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Evaluation of non-linear equations to model different animal growths with mono and bisigmoid profiles.

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37

38 **Abstract**

39 Experimental data of different animals (e.g. cocks, pigs, cats, dogs, cattles, etc.) from
40 recent bibliography were selected to evaluate the capability of five classical sigmoidal
41 equations (*i.e.* Bertalanffy, Weibull, logistic, Gompertz and modified Hill) to model
42 growth. These functions were used in different reparameterised forms in order to
43 define all growth phases and to characterize significant kinetic parameters. The
44 results indicated that logistic and Weibull equations were the best options to simulate
45 the data with mono sigmoid profiles. A subsequent formulation of logistic and
46 Gompertz equations was constructed to describe accurately the biphasic trends for
47 cock and foal growths.

48

49 **Keywords:** animal growth; mathematical modelling; sigmoid equations; biphasic
50 sigmoidal profiles

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1. Introduction

54 In general, the success of animal production is mainly dependent on the minimization
55 of the relationship between growths and nutrient costs. The correct description of
56 growth data is especially important when a rigorous and predictive quantification is
57 necessary in order to establish that animals are ready for market (Kebreab et al.,
58 2007). The most robust tool to address organism live-weight is obtained by the use of
59 sigmoidal equations that permit to evaluate all the characteristic phases of animal
60 growth. It is important that mathematical models not only fit well the experimental
61 data but also contain parameters of clear biological meaningful (Zwietering et al.,
62 1990; France et al., 1996; Vázquez and Murado, 2008a).

63

64 Several equations (e.g. Gompertz, Richards, monomolecular, etc.) have been
65 applied with excellent results for describing a wide number of growth kinetics
66 (Mignon-Grasteau et al., 1999; López et al., 2000; Lambe et al., 2006; Schulin-
67 Zeuthen et al., 2008; Strathe et al. 2010). However, in most cases a full study of
68 important parameters is not performed and, moreover, expressions of those
69 equations with lifetime parameters explicitly formulated are not commonly used. In
70 addition, the modelling of non-conventional profiles (*i.e.*, biphasic sigmoid patterns)
71 has not been previously developed to predict these tendencies and identify possible
72 causes.

73

74 The aim of present work was to evaluate and compare five sigmoid equations for
75 best fit in addressing the growth data of different animals. Subsequently, two
76 equations were selected to formulate bi-sigmoidal models that described accurately
77 the non conventional biphasic trends observed in specific animal growths. In all

78 cases, the mathematical models contained parameters with clear biological and
79 geometrical meaning.

80

81 **2. Materials and methods**

82 *2.1. Experimental data*

83 Animal growth data were collected from results previously reported in the literature
84 and taken from the published figures by means of GetData Graph Digitizer 2.24. The
85 growth kinetics were selected from the following studies: cattles, cats, dogs, goats,
86 rabbits and sheeps (López et al., 2000), Mos and Sasso cocks (Franco et al., 2012a),
87 Castellana cocks and capons (Miguel et al., 2008), and Celta pigs (Franco et al.,
88 2011).

89 On the other hand, the biphasic sigmoid data of Mos cocks and foals were obtained
90 from previous reports (Franco et al., 2012b; Franco et al., submitted). In all cases,
91 the data used for modelling were net growths so that the initial weights at time zero
92 were used to subtract the rest of data values.

93

94 *2.2. Mathematical models*

95 A set of five sigmoid equations (Table 1) was evaluated to model the profiles of
96 animal growth. The selected equations are well-known and applied in a wide range of
97 chemical and biological contexts such as dose-response theory (Murado et al., 2002;
98 Murado and Vázquez, 2007), toxicological assessment (Riobó et al., 2008; Rial et al.,
99 2011; Murado et al., 2011), microbial productions (Vázquez et al., 2006; Vázquez et
100 al., 2008), predictive microbiology (Chhabra et al., 2002; Gil et al., 2006), DNA
101 analysis (Pfaffl, 2001; Goll et al., 2006; Swillers et al., 2008; Rutledge & Stewart,
102 2008) and animal growths (López et al., 2000; Freitas, 2005; Strathe et al. 2010).
103 The formulation of those equations with parameters of clear geometrical and

104 biological meaning (Figure 1 and Table 2) facilitates the perfect description and
105 classification of the growth kinetics. Additionally, the fittings using reparameterised
106 functions help to easily calculate the confidence intervals of the parameters. The
107 algebraic steps required to obtain these reparameterisations are described, for two
108 cases (Bertalanffy and Weibull), in appendix section.

109

110 2.3. *Numerical Methods and Statistical Analysis*

111 The fitting procedures and parametric estimates from the experimental results were
112 performed by minimizing the sum of quadratic differences between the observed and
113 model-predicted values using the nonlinear least-squares (quasi-Newton) method
114 provided by the 'Solver' macro from Microsoft Excel spreadsheet. The confidence
115 intervals of the best-fit values for the parametric estimates (Student's t test, $\alpha = 0.05$),
116 consistency of the mathematical models (Fisher's F test; $p < 0.05$) and covariance
117 and correlation matrices were calculated using the 'SolverAid' macro, which is freely
118 available from Levie's Excellaneous website
119 <http://www.bowdoin.edu/~rdelevie/excellaneous/>. These statistical procedures and
120 residual analysis (Durbin-Watson test, d -value) were confirmed and evaluated,
121 respectively, by DataFit 9 (Oakdale Engineering, Oakdale, PA).

122

123 Subsequently, two criteria based on the information theory (Shannon, 1948), Akaike
124 information criterion (AIC) and Bayesian information criterion (BIC), were used to
125 compare the predictive ability of equations (Table 1) to model animal growth data.
126 The AIC and BIC-equations are a measure of the lack-of-fit of the best model –by
127 taking into account both, bias and variance– and as well as the increased unreliability
128 of the selected model to the increased number of model parameters (in terms of

129 accuracy and complexity of the model) (Yi and Judge, 1988; Shi and Tsai, 2002;
130 Burnham and Anderson, 2002):

131

$$132 \quad AIC = n \ln \left(\frac{SSR}{n} \right) + 2(p+1) + \left[\frac{2(p+1)(p+2)}{n-p-2} \right] \quad (6)$$

133

$$134 \quad BIC = n \ln(SSR) + p \ln(n) \quad (7)$$

135

136 The model with the lowest AIC and BIC is the one with the highest likelihood of being
137 correct. The probability (Pr) of the chosen model being correct between two models
138 m_1 and m_2 (pairwise comparison) can be calculated as indicated below:

139

$$140 \quad Pr_{AIC} = \frac{\exp(-0.5\Delta AIC_{m_2-m_1})}{1 + \exp(-0.5\Delta AIC_{m_2-m_1})} \quad (8)$$

141

$$142 \quad Pr_{BIC} = \frac{\exp(-0.5\Delta BIC_{m_2-m_1})}{1 + \exp(-0.5\Delta BIC_{m_2-m_1})} \quad (9)$$

143

144 Both probabilities vary in a range from 0 to 1 where the maximum probability to select
145 the best equation is 1. In addition, bias and accuracy factors of the equations from
146 Table 1 were calculated to evaluate the fitting of those models to experimental data
147 (Ross, 1996; Vázquez et al., 2011):

148

$$149 \quad B_f = 10^{\frac{\sum \log \left(\frac{\text{predicted}}{\text{observed}} \right)}{n}} \quad (10)$$

150

$$A_f = 10^{\frac{\sum \left| \log \left(\frac{\text{predicted}}{\text{observed}} \right) \right|}{n}} \quad (11)$$

152

153 where $\log(\text{predicted}/\text{observed})$ is the logarithmic relation between the predicted and
 154 the experimental values, and n is the number of data. The nearer the values of B_f
 155 (bias factor) and A_f (accuracy factor) to 1 indicate the better the fitting of the models
 156 to experimental data. A value of 1 indicates that there is perfect agreement among
 157 predicted and observed data.

