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Isospin symmetry violation in mirror $E1$ transitions: Coherent contributions from the giant isovector monopole resonance in the ^{67}As-^{67}Se doublet

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The assumption of an exact isospin symmetry would imply equal strengths for mirror $E1$ transitions (at least, in the long-wavelength limit). Actually, large violations of this symmetry rule have been indicated by a number of experimental results, the last of which is the ^{67}As-^{67}Se doublet investigated at GAMMASPHERE. Here, we examine in detail various possible origins of the observed asymmetry. The coherent effect of Coulomb-induced mixing with the high-lying giant isovector monopole resonance is proposed as the most probable process to produce a large asymmetry in the $E1$ transitions, with comparatively small effect on the other properties of the parent and daughter levels.

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I. INTRODUCTION

The presence of symmetries in physical laws in most cases greatly enhances our understanding of their nature and consequences. Symmetries, either exact or only approximate, have a particular importance in the fields of elementary particle and nuclear physics. The approximate charge independence of nuclear forces, ultimately related to the near degeneracy of up and down quarks [1], permits us to treat protons and neutrons as different states of the same particle (the nucleon) and to classify nuclear states according to the different representation of a symmetry group, the isospin SU(2). In this scheme, protons and neutrons are characterized by the isospin quantum number $T = 1/2$, with a third component $T_3 = +1/2$ and $-1/2$, respectively. States of nuclei with the same mass number A can be grouped, according to the value of the isospin T, in isospin multiplets of $2T + 1$ states belonging to the different nuclei, distinguished by the value of $T_3 = (Z - N)/2$. Isospin symmetry is violated by the electromagnetic interaction (mostly due to Coulomb forces among protons) and, to a lesser extent, also by nuclear forces. However, the most important part of the Coulomb interactions is diagonal with respect to T_3 and mainly contributes to the mass difference among various members of the isospin multiplet. Finer effects of the symmetry-breaking forces can be investigated by measuring the so-called mirror energy differences [2] or, more generally, differences in excitation energies among members of a multiplet. In recent years, this field has become the object of a considerable number of experimental and theoretical studies, as the level schemes of nuclei with $T_3 = +1/2$ (i.e., $Z = N + 1$) could be measured for increasingly larger values of A. Furthermore, when transition probabilities could be determined, their comparison between mirror nuclei opened an important window to investigate the amount and the origin of isospin violation.

Here, we limit our discussion to the relatively simple case of $E1$ transitions [3]. The $E1$ transition operator is expected to be pure isovector, at least in the limit of long wavelengths, where Siegert's theorem [4] holds. This fact implies that (1) $E1$ transitions with $\Delta T = 0$ in nuclei with $Z = N$ are forbidden and that (2) corresponding $E1$ transitions in mirror nuclei have equal reduced strength. Both rules are to some extent violated by isospin-non-conserving (mainly, Coulomb) interactions. In the $Z = N$ case, these violations appear as second-order effects, while in mirror nuclei the effect is of first order. The difference is due to the interference between the *irregular* amplitude (symmetric with respect to the exchange of the two nuclei in the doublet) with the regular amplitude (which is isovector antisymmetric with respect to the exchange).

In the following, we discuss the relative importance of different possible sources of asymmetry in mirror $E1$ transitions. As a simple example, we consider in particular those nuclei which can be described by the nuclear shell model in a limited Hilbert space, containing a full major shell and the unique-parity intruder from the next major shell. Although the particle-hole excitations involving all states of the higher shell must be considered for a reliable description of the $E1$ transitions, we assume that the largest part of the $E1$ amplitudes only involves the intruder orbital j_I and, as a consequence, only the largest-j orbital, $j_N = j_I - 1$, of the lower major shell. It is important to note that the inclusion

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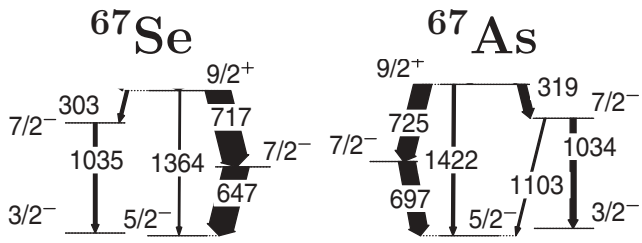

FIG. 1. Partial level scheme of ^{67}Se and ^{67}As, showing the decay of the lowest $9/2^+$ state. Energy labels are in keV. (Adapted from Ref. [6].)

of more orbitals in the calculation, briefly discussed in Appendix C, does not change substantially most of the results.

Actually, strong asymmetries in $B(E1)$ values have been observed in several mirror transitions, e.g., pairs of mirror nuclei of the sd and pf major shells [5,6]. The clearest examples, however, were found in light $N \simeq Z$ nuclei, such as ^{17}O and ^{17}F. Such nuclei often exhibit large differences in the neutron and proton binding energies, and coupling to the continuum needs to be taken into account. The present discussion is limited instead to heavier mirror nuclei, in which the smaller binding energy of the proton is compensated by the larger coulomb barrier.

As a typical example ("benchmark" in this work), we consider the mirror pair ^{67}As-^{67}Se, whose structure involves the pf shell plus the $g_{9/2}$ intruder orbital. This doublet has been investigated in a recent experiment at GAMMASPHERE [6]. Two pairs of mirror transitions with a sizable $E1$ component have been observed, connecting the lowest $9/2^+$ state to lower-lying $7/2^-$ levels (Fig. 1). The measured $E1$ strengths and the absolute value of the corresponding $E1$ matrix elements are reported in Table I. The $9/2^+$ state has presumably a rather pure $g_{9/2}$ character, while the daughter states have a complex structure and contain only a small component that can be reached by the $E1$ transition. As a consequence, the observed values of $B(E1)$ are very small.

All numerical results reported in the following will refer to this particular pair of nuclei. The radial integrals have been obtained with single-particle wave functions in a Woods-Saxon potential with spin-orbit interaction, as specified in Ref. [8]. These integrals change slowly with the atomic number, and for the $f_{7/2} \rightarrow d_{5/2}$ transitions in the middle of the sd shell they would give results very close to those of the $g_{9/2} \rightarrow f_{7/2}$ transitions in mass $A = 67$.

In Sec. II, we derive the expression of the $E1$ transition amplitude from the intruder state a ($J_a = j_I$) to one of the normal-parity states b (J_b). No specific assumptions are made on the structure of these states, apart from the fact that orbits of the higher major shell, different from the intruder, give negligible contribution.

TABLE I. Values of $B(E1)$ for the transitions proceeding from the lowest $9/2^+$ state in ^{67}Se and ^{67}As, as deduced from lifetimes and $M2/E1$ ratios, determined in Ref. [6].

Nucleus	E_γ (keV)	$B(E1)$ (e^2 fm^2)	$\lvert(\frac{9}{2}^+\lVert\mathcal{M}(E1)\rVert\frac{7}{2}^-)\rvert$ (e fm)
^{67}As	725	$1.4 \pm 0.4 \times 10^{-6}$	$3.7 \pm 0.5 \times 10^{-3}$
^{67}Se	717	$0.4 \pm 0.4 \times 10^{-6}$	$2.0 \pm 2.0 \times 10^{-3}$
^{67}As	319	$8.3 \pm 2.5 \times 10^{-6}$	$9.1 \pm 1.3 \times 10^{-3}$
^{67}Se	303	$<1.4(9) \times 10^{-6}$	$<3.7(11) \times 10^{-3}$

In the following sections, we discuss the different processes that can lead to the presence of an (induced) isoscalar $E1$ transition amplitude, in addition to the main isovector term. In Sec. III we consider the effect of higher-order terms in the nucleonic current, in addition to those considered in the Siegert theorem, which are increasingly important when the long-wavelength assumption fails.

In Sec. IV we discuss several simple effects related to the mixing of wave functions: Coulomb mixing between neighboring states (Sec. IV A) and between states of very similar structure, such as analog-antianalog mixing (Sec. IV B). None of the processes considered up to this point seems able to justify the observed asymmetry. We can conclude that the difference in the wave functions of the two mirror nuclei involves mainly (weak) mixing with a large number of states, possibly lying rather far in energy from the levels considered. There are two different approaches to consider this situation. In the most direct treatment, the residual interactions in the two mirror nuclei are assumed from the start to be different and to include the Coulomb interaction (as well as other possible isospin violating terms). It is well known that most of the $E1$ strength is shifted to higher-lying *collective* states, while the low-lying $E1$ transitions remain substantially hindered, due to the destructive interference among the individual contributions. If the residual interactions are not identical in the two mirror nuclei, the negative interference can amplify substantially these differences in the resulting $B(E1)$. This mechanism is easy to understand, but even if a shell-model calculation in this necessarily huge Hilbert space were to become possible, the results would scarcely be transparent with respect to the nature of the processes involved. One could consider, however, the same problem from a different point of view. Namely, let one suppose that a zeroth-order calculation were performed with isospin-conserving residual interaction. As a next approximation, Coulomb interactions could be included to evaluate, to first order, the mixing among zeroth-order states. As in the former approach, one should expect that coherent contributions from collective states play a significant role in producing the $E1$ asymmetry, and the concentration of the $E1$ strength in the collective states has a role in depleting the $E1$ strengths of low-lying transitions. The advantage of this approach is that it can give semiquantitative predictions on the $B(E1)$ asymmetries, even without knowledge of their absolute value. Furthermore, it would elucidate the principal process (or processes) responsible for the largest part of the observed effects. This kind of process, which could in principle account for the magnitude of the observed effects (namely, the coherent contribution of states belonging to the giant isovector monopole resonance), is discussed in detail in Sec. IV C.

II. ISOVECTOR AND ISOSCALAR CONTRIBUTIONS

In the following calculations, the $E1$ transition is assumed to take place from an intruder single-particle orbital j_I to a normal-party orbital $j_N = j_I - 1$ (or vice versa). The parent

state a will be the lowest intruder, with $J_a = j_I$ and parity $\bar{\pi}$. A possible daughter state b must have $J_b = J_a \pm 1$ or J_a and parity $\pi = -\bar{\pi}$. Its wave function can contain pairs inside intruder orbitals coupled to zero: in this case, the transition could proceed from a j_N orbital present in a to a j_I orbital in b.

