

Accelerating Hilbert-Einstein universe without dynamic dark energy

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By using an unmodified Einstein gravity theory it is shown that all of the speeding-up effects taking place in the current universe are entirely due to the quantum effects associated with the background radiation or to the combination of such effects with those derived from the presence of a cosmological constant, without invoking any dynamic dark energy component. We obtain that in both cases the universe accelerates at a rate slightly beyond what is predicted by a cosmological constant but does not induce any big rip singularity in the finite future.

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Nowadays theoretical cosmology appears to confront a rather puzzling situation. Whereas current observations seem to point to less than -1 values for the most probable estimates of the parameter w of the equation of state [1], the most popular theoretical quintessence models are plagued with violent instabilities, violations of the energy conditions, future singularities, so as unphysical scalar field negative kinetic terms, or ghosts, for $w < -1$ [2]. Exorcizing procedures have been therefore considered [3] in order to justify why values $w < -1$ can be compatible with usual quintessence models, or alike, that actually correspond to greater than or equal to -1 w -parameters. None of such procedures have however been successful so far [4]. Time-dependent equations of state associated with tracked quintessence scenarios [5] have also been analyzed to solve the above puzzle but they turned out to fail, too. Even more unsuccessful have been several cosmological models based on the idea that dark energy is not necessary to predict cosmic acceleration. Included among these approaches are, on the one hand, those models using modified theories of gravity whose action integral contains extra terms to the familiar Hilbert-Einstein action [6] and, on the other hand, some descriptions in which late-time acceleration could be explained by inhomogeneities produced during primordial inflation [7]. The first kind of such scenarios is in turn plagued with theoretical inconsistencies and instabilities, and cannot be accommodated with cosmological observations and solar physics experiments [8]. The second type of these scenarios is most interesting in that, besides avoiding the concourse of any mysterious dark energy fluid or field, they do not invoke any modifications of gravity and hence become most economical. Unfortunately, it has been shown [9] that to second order in spatial gradients, the corrections are unable to account for the observed speed-up of the cosmic expansion.

In this letter we shall look at current acceleration by using the same general economical philosophy as in the last kind of the above models, even without invoking, moreover, any effects induced at the primordial inflationary period. The sole ingredients which we shall explicitly include, besides general relativity, are the quantum effects on the trajectories of the particles that make up the background radiation. Such effects will be modeled through the relativistic generalization of the original sub-quantum potential formalism by Bohm [10] and lead by themselves to an accelerating expansion which, consistently, goes beyond what is predicted by a cosmological constant. Thus, we use a version of the sub-quantum model for dark energy [11] which will in principle be motivated by an up-grading-to-scalar field method stemming from the analogy with the classically-interpreted Hamilton-Jacobi equation derived from the Klein-Gordon wave equation for a quasi classical wave function $\Psi = R \exp(iS/\hbar)$, i.e.

$$E^2 - p(v)^2 + \tilde{V}_{SQ}^2 = m_0^2, \quad (1)$$

where

$$\tilde{V}_{SQ} = \hbar \sqrt{\frac{\nabla^2 R - \ddot{R}}{R}} \quad (2)$$

is the sub-quantum potential, $v = \dot{q}(t)$ and $p = \partial \tilde{L} / \partial \dot{q}$, with $\dot{} = d/dt$ and \tilde{L} being the Lagrangian

$$\tilde{L} = \int d\dot{q} p = \int dv \sqrt{\tilde{V}_{SQ}^2 - m_0^2 + \frac{m_0^2}{1 - v^2}}. \quad (3)$$