158

159 **3. Results and discussion**

160 *3.1. Animal growth analysis by proposed equations*

161 The results of parameter estimates and statistical analysis of fittings for all animals
 162 and models are summarized in Tables 3, 4, 5 and 6. In general, all data set were
 163 acceptably fitted with the five equations proposed obtaining coefficients of
 164 determination greater than 0.971, mainly higher than 0.995. When experimental
 165 profiles were not clearly sigmoids but showing hyperbolic forms, the equations with
 166 more clear sigmoid definition (*i.e.* Gompertz and logistic) demonstrated a lack of fit at
 167 baseline. In those cases, lag-phase parameters (λ and λ_{med}) were always not
 168 significant (Student's t test, $\alpha = 0.05$) for any equation applied. On the other hand,
 169 when live weight data did not define a complete plateau phase the most flexible
 170 equations (*i.e.* Bertalanffy and modified Hill) estimated an unrealistic value of the
 171 maximum growth (G_m) and the time to achieve this maximum growth (t_m). In all
 172 cases, F_{ratios} were high and p -values < 0.0001 what indicate the complete consistency
 173 and robustness of equations to adjust the experimental patterns.

174

175 Five out of twelve animals studied, the logistic equation was the best model to
176 describe the growth data selected according to AIC and BIC (Table 7). In those
177 cases, the confidence intervals for the numerical estimations of kinetic parameters
178 were lower than for the rest of equations and therefore less uncertainty in logistic
179 parameter determination was observed. The sum of probabilities produced for each
180 pairwise comparison between sigmoid equations was calculated in Table 7 –the
181 maximum value for the sum of probabilities that can be obtained for the best equation
182 is 4–. In the rest of animal's data the best fittings were distributed for equation (2)
183 (Sasso, cat, sheep and goat), equation (3) (dog and rabbit), equation (4) (Sasso and
184 cat) and equation (5) (cattle). In all cases, agreement among observed and predicted
185 values was revealed.

186

187 Residual analysis by means of the Durbin-Watson test demonstrated, in the best
188 results, a minor positive autocorrelation or lack of residuals autocorrelation and
189 therefore random distribution (d -values ranging between 1.2 to 2.3). Nonetheless in
190 some cases (sheeps, turkeys and cats), all equations generated high positive
191 autocorrelation of residuals (d -values < 1).

192

193 The maximum growth rates derived from median abscissa of Weibull distribution
194 (v_{med} on equation (2)) led to similar numerical values to the conventional maximum
195 growth rates (v_m from equations (1), (4) and (5)). Only in sheeps and cattles data,
196 both parameters were significantly different. However, for v_{med} of equation (3),
197 significant differences in relation to v_m were shown. In general, v_m and τ were the
198 most stable parameters (confidence intervals values, see Tables 3 to 6) and lag
199 phase in the two approaches defined (λ and λ_{med}) was always the most uncertainty
200 parameter (Student's t test, $\alpha = 0.05$). Furthermore, the range of confidence intervals

201 variation of t_i was considerably high depending on the equation used and mainly due
202 to the lower prediction of G_m and t_m in some fittings.

203

204 Finally, our results of Sasso cocks, Castellana cocks and Castellana capons
205 modelling were better than previously exposed by Franco et al. (2012a) and Miguel
206 et al. (2008), respectively, in terms of accuracy between experimental and predicted
207 values (determination coefficients) and statistical significance of parameters
208 (Student's t test, $\alpha = 0.05$).

209

210 3.2. *Biphasic growth profiles and modelling*

211 In some cases, the animal growths do not show the classical sigmoidal profiles
212 described above but biphasic trends defining two sigmoids curves. This type of
213 kinetics could be described by means of sum of two sigmoid equations (Vázquez et
214 al., 2009). Since in previous section the equation (1) was a good candidate to model
215 data from several animal growths, it has considered that a sum of two logistics
216 should be suitable for this purpose. Furthermore, it was compared with the fittings of
217 the sum of two Gompertz models. Both formulations in the two reparameterised
218 expressions are as follows:

219

$$220 \quad G = \frac{G_{m1}}{1 + \exp[\mu_{m1}(\tau_1 - t)]} + \frac{G_{m2}}{1 + \exp[\mu_{m2}(\tau_2 - t)]} \quad (12)$$

221

$$222 \quad G = \frac{G_{m1}}{1 + \exp\left[2 + \frac{4v_{m1}}{G_{m1}}(\lambda_1 - t)\right]} + \frac{G_{m2}}{1 + \exp\left[2 + \frac{4v_{m2}}{G_{m2}}(\lambda_2 - t)\right]} \quad (13)$$

223

224 $G = G_{m1} \exp\left[-\exp(\mu_{m1}(\tau_1 - t) - 0.367)\right] + G_{m2} \exp\left[-\exp(\mu_{m2}(\tau_2 - t) - 0.367)\right]$ (14)

225

226 $G = G_{m1} \exp\left[-\exp\left(\frac{v_{m1} \exp(1)}{G_{m1}}(\lambda_1 - t) + 1\right)\right] + G_{m2} \exp\left[-\exp\left(\frac{v_{m2} \exp(1)}{G_{m2}}(\lambda_2 - t) + 1\right)\right]$

227 (15)

228

229 The graphical meaning of parameters is displayed in Figure 3 and its definitions and
 230 units are listed in Table 2. The experimental data selected from bibliography were
 231 successfully described by both equations (12) and (13) for all live-weight phases.
 232 Only a small lack of fit was observed at initial times using logistics (Figure 4). The
 233 predictive ability of the two models was similar in the cases tested regarding to the
 234 values of statistical analysis and parameter estimations obtained (Table 8). The
 235 parameter estimations from equations (12) and (13) were always statistically
 236 significant ($\alpha=0.05$). However, using (14) and (15) two parameters were not
 237 significant (μ_{m1} on cock and λ_1 on foal). In comparison both parameter sets were
 238 similar and not significant differences were shown ($P>0.05$). Thus, either model from
 239 Table 1 could be interchangeably used without risk of losing effectiveness in
 240 modelling and describing biphasic growth data.

241

242 In addition, a sum of two Weibull equations (2) was also checked but the results were
 243 slightly lower than obtained by (12) and (14) (data not shown). These types of
 244 equations are basically empirical and do not explain the reasons or mechanisms that
 245 generate these profiles. However, they are a mathematical resource that accurately
 246 simulate experimental data and provide parameters with clear geometric and
 247 biological definition.

248

249 **4. Conclusions**

250 Globally, all the sigmoid equations demonstrated good capacities of fitting for
251 describing the live-weight kinetics of several animals. Although in most cases logistic
252 was the best equation, however, when experimental data showed hyperbolic profiles
253 Weibull and modified Hill defined better results. The definition of double sigmoid
254 equations highlighted an accurate tool to model biphasic growths. In all evaluations,
255 equations were explicitly formulated with parameters that have a clear geometric and
256 biological meaning which completely described animal growth characteristics.

