

# History and Coordination Failure \*

**Alicia Adserà**

Universitat Autònoma de Barcelona

**Debraj Ray**

Boston University and Instituto de Análisis Económico (CSIC)

This version published in the *Journal of Economic Growth* **3**, 267–276.

## Abstract

An extensive literature discusses the existence of a virtuous circle of expectations that might lead communities to Pareto-superior states among multiple potential equilibria. It is generally accepted that such multiplicity stems fundamentally from the presence of positive agglomeration externalities. We examine a two-sector model in this class, and look for intertemporal perfect foresight equilibria. It turns out that under some plausible conditions, positive externalities must coexist with external *diseconomies* elsewhere in the model, for there to exist equilibria that break free of historical initial conditions. Our main distinguishing assumption is that the positive agglomeration externalities appear with a time lag (that can be made vanishingly small). Then, in the absence of external diseconomies elsewhere, the long-run behaviour of the economy resembles that predicted by myopic adjustment. This finding is independent of the degree of forward-looking behavior exhibited by the agents.

---

\*We are grateful to Rahul Roy and an anonymous referee for comments that improved the final draft. Ray gratefully acknowledges support under Grant SBR-9709254 from the National Science Foundation. Address correspondence to: Debraj Ray, Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215. Email: [aadsera@volcano.uab.es](mailto:aadsera@volcano.uab.es) and [debraj@bu.edu](mailto:debraj@bu.edu)

# 1 Introduction

An extensive literature discusses the existence of a virtuous circle of expectations that might lead communities to Pareto-superior states among multiple potential equilibria. It is generally accepted that such multiplicity stems fundamentally from the presence of positive agglomeration externalities. The purpose of this paper is to examine the role of agglomeration externalities in a fully dynamic model in which agents exhibit perfect foresight. Under plausible conditions that we describe below, we show that the generation of equilibria that break free of initial historical conditions must critically depend on the presence of *congestion* externalities elsewhere in the model.

The role of externalities in the process of economic development has occupied a central place in theories of growth and development. Perhaps the first study along these lines is due to Paul Rosenstein-Rodan [1943]. Authors such as Tibor Scitovsky [1954], Albert Hirschman [1958] and Gunnar Myrdal [1957] developed these notions further. More recently, Murphy, Shleifer, and Vishny [1989] have formalized some aspects of the Rosenstein-Rodan viewpoint, making precise the conditions that are needed for multiplicity of equilibria. The study of such multiplicity exhibits agglomeration externalities in its own right: a recent issue of the *Journal of Development Economics* concentrated entirely on this topic.

By and large, this literature ignores a central question raised by Rosenstein-Rodan and Hirschman: how does an economy “move from a bad to a good equilibrium”? We place this entire phrase in quotes because it is imprecise: so called transitions from one “equilibrium” to another must themselves be viewed as the equilibria of some encompassing intertemporal process. But this question is problematic for game theorists (and *a fortiori*, for applied economists as well). In pure coordination games (such as those studied by Cooper and John [1988])<sup>1</sup> which essentially underly these models, where is the role of history? Why would an initial coordination failure transmit itself, or persist, over time? This is the fundamental issue raised (though not answered) by Rosenstein-Rodan and Hirschman.

We study this issue by placing the static coordination game in the explicit context of an intertemporal process. Our two-sector migration model has already served as the canonical parable in part of this literature. In one of the sectors (the “traditional sector”), the return to each agent is exogenously given. In the other, “modern” sector, there are agglomeration externalities: the return to each agent depends positively on the number of agents located there. Viewed as a static model of locational choice, the model exhibits two equilibria: one in which everyone is located in the traditional sector, and the other in which everyone is located in the modern sector. Viewed as a dynamic model of locational choice, agents take the intertemporal paths of returns in each sector as given, and then make rational migration decisions to maximize the sum of discounted utility.

---

<sup>1</sup>See also Diamond [1982], Bryant [1993], or Chatterjee and Cooper [1989, 1990].

Such decisions might include, in principle, the option to move back and forth several times. Each migratory step is costly. A perfect foresight equilibrium has the additional property that the joint migration decisions generate precisely the intertemporal path of returns that each agent takes as given.

