Solitary-wave vortices in quadratic nonlinear media

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We find families of vortex solitary waves in bulk quadratic nonlinear media under conditions for second-harmonic generation. We show that the vortex solitary waves are azimuthally unstable and that they decay into sets of stable spatial solitons. We calculate the growth rates of the azimuthal perturbations and show how those affect the pattern of output light. © 1998 Optical Society of America [S0740-3224(98)04402-6]

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Optical vortices, or topological wave-front defects,[1] are ubiquitous entities that display fascinating properties and strikingly rich dynamical behavior, with important fundamental implications and potential applications to the manipulation of light signals.[2–5] An optical vortex is a spiral, or screw, dislocation of the wave front that has a helical phase ramp around a phase singularity. Vortices have been observed to appear spontaneously in several settings, including in speckle fields and in the so-called doughnut laser modes, and otherwise they can be generated with appropriate phase masks or by the transformation of laser modes with astigmatic optical components. To date, optical vortices have been investigated in linear media,[2] cubic nonlinear media,[3,4] and photorefractive crystals,[5] whereas the evolution of focused, intense beams containing phase dislocations in quadratic nonlinear media has been examined only recently.[6]

In this paper we address the existence, stability, and associated dynamics of families of vortex solitary waves in bulk quadratic nonlinear media under conditions for second-harmonic generation. Specifically, we search for phase dislocations nested in the centers of beams with a Gaussian-like shape. Such vortices can appear as higher-order solitary-wave solutions of the governing evolution equations in the quadratic nonlinear crystal.

We discover families of vortices with increasingly higher topological charges that exist above a threshold light intensity for all values of the wave-vector mismatch between the fundamental and second-harmonic waves. We show that the vortex solitary waves self break inside the nonlinear crystal because of an azimuthal instability, and a detailed study of such instability, with calculation of the growth rate of the perturbations with different azimuthal indices, is performed. We find the index of the dominant growth rate and show its dependence with the light intensity, the wave-vector mismatch, and the charge of the phase dislocation. Then we perform a series of numerical simulations to verify the corresponding predictions. The spontaneous azimuthal-beam breaking produces sets of lowest-order spatial solitons that repel each other because of the presence of the phase dislocation, and hence produce different patterns of output light.

We consider cw beams propagating in a bulk medium with a large $\chi^{(2)}$ nonlinearity under conditions for type I second-harmonic generation. In the slowly varying envelope approximation, the beam evolution is described by the equations (see, e.g., Ref. 7)

$$i\,\frac{\partial a_1}{\partial \xi} - \frac{r}{2}\,\nabla_\perp^2 a_1 + a_1^* a_2 \exp(-i\beta\xi) = 0,$$

$$i\,\frac{\partial a_2}{\partial \xi} - \frac{\alpha}{2}\,\nabla_\perp^2 a_2 - i\boldsymbol{\delta}\cdot\nabla_\perp a_2 + a_1^2 \exp(i\beta\xi) = 0, \quad (1)$$

where a_1 and a_2 are the normalized amplitudes of the fundamental and second-harmonic waves, $r = -1$, and $\alpha = -k_1/k_2$. Here $k_{1,2}$ are the linear wave numbers at both frequencies, and in practice $\alpha \simeq -0.5$. The transverse coordinates are normalized to a beam width η, and the propagation coordinate ξ is normalized to the diffraction length $l_d = k_1\eta^2/2$. The parameter β is given by $\beta = k_1\eta^2\Delta k$, where $\Delta k = 2k_1 - k_2$ is the wave-vector mismatch. Typical experimental conditions with focused beams, namely a coherence length, $l_c = \pi/|\Delta k|$, of $\sim$2.5 mm and $\eta \sim 15\ \mu$m, yield $\beta \sim \pm 3$. The parameter δ accounts for the presence of Poynting-vector walk-off. For our present purposes it is convenient to investigate configurations without walk-off, and we hereafter set $\delta = 0$.

Equations (1) define an infinite-dimensional Hamiltonian system with a conserved Hamiltonian that, in the absence of walk-off, is given by

$$\mathscr{H} = -\frac{1}{2}\int\Big[r|\nabla_\perp A_1|^2 + \frac{\alpha}{2}\,|\nabla_\perp A_2|^2 - \beta|A_2|^2 + (A_1^{*2}A_2 + A_1^2 A_2^*)\Big]\mathrm{d}r_\perp\,, \quad (2)$$

where $A_1 = a_1$ and $A_2 = a_2\exp(-i\beta\xi)$. The beam power or energy flow

$$I = \int (|A_1|^2 + |A_2|^2)\mathrm{d}r_\perp \tag{3}$$

is also conserved. We seek stationary solutions of Eqs. (1) with the form $a_{1,2} = U_{1,2}(\rho)\exp(i\kappa_{1,2}\xi + im_{1,2}\varphi)$ where $\kappa_{1,2}$ are nonlinear wave-number shifts, ρ is the radial cylindrical coordinate, and φ is the azimuthal angle. In the case of solutions with a phase dislocation nested in the center of the beams, $m_{1,2}$ are the topological charges of the dislocations and $\mathrm{sgn}(m_{1,2})$ is their chirality. The transverse profiles $U_{1,2}$ are assumed to be real, radially symmetric functions. To avoid power exchange between the fundamental and second-harmonic waves, one needs $\kappa_2 = 2\kappa_1 + \beta$ and $m_2 = 2m_1$. The coupled equations obeyed by U_1 and U_2 are