257

258 **Appendix A**

259 *Reparameterisation of Bertalanffy equation*

260 The most common expression of the Bertalanffy equation is as follows (Bertalanffy,
261 1957):

262

$$263 \quad G = G_m [1 - b \exp(-\mu_m t)]^3 \quad (\text{A.1})$$

264

265 The maximum growth or final asymptote (phase plateau) is calculated from the limit
266 when time tends to infinite:

267

$$268 \quad G_\infty = \lim_{t \rightarrow \infty} G = G_m [1 - b \exp(-\infty)] = G_m \quad (\text{A.2})$$

269

270 The parameter τ is defined as the time required to obtain the semimaximum growth.

271 Thus, when $G = G_m/2$ we have:

272

$$273 \quad \frac{G_m}{2} = G_m [1 - b \exp(-\mu_m \tau)]^3 \Rightarrow \left(\frac{1}{2}\right)^{\frac{1}{3}} - 1 = -b \exp(-\mu_m \tau) \Rightarrow$$

274

$$275 \quad \Rightarrow \tau = \frac{-1}{\mu_m} \ln \left[\frac{1 - \left(\frac{1}{2}\right)^{1/3}}{b} \right] \Rightarrow \tau = \frac{1}{\mu_m} \ln \left[\frac{b}{0.206} \right] \quad (\text{A.3})$$

276

277 We insert the above expression in the conventional form [a1] with the purpose to give
 278 the explicit meaning to that parameter in the following reparameterised form 1 (Table
 279 1):

280

$$281 \quad G = G_m [1 - 0.206 \exp(\mu_m (\tau - t))]^3 \quad (\text{A.4})$$

282

283 Taking the second derivative from (A.1) to zero and isolating the abscissa of the
 284 inflection point ($t = t_i$), we obtain:

285

$$286 \quad \frac{dG}{dt} = 3G_m \mu_m b e^{-\mu_m t} (1 - b e^{-\mu_m t})^2 \Rightarrow$$

287

$$288 \quad \frac{d^2G}{dt^2} = -3G_m \mu_m^2 b e^{-\mu_m t} (1 - b e^{-\mu_m t})^2 + 6G_m \mu_m^2 b^2 e^{-2\mu_m t} (1 - b e^{-\mu_m t}) \Rightarrow$$

289

$$290 \quad \frac{d^2G}{dt^2} = 0 \xrightarrow{t=t_i} -3 + 9b \exp(-\mu_m t_i) = 0 \Rightarrow t_i = \frac{\ln 3b}{\mu_m} \quad (\text{A.5})$$

291

292 The value of growth when $t = t_i$ would be:

293

294
$$G = G_m \left[1 - \frac{b}{3b} \right]^3 = \frac{8G_m}{27} \quad (\text{A.6})$$

295

296 On the other hand, the slope in the inflection point (v_m) is defined by the following
 297 operation:

298

299
$$v_m = \left. \frac{dG}{dt} \right|_{t=t_i} = 3G_m \mu_m b \exp(-\mu_m t_i) [1 - b \exp(-\mu_m t_i)]^2 \xrightarrow[t_i = \frac{\ln 3b}{\mu_m}]{} \rightarrow$$

300

301
$$v_m = \left. \frac{dG}{dt} \right|_{t=t_i} = 3G_m \mu_m b \exp\left(-\mu_m \frac{\ln 3b}{\mu_m}\right) \left[1 - b \exp\left(-\mu_m \frac{\ln 3b}{\mu_m}\right) \right]^2 = \frac{4G_m \mu_m}{9} \quad (\text{A.7})$$

302

303 Taking into account the geometry of the function (Figure 1), we can obtain an
 304 analytical expression for the lag phase of the growth (λ), defined as the intersection
 305 of the tangent at the inflection point with the abscissa axis (Vázquez and Murado,
 306 2008b):

307

308
$$R = G(t_i) + v_m (L - t_i) \text{ with } L = \lambda \text{ when } R = 0:$$

309

310
$$\lambda = t_i - \frac{G(t_i)}{v_m} \quad (\text{A.8})$$

311

312 If mathematical functions (A.5), (A.6) and (A.7) are placed in (A.8), we have:

313

314
$$\lambda = \frac{1}{\mu_m} \left(\ln 3b - \frac{2}{3} \right) \quad (\text{A.9})$$

315

316 Similarly, the time to reach the asymptote phase (t_m) by intersection among R and
317 G_m can be obtained as:

318

319 $R = G(t_i) + v_m(L - t_i)$ with $L = t_m$ when $R = G_m$:

320

$$321 \quad t_m = t_i + \frac{G_m - G(t_i)}{v_m} = \lambda + \frac{2}{3\mu_m} + \frac{G_m - \frac{8G_m}{27}}{\frac{4G_m\mu_m}{9}} = \lambda + \frac{3.375}{\mu_m} \quad (\text{A.10})$$

322

323 Finally, reorganizing terms in (A.9) and inserting both (A.7) and (A.9) in (A.1), the
324 reparameterised form 2 is expressed as follows:

325

$$326 \quad G = G_m \left[1 - \frac{1}{3} \exp\left(\frac{2}{3} + \frac{9\lambda v_m}{4G_m}\right) \exp\left(-\frac{9v_m t}{4G_m}\right) \right]^3 \quad (\text{A.11})$$

327

328 *Reparameterisation of Weibull equation*

329 The conventional equation of cumulative function of the Weibull's distribution is
330 expressed as follows:

331

$$332 \quad G = G_m \left[1 - \exp\left(-\left(t/b\right)^\alpha\right) \right] \quad (\text{A.12})$$

333

334 As in previous case the maximum growth when time tends to infinite is G_m .

335 Therefore, the semimaximum growth (τ) is calculated as:

336

337
$$\frac{G_m}{2} = G_m \left[1 - \exp\left(-\left(t/b\right)^\alpha\right) \right] \Rightarrow \frac{1}{2} = \exp\left[-\left(t/b\right)^\alpha\right] \Rightarrow \tau = b(\ln 2)^{1/\alpha} \quad (\text{A.13})$$

338

339 When (A.13) is inserted in (A.12), the reparameterised form 1 is obtained (Table 1):

340

341
$$G = G_m \left[1 - \exp\left(-\ln 2 \left(t/\tau\right)^\alpha\right) \right] \quad (\text{A.14})$$

342

343 Because of the mathematical expressions of v_m and λ generated by the steps

344 previously described led to equations hardly reparameterised (Murado and Vázquez,

345 2010), we have chosen to use the maximum growth rate at the median abscissa of

346 Weibull density function (v_{med}) and its corresponding lag phase (λ_{med}):

347

348
$$v_{med} = \left. \frac{dG}{dt} \right|_{t=\tau} = G_m \frac{\alpha \ln 2}{\tau^\alpha} \tau^{\alpha-1} \exp\left[-\ln 2 \left(\tau/\tau\right)^\alpha\right] = G_m \frac{\alpha \ln 2}{2\tau} \quad (\text{A.15})$$

349

350
$$\lambda_{med} = \tau - \frac{G(\tau)}{v_{med}} = \tau - \frac{G_m/2}{\alpha G_m \ln 2 / 2\tau} = \tau \left(1 - \frac{1}{\alpha \ln 2} \right) \quad (\text{A.16})$$

351

352 In this case, the time to reach the asymptote phase (t_m) can be calculated as:

353

354
$$t_m = t_i + \frac{G_m - G(t_i)}{v_{med}} = \tau \left(\frac{\alpha - 1}{\alpha \ln 2} \right)^{1/\alpha} + \frac{G_m - \frac{G_m}{2}}{v_{med}} = \tau \left(\frac{\alpha - 1}{\alpha \ln 2} \right)^{1/\alpha} + \frac{2\tau}{\alpha \ln 2} \quad (\text{A.17})$$

355

356 Reorganizing equations (A.15) and (A.16) and replacing terms in (A.14),

357 reparameterised form 2 is achieved (Table 1):

358

$$359 \quad G = \frac{2\lambda_{med}V_{med}}{(\alpha \ln 2 - 1)} \left\{ 1 - \exp \left[-\ln 2 \left(\frac{t(\alpha \ln 2 - 1)}{\alpha \lambda_{med} \ln 2} \right)^\alpha \right] \right\} \quad (A.18)$$

360

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364

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FIGURE CAPTIONS

Figure 1: Graphical description of the kinetic parameters G_m , λ , v_m , t_m y τ from a common sigmoid curve. The line tangent at the inflection point of the sigmoid (R) and $G_m/2$ are also shown.