A similar model has been used by Matsuyama [1991] and Krugman [1991a]<sup>2</sup> to show discuss the role played by discounting in the generation of ahistorical equilibria. If agents discount the future heavily enough, then the intertemporal equilibrium resembles the path obtained through myopic tatonnement. Initial conditions (and thereafter, the current discrepancy in the intersectoral rates of return) determine migratory flows. The sector with the initial advantage will come to dominate. On the other hand, for discount rates close to zero, other perfect foresight equilibria appear. These equilibria can break free of initial conditions, provided that agents harbor common expectations of optimism (or pessimism) about the eventual fate of the modern sector.

There is merit and insight to these arguments. They certainly allow for outcomes that we do sometimes see: a burgeoning, self-fulfilling move away from one type of activity (or location, or technology) to another. Our objective is to make explicit an aspect of these models, and possibly of the coordination framework in general, that remains hidden in the literature that we have seen. The observation is this: *to generate equilibria that break free of historical conditions, negative agglomeration externalities must be present somewhere, in addition to the positive agglomeration externalities that are needed to create the problem in the first place.* Most importantly, this result is *independent of the size of the degree of forward-looking behavior exhibited by the economic agents*, as captured by their discount factors.

To make this point, we make an assumption that we believe to be eminently plausible. We assume that the positive agglomeration externalities must manifest themselves with a time lag.<sup>3</sup> We do not restrict the size of this lag in any way.<sup>4</sup>

In this model, for any positive discount rate, if relocation costs are constant and independent of the intersectoral allocation of agents, the final outcome of any perfect foresight equilibrium depends entirely on initial conditions. The equilibrium paths turn out to be (essentially) the same as if agents were short-sighted. The same result is true if the relocation cost depends *negatively* on the number of agents in the destination sector (which amounts to positive externalities as well). Thus, paradoxically enough, equilibria which break free of initial history

---

<sup>2</sup>See also Fukao and Bénabou [1993] in this context, and in a related scenario, Chamley and Gale [1994] and Gale [1993].

<sup>3</sup>Empirical work on such lags includes the study by Henderson [1994], which reveals the specific lag structure at which changes in own-industry employment affect growth in some manufacturing sectors (at the metropolitan level). Literature on technology investment also stresses the fact that benefits from investment may take time to flow (Farrel and Saloner 1985; Katz and Shapiro 1986, 1992).

<sup>4</sup>To be more precise, our assumption of a continuum of agents (as in the Matsuyama and Krugman studies) allows us to handle arbitrarily small lags. If this is not clear now, it will be as soon as we state our main result.

can exist only if there are diseconomies elsewhere. In this model, such equilibria might exist if relocation costs depend *positively* on the number of agents in the destination sector (congestion). We do not construct such equilibria for our model, as this is not our main focus, but it is easy enough to do so along the lines discussed by Krugman and Matsuyama.

Once stated, the intuition behind such a result is easy enough to see. Suppose, for instance, that the rate of return in the traditional sector is initially higher, and then lower, along some equilibrium path. This implies that some migration must have occurred from the traditional to the modern sector. But because externalities are lagged, the initial migration must have occurred when current returns were unfavorable. Such migrants can benefit from postponing their migration decisions, and they will certainly do so if the cost of relocation is not adversely affected in the process. But this means no one will migrate in this interim period, and so the positive externalities can never be generated in the first place. With diseconomies in relocation costs, this argument is no longer valid and such equilibria can be constructed (under some conditions).

In describing myopic tatonnement, Krugman [1991a, p. 657] writes: “The usefulness of this kind of heuristic approach to dynamics for thinking about models is so great that we would not propose abandoning it.” In this paper, we argue that the prediction of myopic tatonnement may have more than just heuristic appeal on its side. In economies of atomistic agents, the path of myopic adjustment may still be the most likely equilibrium outcome if no congestion cost is added to the model.

## 2 The Model

An economy has two regions,  $A$  and  $B$ . A total capital (or labor) endowment of  $\bar{K}$  is split at date 0 between the two regions. Denote by  $K$  the capital in region  $B$ , so that  $\bar{K} - K$  is the capital stock in region  $A$ . Capital invested in Region  $A$  yields a fixed rate of return, normalized to zero. Region  $B$ 's rate of return  $r$  is taken to depend positively on its capital endowment:

$$r = f(K) \tag{1}$$

where  $f$  is continuous, strictly increasing, and  $f(0) < 0 < f(\bar{K})$ .