If we expand the wave functions of states a and b in terms of products of the one-body wave function times the core wave function (with the proper fractional parentage coefficients C_{fp}) the only terms of the expansion that contribute to the transition are those having a common core state (of isospin $T_c = 0$ or 1) for both states a and b and the single-particle orbit changing from j_I to j_N (with a core state J_μ^+ of positive parity) or vice versa (with a core state J_μ^- of negative parity):

$$|a; J_a, M_a; 1/2, T_3\rangle = \sum_\mu C_{fp}(a|j_I; \mu, J_\mu^+, T_c)[\phi(j_I) \otimes \Phi(\mu, J_\mu^+, T_c)]^{(J_a,1/2)}_{M_a,T_3} + \sum_\mu C_{fp}(a|j_N; \mu, J_\mu^-, T_c)[\phi(j_N) \otimes \Phi(\mu, J_\mu^-, T_c)]^{(J_a,1/2)}_{M_a,T_3} + \cdots, \quad (1)$$

$$|b; J_b, M_b; 1/2, T_3\rangle = \sum_\mu C_{fp}(b|j_N; \mu, J_\mu^+, T_c)[\phi(j_N) \otimes \Phi(\mu, J_\mu^+, T_c)]^{(J_b,1/2)}_{M_b,T_3} + \sum_\mu C_{fp}(b|j_I; \mu, J_\mu^-, T_c)[\phi(j_I) \otimes \Phi(\mu, J_\mu^-, T_c)]^{(J_b,1/2)}_{M_b,T_3} + \cdots. \quad (2)$$

Taking into account the relation, to be used both in ordinary space and in isospin space,

$$([j_1 \otimes j_2]J \| U^{(K)}(1) \| [j_1' \otimes j_2]J') = (-1)^{j_1+j_2+J'+K}\sqrt{(2J+1)(2J'+1)} \begin{Bmatrix} j_1 & J & j_2 \\ J' & j_1' & K \end{Bmatrix} (j_1 \| U^{(K)} \| j_1'), \quad (3)$$

where $U^{(K)}$ is a tensor operator of rank K acting only on the subspace "1," we obtain for the reduced matrix element between states a and b

$$\begin{aligned}
&\big(b, J_b, T_b \big\| \mathcal{M}_E^{(1K)} \big\| a, J_a, T_a\big) \\
&= \sum_{T_c} (-1)^{T_c+K+1} \widehat{T}_a \widehat{T}_b \begin{Bmatrix} 1/2 & T_b & T_c \\ T_a & 1/2 & K \end{Bmatrix} \Bigg[\sum_\mu (-1)^{J_a+j_N+J_\mu^++1} \widehat{J}_a \widehat{J}_b \begin{Bmatrix} j_N & J_b & J_\mu^+ \\ J_a & j_I & 1 \end{Bmatrix} \\
&\times n\, C_{fp}(a|j_I; \mu, J_\mu^+, T_c)\, C_{fp}(b|j_N; \mu, J_\mu^+, T_c) \big(j_N \big\| \mathcal{M}_E^{(1K)} \big\| j_I\big) \\
&+ \sum_\mu (-1)^{J_a+j_I+J_\mu^-+1} \widehat{J}_a \widehat{J}_b \begin{Bmatrix} j_I & J_b & J_\mu^- \\ J_a & j_N & 1 \end{Bmatrix} n\, C_{fp}(a|j_N; \mu, J_\mu^-, T_c)\, C_{fp}(b|j_I; \mu, J_\mu^-, T_c) \big(j_I \big\| \mathcal{M}_E^{(1K)} \big\| j_N\big) \Bigg], \quad (4)
\end{aligned}$$

where n is the number of active nucleons, $\widehat{J} \equiv \sqrt{2J+1}$, and the triple bars indicate a reduced matrix element with respect to ordinary space and to isospin space (with $T_a = T_b = 1/2$). The operator $\mathcal{M}_E^{(1K)}$ is now a tensor of rank 1 in ordinary space and $K = 1$ or 0 in isospin space.

Now,

$$\big(j_N \big\| \mathcal{M}_E^{(1K)} \big\| j_I\big) = (-1)^{j_I - j_N} \overline{\big(j_I \big\| \mathcal{M}_E^{(1K)} \big\| j_N\big)} = +\big(j_I \big\| \mathcal{M}_E^{(1K)} \big\| j_N\big). \quad (5)$$

Here, $j_I - j_N = 1$, both single-particle states have isospin 1/2, and the $E1$ operator is odd under time reversal. We obtain therefore for the reduced matrix element in ordinary space

$$\big(b, J_b; T_b, T_3 \big\| \mathcal{M}_E^{(1K)} \big\| a, J_a; T_a, T_3\big) = (-1)^{1/2-T_3} \begin{pmatrix} T_b & K & T_a \\ -T_3 & 0 & T_3 \end{pmatrix} \big(j_N \big\| \mathcal{M}_E^{(1K)} \big\| j_I\big) (-1)^K \sum_{T_c=0,1} \mathcal{A}(T_c) \begin{Bmatrix} 1/2 & T_b & T_c \\ T_a & 1/2 & K \end{Bmatrix}, \quad (6)$$

where $\mathcal{M}^{(11)} \equiv D_{IV}^{(1)} \vec{\tau}$ and $\mathcal{M}^{(10)} \equiv D_{IS}^{(1)}$ are the isovector and isoscalar parts of the single-particle electric dipole operator:

$$\big(j_N \big\| \mathcal{M}_E^{(10)} \big\| j_I\big) = \big(j_N \big\| D_{IS}^{(1)} \big\| j_I\big)(1/2 \| 1 \| 1/2), \quad \big(j_N \big\| \mathcal{M}_E^{(11)} \big\| j_I\big) = \big(j_N \big\| D_{IV}^{(1)} \big\| j_I\big)(1/2 \| \vec{\tau} \| 1/2) \quad (7)$$

and the core-isospin-dependent coefficients $\mathcal{A}(T_c)$ ($T_c = 0$ or 1) are

$$\begin{aligned}
\mathcal{A}(T_c) = (-1)^{T_c}\, 2\widehat{J}_a \widehat{J}_b \Bigg[&\sum_\mu (-1)^{J_\mu^+ + J_a + j_N} \begin{Bmatrix} j_N & J_b & J_\mu^+ \\ J_a & j_I & 1 \end{Bmatrix} n C_{fp}(a|j_I; \mu, J_\mu^+, T_c) \times C_{fp}(b|j_N; \mu, J_\mu^+, T_c) \\
&+ \sum_\mu (-1)^{J_\mu^- + J_b + J_I} \begin{Bmatrix} j_I & J_b & J_\mu^- \\ J_a & j_N & 1 \end{Bmatrix} n C_{fp}(a|j_N; \mu, J_\mu^-, T_c) \times C_{fp}(b|j_I; \mu, J_\mu^-, T_c) \Bigg]. \quad (8)
\end{aligned}$$

With $T_a = T_b = 1/2$ and $T_c = 0$ or 1, by inserting the numerical values of the coefficients[1] one obtains

$$\begin{aligned}&\left(b, J_b; 1/2, T_3 \middle\| \mathcal{M}_E^{(11)} \middle\| a, J_a; 1/2, T_3\right)\\&\quad= \frac{(-1)^{1/2+T_3}}{6}\left[\mathcal{A}(1) + 3\mathcal{A}(0)\right]\left(j_I \middle\| D_{IV}^{(1)} \middle\| j_N\right), \qquad (9)\\&\left(b, J_b; 1/2, T_3 \middle\| \mathcal{M}_E^{(10)} \middle\| a, J_a; 1/2, T_3\right)\\&\quad= \tfrac{1}{2}\left[\mathcal{A}(1) - \mathcal{A}(0)\right]\left(j_I \middle\| D_{IS}^{(1)} \middle\| j_N\right). \qquad (10)\end{aligned}$$

The leading isovector term in the single-particle operator is, in our case,

$$\left(j_I \middle\| D_{IV}^{(1)} \middle\| j_N\right) = \frac{e}{2}\,\langle j_I | r | j_N\rangle\,(j_I \| Y^{(1)} \| j_N), \qquad (11)$$

where $Y^{(1)}$ is the spherical harmonic for $\ell = 1$.

The different forms of possible isoscalar contributions are discussed in the following sections.

III. HIGHER-ORDER TERMS AFTER SIEGERT

It is well known that the usual expression of electric transition amplitudes, deduced from Siegert's theorem, is only valid in the long-wavelength limit. The complete expression for the electric transition amplitude, in which relativistic corrections are also taken into account, is given by Friar and Fallieros [7] as

$$\begin{aligned}T(E, LM) = {}& \frac{k^{L-1}}{(2L+1)!!}\\&\times \int dr^3 \left[i\sqrt{\frac{L+1}{L}}\,\dot{\rho}(\vec{r}) r^L Y_M^{(L)}(\hat{r}) g_L(kr)\right.\\&\left. + \frac{2k^2 r}{L+2}\vec{\mu}(\vec{r})\cdot\vec{\mathcal{Y}}_M^{(L,1)L}(\hat{r}) h_L(kr)\right], \qquad (12)\end{aligned}$$

where $\vec{\mathcal{Y}}_M^{(L,1)L}$ is the vector spherical harmonic and

$$\begin{aligned}&g_L(z) \approx 1 - \frac{Lz^2}{2(L+2)(2L+3)} + \cdots,\\&h_L(z) = -\frac{L+2}{Lz}\frac{d}{dz}\left\{ z^{-2L}\frac{d}{dz}[z^{2L+1} g_L(z)]\right\} \approx 1 + \cdots.\end{aligned} \qquad (13)$$

For our purposes, it will be sufficient to consider only the first term after the Siegert limit, as given in Eqs. (13). We will consider first the part of the integral (12) containing the time derivative of the charge density $\rho = \rho_0(\vec{r})\exp(-ikct)$, approximating the nucleus to an ensemble of pointlike nucleons:

$$\dot{\rho} = -ikc\rho(\vec{r}) \approx -ikc\sum \frac{1+\tau_3(i)}{2} e\,\delta(\vec{r} - \vec{r}_i). \qquad (14)$$

[1]Namely, $(1/2\|1\|1/2) = \sqrt{2}$; $(1/2\|\vec{\tau}\|1/2) = \sqrt{6}$; $\left(\begin{smallmatrix}1/2 & K & 1/2\\ -T_3 & 0 & T_3\end{smallmatrix}\right) = (-1)^{1/2-T_3}/\sqrt{2}$ for $K = 0$ and $= 1/\sqrt{6}$ for $K = 1$; $\left\{\begin{smallmatrix}1/2 & K & T_c\\ 1/2 & 1/2 & K\end{smallmatrix}\right\} = 1/6$ for $K = T_c = 1$ and $= (-1)^{T_c-1}/2$ for all other cases.