Figure 2: Animal growth kinetics fitted to the best equation in each case. Castellana_cocks: eq. (1), Castellana_capons: eq. (1), Mos_cocks: eq. (1), Sasso_cocks: eq. (2), Celta_pigs: eq. (1), rabbits: eq. (3), turkeys: eq. (1), sheeps: eq. (2), cats: eq. (4), dogs: eq. (3), goats: eq. (2) and cattles: eq. (5).

Figure 3: Graphical description of the kinetic parameters G_{m1} , G_f , λ_1 , λ_2 , v_{m1} , v_{m2} , τ_1 y τ_2 from a bi-sigmoid curve.

Figure 4: Biphasic growths of Mos_cocks (A) and foals (B) fitted to equations (12) (continuous lines) and (14) (discontinuous lines).

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TABLE CAPTIONS

Table 1: Equations used to model the animal growth data obtained from literature. Parameter definitions are summarized in Table 2.

Table 2: Symbolic notations used and corresponding units

Table 3: Parametric estimates and confidence intervals ($\alpha=0.05$) from the equations summarized in Table 1 applied to the growth data of different varieties of cocks. Statistical values of adjusted coefficient of multiple determination (R^2_{adj}), F_{ratio} and p -values from Fisher's F-test ($\alpha=0.05$) and d -values from Duncan test are also listed. NS: non-significant.

Table 4: Parametric estimates and confidence intervals ($\alpha=0.05$) from the equations summarized in Table 1 applied to the growth data of capons, pigs and cattles. Statistical values of adjusted coefficient of multiple determination (R^2_{adj}), F_{ratio} and p -values from Fisher's F-test ($\alpha=0.05$) and d -values from Duncan test are also listed. NS: non-significant.

Table 5: Parametric estimates and confidence intervals ($\alpha=0.05$) from the equations summarized in Table 1 applied to the growth data of turkeys, rabbits and cats. Statistical values of adjusted coefficient of multiple determination (R^2_{aj}), F_{ratio} and p -value from Fisher's F-test ($\alpha=0.05$) and d -value from Duncan test are also listed. NS: non-significant.

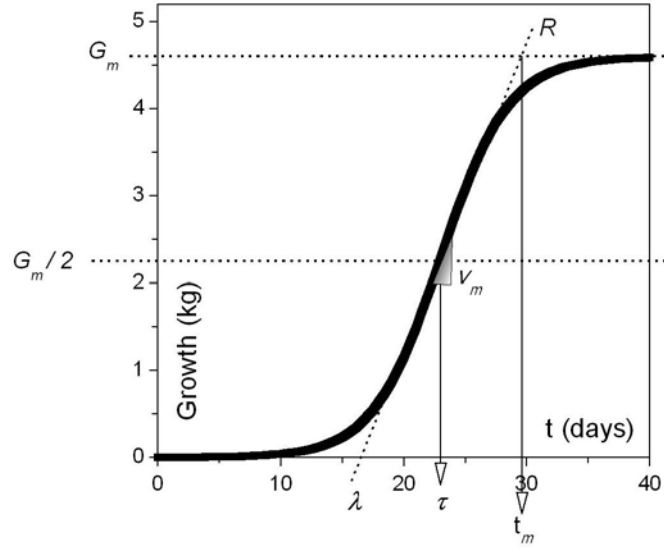
Table 6: Parametric estimates and confidence intervals ($\alpha=0.05$) from the equations summarized in Table 1 applied to the growth data of sheeps, dogs and goats. Statistical values of adjusted coefficient of multiple determination (R^2_{aj}), F_{ratio} and p -value from Fisher's F-test ($\alpha=0.05$) and d -value from Duncan test are also listed. NS: non-significant.

Table 7: Sum of probabilities obtained from AIC and BIC criteria (equations (8) and (9), respectively) applied to the comparison among equations fittings for each animal growth data. It should be noted that the total maximum of each sum is only 4.

Table 8: Parametric estimates and confidence intervals ($\alpha=0.05$) from the biphasic equations (12-15) applied to the growth data of Mos_cocks and foals. Statistical values of adjusted coefficient of multiple determination (R^2_{aj}), F_{ratio} and p -value from Fisher's F-test ($\alpha=0.05$) and d -value from Duncan test are also listed. NS: non-significant.

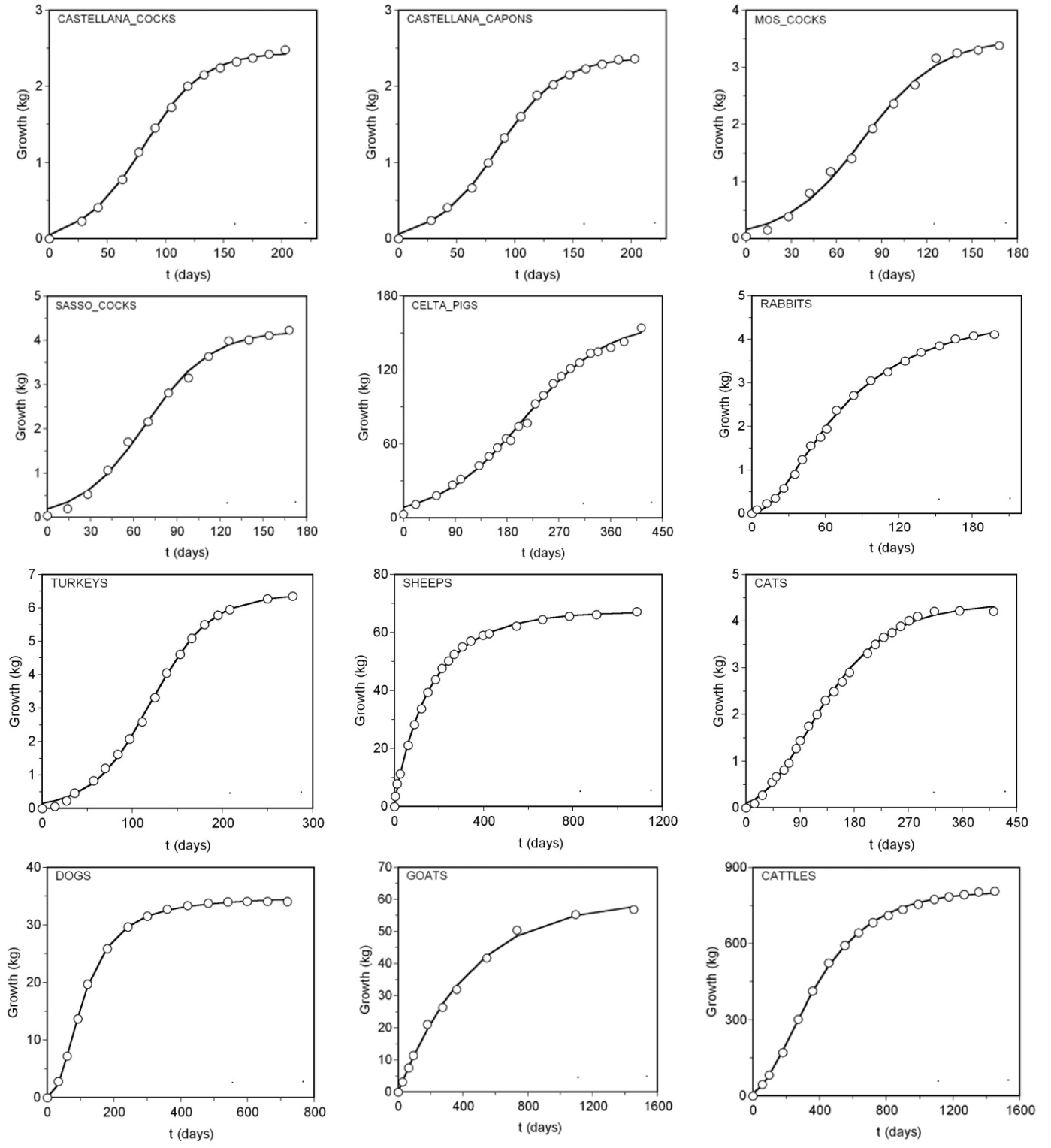
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FIGURE 1