Imagine that there is a continuum of agents, and each agent owns a single unit of capital. Capital is free to move between regions but each relocation entails a nonnegative cost. Assume that the cost of moving from  $B$  to  $A$  is given by a nondecreasing function  $\hat{c}_A(K)$  and that the cost of moving from  $A$  to  $B$  is given by a nonincreasing function  $\hat{c}_B(K)$ , where  $K$  is to be interpreted, in each case, as the population of capital in the destination region. Say that either of these cost functions exhibits *congestion* if it increases, at least over some interval, in  $K$ . Agents make decisions in the way described below.

First let us track the relevant prices. Let  $\gamma \equiv \{r(t), c_A(t), c_B(t)\}_{t=0}^{\infty}$  be some point expectation about the path (measurable in time) of returns and relocation costs in each region. Future returns are discounted in the standard way, using a discount rate  $\rho$ . Denote by  $V(\gamma, i, t)$  the *optimal value* to an agent in region  $i$ ,  $i = A, B$ , beginning at time  $t$ , when the commonly anticipated path of returns is  $\gamma$ . Then by standard dynamic programming arguments, an agent in region  $i$  will switch sectors at time  $t$  if  $V(\gamma, i, t) < V(\gamma, j, t) - c_j(t)$ , will stay if the opposite inequality holds, and will be indifferent if equality holds.

A path  $\gamma$  is an *equilibrium* if it is generated by the optimal decisions of (almost) all agents in response to  $\gamma$ .

To discuss the generation of  $\gamma$ , consider now some exogenously given measurable path  $\{K(t)\}_{t=0}^{\infty}$ .<sup>5</sup> We assume that there is *some* lag (however small) in the speed at which external effects induced by incoming/outgoing factors affect the going rates of return. From this point of view, we regard the return function  $f(K)$  as representing a “long run level” of the rate of return, once the economy has settled at a certain level of capital  $K$ . We assume that at date 0,  $r(0)$  is precisely  $f(K(0))$  (see (1)). Thereafter, we introduce an increasing function  $g$ , with  $g(0) = 0$ , such that

$$\dot{r}(t) = g(f(K(t)) - r(t)). \quad (2)$$

Thus, the rate of return at any date “chases” the “appropriate” rate of return corresponding to the division of the capital endowment at that date. The specific functional form of  $g(\cdot)$  determines the speed at which returns adjust. In any case, capital owners will always get paid  $r(t)$ , in accordance to their real marginal productivity at that point of time.

Several economic situations conform quite naturally to this specification. In models of search or matching, the productivity of some fixed amount of capital may depend on the ability of that capital to find partners (with more capital), say, because of minimum scale requirements in production. This ability, in turn, will depend on the total amount of capital in the economy (see, e.g., Diamond [1982]). Note that a discontinuous jump in the capital stock will lead to a smooth intertemporal increase in productivity as long as the process of “matching partners” takes place in continuous time. Likewise, if one replaces “capital” by “population” and “rate of return” by “utility”, the concentration of population in a particular geographical region may provoke large amounts of productive activity and a variety of goods and services, attracting still more people because of the greater utility to be had (Krugman [1991b]). Again, the degree of productive activity might react smoothly to a sudden influx of population (perhaps because the information regarding a larger market needs time to permeate to all the producers).

---

<sup>5</sup>Note that we do not a priori restrict it to be a continuous path, so that self-fulfilling “jumps” are, in principle, permitted.

Appendix 1 in the paper provides a highly stylized example that makes explicit the process through which intertemporal smoothing can occur.

Thus a path of capital allocations  $\{K(t)\}$  generates a path of returns  $\{r(t)\}$  using (2), and a path  $\{c_A(t), c_B(t)\}$  using the relationships  $c_i(t) = \hat{c}_i(K(t))$  for all  $t$ . Note that we could introduce lags into the  $c$  functions as well. We do not do this: it keeps the notation simple and in any case our focus is primarily on lags in the flow rates of return.