As shown first by Bagla, Jassal and Padmanabhan [12] for the fully classical case and later on by one of the present authors [11] for the case that the Lagrangian contains a sub-quantum potential, upgrading the quantities entering this simple Lagrangian to their field-theory counterparts actually leads to a cosmological tachyonic model which can be used to predict cosmic acceleration. In order to tentatively motivate our cosmic model, following Ref. [11], we shall replace then the quantity q for a scalar field ϕ , the quantity $\dot{q}^2 \equiv v^2$ for $\partial_i \phi \partial^i \phi \equiv \dot{\phi}^2$ and the rest mass m_0 for the potential $\tilde{V}(\phi)$. With these replacements and leaving \tilde{V}_{SQ} constant for the moment, we can then integrate Eq. (3) to have for the field Lagrangian $\tilde{L} = -\tilde{V}(\phi) E(x(\phi), k(\phi))$, with $E(x, k)$ the elliptic integral of the second kind, $x(\phi) = \arcsin \sqrt{1 - \dot{\phi}^2}$ and $k = \sqrt{1 - \tilde{V}_{SQ}^2 / \tilde{V}(\phi)^2}$. At first sight one should also up-grade \tilde{V}_{SQ} to depend on ϕ . However, it will be seen later that such a up-grading would lead to a final expression for \tilde{V}_{SQ} which depends only on $\dot{\phi}$, a dependence that disappears because for the present model it is necessary that $\dot{\phi}$ be constant in order to avoid divergences.

Even though the up-grading-to-field method has been so far used to just suitably motivate the introduction of a cosmic field model, such a method will be in the present scenario shown to be more than a mere motivating procedure devoid of any physical significance [13]. Actually, even after up-grading, the above model can still be interpreted as physically describing pure background radiation equipped with a sub-quantum potential, taking dark energy to be nothing but the effect left in the classical universe by that sub-quantum potential, provided the following two conditions are fulfilled by the field theory that results after up-grading: (1) the field potential $\tilde{V}(\phi)$ is identically

equal to zero, and (2) the time derivative of the scalar field becomes $\dot{\phi}^2 = 1$. In fact, since the sub-quantum potential has not been up-graded to any field-dependent quantity, if such two conditions are either shown to hold or imposed, then the up-grading process can readily be seen to be equivalent to a identity operation, leaving the original particle theory essentially unchanged; i.e. the radiation particles and the sub-quantum potential can also be regarded as the unique physically relevant ingredients for the model. We note that in a FRW framework $\dot{\phi}^2 = 1$ necessarily implies $\phi = q$, and hence the second condition amounts to $V(\phi) = V(q) = m_0 = 0$, so that, restoring the speed of light as c , we have $\dot{q}^2 = v^2 = c^2$. In any event, in what follows we shall eliminate any trace of all classical quantities from our model, thereby representing dark energy by solely the sub-quantum potential, a hidden quantity that has not been up-graded and that by itself should necessarily be associated with the particles described by Lagrangian (3), not with any field quantity. Thus, the resulting dark-energy scenario would not have any classical analog. It follows that the condition that we have to impose to the scalar field theory derived in the sub-quantum model [11] to satisfy the requirement that dark energy disappears once we erase any trace of the background quantum effects is that the Lagrangian, energy density and pressure turn all out to only depend on the sub-quantum potential and will all vanish in the limit where any possible cosmological constant and the sub-quantum potential are both zero, i.e. $\Lambda \rightarrow 0$, $\tilde{V}_{SQ} \rightarrow 0$. It will be seen in what follows that the above conditions are all fulfilled provided that we start with a Lagrangian density given by

