsilon}{2}\right)} \frac{a_n}{|\mathbf{k}|^{(n+1)\epsilon}} \\ &\quad \times \int_0^1 dx \frac{1}{x^{[1+(n+1)\epsilon]/2}(1-x)^{(1+\epsilon)/2}}. \end{aligned} \quad (15)$$

After performing the integral in the x parameter, we find the relation

$$a_{n+1} = p_{n+1}(\epsilon) a_n \quad (16)$$

with

$$p_n(\epsilon) = \frac{1}{2\sqrt{\pi}} (4\pi)^{\epsilon/2} \frac{\Gamma\left(\frac{n\epsilon}{2}\right) \Gamma\left(\frac{1-n\epsilon}{2}\right) \Gamma\left(\frac{1-\epsilon}{2}\right)}{\Gamma\left(\frac{1+(n-1)\epsilon}{2}\right) \Gamma\left(1 - \frac{n+1}{2}\right)}. \quad (17)$$

In this approach, the bare vertex function can be written in compact form as

$$\Gamma(\mathbf{0}; \mathbf{k}) = \gamma_0 + \gamma_0 \sum_{n=1}^{\infty} \lambda^n \frac{\rho^{n\epsilon}}{|\mathbf{k}|^{n\epsilon}} \prod_{j=1}^n p_j(\epsilon), \quad (18)$$

where ρ is a momentum scale introduced to get the dimensionless coupling $\lambda = \rho^{-\epsilon} \lambda_0$.

In the ladder approximation, it can be easily seen that $Z_{\psi^2} = 1$. On the other hand, the renormalization factor Z_{ψ^2} must have the general structure

$$Z_{\psi^2} = 1 + \sum_{n=1}^{\infty} \frac{c_n(\lambda)}{\epsilon^n}. \quad (19)$$

The position of the different poles is determined by requiring the finiteness of $\Gamma_{\text{ren}} = Z_{\psi^2} \Gamma$ in the limit $\epsilon \rightarrow 0$. From the expression in Eq. (18), we can obtain the power series

$$\begin{aligned} c_1(\lambda) &= -\lambda - \log(2)\lambda^2 - 2 \log^2(2)\lambda^3 - \left[\frac{16}{3} \log^3(2) \right. \\ &\quad \left. + \frac{1}{8} \zeta(3) \right] \lambda^4 - \left[\frac{50}{3} \log^4(2) + \log(2)\zeta(3) \right] \lambda^5 \\ &\quad - \left[\frac{288}{5} \log^5(2) + 6 \log^2(2)\zeta(3) + \frac{1}{16} \zeta(5) \right] \lambda^6 + \dots, \\ c_2(\lambda) &= \frac{1}{2} \lambda^2 + \log(2)\lambda^3 + \frac{5}{2} \log^2(2)\lambda^4 + \left[\frac{22}{3} \log^3(2) \right. \\ &\quad \left. + \frac{1}{8} \zeta(3) \right] \lambda^5 + \left[24 \log^4(2) + \frac{9}{8} \log(2)\zeta(3) \right] \lambda^6 + \dots, \\ c_3(\lambda) &= -\frac{1}{6} \lambda^3 - \frac{1}{2} \log(2)\lambda^4 - \frac{3}{2} \log^2(2)\lambda^5 - \left[\frac{29}{6} \log^3(2) \right. \\ &\quad \left. + \frac{1}{16} \zeta(3) \right] \lambda^6 + \dots, \\ c_4(\lambda) &= \frac{1}{24} \lambda^4 + \frac{1}{6} \log(2)\lambda^5 + \frac{7}{12} \log^2(2)\lambda^6 + \dots, \\ c_5(\lambda) &= -\frac{1}{120} \lambda^5 - \frac{1}{24} \log(2)\lambda^6 + \dots, \\ c_6(\lambda) &= \frac{1}{720} \lambda^6 + \dots, \end{aligned} \quad (20)$$

and so on, with the next $c_n(\lambda)$ starting each time with one more power of the coupling.

Of all the poles, only the first can contribute to the anomalous dimension γ_{ψ^2} . This is because the theory at $d \neq 2$ has a finite limit $\Lambda \rightarrow \infty$ and the cutoff only appears from the need to define the units of dimensionful quantities such as ρ . The implicit dependence $\lambda \sim \Lambda^\epsilon \lambda_0$ leads to $\Lambda(\partial\lambda/\partial\Lambda) = \epsilon\lambda$ and, following Eq. (10),

$$\gamma_{\psi^2} = -\Lambda \frac{\partial\lambda}{\partial\Lambda} \frac{\partial \log Z_{\psi^2}}{\partial\lambda} = -\lambda \frac{dc_1}{d\lambda}. \quad (21)$$

In principle, the right-hand side of Eq. (21) can contain contributions from higher order poles in Eq. (19), but these will vanish provided that $dc_{n+1}/d\lambda = c_n(dc_1/d\lambda)$, identically for every n .²⁷ These are key constraints in order to have a renormalizable theory, since they guarantee the finiteness of γ_{ψ^2} in the limit $\epsilon \rightarrow 0$. Quite remarkably, we have checked that those relations are indeed satisfied in our case, up to the order λ^8 for which we have computed the exact expression of Z_{ψ^2} .