Note that the literature on coordination (e.g., Cooper and John [1988]) and on the role of history versus expectations (e.g., Matsuyama [1991], Krugman [1991a]) effectively assumes that  $g$  is “infinitely sensitive”, so that  $r(t)$  is always *exactly* equal to  $f(K(t))$ .<sup>6</sup>

### 3 History and Coordination Failure

If agents migrate only in response to current rate differentials, the resulting equilibrium is equivalent to myopic “tatonnement”. For instance, if  $K(0)$  is such that  $f(K(0)) < 0$ , capital in  $B$  will move to  $A$  to cash in on the rate differential and the process will ultimately lead to the specialization of the economy in sector  $A$ . The reverse will be true whenever  $f(K(0)) > 0$ . With positive relocation costs, there is an interval of initial conditions that will persist; no one finds it worthwhile to switch sectors. However, as Matsuyama [1991] and Krugman [1991a] observe, this strong history-dependence is driven by myopia. There might be “forward-looking” equilibria which lead, for instance, to ultimate specialization in either sector even though initial conditions do not favor this sector. It turns out, though, that such outcomes are impossible in the model of the previous section, irrespective of the degree of farsightedness.

To state this result, say that an intertemporal equilibrium is *exclusively history dependent* if the long-run outcome either equals the initial allocation, or entails migration *only* to the sector that is initially profitable. Note that myopic tatonnement has the same properties, though obviously the exact path may be different. The main similarity is that no room is left for farsighted expectations.

**PROPOSITION 1** *Assume  $f(K(0)) \neq 0$ . Unless the cost of relocation exhibits congestion, every equilibrium must be exclusively history dependent, irrespective of the discount rate.*

This proposition summarizes the main point of the paper: unless rates of return reflect *instantaneously* the external effects of incoming factors, the ability to generate “ahistorical equilibria” and to attain Pareto superior outcomes depends critically on how relocation costs are affected by intersectoral allocation. In particular, Proposition 1 shows that if each agent faces a relocation cost that

---

<sup>6</sup>This is to be contrasted with the more recent work of Gale [1993] and Chamley and Gale [1994] where lags also play a role.

is *nonincreasing* in the number of destination agents, the long-run behavior of the economy is fully determined by initial conditions. Note that this argument includes the case in which movement is costless, or in which the costs of migration are fixed, independent of intersectoral allocation. If rates of return adjust to going factor endowments with a lag, however small, agents will want to postpone their migration decisions to currently unprofitable areas, until returns rise sufficiently to justify the move. With a continuum of agents, this externality cannot be internalized, so that all equilibria with ahistorical steady states are knocked out. In particular, adverse initial conditions are perpetuated. Thus, the only equilibria that survive involve either perpetuation of the initial allocation or movement to initially profitable areas.<sup>7</sup>

Our observation is independent of the magnitude of discounting, and of the degree of responsiveness of returns (as long as it is not instantaneous). Thus by a minor and reasonable weakening of one of the assumptions in the literature, we obtain a class of models where expectations are dwarfed by history, where initial conditions determine the final equilibrium. Of course, if rates of return adjust *instantaneously*, then expectations-driven equilibria are possible.

We reiterate: our claim is not that ahistorical equilibria are impossible. But that, in this class of models, in addition to the intersectoral agglomeration externalities, the “migration technology”, is crucial to understand the sources of such equilibria. The only way in which such outcomes can occur is by introducing a cost to postponement; i.e., by making future relocation costs increase in the stock of settlers in our case.

Thus it appears (though we have not provided a formal proof of this) that in this general approach, expectations can dominate history only through an interesting juxtaposition of *agglomeration* externalities in one sector (production) and *congestion* externalities in another (migration).<sup>8</sup> The reason why migrants might move to a currently unfavored sector, despite the lag in the realization of externalities, is that a later move will involve a higher cost. It is in this sense that the technology of adjustment costs is crucial to the existence of expectations-driven equilibrium.

## 4 Concluding Comments

Why some countries or regions develop and others do not crucially depends on when the benefits from the investment/migration decision will accrue and how

---

<sup>7</sup>This does not rule out multiplicity. It is possible, for instance, that if  $f(K(0)) > 0$ , both the initial allocation and a jump to the B region by everybody in A belong to a set of simultaneously sustainable equilibria. If initially  $f(K(0)) < 0$ , there is always a unique equilibrium: either the initial allocation or the complete abandonment of region B.