$$\frac{r}{2}\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho\frac{\mathrm{d}U_1}{\mathrm{d}\rho}\right) + \left(\kappa_1 - \frac{rm_1^2}{2\rho^2}\right)U_1 - U_1U_2 = 0,$$

$$\frac{\alpha}{2}\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\rho\frac{\mathrm{d}U_2}{\mathrm{d}\rho}\right) + \left(\kappa_2 - \frac{\alpha m_2^2}{2\rho^2}\right)U_2 - U_1^2 = 0. \tag{4}$$

These equations have a rich variety of solutions. Most of the lowest-order, vorticityless bright solutions are stable,[8] and the excitation of the corresponding families, except that for type II phase matching with Gaussian beams, has been observed experimentally.[9] To investigate the existence of families of higher-order solutions and, in particular, vortex solitary waves, we solved Eqs. (4) using a relaxation method for different values of the nonlinear wave-number shift κ_1 at different values of β and m_1.

We found families of stationary solutions of two types: higher-order solutions without vorticity, which are characterized by the number of nodes or field zeros of the beams, and solutions without nodes but with phase dislocations of increasingly higher charge. Here we focus on the latter. A convenient way to represent the new families of solitons is to plot the Hamiltonian of the solutions as a function of the energy flow. Figure 1 shows the outcome of the calculations at positive and negative wave-vector mismatches for the two lowest values of the charge of the phase dislocation. Vortices also exist at exact phase matching, even though they are not shown here. Figure 2 shows the typical shape of the vortex solitary waves with charges $m_1 = 1$ and $m_1 = 2$.

The salient point of Fig. 1 is that for a given energy flow the value of the Hamiltonian of the families of vortices is higher than H of the lowest-order solitons, which realize the absolute minimum of H. Therefore the higher-order solutions might be unstable. In general, such instability can produce the reshaping of the beams to a single stable soliton, or it can produce the self-breaking of the beams in the azimuthal direction. One expects the latter to be the case for the vortices because of the presence of the phase dislocation. On physical grounds, such self-breaking can be viewed as being due to the modulational instability of the top of the ring-shaped beams, somehow analogous to other spatial and temporal modulational instabilities arising in $\chi^{(2)}$ media.[10]

To examine the stability of the vortex solitary waves against azimuthal perturbations, we seek solutions of the form

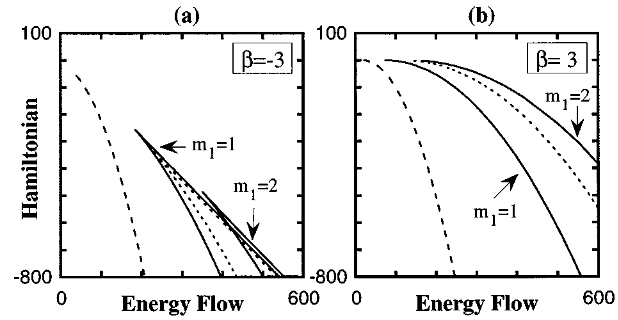

Fig. 1. Families of solitary wave solutions represented in an energy-flow–Hamiltonian diagram at $\beta = \pm 3$. Dashed curves, families of lowest-order, stable solitons; dotted curves, higher-order vorticityless solutions; solid curves, vortex solutions. (a) $\beta = -3$; (b) $\beta = 3$.

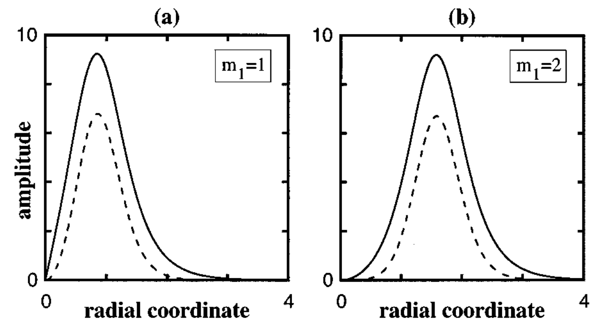

Fig. 2. Shape of typical vortex solutions with two different charges. Solid curves, fundamental beam; dashed curves, second-harmonic beam. Conditions: $\beta = 3$, $\kappa_1 = 3$. (a) $m_1 = 1$; (b) $m_2 = 2$.