The isovector part of the $E1$ transition operator is, for a single-particle transition,

$$D_{IV}(E1)\tau_3 = \frac{\sqrt{2}}{3}\frac{e}{2} kc\, r Y^{(1)}(\hat{r})\tau_3. \qquad (15)$$

To a first approximation, the isoscalar part of the transition amplitude only comes from the second term of the series expansion of $g_L(kr)$. As we have to deal with a one-body operator, we can easily obtain the amount of this correction with respect to the main (Siegert) term, for each single-particle transition:

$$\frac{\left\langle \ell_1, j_1 \middle\| D_{IS}^E(E1) \middle\| \ell_2, j_2\right\rangle}{\left\langle \ell_1, j_1 \middle\| D_{IV}(E1) \middle\| \ell_2, j_2\right\rangle} = -\frac{k^2}{30}\frac{\langle \ell_1, j_1 | r^3 | \ell_2, j_2\rangle}{\langle \ell_1, j_1 | r | \ell_2, j_2\rangle}. \qquad (16)$$

Now we can estimate the numerical value of this ratio for the case of the $A = 67$ doublet [6] chosen as a suitable benchmark, and for a $g_{9/2} \to f_{7/2}$ transition. With Woods-Saxon radial wave functions one obtains

$$\frac{\langle f_{7/2}|(r/R_0)^3|g_{9/2}\rangle}{\langle f_{7/2}|r/R_0|g_{9/2}\rangle} = 0.834. \qquad (17)$$

Assuming $R_0 = 1.27A^{1/3}$ fm $= 5.158$ fm, one obtains

$$\begin{aligned}\frac{k^2}{30}\frac{\langle f_{7/2}|r^3|g_{9/2}\rangle}{\langle f_{7/2}|r|g_{9/2}\rangle} &= \frac{1}{30}(kR_0)^2\frac{\langle f_{7/2}|(r/R_0)^3|g_{9/2}\rangle}{\langle f_{7/2}|r/R_0|g_{9/2}\rangle}\\&= 1.90\times 10^{-5}(E_\gamma\ [\mathrm{MeV}])^2. \qquad (18)\end{aligned}$$

The evaluation of the second part of Eq. (12) can be easily performed if we substitute the continuous magnetic density $\vec{\mu}(\vec{r})$ with that of an ensemble of pointlike nucleons with spin:

$$\begin{aligned}\vec{\mu}(\vec{r}) &= \mu_n\sum_i\left\{\frac{1+\tau_3(i)}{2}[\vec{\ell}_i + g_p\vec{s}_i] + \frac{1-\tau_3(i)}{2}g_n\vec{s}_i\right\}\delta(\vec{r}-\vec{r}_i)\\&= \frac{\mu_n}{2}\sum\{[\vec{j}_i + (g_p - 1 + g_n)\vec{s}_i]\\&\quad + [\vec{j}_i + (g_p - 1 - g_n)\vec{s}_i]\tau_3(i)\}, \qquad (19)\end{aligned}$$

where $\mu_n = e\hbar/(2M_p)$ is the nuclear magneton (and M_p is the proton mass). On the basis of Eq. (18), we can observe that the magnitude of this term in comparison to the first term of Eq. (12) is given by

$$\frac{k^2\mu_n}{kec} = \frac{\hbar kc}{2M_pc^2} \approx 0.53\ 10^{-3}E_\gamma\ [\mathrm{MeV}]. \qquad (20)$$

Here, we are only interested in the isoscalar part, where the contribution of the term $\vec{s}$ is hindered due to the numerical factor $g_p - 1 + g_n \approx 0.76$. The evaluation of the matrix elements of $\vec{s}\cdot\vec{\mathcal{Y}}_M^{(L,1)L}$ and $\vec{j}\cdot\vec{\mathcal{Y}}_M^{(L,1)L}$ is performed in detail in Appendix A. For our benchmark, corresponding to a $g_{9/2} \to f_{7/2}$ single-particle transition, one obtains

$$\begin{aligned}\frac{\left(g_{9/2} \middle\| D_{IS}^M \middle\| f_{7/2}\right)}{\left(g_{9/2} \middle\| D_{IV} \middle\| f_{7/2}\right)} &\approx -\frac{2\sqrt{2}}{3}\frac{k\mu_n}{ec}\left[\frac{(g_p - 1 + g_n)}{\sqrt{6}} - \frac{1}{\sqrt{2}}\right]\\&\approx 2\times 10^{-4}\ (E_\gamma\ [\mathrm{MeV}]) \qquad (21)\end{aligned}$$

if we assume that the above description of the magnetic density is approximately correct.

For γ-ray energies around 1 MeV, both correction terms are far too small to justify the observed asymmetries in $A = 67$.

IV. THE COULOMB MIXING OF WAVE FUNCTIONS

If one takes into account the level mixing due to the Coulomb interaction V_c, the wave function of a pure eigenstate $|a_0\rangle$ of the charge-invariant Hamiltonian is changed into a new one, $|a'\rangle$. To first order,

$$|a'\rangle = |a_0\rangle + \sum_k \frac{\langle a_k|V_c|a_0\rangle}{E(a_0) - E(a_k)}|a_k\rangle, \tag{22}$$

where the sum is extended over all states $|a_k\rangle$ having the same J^π as $|a_0\rangle$, and which may or may not have the same isospin. The $E1$ transition matrix element between the modified states a' and b' is, again to first order,

$$\begin{aligned}\langle b'|\mathcal{M}(E1)|a'\rangle &= \langle b_0|\mathcal{M}(E1)|a_0\rangle \\ &\quad + \sum \frac{\langle a_k|V_c|a_0\rangle}{E(a_0) - E(a_k)} \langle b_0|\mathcal{M}(E1)|a_k\rangle \\ &\quad + \sum \frac{\langle b_0|V_c|b_k\rangle}{E(b_0) - E(b_k)} \langle b_k|\mathcal{M}(E1)|a_0\rangle \\ &\equiv \langle b_0|\mathcal{M}(E1)|a_0\rangle + \langle b_0|\widetilde{\mathcal{M}}(E1)|a_0\rangle. \end{aligned} \tag{23}$$

It was assumed, here, that the $\mathcal{M}(E1)$ operator is pure isovector. The ensemble of first-order corrections [indicated as $\langle b_0|\widetilde{\mathcal{M}}(E1)|a_0\rangle$] transforms as an even tensor in isospin space. In the $T = 1/2$ or $T = 0$ subspaces, it can be considered as an *induced isoscalar* amplitude.

If $T_3 = 0$ and the *unperturbed* states a_0 and b_0 have the same isospin, the first term of the sum (23) vanishes and only the induced part contributes. Instead, if $T_3 = \pm 1/2$, the first term is the leading one and the other two are only first-order corrections.

The Coulomb potential can be written as the sum of isoscalar, isovector, and rank-2 isotensor terms:

$$\begin{aligned} V_c &= \frac{1}{2}\sum_i \sum_{j\neq i} \frac{e^2}{r_{ij}} \frac{1+\tau_3(i)}{2}\frac{1+\tau_3(j)}{2} \\ &= \frac{1}{8}\sum_i \sum_{j\neq i} \frac{e^2}{r_{ij}} \left[1 + \frac{1}{3}[\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)]\right] \\ &\quad + \sum_i \frac{e}{2}\tau_3(i) \sum_{j\neq i} \frac{e}{2}\frac{1}{r_{ij}} \\ &\quad + \frac{1}{8}\sum_i \sum_{j\neq i} \frac{e^2}{r_{ij}} \left[\tau_3(i)\tau_3(j) - \frac{1}{3}[\boldsymbol{\tau}(i)\cdot\boldsymbol{\tau}(j)]\right]. \end{aligned} \tag{24}$$

The isoscalar part can be included in the charge-invariant Hamiltonian. The matrix elements of the isotensor term vanish in the $T = 1/2$ subspace. They could contribute to the mixing with a $T = 3/2$ state but would produce, in any case, equal effects in two mirror nuclei.

Therefore, any difference between mirror nuclei has to be attributed to the mixing induced by the isovector term $V_c^{(1)}$:

$$V_c^{(1)} = \sum_i \frac{e}{2}\tau_3(i) \sum_{j\neq i} \frac{e}{2}\frac{1}{r_{ij}}, \tag{25}$$

where $V_c^{(1)}$ is, obviously, a two-body operator. It is possible, however, to approximate its matrix elements with those of a suitable one-body operator (see [8], Eqs. (2)–(104). Actually, the second sum in Eq. (25) corresponds to the Coulomb potential of a system of $A - 1$ pointlike charges $e/2$ associated with all nucleons j different from the nucleon i, and we can approximate it with the electrostatic potential of a uniformly charged sphere of radius R, i.e., for $r < R$

$$\varphi_c(r) \equiv \frac{e(A-1)}{2R} f_c(r/R) \approx \frac{e(A-1)}{2R}\,\frac{3R^2 - r^2}{2R^2}. \tag{26}$$

(Slightly different forms of the function f_c will be considered in the following.) With these approximations,

$$\begin{aligned} V_c^{(1)} &\approx \sum_i \frac{e}{2}\tau_3(i)\varphi_c(r_i) \\ &= eT_3\varphi_c(0) - \sum_i \frac{e}{2}\tau_3(i)[\varphi_c(0) - \varphi_c(r)] \\ &\equiv eT_3\varphi_c(0) + \widetilde{V}_c^{(1)} \end{aligned} \tag{27}$$

and, for the potential φ_c of a uniformly charged sphere,

$$\widetilde{V}_c^{(1)} = -\frac{e(A-1)}{R^3}\sum_i \frac{e r_i^2 \tau_3(i)}{8}. \tag{28}$$

The first term of Eq. (27) is diagonal and does not contribute to the mixing. The second term is proportional to the isovector monopole operator

$$\mathcal{M}^{(1)}(E0) = \sum_i \frac{e\, r_i^2 \tau_3(i)}{2}. \tag{29}$$

This result will be exploited again in Sec. IV C.

Actually, the use of a constant charge density inside a sphere to evaluate the electrostatic potential φ_c is somewhat inconsistent with the Woods-Saxon distribution of matter density assumed to calculate the radial wave functions. Moreover, the tails of these wave functions extend outside the nuclear radius, in a region where φ_c would decrease as $1/r$. Calculations of the electrostatic potential for a Woods-Saxon density of charge are given in Appendix B. For small values of r—i.e., as long as the charge density of the Woods-Saxon distribution is substantially constant and equal to that of the sphere—the values of $\widetilde{V}_c^{(1)}$ are equal in the two cases, and the differences in the calculated integral are always rather small. To obtain the same charge density at the center, the radius R of the uniformly charged sphere must take a slightly different value from the parameter R_0 of the Woods-Saxon distribution. Adopting for the Woods-Saxon parameters the values suggested by Bohr and Mottelson [8], $R_0 = 1.27A^{1/3}$ fm and $a = 0.67$ fm, for $A = 67$ one obtains $R_0 = 5.158$ fm and $R = 5.430$ fm.

