Business School

Department of Real Estate & Planning

University of Reading

Working Papers in Real Estate & Planning 05/08

Hedging Effectiveness of Total Returns Swaps: Application to the Japanese Market

Paper Prepared for the European Real Estate Society Conference
Krakow, June 2008

Yoshiki Kago and Charles W. R. Ward

Department of Real Estate & Planning

University of Reading Business School

Whiteknights, Reading, RG6 6AW, UK

June 2008

ABSTRACT

The development of the real estate swap market offers many opportunities for investors to adjust the exposure of their portfolios to real estate. A number of OTC transactions have been observed in markets around the world. In this paper we examine the Japanese commercial real estate market from the point of view of an investor holding a portfolio of properties seeking to reduce the portfolio exposure to the real estate market by swapping an index of real estate for LIBOR.

This paper explores the practicalities of hedging portfolios comprising small numbers of individual properties against an appropriate index. We use the returns from 74 properties owned by Japanese Real Estate Investment Trusts over the period up to September 2007. The paper also discusses and applies the appropriate stochastic processes required to model real estate returns in this application and presents alternative ways of reporting hedging effectiveness

We find that the development of the derivative does provide the capacity for hedging market risk but that the effectiveness of the hedge varies considerably over time. We explore the factors that cause this variability.

Introduction

The development of real estate derivatives offers investors considerable flexibility when seeking to include real estate in a mixed asset portfolio. A market in which derivatives could be traded would seem to enhance the value of real estate since it would allow investors to alter their exposure to real estate without the large transaction costs and illiquidity endemic to the private real estate market. There are, however, two issues that need to be considered before arguing for an increased holding of real estate within institutional mixed asset portfolios. First, the pricing and efficiency of the derivatives market might limit its usefulness if spreads in, say, the swap market for real estate were very large and/or volatile. This is mainly an empirical question that requires more trading and market data to be researched. The second is the ability of investors to hedge their individual real estate assets by operating in the swap market. It is this second issue that is explored in this paper.

The object is to estimate, using data from individual properties, how effectively might investors modify their portfolio exposure to real estate by operating in the total return swap market. The study is based on Japanese properties, but the principles, techniques and findings apply in general terms to real estate markets internationally. In Section 1, we place this study within the context of related literature. In Section 2, we develop a model for analyzing the returns from the real estate portfolio hedged with total returns swaps (hereafter TRS). We also discuss and define hedging effectiveness. In Section 3 we discuss the data applied to this study and the model estimation. In Section 4, we show the results of the hedging effectiveness and consider the factors that affect hedging effectiveness. In Section 5, we assess the actual performance of hedged portfolios TRS. Finally we draw our conclusions in Section 6.

1. Swap Market for Real Estate

1.1 Hedging Real Estate Investment Risk

When investors hold assets such as real estate, they may be faced with the need to adjust their portfolio exposure to the underlying market. With illiquid markets, portfolio adjustment may be very costly and in heterogeneous markets, once a specific asset is sold, it may be difficult or impossible to replace. Thus the need for some process by which market exposure might be adjusted would seem to be a necessary condition of creating a successful environment for holding real estate. This, in turn, would seem to be provided by the development of derivative products such as futures, options and swaps. It is therefore logical for researchers to address the opportunities created by establishing derivative markets in real estate.

One obvious problem with a heterogeneous asset is basis risk and the correlation between the returns of the asset held in the portfolio and the hedging instrument. The first study focusing on the availability of TRS for real estate investment was Park and Switzer (1995, 1996) but other authors have also addressed similar issues. Case and Shiller (1996) show that the mortgage default risk can be hedged by the futures and options based on a real estate index. They focus on the correlation between the change of default probability and the real estate index. Shiller and Weiss (1999) propose insurance policies to enable individuals to protect themselves against the risks of falls in the price of their homes. Other studies include empirical analysis of the real estate market, Englund et al (2002) and Iacoviello and Ortalo-Magne (2003) analyze the risk and the expected return of the hedge created by shorting real-estate stocks and an index whilst Syz et al. (2007) address the hedging of real estate using an index, and indicate the importance of correlation between the real asset and the real estate index.