$$L = -V \left(E(x, k) - \sqrt{1 - \dot{\phi}^2} \right), \quad (4)$$

where again $x = \arcsin \sqrt{1 - \dot{\phi}^2}$ and now $k = \sqrt{1 - V_{SQ}^2/V^2}$, with $V \equiv V(\phi)$ the density of potential energy associated to the field ϕ . We do not expect \tilde{V}_{SQ} to remain constant along the universal expansion but to increase like the volume of the universe V does. It is the sub-quantum potential density $V_{SQ} = \tilde{V}_{SQ}/V$ appearing in Eq. (4) what should be expected to remain constant at all cosmic times. In fact, from the imaginary part of the Klein-Gordon equation applied to the wave function Ψ we can get $v \cdot \nabla R - \dot{R}$ and hence the continuity equation for the probability flux $J = \hbar \text{Im}(\Psi^* \nabla \Psi)/(mV)$, $\nabla \cdot J - \dot{P} = 0$, where P is the probability density $P = \text{Probability}/V$. This continuity equation is the mathematical equivalent of a probability conservation law. Up-grading then the velocity v to $\dot{\phi}$ and noting that $\dot{\phi} = \pm 1$ (see later) it follows that $(\nabla^2 R - \dot{R})/R = (\nabla^2 P - \dot{P})/(2P)$, with $P = R^2$. Assuming that the particles move locally according to some causal law [10], one can now average Eq. (1) with the probability weighting function $P = R^2$, so that one obtains for the averaged sub-quantum potential squared, $\langle \tilde{V}_{SQ}^2 \rangle_{av} = \int \int \int dx^3 P \tilde{V}_{SQ}^2 = \hbar^2 \int \int \int dx^3 (\nabla^2 P - \dot{P}) \equiv \hbar^2 (\langle \nabla^2 P \rangle_{av} - \langle \dot{P} \rangle_{av})$. Since the universe is isotropic and homogeneous, the corresponding cosmic conserved quantity can then be obtained by simply taking $\langle \tilde{V}_{SQ}^2 \rangle_{av}^{1/2}/V = \langle V_{SQ}^2 \rangle_{av}^{1/2}$, that is, renaming for the aim of simplicity all the quantities $\langle f^2 \rangle_{av}^{1/2}$ involved in the averaged version of Eq. (1) as f , we can again derive Eq. (4), now with V_{SQ} a constant conserved quantity when referred to the whole volume V of the isotropic and homogeneous universe.

It is easy to see that in the limit of vanishing V_{SQ} , $VE(x, k)$ reduces to $\sqrt{1 - \dot{\phi}^2}$ so that the Lagrangian (4) vanishes as required. The pressure and energy density are then obtained from Eq. (4) to read

$$p_\phi = -V \left(E(x, k) - \sqrt{1 - \dot{\phi}^2} \right) \quad (5)$$

$$\rho_\phi = V \left(\frac{\sqrt{\dot{\phi}^2 + \frac{V_{SQ}^2}{V^2} (1 - \dot{\phi}^2)} \dot{\phi}}{\sqrt{1 - \dot{\phi}^2}} + E(x, k) - \frac{1}{\sqrt{1 - \dot{\phi}^2}} \right), \quad (6)$$

where we have considered $V \equiv V(\phi)$. In any case, for a source with parameter $w(t) = p_\phi/\rho_\phi$ we must always have

$$\frac{\dot{\rho}_\phi}{\rho_\phi} = -3H(1 + w(t)) = \frac{2\dot{H}}{H}. \quad (7)$$

By itself this expression can generally determine the solution for the scale factor $a(t)$, provided $w = \text{const}$. In such a case, we obtain after integrating Eq. (7) for the scale factor

$$a = \left(a_0^{3(1+w_0)/2} + \frac{3}{2}(1+w_0)\kappa t \right)^{2/[3(1+w_0)]},$$

in which a_0 is the initial value of the scale factor and κ is a constant. However, we shall not restrict ourselves in this letter to a constant value for the parameter w of the equation of state but leave it as a time-dependent parameter whose precise expression will be determined later on. Combining now Eq. (7) with the expression for $w(t)$ we can then obtain an expression for $d(H^{-1})/dt$ by using Eqs. (5) and (6) as well. Moreover, multiplying Eqs. (5) and (6) and using Eq. (7), a relation between the potential density V and the elliptic integral E can be derived from the Friedmann equation $H^2 = 8\pi G\rho_\phi/3$. These manipulations allow us to finally obtain

$$E = - \left[\frac{A(\dot{\phi}, V, V_{SQ})\dot{\phi} \left(1 + \frac{3H^2}{2\dot{H}}\right) - 1 - \frac{3H^2\dot{\phi}^2}{2\dot{H}}}{\sqrt{1 - \dot{\phi}^2}} \right] = - \left\{ \frac{\frac{3H^2\dot{\phi}^4 V_{SQ}^2}{\dot{H}} - \left(\frac{2\dot{H}}{G}\right)^2 (1 - \dot{\phi}^2) + \dot{\phi}^2 V_{SQ}^2 (1 + \dot{\phi}^2)}{\sqrt{1 - \dot{\phi}^2} \left[\left(\frac{\dot{H}}{4\pi G}\right)^2 - \dot{\phi}^2 V_{SQ}^2 \right]} \right\} \quad (8)$$