<sup>8</sup>In the case where the adjustment cost depends on the *flow* of migration, it turns out that the existing literature that we are aware of (see, for example, Kemp and Wan [1974], Mussa [1978], and Krugman [1991a]) *also* invoke congestion (rather than agglomeration) in migration costs.

costly the move is. It is unclear that congestion in relocation is always a good assumption. To the extent that this is the case, it may be difficult to invoke coordination games to explain some great migrations. For instance, being one of the first settlers in the Far West was possibly far more costly than arriving there later by train. In other cases, such as in the sudden growth of new cities in developing countries, congestion may well be a reasonable postulate.

In a similar way, the results of the paper shed some light on the problem of technology adoption (see, e.g., Arthur [1984], David [1985], Chamley and Gale [1993] and Gale [1993]). A technology already in place may be dominated by an alternative technology if the latter is adopted by a significant group of firms. However, if these potential returns only appear over time and the cost of adoption decreases with the number of entrants, everyone waits for others to move first, negating the adoption of the technology even with forward-looking agents. If, on the other hand, the costs of adoption increase with the spread of the new technology, such equilibria are possibly sustainable.<sup>9</sup> Thus society may gain by being organized in such a way that being the first in the business matters.

The standard role for intervention, which is weakened by the possibility of ahistorical equilibria, may need to be reexamined. As usual, we may think of the government as involved in the task of building up a critical mass in cases where history acts as a constraint. But the focus of this paper reveals another role as well. The initial losses that migrating agents incur in moving to a poor region may be partially alleviated by subsidies (such as tax holidays) in the hope of more than recovering the cost in future fiscal revenues. The idea is to mimic or create the equivalent of congestion in the initial costs of relocation.

---

<sup>9</sup>Perhaps this “explains” why new software, such as Internet browsers, need to be given away, with prices rising later as the number of devotees to that software swell.



# Appendix 1

Suppose that  $n$  workers in a region have productivity  $a_1, a_2, \dots, a_n$ . Let  $a$  be the average productivity. If individual productivity is not observed by competitive employers and the production technology is linear with production coefficient normalized to one, then the equilibrium wage will be  $\tilde{a}$ , where this is the expectation of average productivity that employers calculate given common beliefs about the probability distribution of each individual's productivity.

Now suppose that the productivity of an individual is positively affected by meeting other individuals that he has not met before. Specifically, suppose that every encounter with a new individual adds  $\delta$  to productivity.

Encounters take place in the following way. There is a Poisson process with given parameter: whenever a realization occurs, two (or more pairs of) individuals are taken randomly from the population and matched. A productivity upgrade occurs if and only if a match is new.

Take as given the following initial conditions. Each individual located in the region at date 0 has the an initial productivity  $b$ . There is some initial number  $n(0)$  of such individuals. Thereafter migration occurs as a function  $n(t)$  which starts at time 0, jumps upwards periodically, and finally flattens out when  $n(t) = N$  (think of  $N$  as total population).

The productivity of each individual is a random variable: for any such individual  $i$  at date  $t$ ,  $a_i(t) = b_i + \delta m_i(t)$ , where  $b_i$  is the initial productivity of the migrant (which is  $b$  for the original settlers at date 0), and  $m_i(t)$  is the number of new meetings  $i$  has had with other incumbents in the region up to date  $t$ . Clearly,  $a_i(t)$  is a random variable that begins at the initial productivity  $b_i$ , exhibits jumps at random dates along its sample paths, and converges almost surely to the value  $b(t_0) + \delta(N - 1)$ .

We assume that each new migrant (after date 0) is initially housed by an incumbent "relative". These relatives are uniformly distributed among the incumbent settlers. The going productivity of the relative is assumed to be instantly transmitted to the newcomer at the time of migration. Thereafter, the new migrant becomes an incumbent with something to teach as well as something to learn, exactly the same way as described in the previous paragraph.

Let  $a(t)$  be the average productivity of all members of the region at date  $t$ ; of course,  $a(t)$  is still a random variable in its own right (with jumps on its sample paths). It is easy to check that  $a(0) = b$  and (given the assumptions about new arrivals),  $a(t)$  increases up to the limit  $b + \delta(N - 1)$ . But employers do not know the average productivity at date  $t$  for sure because they do not know about the pattern of meetings that have occurred economy-wide, neither do they know the pattern of initial matching between migrants and relatives, which they take to be random. Take the expectation of  $a(t, n)$  at each date  $t$  using the characteristics of the underlying Poisson process; call this  $\tilde{a}(t)$ . The (straightforward) proof of the following fact is omitted but is available on request:

$\tilde{a}(t)$  is a deterministic, continuous function of  $t$ , which starts at  $b$  and converges monotonically to  $b + \delta(N - 1)$ .