The matrix elements of $\widetilde{V}_c^{(1)}$ are in any case very small. To produce a sizable mixing of states, it is necessary that the effect be amplified due to some particular conditions. This can happen, in particular, when (i) two levels with equal J^π are very close in energy or (ii) have very similar wave functions or (iii) when many different levels contribute *coherently* to the mixing. We will consider these three cases in the following sections.

A. Close-lying states

The simplest possible case is the mixing of two states which lie close in energy. As an example, we can consider the $E1$ decay of a given state a (of spin J_a) toward two states b_1 and b_2 of equal angular momentum J_b, and rather close in energy. In this case, taking into account only the Coulomb mixing between b_1 and b_2 (and neglecting small isoscalar terms in the $E1$ operator) we obtain up to first order

$$(b_1', J_b\|\mathcal{M}(E1)\|a, J_a) = (b_1, J_b\|\mathcal{M}(E1)\|a, J_a) + \alpha T_3(b_2, J_b\|\mathcal{M}(E1)\|a, J_a) \quad (30)$$

$$(b_2', J_b\|\mathcal{M}(E1)\|a, J_a) = (b_2, J_b\|\mathcal{M}(E1)\|a, J_a) - \alpha T_3(b_1, J_b\|\mathcal{M}(E1)\|a, J_a) \quad (31)$$

with

$$\alpha T_3 = \langle b_1|\widetilde{V}_c^{(1)}|b_2\rangle/[E(b_1) - E(b_2)]. \quad (32)$$

In fact, as a consequence of the Wigner-Eckart theorem, the matrix element of $\widetilde{V}_c^{(1)}$ must be proportional to that of T_3.

The reduced transition probabilities become, up to first order,

$$B(E1; a \to b_1') = \frac{1}{2J_a+1}[(b_1, J_b\|\mathcal{M}(E1)\|a, J_a)^2 + 2\alpha T_3(b_1, J_b\|\mathcal{M}(E1)\|a, J_a) \times (b_2, J_b\|\mathcal{M}(E1)\|a, J_a)] \quad (33)$$

$$B(E1; a \to b_2') = \frac{1}{2J_a+1}[(b_2, J_b\|\mathcal{M}(E1)\|a, J_a)^2 - 2\alpha T_3(b_2, J_b\|\mathcal{M}(E1)\|a, J_a) \times (b_1, J_b\,|\mathcal{M}(E1)\|a, J_a)]. \quad (34)$$

Hence, the sum of the two reduced strengths,

$$B(E1; a \to b_1') + B(E1; a \to b_2') = \frac{1}{2J_a+1}[(b_1, J_b\|\mathcal{M}(E1)\|a, J_a)^2 + (b_2, J_b\|\mathcal{M}(E1)\|a, J_a)^2], \quad (35)$$

is independent of T_3 and consequently identical in the two mirror nuclei. If one of the two unperturbed transition strengths (either for $a \to b_1$ or $a \to b_2$) is much smaller than the other, a large percentage difference between mirror values can be found, but only for the weaker transition.

B. Analog-antianalog mixing

A second interesting case concerns the mixing between two very similar wave functions, as for a pair of *analog-antianalog states* (which would be a very favorable case of the mixing of $T = 1/2$ and $T = 3/2$ states discussed in Ref. [9]). Let us consider, as a simple example, the state obtained with the coupling of a $j_I = 9/2$ nucleon to the lowest state ϕ_0 ($J^\pi = 0^+$, $T = 1$) of the isospin triplet $A = 66$. Isospin 3/2 states are obtained in the two $|T_3| = 3/2$ nuclei. In the $|T_3| = 1/2$ nuclei ^{67}As and ^{67}Se two independent wave functions will result from the coupling, and two pure isospin states can be constructed by proper linear combinations: a $T = 3/2$ state $|a_3\rangle$, which is the *isospin analog* of those in the $|T_3| = 3/2$ nuclei, and a $T = 1/2$ state $|a_1\rangle$, sometime referred to as the *antianalog* of them. Here we will give the results for the $T_3 = +1/2$ nucleus (from which, those for $T_3 = -1/2$ can be easily deduced by means of the Wigner-Eckart theorem):

$$|a_3\rangle = |[\phi_j(T = 1/2) \otimes \Phi_0(T_c = 1)]j, T = 3/2\rangle = c_1|\phi_\pi(g_{9/2})\,\Phi_0(T_3 = 0)\rangle + c_2|\phi_\nu(g_{9/2})\,\Phi_0(T_3 = 1)\rangle, \quad (36)$$

$$|a_1\rangle = |[\phi_j(T = 1/2) \otimes \Phi_0(T_c = 1)]j, T = 1/2\rangle = c_2|\phi_\pi(g_{9/2})\,\Phi_0(T_3 = 0)\rangle - c_1|\phi_\nu(g_{9/2})\,\Phi_0(T_3 = 1)\rangle, \quad (37)$$

where, for $T_3 = +1/2$,

$$c_1 = (1/2, 1/2, 1, 0 \mid 3/2, 1/2) = -(1/2, -1/2, 1, 1 \mid 1/2, 1/2) = \sqrt{2/3} \quad (38)$$

and

$$c_2 = (1/2, -1/2, 1, 1 \mid 3/2, 1/2) = (1/2, 1/2, 1, 0 \mid 1/2, 1/2) = \sqrt{1/3}. \quad (39)$$

We now use Eqs. (27) and (28) to approximate the nondiagonal part of the isovector Coulomb interaction $V_C^{(1)}$ with a one-body operator $\widetilde{V}_C^{(1)}$, whose matrix element between analog and antianalog states is, for $T_3 = +1/2$,

$$\begin{aligned}\langle a_3|\widetilde{V}_C^{(1)}|a_1\rangle &= c_1c_2\langle\phi_\pi\Phi_0(1,0)|\widetilde{V}_C^{(1)}|\phi_\pi\Phi_0(1,0)\rangle - c_2c_1\langle\phi_\nu\Phi_0(1,1)|\widetilde{V}_C^{(1)}|\phi_\nu\Phi_0(1,1)\rangle \\ &= c_1c_2\big[\langle\phi|\widetilde{V}_C^{(1)}(\pi)|\phi\rangle - \langle\phi|\widetilde{V}_C^{(1)}(\nu)|\phi\rangle - \langle\Phi_0(1,1)|\widetilde{V}_C^{(1)}|\Phi_0(1,1)\rangle\big]. \end{aligned} \quad (40)$$

The diagonal matrix element of the *isovector* operator $\widetilde{V}_C^{(1)}$ over the core state $T = 1$, $T_3 = 0$ is zero.

Starting from Eq. (40) and assuming an energy spacing $E(a_3) - E(a_1) = \Delta E$, we can now estimate at least the order of magnitude of the mixing coefficient. For $T_3 = +1/2$,

$$\alpha = \frac{\langle a_3|\widetilde{V}_C^{(1)}|a_1\rangle}{(-\Delta E)} = c_1c_2\frac{(A-1)e^2}{8R\,\Delta E}\left[2\langle g_{9/2}|\frac{r^2}{R^2}|g_{9/2}\rangle - \langle\Phi_0(1,1)|\sum_i \tau_3(i)\frac{r_i^2}{R^2}|\Phi_0(1,1)\rangle\right]. \quad (41)$$

In the second term, the contributions of a proton and of a neutron *in the same orbit* cancel one another, due to the opposite eigenvalue of τ_3. There are, however, two excess protons in the $T_3 = 1$ core state. If all the radial wave functions of active nucleons in the core were equivalent to that of the j_I orbit, the second term in the sum of Eq. (41) would exactly cancel the first one. We can expect, therefore, a resulting matrix element substantially smaller than the first term alone, due to the effect of the core term. However, the expectation value of r^2/R^2 in the $j_I = g_{9/2}$ orbit is certainly larger than those for the lower orbits in the core. For $A = 67$, and with Woods-Saxon wave functions, the radial integral of $(r/R)^2$ in

the $0g_{9/2}$ orbit is 0.7495, while in the normal-party orbits $0f_{7/2}$. $0f_{5/2}$, $1p_{3/2}$, and $1p_{1/2}$ it is, respectively, 0.6251, 0.5922, 0.6251, and 0.6359. In Eq. (41), we will use the average of these values, $\langle r^2/R^2\rangle = 0.6119$, and the above estimate of the matrix element in the $g_{9/2}$ orbit, to evaluate an order of magnitude for the analog-antianalog mixing.[2] Numerically, with $c_1c_2 = \sqrt{2}/3$, $A = 67$, and $R = 5.43$ fm and assuming $\Delta E \approx 4$ MeV as in ^{59}Cu [10], we obtain $\alpha \approx 0.071$. As the matrix element of the isovector interaction $\widetilde{V}_C^{(1)}$ between a state of isospin 3/2 and a state of isospin 1/2 is

$$\langle 3/2, T_3|\widetilde{V}_C^{(1)}|1/2, T_3\rangle = (-1)^{3/2-T_3}\begin{pmatrix} 3/2 & 1 & 1/2 \\ -T_3 & 0 & T_3 \end{pmatrix} \times \left(3/2\middle\|\widetilde{V}_C^{(1)}\middle\|1/2\right) \tag{42}$$

the value of α has equal sign in both nuclei of the doublet.