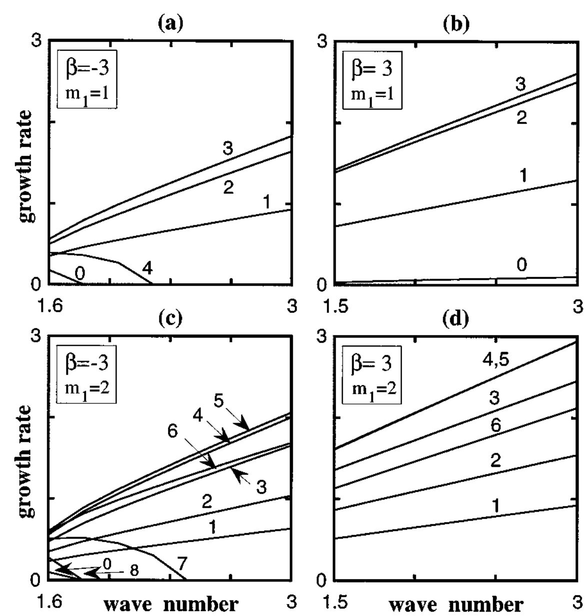

Fig. 3. Growth rate of perturbations with different azimuthal indices as a function of κ_1 for two different values of the wave-vector mismatch and two different charges. (a) $\beta = -3$, $m_1 = 1$; (b) $\beta = 3$, $m_1 = 1$; (c) $\beta = -3$, $m_1 = 2$; (d) $\beta = 3$, $m_1 = 2$.

$$a_j = [U_j(\rho) + f_j(\rho, \xi)\exp(in\varphi) + g_j(\rho, \xi)\exp(-in\varphi)]\exp(im_j\varphi)\exp(i\kappa_j\xi), \quad j = 1, 2. \quad (5)$$

Inserting Eq. (5) into Eq. (1) and linearizing around the perturbations, one obtains the set of four coupled, linear, partial differential equations obeyed by $f_j(\rho, \xi)$ and $g_j(\rho, \xi)$. Such equations, not written here for brevity, have many possible types of solutions. Here we are interested in those solutions that display exponential growth along ξ, and to obtain them we used the method described in Refs. 11. Figure 3 shows the outcome of such a stability analysis. The main result predicted by the plots is that all the vortex solitary waves are unstable against azimuthal perturbations, and with the larger parameter κ_1, comes the stronger light intensity and hence the stronger the instability. For vortices with charge $m_1 = 1$, the perturbations with $n = 3$ and $n = 2$ exhibit the largest growth rate. For $m_1 = 2$, the perturbations that tend to dominate are $n = 5$ together with $n = 4$. Therefore, under ideal conditions when all the perturbations are excited with equal strength, the vortex solitary waves should tend to split into the corresponding number of beams. This is so at the initial states of the evolution, where the ansatz (5) is justified. At further distances, the mutual interaction of the beams comes into play, and such interaction might affect significantly the eventual pattern of output light.

To elucidate the actual decay of the vortex solitary waves in the far field, we performed a series of simulations by solving Eqs. (1) with the initial conditions given by the stationary solutions. To seed any instabilities present in the system, we multiply the input by the factor $[1 + R(r_\perp)]$, where R is a uniformly distributed, complex random quantity with zero mean. Figure 4 shows the typical outcome of the numerical experiments. We observed that the result of the simulations agreed with the general predictions of Fig. 3. Namely, the majority of vortex solitary waves with charge $m_1 = 1$ self-break into three solitons, and vortices with charge $m_1 = 2$ tend to produce five solitons. The output solitons are not equal to each other, neither in amplitude nor in phase. This is partially because the beam evolution is dictated by the interplay between perturbations with different azimuthal indices, although the mode with the largest growth rate tends to dominate. Also, some simulations gave two solitons for vortices with $m_1 = 1$ and four solitons in the case $m_1 = 2$. This is because of the specific perturbation seeded by the numerics and because of the nonlinear dynamics that the beams undergo after the initial stages. A detailed study of such dynamics shall be reported in the future, but the main conclusion is the dominance of the $n = 2, 3$ and $n = 4, 5$ outputs. This dominance appears to be responsible for the recent numerical observation of the breakup into three solitons of fundamental input beams that contain a charge $m_1 = 1$.[6] This is remarkable, provided that those input conditions fall far from the stationary vortex solitary waves, which shows the robustness of the azimuthal instability reported here.

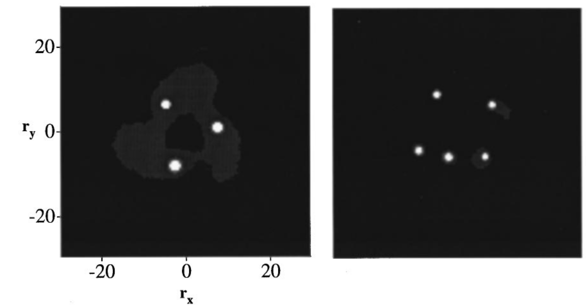

Fig. 4. Typical decay of two vortex solitary waves; the plots show the fundamental beams at $\xi = 10$. The second harmonic exhibits identical features. The inputs are the vortices of Fig. 2.

To summarize, we have found families of optical vortex solitary waves that exist in quadratic nonlinear media. The new vortex solitary waves are azimuthally unstable on propagation, and their decay produces sets of stable spatial solitons that emerge separating from each other. We have developed a linear azimuthal stability analysis and compared its predictions with the numerical simulations of the full governing equations. The azimuthal self-breaking of the vortex solitary waves requires currently available laboratory ingredients; hence it holds promise for experimental demonstration.

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