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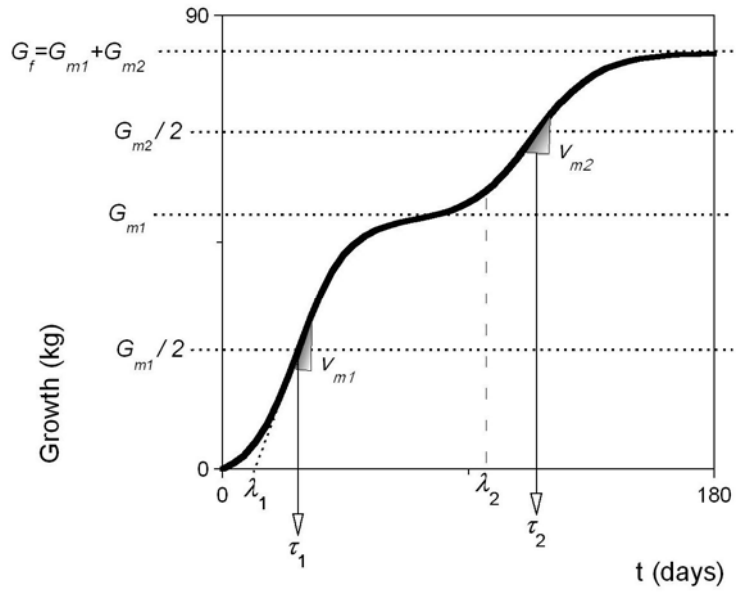
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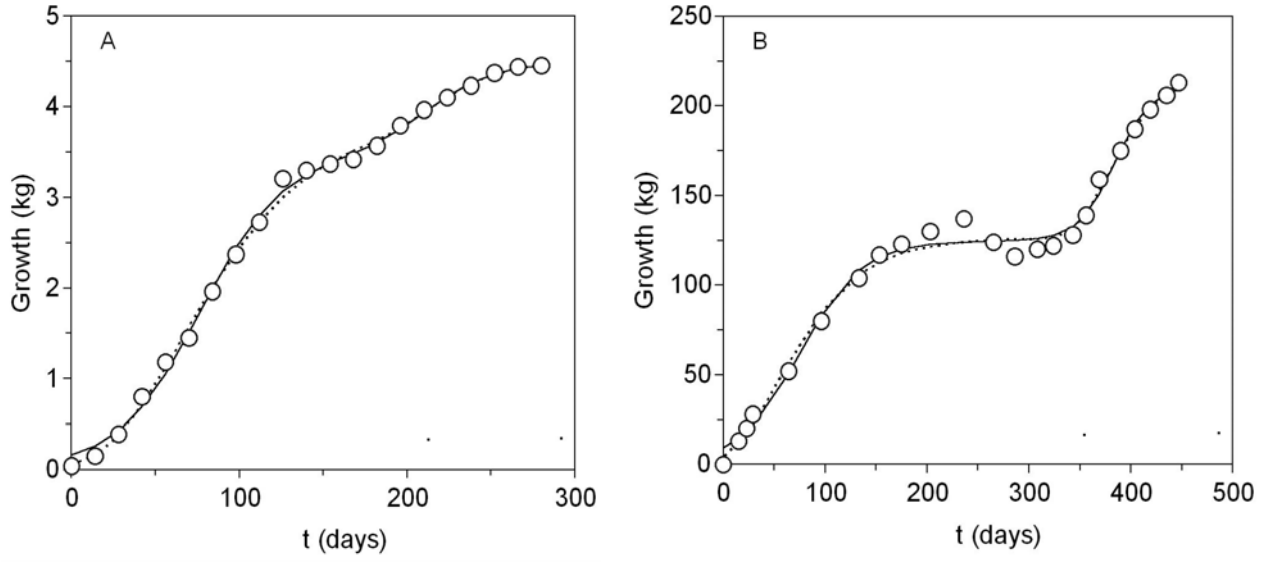
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FIGURE 3



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661 **FIGURE 4**
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Table 1

Equation	Reparameterised form 1	Parameters	Reparameterised form 2
Logistic (1)	$G = \frac{G_m}{1 + \exp[\mu_m(\tau - t)]}$	$v_m = \frac{G_m \mu_m}{4} ; t_i = \tau$ $\lambda = \tau - \frac{2}{\mu_m} ; t_m = \tau + \frac{G_m}{2v_m}$	$G = \frac{G_m}{1 + \exp\left[2 + \frac{4v_m}{G_m}(\lambda - t)\right]}$
Weibull (2)	$G = G_m \left[1 - \exp\left(-\ln 2 \left(\frac{t}{\tau}\right)^\alpha\right) \right]$	$v_{med} = \frac{G_m \alpha \ln 2}{2\tau} ; \lambda_{med} = \tau \left(1 - \frac{1}{\alpha \ln 2}\right)$ $t_i = \tau \left(\frac{\alpha - 1}{\alpha \ln 2}\right)^{1/\alpha} ; t_m = \tau \left(\frac{\alpha - 1}{\alpha \ln 2}\right)^{1/\alpha} + \frac{G_m}{2v_{med}}$	$G = \frac{2\tau v_{med}}{\alpha \ln 2} \left\{ 1 - \exp\left[-\ln 2 \left(\frac{t \left(1 - \frac{1}{\alpha \ln 2}\right)}{\lambda_{med}}\right)^\alpha\right] \right\}$
modified Hill (3)	$G = \frac{G_m t^a}{\tau^a + t^a}$	$v_{med} = \frac{G_m a}{4\tau} ; t_i = \tau \left(\frac{a-1}{a+1}\right)^{1/a}$ $\lambda_{med} = \tau \left(1 - \frac{2}{a}\right) ; t_m = \tau \left(\frac{a-1}{a+1}\right)^{1/a} + \frac{G_m}{2v_{med}}$	$G = \frac{4\tau v_{med}}{a} \left\{ 1 - \frac{\lambda_{med}^a}{\lambda_{med}^a + \left[t \left(1 - \frac{2}{a}\right)\right]^a} \right\}$
Gompertz (4)	$G = G_m \exp\left[-\exp(\mu_m(\tau - t) - 0.367)\right]$	$v_m = \frac{G_m \mu_m}{\exp(1)} ; t_i = \lambda + \frac{1}{\mu_m}$ $\lambda = \tau - \frac{1.367}{\mu_m} ; t_m = \lambda + \frac{2.35}{\mu_m}$	$G = G_m \exp\left[-\exp\left(\frac{v_m \exp(1)}{G_m}(\lambda - t) + 1\right)\right]$
Bertalanffy (5)	$G = G_m \left[1 - 0.206 \exp(\mu_m(\tau - t))\right]^3$	$v_m = \frac{4G_m \mu_m}{9} ; t_i = \lambda + \frac{2}{3\mu_m}$ $\lambda = \tau - \frac{1.148}{\mu_m} ; t_m = \lambda + \frac{3.375}{\mu_m}$	$G = G_m \left[1 - \frac{1}{3} \exp\left(\lambda + \frac{2}{3}\right) \exp\left(-\frac{9v_m t}{4G_m}\right)\right]^3$

