An adaptive scheme for homogenised domains

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ABSTRACT

In this paper, we extend the concept of modelling error estimation for the homogenisation of elliptic PDEs. In order to do so, we fully acknowledge that the rapid spatial variation of microscopic diffusion constants cannot be known exactly. Therefore, we represent the microscopic diffusion coefficients as a random field. In this context, the accuracy of surrogate models, such as homogenisation schemes, can be quantified by estimating the error in the first moments of the probability density function of a quantity of interest.

We propose a way to bound the error in the two first moments, following and extending the seminal work of [1]. Our derivations rely on the Constitutive Relation Error[2] (CRE), which states that a certain distance between the solutions delivered by the primal and a dual surrogates of the original stochastic problem is equal to some measure of the exact and unaffordable errors. We further assume that these surrogates are deterministic, consistently with the theory of homogenisation. Minimising the CRE in this subset of homogenisation schemes leads us to an optimal surrogate that is closely related to the classical Voigt and Reuss models. This result is used in a goal-oriented setting to establish upper and lower bounds for the first two moments of the quantity of interest.

We show that the method respect the numerical separation of scales, and is therefore affordable and easy to implement, and that it produces useful results as long as the mismatch between the diffusion coefficients of the microstructure remains small. We will propose extensions for the case of high mismatch, by allowing the surrogate solutions to fluctuate in the stochastic domain.

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