**Remark 1.** The assumption that new migrants costlessly acquire the productivity of the incumbents they go and live with, and thereafter proceed to teach and learn just as the incumbents do, is only used to ensure that a jump in  $n$  does not instantly (and temporarily) lower expected average productivity at the date of the jump (as it would, for instance, if new migrants came in with a productivity of  $b$ ). [This sort of temporary downward jump, it should be added, would only strengthen the results of the main model.]

**Remark 2.** As written down, this example is unrealistic and is only meant to capture the process of “intertemporal smoothing” in the rates of return in the simplest possible way. In particular, to completely fit our model, it should be the case that if there is migration *out* of the region, then productivity in the region should fall over time. This sort of extension is easy enough to accomplish by putting in a depreciation factor for productivity that is constantly compensated for by new encounters with other incumbents. Then it will be the case that a lowering of  $n$  will cause an economy-wide depletion of average productivity, as encounters become fewer, and the economy must move down to its new steady state level of productivity.

## Appendix 2

**Proof of Proposition 1.** Consider the case in which  $f(K(0)) < 0$ . The case  $f(K(0)) > 0$  can be settled by a parallel argument.

Fix any equilibrium  $\gamma$ . We claim that  $K(t) \leq K(s)$  for all  $t \geq s$ , which settles exclusive history-dependence.

Suppose this is false for some  $t'$  and  $s'$ . Then, indeed, there is some  $t$  and  $s$  with  $t > s$ ,  $K(t) > K(s)$ , and  $r(\tau) < 0$  for all  $s \leq \tau \leq t^*$  for some  $t^* > t$  (this last observation uses the assumption that returns do not adjust instantaneously).

First note that

$$K(t^*) \geq K(\tau) \text{ for all } \tau \in [s, t^*]. \quad (3)$$

For suppose this is not true; then  $K(t^*) < K(\tau)$  for some  $\tau \in [s, t^*]$ . Because  $K(t) > K(s)$  by assumption and  $t \in [s, t^*]$ , we may conclude that there exist  $\{t_1, t_2, t_3\}$  in the interval  $[s, t^*]$  such that  $K(t_1) < K(t_2) > K(t_3)$ , while  $r(\tau) < 0$  for all  $\tau \in [s, t^*]$ . This means that (at least) two costly switches between sector  $A$  and  $B$  are optimal while sector  $A$  earns a higher return throughout, a contradiction.

Now return to the main proof. So far, we know that there is a positive measure of agents between  $s$  and  $t$  such that for each of them, at the time  $\tau$  of their move,  $V(\gamma, B, \tau) - \hat{c}_B(K_\tau) \geq V(\gamma, A, \tau)$ . But for each such person, denoting the discount rate by  $\rho$ , we see that

$$V(\gamma, A, \tau) \geq e^{-\rho(t^* - \tau)} [V(\gamma, B, t^*) - \hat{c}_B(K(t^*))]$$

$$\begin{aligned}
&> \int_{\tau}^{t^*} e^{-\rho(z-\tau)} r(z) dz + e^{-\rho(t^*-\tau)} [V(\gamma, B, t^*) - \hat{c}_B(K(t^*))] \\
&> \int_{\tau}^{t^*} e^{-\rho(z-\tau)} r(z) dz + e^{-\rho(t^*-\tau)} V(\gamma, B, t^*) - \hat{c}_B(K(\tau)) \\
&= V(\gamma, B, \tau) - \hat{c}_B(K(\tau)).
\end{aligned}$$

The first inequality follows from the agent's option to stay in Sector  $A$  until  $t^*$ , and then switch. The second inequality follows from the fact that  $t^* > \tau$  for all  $s \leq \tau \leq t$  and  $r(z) < 0$  for all  $\tau \leq z \leq t^*$ . The third inequality follows from discounting and (3), so that  $\hat{c}_B(K_{\tau}) > e^{-\rho(t^*-\tau)} \hat{c}_B(K_{t^*})$ , and the last equality follows from the observation that that two or more switches between  $\tau$  and  $t^*$  are clearly suboptimal.