Table 2

Mono-sigmoid profile

G :	Animal net growth, kg
t :	Time, days
G_m :	Maximum animal net growth, kg
τ :	Time required to reach 50% of maximum growth, days
μ_m :	Specific maximum net growth rate, days ⁻¹
t_i :	Time at inflection point, days
v_m :	Maximum net growth rate, kg days ⁻¹
t_m :	Time required to achieve the beginning of asymptote phase (G_m), days
λ :	Lag phase of net growth, days
v_{med} :	Maximum net growth rate obtained at the median abscissa of equations (2) and (3), kg days ⁻¹
λ_{med} :	Lag phase obtained at the median abscissa of equations (2) and (3), days
α :	Parameter related with the maximum slope of the net growth (equation (2)), dimensionless
a :	Parameter related with the maximum slope of the net growth (equation (3)), dimensionless

Bi-sigmoid profile

G :	Animal net growth, kg
t :	Time, days
G_{m1} :	Maximum animal net growth in the first sigmoid of the biphasic pattern, kg
τ_1 :	Time required to reach 50% of maximum growth in the first sigmoid of the biphasic pattern, days
μ_{m1} :	Specific maximum net growth rate in the first sigmoid of the biphasic pattern, days ⁻¹
v_{m1} :	Maximum net growth rate in the first sigmoid of the biphasic pattern, kg days ⁻¹
λ_1 :	Lag phase for the first sigmoid, days
G_{m2} :	Maximum animal net growth in the second sigmoid of the biphasic pattern, kg
τ_2 :	Time required to reach 50% of maximum growth in the second sigmoid of the biphasic pattern, days
μ_{m2} :	Specific maximum net growth rate in the second sigmoid of the biphasic pattern, days ⁻¹
v_{m2} :	Maximum net growth rate in the second sigmoid of the biphasic pattern, kg days ⁻¹
λ_2 :	Lag phase for the second sigmoid, days
G_f :	Final maximum animal net growth in the biphasic process (value of G when $t \rightarrow \infty$; $G_f = G_{m1} + G_{m2}$), kg

1 Table 3

Castellana cocks	Equation (1)	Equation (2)	Equation (3)	Equation (4)	Equation (5)
G_m	2.444±0.036	2.464±0.066	2.755±0.188	2.566±0.077	2.652±0.139
μ_m	0.039±0.002	-	-	0.025±0.002	0.020±0.003
τ	81.782±1.591	80.654±2.491	85.956±6.097	82.303±3.086	83.570±5.469
λ	31.079±2.715	-	-	27.082±3.880	25.851±5.372
v_m	0.024±0.001	-	-	0.023±0.002	0.023±0.003
t_i	81.782±1.591	70.791±3.581	63.561±4.226	67.478±2.364	59.372±3.367
t_m	132.49±3.73	125.91±7.31	128.68±9.96	122.01±6.30	195.55±21.38
λ_{med}	-	25.528±4.368	20.839±8.669	-	-
v_{med}	-	0.022±0.002	0.021±0.003	-	-
α	-	2.111±0.169	-	-	-
a	-	-	2.641±0.392	-	-
R^2_{adj}	0.9990	0.9983	0.9967	0.9979	0.9956
F -ratio	4371.9	2072.2	1074.4	1967.5	926.2
p -value	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
d -value	1.1637	0.8830	0.7230	0.8757	0.7496
A_f	1.0194	1.0341	1.0665	1.0354	1.0610
B_f	1.0087	0.9814	0.9591	0.9834	0.9701
Mos-cocks					
G_m	3.504±0.193	3.666±0.383	4.612±1.093	3.846±0.338	4.121±0.540
μ_m	0.039±0.006	-	-	0.022±0.004	0.017 (NS)
τ	78.159±4.957	78.702±8.082	95.695±25.72	81.883±8.230	86.622±13.20
λ	26.963±6.122	-	-	20.654±6.082	18.433±6.929
v_m	0.034±0.004	-	-	0.032±0.004	0.031±0.004
t_i	78.159±4.957	63.662±6.734	57.114±7.367	65.445±5.772	58.033±6.909
t_m	129.36±11.24	124.30±24.94	150.23±30.61	125.19±15.95	218.91±46.27
λ_{med}	-	18.066±8.859	2.586 (NS)	-	-
v_{med}	-	0.030±0.005	0.025±0.007	-	-
α	-	1.873±0.306	-	-	-
a	-	-	2.057±0.540	-	-
R^2_{adj}	0.9937	0.9937	0.9911	0.9934	0.9917
F -ratio	675.9	663.6	477.4	674.0	524.9
p -value	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
d -value	1.4226	1.3696	1.1670	1.4695	1.2261
A_f	1.0900	1.0789	1.0929	1.0423	1.0620
B_f	1.0351	0.9548	0.9460	0.9994	0.9726
Sasso-cocks					
G_m	4.214±0.177	4.334±0.169	5.023±0.476	4.466±0.116	4.643±0.252
μ_m	0.044±0.006	-	-	0.027±0.003	0.021±0.020
τ	68.591±3.844	68.046±2.950	75.755±8.217	69.476±3.328	71.140±4.985
λ	23.286±5.648	-	-	18.877±3.439	17.749±4.045
v_m	0.047±0.005	-	-	0.044±0.003	0.044±0.004
t_i	68.591±3.844	55.229±3.713	49.462±4.091	55.892±2.434	48.756±2.808
t_m	113.90±8.79	107.44±16.61	116.93±17.84	105.86±6.65	174.72±18.85
λ_{med}	-	15.833±4.341	8.288 (NS)	-	-
v_{med}	-	0.042±0.003	0.037±0.006	-	-
α	-	1.880±0.164	-	-	-
a	-	-	2.246±0.350	-	-
R^2_{adj}	0.9947	0.9979	0.9965	0.9977	0.9968
F -ratio	751.5	2078.7	1195.4	1922.9	1349.1
p -value	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
d -value	1.2230	2.3608	1.5596	2.2371	1.6964
A_f	1.0796	1.0403	1.0700	1.0203	1.0579
B_f	1.0431	0.9790	0.9511	1.0002	0.9625