The $E1$ transition matrix element from the state $|a_1'\rangle$ to a given state $|b\rangle$ will be, at first order,

$$\langle b\|\mathcal{M}(E1)\|a_1'\rangle = \langle b\|\mathcal{M}(E1)\|a_1\rangle + \alpha\langle b\|\mathcal{M}(E1)\|a_3\rangle. \tag{43}$$

We assume, for sake of simplicity, that state b has pure isospin 1/2. If, as we have supposed, the $E1$ transition proceeds from a $j_I = 9/2$ to a $j_N = 7/2$ single-particle state, we can use for state $|b\rangle$ a fractional parentage expansion in the style of the first line of Eq. (2). But only the terms corresponding to the coupling of a nucleon in the state $j_N = 7/2$ to the core configuration Φ_0 with $J = 0, T = 1$ can be reached by the $E1$ transition. We can write the (presumably small) part of the wave function of state b which is relevant for the $E1$ transition in the form of Eq. (37):

$$\begin{aligned}|b\rangle &= |[\phi_j(T = 1/2)\otimes\Phi_0(T_c = 1)]j, T = 1/2\rangle + \cdots \\ &= c_2|\phi_\pi(f_{7/2})\,\Phi_0(T_3 = 0)\rangle \\ &\quad - c_1|\phi_\nu(f_{7/2})\,\Phi_0(T_3 = 1)\rangle + \cdots.\end{aligned} \tag{44}$$

Taking into account the effective charges for the $E1$ transition, $\epsilon_\pi = 1/2$ and $\epsilon_\nu = -1/2$, from Eq. (43) we obtain

$$\begin{aligned}(b\|\mathcal{M}(E1)\|a_1') &= [(|c_2|^2\epsilon_\pi + |c_1|^2\epsilon_\nu) + \alpha c_1c_2(\epsilon_\pi - \epsilon_\nu)] \\ &\quad\times (f_{7/2}\|erY^{(1)}\|g_{9/2}) \\ &= \frac{-1 + 2\sqrt{2}\,\alpha}{6}(f_{7/2}\|erY^{(1)}\|g_{9/2}).\end{aligned}$$

For $T_3 = \pm 1/2$, using Eq. (42) we obtain the numerical coefficient $(\mp 1 + 2\sqrt{2}\,\alpha)/6$. In conclusion, the $E1$ strength in the two mirror transitions is proportional to $(\mp 1 + 2\sqrt{2}\alpha)^2$. The mirror asymmetry in the $E1$ strength is therefore, approximately,

$$\frac{B(E1, \mathrm{As}) - B(E1, \mathrm{Se})}{B(E1, \mathrm{As}) + B(E1, \mathrm{Se})} = \frac{4\sqrt{2}\,\alpha}{1 + 8\alpha^2} \approx 0.386, \tag{45}$$

and the ratio $B(E1, \mathrm{As})/B(E1, \mathrm{Se}) \approx 2.26$. We note, however, that such a large asymmetry has been obtained for a pure configuration of the analog and antianalog states, while the antianalog strength is usually spread over a number of final states [11], a situation which will strongly reduce the mirror asymmetry in the $E1$ strength. A detailed shell-model investigation would possibly elucidate the role of analog-antianalog mixing in the $E1$ asymmetry between mirror nuclei, as the analog and the antianalog states can be described in the same shell-model space.

[2]See Table II in Appendix B. With a Woods-Saxon charge distribution, the estimate does not change by more than a few percent.

C. Coherent enhancement of induced isoscalar $E1$

The Coulomb mixing discussed in the previous sections involves states belonging to the same set of shell-model orbits necessary for the (unperturbed) parent and daughter state of the $E1$ transitions (presumably limited to two major shells). However, it is well known that a comparatively large contribution to the isospin mixing comes from states outside this model space, such as those belonging to the giant isovector monopole resonance [12]. Obviously, the mixing with any of these higher-lying states, induced by the isovector part of the Coulomb interaction, is expected to be very small. The combined effect of many higher-lying states on the $E1$ transition amplitude can however become appreciable if their individual contributions combine coherently. We shall see how this can be the case.

We have seen [Eqs. (27) and (28)] that the nondiagonal isovector part of the Coulomb interaction V_c can be approximated with a one-body operator $\widetilde{V}_C^{(1)}$, having the same form of the isovector monopole operator $\mathcal{M}^{(1)}(E0)$. Therefore, it is a sensible approximation [13,14] to consider in the ensemble of states a_k, b_k (with $k \neq 0$) of Eqs. (22) and (23) only those of the isovector monopole resonances built over a_0 and b_0, and to use the mean excitation energy ΔE_a (or ΔE_b) of the giant resonance over the state a_0 (or b_0) in the place of those of individual states. In this case, Eq. (23) becomes

$$\langle b'|\mathcal{M}(E1)|a'\rangle = \langle b_0|\mathcal{M}(E1)|a_0\rangle + \langle b_0|\widetilde{\mathcal{M}}(E1)|a_0\rangle, \tag{46}$$

where

$$\begin{aligned}\langle b_0|\widetilde{\mathcal{M}}(E1)|a_0\rangle \approx{}& \frac{-1}{\Delta E_a}\sum\langle b_0|\mathcal{M}(E1)|a_k\rangle\langle a_k|V_c|a_0\rangle \\ &+ \frac{-1}{\Delta E_b}\sum\langle b_0|V_c|b_k\rangle\langle b_k|\mathcal{M}(E1)|a_0\rangle.\end{aligned} \tag{47}$$

We are only interested in the isoscalar part of $\widetilde{\mathcal{M}}(E1)$, which results from the isovector part $V_c^{(1)}$ of the Coulomb interaction. By approximating the nondiagonal part of $V_c^{(1)}$ with the one-body potential of Eq. (28), the *closure approximation* gives

$$\begin{aligned}\sum\langle b_0|\widetilde{V}_c^{(1)}|b_k\rangle\langle b_k|\mathcal{M}(E1)|a_0\rangle &\approx \langle b_0|\widetilde{V}_c^{(1)}\,\mathcal{M}(E1)|a_0\rangle, \\ \sum\langle b_0|\mathcal{M}(E1)|a_k\rangle\langle a_k|\widetilde{V}_c^{(1)}|a_0\rangle &\approx \langle b_0|\mathcal{M}(E1)\,\widetilde{V}_c^{(1)}|a_0\rangle,\end{aligned} \tag{48}$$

and therefore [as $\mathcal{M}(E1)$ and $\widetilde{V}_c^{(1)}$ commute]

$$\langle b_0|\widetilde{\mathcal{M}}^{(0)}(E1)|a_0\rangle \approx 2\frac{\langle b_0|\mathcal{M}(E1)\widetilde{V}_c^{(1)}|a_0\rangle}{(-\Delta E_0)}, \tag{49}$$

where we have assumed $\Delta E_a \approx \Delta E_b \Rightarrow \Delta E_0$, and so

$$\langle b_0|\widetilde{\mathcal{M}}^{(0)}(E1)V_c^{(1)}|a_0\rangle \approx \frac{2}{(-\Delta E_0)}\langle b_0|\sum_i \frac{e}{2} r_i Y^{(1)}(\hat{r}_i)\tau_3(i) \times \sum_j \frac{e}{2}[\varphi_c(r_j)-\varphi_c(0)]\tau_3(j)|a_0\rangle \equiv \langle b_0|\widetilde{\mathcal{M}}^{(0)}_{1-b}+\widetilde{\mathcal{M}}^{(0)}_{2-b}|a_0\rangle, \quad (50)$$

where $\widetilde{\mathcal{M}}^{(0)}_{1-b}$ is the one-body operator resulting from the term with $j = i$ in the second sum, and $\widetilde{\mathcal{M}}^{(0)}_{2-b}$ is a two-body operator resulting from all other terms. As $\tau_3^2 = 1$, the first term is

$$\widetilde{\mathcal{M}}^{(0)}_{1-b} = \frac{1}{\Delta E_0}\sum_i e[\varphi_c(0)-\varphi_c(r_i)]\frac{e}{2} r_i Y^{(1)}(\hat{r}_i). \quad (51)$$

With the expression of φ_c corresponding to the uniformly charged sphere, given in Eq. (26) (and extrapolated also for $r > R$), one obtains for the one-body operator

$$\widetilde{\mathcal{M}}^{(0)}_{1-b} \equiv C\sum_i \frac{r_i^3}{R^2}\frac{e}{2} Y^{(1)}(\hat{r}_i), \quad (52)$$

which has the same structure as the one coming from the second-order term in the series expansion of Eq. (12), with a different coefficient,

$$C = +(A-1)e^2/(8R\Delta E_0). \quad (53)$$

An alternative calculation using a Woods-Saxon charge distribution is reported in Appendix B.

As the one-body operator (49) is isoscalar, its matrix elements can be expressed in the form anticipated in Eq. (10):

$$\left(b, J_b; \frac{1}{2}, T_3 \left\| \sum_i \frac{e}{2}\frac{r_i^2}{R^3} r_i Y^{(1)}(\hat{r}_i)\right\| a, J_a; \frac{1}{2}, T_3\right) = \frac{1}{2}[\mathcal{A}(1)-\mathcal{A}(0)]\frac{e}{2}(j_N\|r\,Y^{(1)}(\hat{r})\|j_I)\frac{\langle j_N|(r/R)^3|j_I\rangle}{\langle j_N|(r/R)|j_I\rangle}. \quad (54)$$

Again, we can evaluate the numerical results for our benchmark doublet. For $A = 67$, we assume $R = 5.30$ fm. The energy difference is $\Delta E_0 \approx 20$ MeV in ^{60}Ni (according to [15]). As ΔE_0 is expected to scale as $A^{-1/3}$ [12], we assume $\Delta E_0 \approx 19.3$ MeV for $A = 67$. With these assumptions, the numerical value of the adimensional coefficient C in Eq. (53) is $C = 0.116$. For the ratio of radial integrals [the last factor of Eq. (54)], with the radial wave functions corresponding to the Woods-Saxon potential one obtains $\langle g_{9/2}|(r/R)^3|f_{7/2}\rangle/\langle g_{9/2}|r/R|f_{7/2}\rangle = 0.752$.

