# Geometrical Accuracy and Numerical Properties of High-Order Meshes

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## ABSTRACT

In the computational physics community, a consensus is forming on the superior efficiency of high-order numerical schemes for problems with high resolution requirements. However, many contributions show that a linear discretization of the geometrical model can limit the accuracy of high-order methods. Thus, it becomes necessary to develop robust and efficient methods for curvilinear mesh generation and adaptation.

In a previous work [1], a method for generating valid high-order meshes has been presented. The meshing procedure starts with a mesh that is straight-sided everywhere in the domain, except on the boundary, where high-order nodes are placed on the real geometry. An optimization procedure is used to untangle the invalid elements that often appear in the vicinity of curved boundaries, while preserving the original mesh as much as possible.

In this talk, we focus on the problem of estimating and controlling the geometrical accuracy of high-order meshes as well as their impact on the stability and accuracy of the simulation.

The geometrical accuracy of curvilinear meshes can be estimated by measuring a distance between a mesh boundary and the corresponding geometrical model entity. Distance measures defined in the literature, namely the Hausdorff and the Fréchet distances, rely on a solid mathematical basis. However, they are complex to implement, and their computational cost is prohibitive in the present context, even in an approximate form. We present an alternative estimate for the geometrical error, that is simple and computationally efficient enough to be used in 2D and 3D applications.

The curvilinear character of high-order meshes also impacts the performance of the numerical scheme. In most Finite Element methods, the functional space of approximation locally defined on a reference element is affected by curved elements in physical space, which may lead to a degraded accuracy [2]. The conditioning of the spatial discretization is also influenced, with possibly negative consequences on the maximum time step that can be used with explicit time integration schemes [1]. We put these effects in evidence and identify Jacobian-based measures that quantify them.

Finally, we describe how the existing mesh optimization procedure can be adapted to take into account the geometrical error or the numerical effects of the mesh. We illustrate the benefits of this method with several examples.

## REFERENCES

[1] T. Toulorge, C. Geuzaine, J.-F. Remacle, and J. Lambrechts, "Robust untangling of curvilinear meshes", *J. Comput. Phys.*, Vol. **254**, pp. 8-26 (2013).

[2] L. Botti, "Influence of Reference-to-Physical Frame Mappings on Approximation Properties of Discontinuous Piecewise Polynomial Spaces", *J. Sci. Comput.*, Vol. **52**, pp. 675-703 (2012).