Following Park and Switzer (1995, 1996), we take as our starting point, the view of real estate owners who wish to modify their exposure to future fluctuations in the market by using TRS. Conventionally, this could be managed by swapping the total return on a nominal amount of real estate (or a nominal value of a specified real estate index) for short-term interest rates (adjusted by a premium or discount to reflect market conditions in the swap market), This transaction could reduce the exposure of the portfolio to future fluctuations in the real estate market over the designated period, leaving the real estate owner to bear only the basis risk of their portfolio. In financial terms, this would imply that real estate owners could hold onto the “alpha” but eliminate the “beta” or systematic risk of their real estate assets. But such a result critically depends on the relative sizes of the volatility of the swap spread and the basis risk of real estate portfolios.

1.2 Spread of TRS

The pricing of TRS is derived by Buttimer et al (1997) and evolved by Bjork and Clapham (2002). Though Bjork and Clapham (2002) indicate the theoretical price of the TRS is zero, Patel and Pereira (2008) show that the price is non-zero under the existence of counterparty default risk. They argue that TRS payers must charge a spread over the market interest rate that compensates them for the exposure to this additional risk. However they also indicate that computed spreads on IPD indices are much lower compared to a sample of quotes they obtained from one of the traders in the market. As they point out, the actual spreads observed in the swap market for real estate are larger than their counterparts in the equity market, partly because the swap market is a new market and the spreads are (as was the case for equity market spreads), both more variable and larger than those observed in more mature markets. Partly also because the swap pricing for real estate assets is more difficult to arbitrage because of the high transaction costs for buying and selling the underlying asset .Amihud and Mendelson (1989) estimate the effect of illiquidity on stock returns, and Benveniste et al (2001) show that creating liquid equity claims on relatively illiquid property asset increases value by 12-22% through their analysis on the REIT market. Moreover Collett et al (2003) indicate that the holding period for U.K. real estate is considerably longer than the holding periods reported for equities, and those holding periods have varied over the time period giving rise to illiquidity and high transaction costs. It not only makes TRS pricing difficult but also makes the planning of the hedging strategy more complicated. Bond et al (2007) show that marketing time uncertainty can be reduced by constructing a portfolio, but that at least 10 properties are necessary to reduce the risk.

1.3 The Japanese Real Estate Market

After the ‘Bubble’ from 1986 until 1992 Japanese real estate market plunged into a serious depression which has been called the ‘lost decade’. However, the market condition after 2000 was much improved to the extent that a Japanese REIT (J-REIT) market was established in 2001. The J-REIT market has since expanded until the number of companies traded exceeded 40 with a total market value of around 5 trillion JPY in the beginning of 2007.

IPD (Japan) started to publish a monthly indicator of the Japanese real estate market in 2006, and the first deal based on the index of IPD Japan was executed offshore in 2007. We can therefore claim that the real estate derivatives market in Japan has started although in a limited way. However this market is expected to grow and the number of participants will also increase. The Ministry of Land, Infrastructure, Transport and Tourism in Japan held the first workshop of real estate derivatives and published a report in 2007. The report highlighted the conditions and policy required to expand the use of real estate derivatives in Japan. This has also been accompanied by the publications by Japanese researchers interested in real estate derivatives. Moridaira (2006) proposed the pricing model of real estate index derivatives applying the Esscher Transform. However an empirical study about real estate derivatives based on the Japanese market has not yet been published. Thus one motivation for our paper is the need to fill this gap. The second motivating factor is the availability of data. Though the length of time is limited, not only real estate indices but also individual property data based on the J-REIT report have become available.