It remains to considerd the two-body term [the second term of Eq. (50)]. Again, we can use the fractional parentage expansion of Eqs. (1) and (2). Here, however, the tensor operator is the product of two factors: a vector isovector term acting on the single-particle state and a scalar isovector one acting on the core state. The product $\tau_3(i)\tau_3(j)$ contains isoscalar and isotensor parts:

$$\tau_3(i)\tau_3(j) = \left[\tau_3(i)\tau_3(j) - \tfrac{1}{3}(\vec{\tau}(i)\cdot\vec{\tau}(j))\right] + \tfrac{1}{3}(\vec{\tau}(i)\cdot\vec{\tau}(j)), \quad (55)$$

but only the isoscalar is effective if the states a_0 and b_0 have $T = 1/2$. To evaluate the reduced matrix element for the isoscalar part of the two-body operator

$$\widetilde{\mathcal{M}}^{(0)}_{2-b} = \frac{e^2}{6\Delta E_0}\sum_i r_i Y^{(1)}(\hat{r}_i) \times \left(\vec{\tau}(i)\cdot\sum_{j\neq i}\vec{\tau}(j)[\varphi(0)-\varphi_c(r_j)]\right) \quad (56)$$

we can use the standard relations of tensor algebra for the matrix elements of tensor products to obtain the reduced matrix element (in ordinary space):[3]

$$\begin{aligned}
&\left(b_0; J_b; 1/2, T_3\left\|\widetilde{\mathcal{M}}^{(0)}_{2-b}\right\|a_0; J_a, 1/2, T_3\right) \\
&= \frac{e}{2}(j_N\|rY^{(1)}(\hat{r})\|j_I)\frac{C}{\sqrt{3}}\sum_{T_c,T_c'}(-1)^{T_c+1}\begin{Bmatrix} 1/2 & T_c & 1/2 \\ T_c' & 1/2 & 1\end{Bmatrix}\left[\sum_{\mu,\mu'}(-1)^{j_N+J_a+J_\mu^++1}\frac{\widehat{J}_a\widehat{J}_b}{\widehat{J}_\mu^+}\begin{Bmatrix} J_b & j_N & J_\mu^+ \\ j_I & J_a & 1\end{Bmatrix} C_{fp}(a|j_I;\mu,J_\mu^+,T_c)\right. \\
&\times C_{fp}(b|j_N;\mu',J_\mu^+,T_c')\left(\mu,J_\mu^+,T_c\left\|\sum\frac{r_j^2}{R^2}\vec{\tau}(j)\right\|\mu',J_\mu^+,T_c'\right) + \sum_{\mu,\mu'}(-1)^{j_I+J_a+J_\mu^-+1}\frac{\widehat{J}_a\widehat{J}_b}{\widehat{J}_\mu^-}\begin{Bmatrix} J_b & j_I & J_\mu^- \\ j_N & J_a & 1\end{Bmatrix} \\
&\left.\times C_{fp}(a|j_N;\mu,J_\mu^-,T_c)C_{fp}(b|j_I;\mu',J_c^-,T_c')\left(\mu,J_\mu^-,T_c\left\|\sum\frac{r_j^2}{R^2}\vec{\tau}(j)\right\|\mu',J_\mu^-,T_c'\right)\right] \quad (57)
\end{aligned}$$

as $J_{\mu'} = J_\mu$. As the $\widetilde{\mathcal{M}}^{(0)}_{2-b}$ operator transforms as a scalar in isospin space, its matrix elements have the same sign in both nuclei of the isospin doublet.

The parent state can have $T = 0$ or 1, and in principle we have to consider both diagonal and nondiagonal matrix elements (in the parent-state variables) of the isovector operator $\sum(r_j^2/R^2)\,\vec{\tau}(j)$. Obviously, its matrix elements vanish when T or T' is equal to zero. Otherwise, we can use again a fractional parentage expansion. Only terms having the same parent can contribute to

[3] In fact, $\langle 1/2, T_3; 1/2, T_3|(\vec{\tau}(i)\cdot\vec{\tau}(j)|1/2, T_c'; 1/2, T_3\rangle = (-1)^{T_c+1}\begin{Bmatrix} 1/2 & T_c & 1/2 \\ T_c' & 1/2 & 1\end{Bmatrix}(1/2\|\vec{\tau}(i)\|1/2)(T_c\|\vec{\tau}(i)\|T_c')$ and $(j_N, J_\mu, J_b\|r_iY^{(1)}(i)(r_j/R)^2)\|j_I.J_\mu, J_a) = \widehat{J}_a\widehat{J}_b \times \begin{Bmatrix} j_N & J_\mu & J_b \\ j_I & J_\mu & J_a \\ 1 & 0 & 1\end{Bmatrix}(j_N\|r_iY^{(1)}(i)\|j_I)(J_\mu\|(r_j/R)^2\|J_\mu)$.

the matrix element and, in addition, the one-body operator has nondiagonal terms only between single-particle states (with equal j^π) differing by at least two units of the principal quantum number: i.e., it does not possess nondiagonal matrix elements inside our model space. As for the diagonal ones, shells (or subshells) completely filled with protons and neutrons do not contribute to the sum, as they necessarily have $T = 0$. If the valence nucleons are all in the same subshell [or, approximately, in subshells with similar $\langle (r_j/R)^2 \rangle \approx \langle (r/R)^2 \rangle_v$], the integral over the radial coordinates can be factorized, $\sum \vec{\tau}(i) = 2\vec{T}$, and only the diagonal terms with $\mu' = \mu$ survive. Therefore, the matrix element takes the form

$$\left(\mu, J_\mu, T_c \left\| \sum_j \left(r_j^2/R^2 \right) \vec{\tau}(j) \right\| \mu', J_\mu, T_c' \right) \approx \langle (r/R)^2 \rangle_v (J_\mu \| 1 \| J_\mu)(T_c \| 2\vec{T} \| T_c') \, \delta_{\mu,\mu'}, \tag{58}$$

where $\langle (r/R)^2 \rangle_v$ is the average over active valence nucleons, and $(J_\mu \| 1 \| J_\mu) = \widehat{J}_\mu$. For $T_c = T_c' = 1$, $(T_c \| \vec{T} \| T_c) = \sqrt{T_c(T_c+1)(2T_c+1)} = \sqrt{6}$. By comparing the result with Eq. (8), we obtain approximately [as the first 6-J coefficient of Eq. (57) has the value $-1/3$]:

$$\left(b_0; J_b; 1/2, T_3 \left\| \widetilde{\mathcal{M}}_{2-b}^{(0)} \right\| a_0; J_a, 1/2, T_3 \right) \approx -C \, \frac{2}{3} \, \mathcal{A}(1) \frac{e}{2} (j_N \| r Y^{(1)}(\hat{r}) \| j_I) \langle (r/R)^2 \rangle_v. \tag{59}$$

Actually, the expectation values of r^2/R^2 for the different orbitals of the pf shell (estimated with Woods-Saxon wave functions) do not differ by more than 3% from their average value 0.615, as we obtain in Appendix B. By using this average value, one obtains for the numerical coefficient of the two-body term $(2/3)\langle (r/R)^2 \rangle_v = 0.410$. As this value is not negligible in comparison to that of the one-body term (0.752), a sizable quenching of the isoscalar transition amplitude corresponding to the one-body term results from the negative interference of the two-body term. A similar effect is found for the $E1$ transitions with $\Delta T = 0$ in the $N = Z$ nuclei [13]. However, in the present case the quenching only concerns the parent $T = 1$ term. As the parent $T = 0$ term of Eq. (54) has no counterpart in the two-body matrix element, its contribution remains unaltered.

If we assume that the most important contribution to the asymmetry is due to the effect of coherent mixing, as approximated in this paragraph, we obtain

$$\epsilon(T_3) \equiv \frac{\left(b, 7/2^-; 1/2, T_3 \left\| \widetilde{\mathcal{M}}_E^{(10)} \right\| a, 9/2^+; 1/2, T_3 \right)}{\left(b, 7/2^-; 1/2, T_3 \left\| \mathcal{M}_E^{(11)} \right\| a, 9/2^+; 1/2, T_3 \right)} = (-1)^{1/2+T_3} \, 3C \, \frac{\langle j_N | (r/R)^3 | j_I \rangle}{\langle j_N | r/R | j_I \rangle} \times \frac{\eta \mathcal{A}(1) - \mathcal{A}(0)}{\mathcal{A}(1) + 3\mathcal{A}(0)}, \tag{60}$$

where the quenching factor η takes into account the negative interference with the two-body term of Eq. (50).

Equation (60) only gives an approximate estimate of the effect, due to the many simplifying assumptions (notably, the closure approximation) that have been introduced to obtain this result. Moreover, inclusion in the model space of other orbitals of the upper major shell (as discussed in Appendix C) would somewhat alter this result. However, it could be instructive to evaluate some numerical results, also in the limited space considered, to show that the coherent mixing with the isovector giant monopole resonance can explain the large values of the $E1$ asymmetries observed in our example of the $A = 67$ doublet, while the simplest processes discussed in the previous sections were unable to do.

With the above estimate, $\eta = (0.752 - 0.410)/0.752 = 0.458$ and the asymmetry ratio for the mirror $E1$ strengths is

$$\mathcal{R} \equiv \frac{B(E1, T_3 = -1/2)}{B(E1, T_3 = +1/2)} = \left[\frac{1+\epsilon^-}{1-\epsilon^-} \right]^2, \tag{61}$$

where we have put $\epsilon^- \equiv \epsilon(-1/2) = -\epsilon(+1/2)$. Now, to obtain a more accurate estimate one should know the ratio $\mathcal{A}(0)/\mathcal{A}(1)$, which in turn depends on the C_{fp} coefficients.

The relative sign of $\mathcal{A}(0)$ and $\mathcal{A}(1)$ depends on the combined effect of all terms in the sum of Eq. (8). However, we can notice that each of them contains a factor $(-1)^T$. If any of these terms dominates, the relative sign of $\mathcal{A}(0)$ and $\mathcal{A}(1)$ is well defined and negative. Actually, this is very probably the case also under somewhat broader conditions. Most probably, the second line of Eq. (8) (corresponding to negative-parity parents) is only a small correction in comparison to the first one. Let us consider, from now on, the numerical values corresponding to the $A = 67$ doublet. We can note that the expression

$$(-1)^J \begin{Bmatrix} 9/2 & 9/2 & J \\ 7/2 & 7/2 & 1 \end{Bmatrix}$$

has always the same (negative) sign for all J values (from 0 to 7) and its value changes very slowly as long as $J \leqslant 3$. Therefore, unless the parentage coefficients have a very singular behavior, the relative sign is determined only by the factor $(-1)_c^T$ [see also Eq. (4)].

To obtain just an order-of-magnitude estimate of the expected effect, we could evaluate the asymmetry in the $A = 67$ doublet, for two limiting cases in which one of the two coefficients $\mathcal{A}(1)$ and $\mathcal{A}(0)$ is negligible in comparison to the other. Neglecting $\mathcal{A}(1)$ one obtains $\epsilon^- \approx -0.753C \approx -0.0872$ and the asymmetry ratio $\mathcal{R} \approx 0.705$.

Taking into account also $\mathcal{A}(1)$ would bring smaller asymmetry (larger $\mathcal{R}$) if $\mathcal{A}(1)$ and $\mathcal{A}(0)$ have the same sign, but it can also result in a larger asymmetry if—as is most probable—they have opposite sign. If, instead, $\mathcal{A}(0)$ is negligible in comparison to $\mathcal{A}(1)$, ϵ^- is positive and its value depends on the coefficient η, which takes into account the negative interference of the core terms. With $\eta = 0.458$, for $\mathcal{A}_0 \ll \mathcal{A}_1$ one obtains $\epsilon^- \approx +0.120$ and $\mathcal{R} \approx 1.62$. Again, a larger asymmetry could be obtained if also a contribution from $\mathcal{A}_0$ (having opposite sign) is included.