with $A(\dot{\phi}, V, V_{SQ}) = \sqrt{\dot{\phi}^2 + \frac{V_{SQ}^2}{V^2}(1 - \dot{\phi}^2)}$, and

$$V = - \frac{2\pi G \sqrt{1 - \dot{\phi}^2}}{\dot{H}\dot{\phi}^2} \left[\left(\frac{\dot{H}}{4\pi G}\right)^2 - \dot{\phi}^2 V_{SQ}^2 \right]. \quad (9)$$

Thus, simple general expressions for the energy density and pressure can be finally derived to be

$$\rho_\phi = 6\pi G \left(\dot{H}^{-1} H \dot{\phi} V_{SQ} \right)^2 \quad (10)$$

$$p_\phi = -4\pi G \dot{H}^{-1} \dot{\phi}^2 V_{SQ}^2 \left(1 + \frac{3H^2}{2\dot{H}} \right) = w(t)\rho_\phi, \quad (11)$$

where

$$w(t) = - \left(1 + \frac{2\dot{H}}{3H^2} \right). \quad (12)$$

The Friedmann equation $H^2 = 8\pi G\rho_\phi/3$, derived from the action integral with the Lagrangian (4), corresponds to a universe dominated by sub-quantum energy. Using Eq. (10) this Friedmann equation leads to

$$\dot{H} = \pm 4\pi G \dot{\phi} V_{SQ}, \quad (13)$$

with a slowly-varying $w(t)$ that should probably be quite close, but still less than -1 (that is, the case that current observations each time more clearly are pointing to [1]). We have also

$$H = \pm 4\pi G \phi V_{SQ} + C_1, \quad (14)$$

with C_1 an integration constant. Note that from Eqs. (9) and (13) it follows that $V(\phi) = 0$, which is just one of the two conditions required to make consistent our interpretation. Moreover, if we assume that $\dot{\phi}$ is constant (an assumption which would indeed be demanded by the fact that $v^2 = 1$ for radiation), then from the equation of motion that corresponds to the Lagrangian for the field ϕ alone [12] $\ddot{\phi} + (1 - \dot{\phi}^2)(3H\dot{\phi} + dV/Vd\phi) = 0$, we have $\dot{\phi}^2 = +1$. Actually, from the Lagrangian density $L_{SQ} = -V(\phi)E(x, k)$ we can also obtain,

$$\dot{\phi}\ddot{\phi} = (1 - \dot{\phi}^2) \left\{ -3H \left[\dot{\phi}^2 + \frac{V_{SQ}^2}{V(\phi)^2}(1 - \dot{\phi}^2) \right] + \sqrt{1 - \dot{\phi}^2} \sqrt{\dot{\phi}^2 + \frac{V_{SQ}^2}{V(\phi)^2}(1 - \dot{\phi}^2)} \frac{\partial L_{SQ}}{V(\phi)\partial\phi} - \frac{\partial V}{V\partial\phi} \dot{\phi}^3 \right\},$$

from which we again derive the conclusion that $\ddot{\phi} = 0$ implies $\dot{\phi}^2 = 1$. Indeed, the assumption that $\dot{\phi}^2 = 1$ can be really regarded as a regularity requirement for $\ddot{\phi}$ because if $\dot{\phi}^2 \neq 1$ then $\ddot{\phi}$ would necessarily diverge since $V(\phi)$ vanishes even when $\dot{\phi}^2 \neq 1$, as it can be checked from Eqs. (9) and (13). The same result can then be obtained from the equation of motion derived from Lagrangian (4). Hence a vanishing $\ddot{\phi}$ implies that strictly $\dot{\phi}^2 = 1$ and since in addition $V = 0$ the present model can be interpreted to describe the cosmic sub-quantum effects necessarily associated with an isotropic