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3 **Table 4**

Castellana capons	Equation (1)	Equation (2)	Equation (3)	Equation (4)	Equation (5)
G_m	2.374±0.033	2.393±0.087	2.791±0.261	2.518±0.111	2.621±0.194
μ_m	0.038±0.002	-	-	0.023±0.003	0.020±0.004
τ	85.739±1.468	84.624±3.307	91.464±8.928	87.054±4.520	89.076±7.877
λ	33.140±2.396	-	-	28.207±5.192	26.575±6.900
V_m	0.023±0.001	-	-	0.022±0.002	0.021±0.003
t_i	85.739±1.468	74.831±4.391	67.146±5.354	71.256±3.339	62.873±4.525
t_m	138.34±3.46	131.88±7.11	137.22±10.00	129.37±9.23	210.33±30.37
λ_{med}	-	27.552±5.554	21.394±11.67	-	-
V_{med}	-	0.021±0.002	0.019±0.004	-	-
α	-	2.139±0.213	-	-	-
a	-	-	2.611±0.491	-	-
R^2_{adj}	0.9992	0.9973	0.9945	0.9963	0.9929
$F\text{-ratio}$	5738.3	1356.5	693.9	1163.6	612.9
$p\text{-value}$	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
$d\text{-value}$	2.3483	1.1289	0.9345	1.0865	0.9095
A_f	1.0161	1.0599	1.0946	1.0585	1.0857
B_f	0.9977	0.9647	0.9442	0.9732	0.9602
Celta pigs					
G_m	159.46±6.310	184.74±33.55	265.82±90.06	187.98±16.10	215.83±31.45
μ_m	0.014±0.001	-	-	0.007±0.001	0.005±0.001
τ	208.61±8.079	232.14±40.41	335.45±131.86	236.27±20.66	269.72±41.29
λ	61.677±7.990	-	-	45.500±9.711	37.99±11.55
V_m	0.543±0.029	-	-	0.496±0.031	0.475±0.036
t_i	208.61±8.079	167.29±15.35	155.74±17.84	185.06±14.38	172.56±22.18
t_m	355.54±18.50	367.87±69.71	476.02±169.31	373.46±38.61	719.34±133.13
λ_{med}	-	31.54±24.98	-53.478 (NS)	-	-
V_{med}	-	0.460±0.064	0.342±0.088	-	-
α	-	1.670±0.246	-	-	-
a	-	-	1.725±0.361	-	-
R^2_{adj}	0.9960	0.9918	0.9904	0.9941	0.9919
$F\text{-ratio}$	1903.9	813.2	717.3	1297.5	927.5
$p\text{-value}$	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
$d\text{-value}$	1.4452	0.6683	0.6038	0.9506	0.7106
A_f	1.0776	1.1332	1.1386	1.0746	1.0936
B_f	1.0428	0.9140	0.9145	0.9909	0.9523
cattles					
G_m	775.02±25.43	801.99±10.53	878.88±20.19	794.50±13.24	806.74±7.67
μ_m	0.007±0.001	-	-	0.004±0.000	0.004±0.000
τ	363.67±27.68	350.93±8.87	374.48±13.69	354.79±13.34	353.11±7.48
λ	72.27±46.51	-	-	49.94±18.08	44.903±8.685
V_m	1.330±0.194	-	-	1.311±0.080	1.335±0.042
t_i	363.67±27.68	192.39±19.87	194.72±13.39	272.95±11.29	223.89±5.61
t_m	655.07±60.20	549.13±26.81	563.20±25.16	574.01±25.31	951.23±27.99
λ_{med}	-	-5.860 (NS)	-30.363 (NS)	-	-
V_{med}	-	1.124±0.047	1.085±0.086	-	-
α	-	1.419±0.063	-	-	-
a	-	-	1.850±0.107	-	-
R^2_{adj}	0.9893	0.9991	0.9993	0.9979	0.9994
$F\text{-ratio}$	501.1	6870.0	7123.1	2670.2	10037.0
$p\text{-value}$	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
$d\text{-value}$	0.3259	0.9015	1.1454	0.3972	1.1176
A_f	1.0828	1.0211	1.0532	1.0295	1.0124
B_f	1.0389	0.9980	0.9600	1.0123	0.9974

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6 Table 5

turkeys	Equation (1)	Equation (2)	Equation (3)	Equation (4)	Equation (5)
G_m	6.391±0.124	6.421±0.193	7.208±0.582	6.825±0.332	7.157±0.554
μ_m	0.031±0.002	-	-	0.018±0.002	0.014±0.002
τ	121.51±2.32	120.25±3.29	127.52±9.25	123.62±5.96	127.15±10.15
λ	56.155±3.127	-	-	48.235±5.737	45.238±7.003
v_m	0.049±0.002	-	-	0.046±0.004	0.045±0.005
t_i	121.51±2.32	112.42±3.60	100.99±5.22	103.38±4.35	92.809±5.960
t_m	186.87±5.12	183.79±6.46	186.39±15.71	177.83±11.73	286.12±35.47
λ_{med}	-	48.875±4.804	42.122±10.619	-	-
v_{med}	-	0.045±0.003	0.042±0.006	-	-
α	-	2.431±0.169	-	-	-
a	-	-	2.987±0.444	-	-
R^2_{adj}	0.9987	0.9946	0.9946	0.9957	0.9918
F -ratio	4389.8	970.6	970.6	1319.8	683.7
p -value	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
d -value	0.6864	0.4793	0.4793	0.5306	0.4666
A_f	1.1232	1.1692	1.3165	1.1960	1.2542
B_f	1.0858	0.8716	0.7827	0.9062	0.8346
rabbits					
G_m	3.945±0.171	4.182±0.120	4.793±0.205	4.133±0.111	4.264±0.092
μ_m	0.042±0.007	-	-	0.026±0.002	0.021±0.001
τ	63.163±4.395	64.591±2.493	72.687±4.331	63.928±2.688	65.347±2.213
λ	16.044±6.301	-	-	11.952±2.869	10.841±1.914
v_m	0.042±0.006	-	-	0.040±0.003	0.040±0.002
t_i	63.163±4.395	41.336±3.514	38.631±2.239	49.974±1.969	42.495±1.276
t_m	110.28±10.35	101.99±7.40	110.42±9.73	101.30±5.41	171.13±8.47
λ_{med}	-	3.916±3.512	-4.722 (NS)	-	-
v_{med}	-	0.034±0.002	0.031±0.003	-	-
α	-	1.536±0.091	-	-	-
a	-	-	1.878±0.130	-	-
R^2_{adj}	0.9883	0.9982	0.9987	0.9972	0.9986
F -ratio	513.1	3804.5	5066.9	2254.9	4817.9
p -value	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
d -value	0.3835	1.5154	1.9368	0.9485	1.8000
A_f	1.1639	1.0755	1.1133	1.0688	1.0430
B_f	1.1056	0.9717	0.9159	1.0423	0.9897
cats					
G_m	4.179±0.130	4.354±0.080	4.973±0.270	4.382±0.083	4.514±0.102
μ_m	0.021±0.002	-	-	0.013±0.001	0.011±0.001
τ	127.79±5.86	127.41±2.96	141.53±10.06	128.48±3.55	130.54±4.33
λ	32.518±7.987	-	-	25.494±3.567	23.913±3.562
v_m	0.022±0.002	-	-	0.021±0.001	0.022±0.001
t_i	127.79±5.86	89.390±2.003	81.198±5.266	100.83±2.60	85.837±2.485
t_m	233.06±13.54	201.85±8.06	223.45±9.10	202.54±7.06	337.40±16.28
λ_{med}	-	14.954±3.989	-0.711 (NS)	-	-
v_{med}	-	0.019±0.001	0.018±0.002	-	-
α	-	1.635±0.060	-	-	-
a	-	-	1.990±0.174	-	-
R^2_{adj}	0.9933	0.9989	0.9967	0.9983	0.9982
F -ratio	1114.2	7472.5	2419.8	4853.9	4545.9
p -value	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
d -value	0.2240	0.8764	0.5669	0.6767	0.7371
A_f	1.1160	1.0320	1.0762	1.0587	1.0344
B_f	1.0747	0.9857	0.9489	1.0291	0.9929