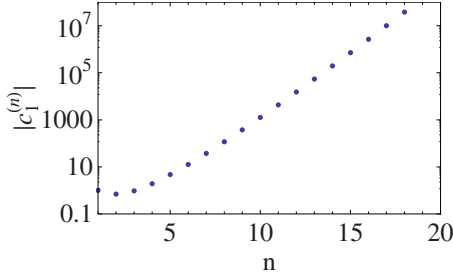


FIG. 4. (Color online) Plot of the absolute value of the coefficients $c_1^{(n)}$ in the expansion of $c_1(\lambda)$ as a power series in the coupling λ .

The other important check we have made along the way is that Z_{ψ^2} does not contain nonlocal divergences proportional to $\log(|\mathbf{k}|/\rho)$, which appear at intermediate stages of the calculation. In the case of the simple pole, we have the result to order λ^8

$$\begin{aligned}
c_1(\lambda) = & -\lambda - \log(2)\lambda^2 - 2\log^2(2)\lambda^3 - \left(\frac{16}{3}\log^3(2) \right. \\
& + \left.\frac{1}{8}\zeta(3)\right)\lambda^4 - \left(\frac{50}{3}\log^4(2) + \log(2)\zeta(3)\right)\lambda^5 \\
& - \left(\frac{288}{5}\log^5(2) + 6\log^2(2)\zeta(3) + \frac{1}{16}\zeta(5)\right)\lambda^6 \\
& - \left(\frac{9604}{45}\log^6(2) + \frac{98}{3}\log^3(2)\zeta(3) + \frac{1}{8}\zeta^2(3) \right. \\
& + \left.\frac{3}{4}\log(2)\zeta(5)\right)\lambda^7 - \left(\frac{262}{315}\log^7(2) \right. \\
& + \frac{512}{3}\log^4(2)\zeta(3) + 2\log(2)\zeta^2(3) + 6\log^2(2)\zeta(5) \\
& + \left.\frac{9}{256}\zeta(7)\right)\lambda^8 + O(\lambda^9). \quad (22)
\end{aligned}$$

The general term of this series does not have a simple expression, but one can still obtain numerically higher orders of the perturbative expansion to determine the behavior of the function $c_1(\lambda)$. Thus, we have computed the coefficients $c_1^{(n)}$ of the power series in λ up to order λ^{18} , what is enough to establish their exponential growth with n . The results are displayed in Fig. 4, showing that

$$-c_1(\lambda) = \sum_{n=1}^{\infty} \alpha^n \lambda^n + \text{regular terms.} \quad (23)$$

A best fit of the asymptotic behavior at large n gives the value $\alpha \approx 4.5$.

The important point is the evidence that the anomalous dimension γ_{ψ^2} obtained from Eq. (21) must have a singularity at a finite value of the effective coupling $\lambda^* = 1/\alpha$. As one approaches this value from below, the anomalous dimension gets arbitrarily large, meaning that the opening of a gap is the

effect that has to prevail in the system near λ^* , in spite of the upward renormalization of the Fermi velocity at low energies.²⁸ This applies, in particular, to the theory with small number of flavors, implying that, to the right of the phase diagram in Fig. 2, CSB should take place above a critical coupling $e^2/8\pi v_F = 1/\alpha (\approx 0.2)$.

IV. CONCLUSION

In this paper we have applied renormalization group methods to analyze the development of a gapped phase in graphene, taking advantage of the scaling properties of the theory of interacting Dirac fermions in the two-dimensional (2D) system. In this regard, an important effect that may question the breakdown of the chiral symmetry is the divergence of the renormalized Fermi velocity at low energies. In principle, the downward scaling of the effective coupling e^2/v_F can prevent to remain above the line of the transition in Fig. 2, even in cases where the nominal value of the coupling places the model inside the region with $m \neq 0$. Similar objection for the CSB can be applied to additional local four-fermion interactions, as their relative strength is always to be measured with respect to the scale of the kinetic energy. One may argue, however, that, in a statistical formulation of the problem, there has to be a critical temperature for the transition to the gapped phase. The temperature is also a relevant scale arresting the renormalization of the Fermi velocity at low energies. Then, it is feasible that the renormalized coupling e^2/v_F may still keep a sufficiently large value to force the transition at the critical energy scale.

We note that our results in the large- N approach establish that, for the physical value $N=4$, graphene would remain in the gapless phase even for the largest values of the effective coupling attained in vacuum ($e^2/4\pi v_F \approx 2.2$). This is in agreement with the fact that no evidence of transition to an insulating state has been found in free-standing graphene. The other important conclusion is that CSB must exist anyhow at sufficiently small values of N , given the evidence we have obtained of a critical coupling at which the anomalous dimension of the order parameter diverges. This result could explain the observation of a transition in Monte Carlo simulations of the long-range Coulomb interaction in the 2D system.^{20,21}

A natural prediction from our analysis is that the gapped phase should emerge at some point in the way from $N=4$ to $N=1$. The spin projection can be frozen, for instance, by applying a magnetic field, and it is actually very appealing to think that the metal-insulator transition observed in that case in graphene may rest on this effect of CSB. It remains to be seen whether quenching also the Dirac-valley degree of freedom could lead to an insulating state for accessible values of e^2/v_F , in accordance with the results of this study.

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