Thus we have a contradiction to our presumption that  $V(\gamma, B, \tau) - \hat{c}_B(K_{\tau}) \geq V(\gamma, A, \tau)$ , which completes the proof.  $\blacksquare$

## References

1. Arthur, B. (1984), “Competing Technologies and Lock-In by Historical small Events: The Dynamics of Allocation under Increasing returns” (CEPR Publication, Stanford University).
2. Bryant, J. (1982), “A Simple Rational Expectations Keynes-Type Model”, *Quarterly Journal of Economics* **97**, 525–529.
3. Chamley, C. and Gale, D. (1994), “Information Revelation and Strategic Delay in a Model of Investment”, *Econometrica* **62**, 1065–1086.
4. Chatterjee, S. and Cooper, R. (1989), “Multiplicity of Equilibria and Fluctuations in Dynamic Perfectly Competitive Economies”, *AEA Papers and Proceedings* **79**, 353–357.
5. Chatterjee, S., Cooper, R. and Ravikumar, B. (1990), “Participation dynamics: Sunspots and Cycles”, *NBER Working Paper Series* N. 3438.
6. Cooper, R. and John, A. (1988) “Coordinating Coordination Failures in a Keynesian Model”, *Quarterly Journal of Economics* **103**, 441–464.
7. Diamond, P. (1982), “Aggregate Demand Management in Search Equilibrium”, *Journal of Political Economy* **90**, 881–894.
8. Farrell, J. and Saloner, G. (1985) “Standardization, Compatibility and Innovation”, *Rand Journal of Economics* **16**, 70–83.
9. David, P. (1985), “Clio and the Economics of QWERTY”, *American Economic Review, Papers and Proceedings* **75**, 332–337.
10. Fukao, K. and Benabou, R. (1993), “History versus Expectations: A Comment”, *Quarterly Journal of Economics* **108**, 535–542.
11. Gale, D. (1995), “Dynamic Coordination Games”, *Economic Theory* **5**, 1–18.
12. Henderson, J.V. (1994) “Externalities and Industrial Development”, *NBER Working Paper* N. 4730.
13. Hirschman, A. (1958), *The Strategy of Economic Development*, New Haven: Yale University Press.
14. Katz, M.L. and C. Shapiro (1986), “Technology adoption in the presence of network externalities”, *Journal of Political Economy* **94**, 822–841.
15. Katz, M.L. and C. Shapiro (1992), “Product introduction with network externalities”, *Journal of Industrial Economics* **40**, 55–83.
16. Kemp, M. C. and Wan, H. Y. (1974), “Hysteresis of Long-Run Equilibrium from Realistic Adjustment Costs.” In *Trade, Stability and Macroeconomics*, edited by George Horwich and Paul A. Samuelson. New York: Academic Press.

17. Krugman, P. (1991a), “History versus Expectations”, *Quarterly Journal of Economics* **106**, 651–667.
18. Krugman, P. (1991b), “Increasing Returns and Economic Geography,” *Journal of Political Economy* **99**, 483–499.
19. Matsuyama, K. (1991), “Increasing Returns, Industrialization and, Indeterminacy of Equilibrium”, *Quarterly Journal of Economics* **106**, 616–650.
20. Murphy, K., Shleifer, A. and R. Vishny (1989), “Industrialization and the Big Push”, *Journal of Political Economy* **97**, 1003–1026.
21. Mussa, M. (1978), “Dynamic Adjustment in the Heckscher-Ohlin-Samuelson Model”, *Journal of Political Economy* **68**, 775–791.
22. Myrdal, G. (1957), *Economic Theory and Underdeveloped Regions* London, Duckworth.
23. Rosenstein-Rodan, P. (1943), “Problems of Industrialization of Eastern and Southern Europe”, *Economic Journal* **55**, 202–211.
24. Scitovsky, T. (1954), “Two Concepts of External Economies,” *Journal of Political Economy* **62**, 143–151.
25. Shleifer, A. (1986), “Implementation Cycles”, *Journal of Political Economy* **94**, 1163–1190.