These results do not change appreciably if one assumes a charge distribution of Woods-Saxon shape (Appendix B): one obtains $\eta = 0.445$; for $\mathcal{A}_1 \ll \mathcal{A}_0$, $\epsilon^- \approx -0.0852$ and $\mathcal{R} \approx 0.710$; for $\mathcal{A}_1 \gg \mathcal{A}_0$, $\epsilon^- \approx +0.116$ and $\mathcal{R} \approx 1.58$.

A last comment concerns the expected sign of ϵ^-. If the dominant term in Eq. (60) is the one with the $T = 1$ parent, $\epsilon^- > 0$ and the reduced strength should be larger in the nucleus with $N = Z + 1$, for all transitions between $g_{9/2}$ and $f_{7/2}$.

The opposite is true if the $T = 0$ parent dominates. Again, qualitative considerations can help in predicting the relative importance of the two terms. It is likely, in fact, that one of the most important parents be the lowest $J = 0$. Now, if $A = 4n + 1$, the lowest $J = 0$ parent state is the ground state of the even-even self-conjugate nucleus with $A - 1$ nucleons. Instead, if $A = 4n - 1$ (as in the case ^{67}As-^{67}Se), the self-conjugate parent nucleus is odd-odd and the lowest $J = 0$ parent has $T = 1$. If this consideration is correct, the predicted sign of the asymmetry is consistent with the experimental results in the $A = 67$ mirror pair.

V. CONCLUSIONS

It seems worth summarizing the results obtained for the different processes which could, in principle, produce an asymmetry in the $E1$ transition strength, as observed in the case of the ^{67}As-^{67}Se mirror pair. Higher-order terms, either of "electric" or "magnetic" origin, usually excluded from calculations by the approximation linked to Siegert's theorem, in the case considered are three orders of magnitude lower than the leading one. We note that these corrections apply to the transition operator and not to the level wave functions. Therefore, as long as—as was assumed here—most of the shell-model terms contributing to the $E1$ transition involve the same pair of single-particle states, the same combination of fractional parentage coefficients is involved for both the isoscalar and the isovector terms. Thus if the isovector term is hindered as a consequence of accidental cancellation, a similar hindrance factor can be expected also for the isoscalar term, leaving the ratio almost unchanged. Only meson currents, neglected in our approximate estimation of the magnetic term, could break, to some extent, the above conclusion.

The Coulomb interaction, mixing in a different way the level wave functions in the two mirror nuclei, is presumably at the origin of the observed asymmetries. Its effect could be enhanced when a pair of levels having equal J^π lie, accidentally, close together. For example, this could have been the case for the two $7/2^-$ levels lying between 640 and 1100 keV in ^{67}As and ^{67}Se. However, if the asymmetry originated uniquely from the mixing between the two daughter levels, the total sum of the reduced strengths of the $E1$ transitions feeding these levels ought to be equal in the mirror nuclei, in contrast with the experimental evidence.

The Coulomb mixing could also be enhanced if it took place between states with two "very similar" wave functions. In Sec. IV B we considered a hypothetical mixing between an "isospin analog" state and its corresponding "antianalog." In the case of mass $A = 67$, this mixing would lead to an asymmetry similar in size to the observed effect. It would also give the right sign for the asymmetries. However, this would only happen if our $T = 1/2$, $J^\pi = 9/2^+$ state would be the exact antianalog of the lowest $T = 3/2$ state with the same J^π, while some spread of the antianalog strength among different levels is expected also in this region of nuclei [16,17].

The effects of Coulomb mixing considered thus far only involved states in the same Hilbert subspace needed to describe the parent and daughter states of the $E1$ transition: in the simplest case, a full major shell and at least one particle-hole excitation to the next major shell. A shell-model calculation in this Hilbert space could treat, on the same footing, both the regular (isovector) part of the $E1$ transition amplitude and the "induced-isoscalar" term originating from the mixing. In such a calculation, the isovector part of the two-body Coulomb interaction could be added to the empirical residual interactions, which could also include the symmetry-violating part necessary to account for the Coulomb energy differences [18].

Finally, we have considered the possible effect of mixing with states outside the truncated shell-model space, such as those belonging to the giant isovector monopole resonance. With the approximations discussed in Sec. IV C, this effect could also be expressed in a form that could be treated in the truncated space, if the mean excitation energy of the monopole resonance were at least approximately known.

A shell-model calculation in such a restricted basis could therefore be able to identify the origin of the observed asymmetry in $E1$ transition strengths. At the moment, the coherent contribution of states belonging to the giant isovector monopole resonance appears as the most probable candidate.

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APPENDIX A: EVALUATION OF THE REDUCED MATRIX ELEMENTS FOR THE MAGNETIC TERM

Here we evaluate the reduced matrix elements of the operators entering in the second line of Eq. (12), between single-particle states ℓ_1, j_1 and ℓ_2, j_2. For this purpose, the following property [19] of vector spherical harmonics is exploited:

$$\vec{\mathcal{Y}}^{(L,1)J}_M \cdot \vec{v} = [Y^{(L)} \otimes v^{(1)}]^{(J)}_M, \tag{A1}$$

where $\vec{v}$ is a generic vector. Here the cases $\vec{v} = \vec{s}$ and $\vec{v} = \vec{j}$ are considered.

In the first case, the reduced matrix element of the tensor product can be obtained easily, because s and $Y^{(L)}$ operate on different Hilbert spaces

$$(\ell_1, J_1 \| [Y^{(L)} \otimes s^{(1)}]^{(J)} \| \ell_2, J_2) = \widehat{J_1}\widehat{J}\widehat{J_2} \begin{Bmatrix} \ell_1 & 1/2 & J_1 \\ \ell_2 & 1/2 & J_2 \\ L & 1 & J \end{Bmatrix} (\ell_1 \| Y^{(L)} \| \ell_2)(1/2 \| s^{(1)} \| 1/2), \tag{A2}$$

where $J = L$ and $(1/2 \| s^{(1)} \| 1/2) = \sqrt{3/2}$. The relation

$$(\ell_1, J_1 \| Y^{(L)} \| \ell_2, J_2) = (-1)^{J_2+\ell_1+L+1/2} \frac{\sqrt{3}}{2} \widehat{J_1}\widehat{J_2} \begin{Bmatrix} J_1 & J_2 & L \\ \ell_2 & \ell_1 & 1/2 \end{Bmatrix} (\ell_1 \| Y^{(L)} \| \ell_2) \tag{A3}$$

can be exploited to express the result as a function of $(\ell_1, J_1 \| Y^{(L)} \| \ell_2, J_2)$ as in Eq. (11):

$$(\ell_1, J_1 \| [Y^{(L)} \otimes s^{(1)}]^{(L)} \| \ell_2, J_2) = (-1)^{J_2+\ell_1+L+1/2} \sqrt{2}\, \widehat{L} \begin{Bmatrix} \ell_1 & 1/2 & J_1 \\ \ell_2 & 1/2 & J_2 \\ L & 1 & L \end{Bmatrix} \times \begin{Bmatrix} J_1 & J_2 & L \\ \ell_2 & \ell_1 & 1/2 \end{Bmatrix}^{-1} (\ell_1, J_1 \| Y^{(L)} \| \ell_2, J_2). \quad \text{(A4)}$$

The second case is not so simple, because the operators j and $Y^{(L)}$ do not commute, so that the symmetrized form of the operator must be employed. Furthermore, they operate on the same Hilbert space, but one can exploit the fact that $\vec{j}$ has no matrix elements between different single-particle states to obtain

$$\frac{1}{2} (\ell_1, J_1 \| [Y^{(L)} \otimes j^{(1)}]^{(J)} + [j^{(1)} \otimes Y^{(L)}]^{(J)} \| \ell_2, J_2) = \frac{1}{2} (\ell_1, J_1 \| Y^{(L)} \| \ell_2, J_2)\, (-1)^{J_1+J+J_2} \widehat{J} \times \left[\begin{Bmatrix} 1 & L & J \\ J_2 & J_1 & J_2 \end{Bmatrix} (J_2 \| j^{(1)} \| J_2) + \begin{Bmatrix} L & 1 & J \\ J_1 & J_2 & J_1 \end{Bmatrix} (J_1 \| j^{(1)} \| J_1) \right], \quad \text{(A5)}$$

where $(j \| j^{(1)} \| j) = \sqrt{j(j+1)(2j+1)}$.

In the present case, $L = J = 1$, $\ell_1 = 4$, $J_1 = 9/2$, $\ell_2 = 3$, and $J_2 = 7/2$. With these numerical values, the coefficients of the reduced matrix element of $Y^{(L)}$ in Eqs. (A4) and (A5) are, respectively, $\sqrt{1/6}$ and $-\sqrt{1/2}$.

APPENDIX B: RADIAL WAVE FUNCTIONS AND COULOMB POTENTIAL WITH A WOODS-SAXON DISTRIBUTION

The radial wave functions have been calculated by assuming a Woods-Saxon potential plus spin-orbit part:

$$V(r) = \left[V_0 + V_s \vec{\ell} \cdot \vec{s}\, \frac{r_0^2}{r} \frac{d}{dr} \right] \frac{1}{1 + e^{(r-R_0)/a}}, \quad \text{(B1)}$$

with the values of the constants consistent with those of Bohr and Mottelson [8]: $V_0 = -51$ MeV, $V_s = 22$ MeV, $R_0 = r_0 A^{(1/3)}$, $r_0 = 1.27$ fm, and $a = 0.67$ fm.