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10 Table 6

sheeps	Equation (1)	Equation (2)	Equation (3)	Equation (4)	Equation (5)
G_m	62.627±2.984	67.063±1.055	74.596±3.500	63.802±2.234	64.373±1.845
μ_m	0.014±0.003	-	-	0.010±0.001	0.008±0.001
τ	126.08±17.01	114.11±4.54	131.36±14.03	121.48±12.21	119.66±9.95
λ	-12.206 (NS)	-	-	-17.261 (NS)	-16.475 (NS)
V_m	0.226±0.043	-	-	0.231±0.028	0.241±0.023
t_i	126.08±17.01	89.390±2.003	11.353±10.386	84.23±10.37	62.582±7.632
t_m	264.37±37.60	270.81±27.01	243.21±21.03	221.24±22.97	383.88±36.30
λ_{med}	-	-70.094 (NS)	-100.52 (NS)	-	-
V_{med}	-	0.182±0.009	0.161±0.015	-	-
α	-	0.894±0.040	-	-	-
a	-	-	1.133±0.117	-	-
R^2_{adj}	0.9709	0.9989	0.9821	0.9806	0.9911
F -ratio	223.3	7472.5	233.0	479.7	770.1
p -value	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
d -value	0.2631	0.8764	0.2579	0.3204	0.3778
A_f	1.1171	1.0452	1.0937	1.0743	1.0586
B_f	1.0395	0.9685	0.9325	1.0198	1.0096
dogs					
G_m	33.242±1.057	33.737±0.532	34.912±0.347	33.638±0.554	33.840±0.360
μ_m	0.023±0.005	-	-	0.015±0.002	0.013±0.001
τ	113.51±10.15	111.04±4.918	109.80±2.34	110.96±5.18	109.94±3.32
λ	27.52±17.73	-	-	21.259±7.901	20.981±4.627
V_m	0.193±0.040	-	-	0.189±0.018	0.194±0.011
t_i	113.51±10.15	69.61±36.15	70.361±2.956	86.875±4.476	72.646±2.749
t_m	199.51±24.00	175.14±8.61	170.32±8.93	175.46±10.21	282.54±13.20
λ_{med}	-	5.333 (NS)	9.837±4.483	-	-
V_{med}	-	0.160±0.013	0.175±0.008	-	-
α	-	1.518±0.129	-	-	-
a	-	-	2.202±0.100	-	-
R^2_{adj}	0.9879	0.9974	0.9995	0.9970	0.9987
F -ratio	379.8	1937.1	9931.6	1596.9	4114.8
p -value	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
d -value	0.7276	1.1298	2.1438	0.8478	1.3153
A_f	1.0657	1.0342	1.0222	1.0325	1.0159
B_f	1.0242	1.0182	0.9878	1.0122	1.0044
goats					
G_m	54.705±5.462	59.650±3.694	71.321±10.275	56.106±4.242	56.905±3.657
μ_m	0.007±0.002	-	-	0.004±0.001	0.004±0.001
τ	309.51±65.01	308.63±36.87	400.61±121.93	301.79±49.70	300.66±42.85
λ	14.043 (NS)	-	-	-9.104 (NS)	-12.565 (NS)
V_m	0.093±0.028	-	-	0.091±0.018	0.093±0.015
t_i	309.51±65.01	12.399 (NS)	-46.832 (NS)	218.32±37.43	169.34±26.41
t_m	604.97±147.00	446.30±76.41	728.79±140.13	525.35±95.93	908.34±155.32
λ_{med}	-	-125.057 (NS)	-281.28 (NS)	-	-
V_{med}	-	0.069±0.010	0.052±0.021	-	-
α	-	1.027±0.113	-	-	-
a	-	-	1.175±0.221	-	-
R^2_{adj}	0.9679	0.9968	0.9912	0.9847	0.9902
F -ratio	109.0	1183.0	356.0	236.9	640.9
p -value	<0.0001	<0.0001	<0.0001	<0.0001	<0.0001
d -value	0.6798	1.8718	1.0618	0.7889	0.9334
A_f	1.1554	1.0290	1.0401	1.1035	1.0788
B_f	1.0503	0.9990	0.9854	1.0259	1.0149

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Table 7

	Equation (1)		Equation (2)		Equation (3)		Equation (4)		Equation (5)	
	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC	AIC	BIC
Castellana cocks	3.991	3.996	2.581	1.604	0.762	1.050	2.395	2.217	0.271	0.399
Mos cocks	2.779	3.217	2.662	0.451	0.646	1.300	2.762	3.204	1.152	1.828
Sasso cocks	0.073	0.107	3.540	1.979	1.347	1.749	3.241	3.806	1.800	2.358
Castellana capons	4.000	4.000	2.731	1.641	0.735	1.006	2.217	2.897	0.316	0.456
Celta pigs	3.987	3.987	0.994	0.002	0.245	1.050	2.985	2.990	1.789	1.971
cattles	0.000	0.000	2.444	1.952	2.620	3.035	1.000	1.050	3.936	3.963
turkeys	3.994	4.000	3.002	2.462	1.018	1.074	1.941	2.417	0.044	0.047
rabbits	0.000	0.000	2.134	1.211	3.566	3.620	1.006	1.790	3.294	3.379
cats	0.000	0.000	3.993	2.085	1.000	1.006	2.695	3.662	2.311	3.246
sheeps	0.000	0.000	4.000	3.988	2.695	3.662	1.007	1.007	1.993	1.993
dogs	0.000	0.000	1.813	1.061	3.999	3.999	1.192	1.940	2.997	3.000
goats	0.016	0.016	3.935	3.542	3.033	3.405	1.062	1.063	1.954	1.974

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35 Table 8

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	Mos_cocks		Foals	
	Eqs. (12) and (13)	Eqs. (14) and (15)	Eqs. (12) and (13)	Eqs. (14) and (15)
G_{m1}	3.501±0.263	3.871±0.239	124.494±5.834	126.531±5.877
μ_{m1}	0.040±0.006	0.023±0.003	0.033±0.007	0.022±0.005
τ_1	77.206±5.397	81.211±5.832	75.698±9.111	70.763±9.142
λ_1	26.941±4.885	21.014±4.613	15.416±13.066	8.545 (NS)
V_{m1}	0.035±0.003	0.032±0.003	1.033±0.213	1.023±0.192
G_{m2}	0.976±0.392	0.657±0.373	87.277±16.223	93.160±24.979
μ_{m2}	0.053±0.041	0.042 (NS)	0.055±0.026	0.032±0.018
τ_2	213.405±14.033	222.105±20.607	384.281±10.734	387.369±16.753
λ_2	175.827±33.215	189.244±28.370	347.928±15.953	344.984±14.485
V_{m2}	0.013±0.006	0.010±0.007	1.200±0.416	1.105±0.384
G_f	4.478±0.195	4.528±0.242	211.769±13.882	219.685±23.263
R^2_{adj}	0.9969	0.9971	0.9890	0.9896
$F\text{-ratio}$	1125.5	1225.5	346.0	372.3
$p\text{-value}$	<0.0001	<0.0001	<0.0001	<0.0001
$d\text{-value}$	1.3442	1.4417	1.1847	1.2263
A_f	1.0558	1.0255	1.0465	1.0533
B_f	1.0217	0.9983	0.9899	0.9843

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