Interpolation of the inverse of parameter dependent operator for preconditioning parametric and stochastic equations.

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ABSTRACT

When solving partial differential equations with parametrized (or random) coefficents, one usually needs to solve high dimensional problems for a large number of realizations of the coefficients, which is computationaly expensive. Model reduction techniques such as the Reduced Basis Method [4] or the Proper Generalized Decomposition [2] are now comonly used for the construction of approximation of the solution of such problems. The idea is to build a subspace on which a projection of the solution can be computed with a low computational cost. From a practical point of view, that subspace is constructed so that it minimizes some norm of the residual associated with the equation. In practice, we observe that a bad condition number of the operator leads to a poor approximation : preconditioning is necessary to achieve efficient model reduction.

There exist in the literature different definitions for the preconditioner. A widely used preconditioner is the inverse of the operator at a given parameter value [4], or the inverse of the mean operator in the context of uncertainty quantification. In [1], the authors propose an analytical interpolation of the inverse of the operator, and show the benefits of using a parameter dependent preconditioner.

We propose here different interpolation methods of the inverse of the operator for the construction of the preconditioner, and compare them. In particular, we show that the interpolation based on the projection of the identity matrix [3] with respect to the Frobenius norm seems to be the most appropriated strategy. In addition, we introduce a greedy algorithm for the construction of the preconditioner: the corresponding set of interpolation points results in a better preconditioner. Finally, numerical examples show that the quality of the model reduction (Reduced Basis or PGD) is significantly better when using the proposed preconditioner.

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