and homogeneous sea of bosonic particles with zero rest mass which move at the speed of light, i.e. photons - identifying that photon sea with the CMB is just a reasonable assumption. It then follows that the condition $\dot{\phi}^2 = \dot{q}^2 = 1$ becomes a regularity requirement, and the condition $V(\phi) = V(q) = m_0 = 0$ results from the combined effect of the Friedmann equations and the very nature of the model. We have now $\rho_\phi = \rho_q = 6\pi G(\dot{H}^{-1}HV_{SQ})^2 = p_q/w(t)$ which in fact does not depend on any field quantity, such as it was required for interpreting dark energy as the sub-quantum energy associated with radiation particles. The use of an up-grading-to-field motivating method becomes thus rather superfluous in the present theory. We had indeed obtained identical results and conclusions if we had replaced ϕ and $V(\phi)$ for q and m_0 , respectively, leaving V_{SQ} unchanged, in Eqs. (4) - (14).

It follows then

$$H = \pm 4\pi GV_{SQ}t + C_0, \quad (15)$$

in which C_0 is another integration constant, and for the scale factor

$$a_\pm = a_0 e^{\pm 2\pi GV_{SQ}t^2 + C_0 t}. \quad (16)$$

The solution a_- would predict a universe which initially expands but that immediately started to contract, tending to vanish as $t \rightarrow \infty$. An always accelerating solution slightly beyond the speeding-up predicted by a De Sitter universe is given by the scale factor a_+ . In what follows we shall consider the latter solution as that representing the evolution of our current universe and restrict ourselves to deal with that solution only for the branch $t > 0$, denoting $a_+ \equiv a$ and taking then H and \dot{H} to be definite positive.

Thus, the time-dependent parameter of the equation of state will be given by

$$w(t) = -1 - \frac{8\pi GV_{SQ}}{3(4\pi GV_{SQ}t + C_0)^2}, \quad (17)$$

which takes on values very close, though slightly less than -1 on the regime considered so far.

Notice that in the limit $V_{SQ} \rightarrow 0$, H becomes a constant $H_0 = C_0$, and hence $\rho_\phi \rightarrow 3C_0/(8\pi G)$ and $w \rightarrow -1$. Clearly, $H_0^2 = \Lambda$ must be interpreted as the cosmological constant associated with the De Sitter solution $a = a_0 e^{H_0 t}$. When we set $C_0 = 0$ instead, then all remaining quantities have the following limiting values

$$\rho_\phi = \frac{p_\phi}{w(t)} = 6\pi GV_{SQ}^2 t^2 \rightarrow 0, \quad (18)$$

$$w(t) = -1 - \frac{1}{6\pi GV_{SQ}t^2} \rightarrow -\infty \quad (19)$$

and

$$a = a_0 e^{2\pi GV_{SQ}t^2} \rightarrow a_0, \quad (20)$$

as $V_{SQ} \rightarrow 0$. That is precisely the result we wanted to have and means that all the cosmic speed-up effects currently observed in the universe can be attributed to the purely sub-quantum dynamics that one can associate to the background radiation, rather than to the presence of a dark energy component or any modifications of Hilbert-Einstein gravity. In fact, it can be readily checked that the expression obtained for \dot{H} inexorably leads to a vanishing value for the potential $V(\phi)$, and hence to $\dot{\phi}^2 = 1$, which correspond to pure radiation. Consistency for the present theory is ensured by noticing that: (i) $\dot{\phi}^2 = 1$ does clearly satisfy the Friedmann equation $H^2 = 8\pi G\rho$, with $\rho = 6\pi G(\dot{H}^{-1}HV_{SQ})^2$ and that for the field ϕ from which that condition was derived, and (ii) if we substitute $\dot{\phi}^2 = \dot{q}^2 = 1$ and $V(\phi) = V(q) = 0$ back into Eqs. (5) and (6) and we use Eqs. (8) and (9), we recover the regular values for energy density and pressure given by Eqs. (10) and (11) for $\dot{\phi}^2 = \dot{q}^2 = 1$, which in fact show no dependence whatsoever on any field quantity.