For a consistent evaluation of Coulomb interactions, one needs the average electrostatic potential $\varphi_c(r)$ of a distribution of $A-1$ point charges $e/2$, which will be approximated with a continuous charge distribution having a Woods-Saxon shape:

$$\rho_e(r) = \frac{\rho_0}{1 + e^{\frac{r-R_0}{a}}}, \quad \text{(B2)}$$

where

$$\rho_0 = (A-1)\frac{e}{2} \left[\int \frac{1}{1 + e^{\frac{r-R_0}{a}}} 4\pi r^2 dr \right]^{-1}. \quad \text{(B3)}$$

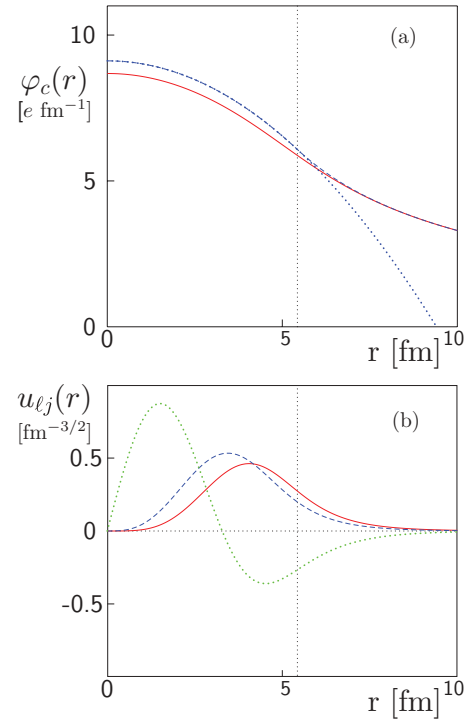

FIG. 2. (Color online) (a) Electrostatic potential for a uniformly charged sphere (dashed curve) and with a Woods-Saxon charge distribution for $A = 67$ (continuous curve). The dotted curve shows the continuation outside the sphere of the expression for the uniform distribution (dashed curve) in the internal region. (b) Examples of radial wave functions for the Woods-Saxon potential (+ spin orbit) with the parameters suggested in Ref. [8]: $0g_{9/2}$ (continuous curve), $0f_{5/2}$ (dashed curve), and $1p_{1/2}$ (dotted curve). The vertical dotted line corresponds to the value of the nuclear radius R.

With the condition that $\varphi_c(r) \to 0$ for $r \to \infty$, we obtain

$$\varphi_c(r) = \int_r^\infty \frac{dy}{y^2} \int_0^y \frac{\rho_0}{1 + e^{\frac{x-R_0}{a}}} x^2\, dx. \quad \text{(B4)}$$

This integral has been evaluated numerically, for $A = 67$, with the parameter values suggested in Ref. [8]. In Fig. 2 , the result is compared with the potential of a uniformly charged sphere of charge density equal to ρ_0 and total charge $(A-1)e/2$. The radius R of the sphere is determined by the condition

$$\frac{4\pi}{3} R^3 = \frac{(A-1)e}{2\rho_0} = \int_0^\infty \frac{1}{1 + e^{\frac{r-R}{a}}} 4\pi r^2 dr. \quad \text{(B5)}$$

To simplify the comparison of the results, $\varphi_c(r)$ is expressed in terms of the adimensional function $f_c(r/R)$:

$$\varphi_c(r) \equiv \frac{(A-1)e}{4R} f_c(r/R), \quad \text{(B6)}$$

and we define $\Delta f_c(r) = f_c(0) - f_c(r)$. For the (extrapolated) potential of the uniformly charged sphere, one obtains $\Delta f_c(r) = (r/R)^2$. One must now calculate the matrix elements of the operators $\widetilde{\mathcal{M}}^{(0)}_{1-b}$ and $\widetilde{\mathcal{M}}^{(0)}_{2-b}$ defined in Sec. IV C. For the one-body term, we consider the ratio

$$\frac{\langle f_{7/2} | (r/R) f_c(r/R) | g_{9/2} \rangle}{\langle f_{7/2} | r/R | g_{9/2} \rangle}, \quad \text{(B7)}$$

TABLE II. Values of radial integrals for different assumptions on the Coulomb potential. In all cases, $\langle f_{7/2}|r/R|g_{9/2}\rangle = 0.8285$.

	Constant ρ_c		Woods-Saxon
	sphere	extrapol.	distribution
$\frac{\langle f_{7/2}\|r\Delta f_c\|g_{9/2}\rangle}{\langle f_{7/2}\|r\|g_{9/2}\rangle}$	0.700	0.752	0.739
$\langle g_{9/2}\|\Delta f_c\|g_{9/2}\rangle$	0.697	0.749	0.735
$\langle f_{7/2}\|\Delta f_c\|f_{7/2}\rangle$	0.594	0.625	0.620
$\langle f_{5/2}\|\Delta f_c\|f_{5/2}\rangle$	0.564	0.592	0.587
$\langle p_{3/2}\|\Delta f_c\|p_{3/2}\rangle$	0.572	0.625	0.608
$\langle p_{1/2}\|\Delta f_c\|p_{1/2}\rangle$	0.580	0.636	0.617

while for the two-body term (and also for the calculations of Sec. IV B), it is sufficient to evaluate the diagonal matrix elements of $f_c(r/R)$.

By numerical integration, with the parameters of Ref. [8] one obtains the values of the necessary integrals reported in the last column of Table II. In the other columns, the corresponding values are calculated, with the Woods-Saxon wave functions, for the potential of the uniformly charged sphere and for the extrapolation of the inner potential outside the sphere [dotted line in Fig. 2(a)].

APPENDIX C: EFFECT OF THE INCLUSION OF MORE ORBITALS

Until now, we have assumed that only the intruder orbit $g_{9/2}$ is significant for the description of the relevant states. As a consequence, only the transitions between $g_{9/2}$ and $f_{7/2}$ contribute to $E1$. If other orbitals of the upper major shell (e.g., $1d_{5/2}$) are taken into account, other orbitals of the lower major shell can be involved in the $E1$ transitions. We consider now the changes that must be introduced in our calculations as a consequence of the inclusion in the model space of the two complete major shells.

Equation (1) must be modified as follows:

$$|a; J_a, M_a; 1/2, T_3\rangle = \sum_{j_I}\sum_{\mu} C_{fp}(a|j_I;\mu, J^+_\mu, T_c)[\phi(j_I)\otimes\Phi(\mu, J^+_\mu, T_c)]^{(J_a,1/2)}_{M_a,T_3} + \sum_{j_N}\sum_{\mu} C_{fp}(a|j_N;\mu, J^-_\mu, T_c)[\phi(j_N)\otimes\Phi(\mu, J^-_\mu, T_c)]^{(J_a,1/2)}_{M_a,T_3} \tag{C1}$$

and similarly for Eq. (2). Equation (6) becomes

$$\begin{aligned}&\big(b, J_b; T_b, T_3\big\|\mathcal{M}^{(1K)}_E\big\|a, J_a; T_a, T_3\big)\\ &= (-1)^{1/2-T_3}\begin{pmatrix} T_b & K & T_a\\ -T_3 & 0 & T_3\end{pmatrix}\sum_{j_I,j_N}\big(j_N\big\|\mathcal{M}^{(1K)}_E\big\|j_I\big)(-1)^K\sum_{T_c=0,1}\mathcal{A}_{j_I,j_N}(T_c)\begin{Bmatrix} 1/2 & T_b & T_c\\ T_a & 1/2 & K\end{Bmatrix},\end{aligned} \tag{C2}$$

with $\mathcal{A}_{j_I,j_N}(T_c)$ given by Eq. (8). Finally, Eqs. (9) and (10) become

$$\big(b, J_b; 1/2, T_3\big\|\mathcal{M}^{(11)}_E\big\|a, J_a; 1/2, T_3\big) = \frac{(-1)^{1/2+T_3}}{6}\sum_{j_I,j_N}[\mathcal{A}_{j_I,j_N}(1) + 3\mathcal{A}_{j_I,j_N}(0)]\big(j_I\big\|D^{(1)}_{IV}\big\|j_N\big), \tag{C3}$$

$$\big(b, J_b; 1/2, T_3\big\|\mathcal{M}^{(10)}_E\big\|a, J_a; 1/2, T_3\big) = \frac{1}{2}\sum_{j_I,j_N}[\mathcal{A}_{j_I,j_N}(1) - \mathcal{A}_{j_I,j_N}(0)]\big(j_I\big\|D^{(1)}_{IS}\big\|j_N\big) \tag{C4}$$

With these modifications the possible consequences of the inclusion of more orbitals on the results of the different sections can now be considered.

Section III only concerns the form of the $E1$ *operator*, and it does not depend on the assumed form of the wave functions.

Section IV A also is completely valid, as the considerations reported there do not depend on the details of the wave functions.

Section IV B depends on the assumed structure of the analog and anti-analog states. The choice given there presumably corresponds to an upper limit of the mixing. For example, in Eq. (41), the choice of a pure $g_{9/2}$ orbit corresponds to the maximum possible value of the expectation value of r^2/R^2. Our conclusion, i.e., that this process is not able to explain the observed effect, is therefore even stronger if other orbitals are considered.

It remains to consider Sec. IV C. The sum on j_I, j_N must be included in Eqs. (54) and (59) to obtain the one-body and the two-body contributions to the induced isoscalar $E1$:

$$\begin{aligned}&\Big(b, J_b; \frac{1}{2}, T_3\big\|\widetilde{\mathcal{M}}^{(0)}_{1-b}\big\|a, J_a; \frac{1}{2}, T_3\Big)\\ &= \frac{1}{2}\sum_{j_I,j_N}[\mathcal{A}_{j_I,j_N}(1) - \mathcal{A}_{j_I,j_N}(0)]\frac{e}{2}(j_N\|r\,Y^{(1)}(\hat r)\|j_I)\\ &\quad\times\frac{\langle j_N|(r/R)^3|j_I\rangle}{\langle j_N|(r/R)|j_I\rangle}\end{aligned} \tag{C5}$$

and

$$\big(b_0; J_b; 1/2, T_3\big\|\widetilde{\mathcal{M}}^{(0)}_{2-b}\big\|a_0; J_a, 1/2, T_3\big) \approx -C\,\frac{2}{3}\sum_{j_I,j_N}\mathcal{A}_{j_I,j_N}(1)\frac{e}{2}(j_N\|rY^{(1)}(\hat r)\|j_I)\langle(r/R)^2\rangle_v. \tag{C6}$$

We obtain therefore

$$\epsilon(T_3) \equiv \frac{\left(b, 7/2^-; 1/2, T_3 \middle\| \widetilde{\mathcal{M}}_E^{(10)} \middle\| a, 9/2^+; 1/2, T_3\right)}{\left(b, 7/2^-; 1/2, T_3 \middle\| \mathcal{M}_E^{(11)} \middle\| a, 9/2^+; 1/2, T_3\right)} = (-1)^{1/2+T_3}\, 3C \frac{\sum_{j_I, j_N} \langle j_N | (r/R)^3 | j_I \rangle [\eta_{j_I, j_N} \mathcal{A}_{j_I, j_N}(1) - \mathcal{A}_{j_I, j_N}(0)]}{\sum_{j_I, j_N} \langle j_N | r/R | j_I \rangle [\mathcal{A}_{j_I, j_N}(1) + 3\mathcal{A}_{|_{\mathcal{I}}, |_{\mathcal{N}}}(0)]}, \tag{C7}$$

where η_{j_I, j_N} has the same meaning as in Eq. (60).

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