The result that, if there is not constant cosmological term, then they are the considered sub-quantum effects associated with the background radiation which are responsible for a current accelerating expansion of the universe that goes beyond the cosmological constant limit, implies, on the other hand, that (i) the parameter of the equation of state is necessarily less than -1, though probably very close to it, (ii) the energy density increases with time, (iii) $\rho_\phi + p_\phi < 0$, that is the dominant energy condition is violated, and (iv) the kinetic term of the equivalent field theory turns out to be $\dot{\phi}^2 > 0$. Whereas the first three properties are shared by the so called phantom models [2], unlike such models, the fourth one guarantees stability of the resulting universe because $V(\phi) = 0$. Also unlike the usual phantom scenarios, the present model does not predict, moreover, any big rip singularity in the future. Finally, the considered quantum effects may justify violation of the dominant energy condition.

On the other hand, if we place a Schwarzschild black hole with initial mass M_0 in the universe described by the suggested model, the mechanism advanced by Babichev, Dokuchaev and Eroshenko [14] would imply that the black hole will accrete this sub-quantum phantom energy so that it would progressively lose mass down to finally vanish at $t = \infty$, according to the equation

$$M = \frac{M_0}{1 + \pi^2 DV_{SQ} M_0 t}, \quad (21)$$

with D a constant. If we place a Morris-Thorne wormhole with initial throat radius b_0 instead, the corresponding accretion mechanism [15] leads now to a progressive increase of the wormhole size governed by

$$b = \frac{b_0}{1 - \pi^2 D' V_{SQ} b_0 t}, \quad (22)$$

with D' another constant, bringing us to consider the existence of a big trip process [15] by which, relative to an asymptotic observer at $r = \infty$, the wormhole will quickly grows up to engulf the universe itself, blowing up at a finite time in the future given by

$$\tilde{t} = \frac{1}{\pi^2 D' V_{SQ} b_0}. \quad (23)$$

In this case, on times $t > \tilde{t}$ the wormhole converts into an Einstein-Rosen bridge which decays into a black hole plus a white hole that will in this case progressively lose mass to vanish at $t = \infty$ [15]. This result holds both for a static wormhole metric and when the throat radius is allowed to be time-dependent [15].

Before closing up we shall briefly consider solution a_- . As it has already been pointed out before, if $C_0 = H_0 = \Lambda^{1/2} \gg \sqrt{4\pi G V_{SQ}}$, then this solution corresponds to an initial period of accelerating expansion with an equation-of-state parameter w greater, though very close to -1. This situation would stand until a time

$$t_a = \frac{H_0 - \sqrt{4\pi G V_{SQ}}}{4\pi G V_{SQ}}, \quad (24)$$

which corresponds to $w = -1/3$. After t_a the universe would keep expanding but now in a decelerating way until a time

$$t_c = \frac{H_0}{4\pi G V_{SQ}}, \quad (25)$$

after which the universe entered a contracting phase which would be maintained until $t = \infty$. If $H_0 \leq \sqrt{4\pi G V_{SQ}}$, then the present model would no longer be valid.

It could be at first sight thought that the universe might now be in the phase $t < t_a$ of solution a_- , but current constraints on w [1] seem to preclude that it can be greater than -1. Perhaps another argument against solution a_- be the fact that for this kind of solution, while the accretion of the sub-quantum energy onto a Morris-Thorne wormhole leads to a progressive decrease of the wormhole size according to the law $b = b_0/(1 + \pi^2 D' V_{SQ} t)$, the size of a black hole of initial mass M_0 will progressively increase with sub-quantum energy accretion so that $M = M_0/(1 - \pi^2 DV_{SQ} M_0 t)$. In this way, at a time $t_* = 1/(\pi^2 DV_{SQ} M_0)$ the black hole would blow up. Clearly, for a supermassive black hole at a galactic center one would then expect that by the present time the black hole had grown up so big that its astronomical effects would be probably observable.

All the above results have been obtained in the case that the energy density associated with the sub-quantum potential would dominate over any other type of energy. More realistic models where contributions from dark and observable matters are taken into account as well will be considered elsewhere.

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