Based on Japanese real estate data, we focus on the following three areas in this study. The first is to examine the modeling of real estate returns using a Wiener process or an autoregressive process. Another approach, the GARCH model, is often applied to many financial products, for instance Baillie and Myers (1991) consider the optimal hedge for commodity futures based on GARCH model and they show that the estimated models provide a good description of the distribution of changes in commodity prices. Unfortunately we can't apply GARCH in this study because of the restrictions of data. Our individual property data are initially semiannual and the maximum length is 5 years (see Section 3.1 for details). This data set would be far too small to estimate a GARCH model. Park and Switzer (1995, 1996) apply a Wiener process to describe the process of returns but we doubt whether such diffusion process is appropriate for real estate returns. Thus we also apply an autoregressive process for the real estate index and the individual real estate returns, and we compare the standard error of estimates for both approaches.

Our second focus is the hedging effectiveness of TRS. Park and Switzer (1995, 1996) assessed the optimal position of real estate swaps for risk management for the real estate owner by transforming the problem into a mean-variance framework. We assess the hedging effectiveness of TRS in the same framework but while Park and Switzer (1995, 1996) determine the hedge ratio to maximize the investor's utility function, we apply a minimum-variance hedging strategy (see Section 2.3 for details). Our analysis is based on the conditional variance and covariance for different specific periods, because the time period covered by each property differs, so properties that can be incorporated into the portfolio depend on the period selected. Harris and Stoja (2008) analyze both the unconditional minimum-variance hedge ratio and the conditional one for the currency market. They conclude that conditional minimum-variance hedge ratio does not perform significantly better than the unconditional one in terms of either hedge portfolio variance reduction, or utility maximization.

The third and final focus of this study is the actual performance of the hedged portfolio. The investor is planning a hedging strategy based on the market movement experienced. If the investor determines the hedge ratio expected to minimize the variance of return, we explore whether the actual performance in the succeeding period is better than the non-hedged portfolio. We use a fixed number of observations to estimate each model and performance assessment. The following period is used for the actual assessment (see Section 3.2 for details).

Our study is based on some strong assumptions. As mentioned above, the spread of TRS would critically affect the performance of hedge. We assume a single period investment in this study. The investor and the counterparty make a contract of TRS for the period $t-1$, and settle at time t in this single period investment. We take the spread as fixed at the contract time $t-1$, and the fixed spread is unchanged at settlement. Under this assumption, as the spread is not a stochastic variable, it does not affect the variance of return of portfolio. Of course the fixed spread still affects the return on the portfolio, but it just provides the same change to both the expected return and the actual return. Initially we set the spread equal to zero in the empirical analysis and the investment horizon for 6 months or 1 year.

2. The Model

2.1 Naked Portfolio and Hedged Portfolio

The return of portfolio without hedging is given in equation (1). We call this, the naked portfolio. $y_{i,t}$ is the total return of the individual real estate i at the period t while w_i is the weight in the portfolio of the property i under the constraint of $\sum w_i = 1$.

$$R_{N,t} = \sum w_i y_{i,t} \tag{1}$$

The return of a portfolio hedged with TRS is given in equation (2). Fig.1 expresses the exchanges of return of this hedged portfolio. The investor contracts with the counterparty at the period $t-1$ to swap the total return of the real estate index (I_t) with LIBOR (r_t) plus spread (s) at the period t. h is the hedge ratio given as the ratio of the principle amount of swap to the exposure of real estate portfolio at the period $t-1$.

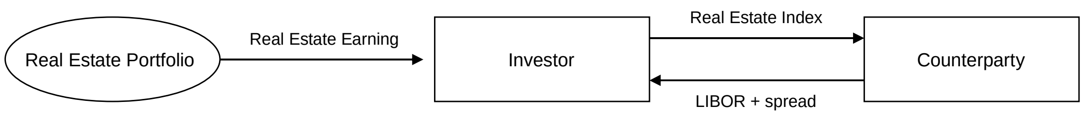


Fig.1 Exchange flows of the hedged portfolio

$$R_{H,t} = \sum w_i y_{i,t} + h\left(r_t + s - I_t\right) \qquad (2)$$

2.2 Expectation and Variance of Returns

We apply the autoregressive process for the returns of the real estate index and the individual real estate returns (see equation (3) and (4) respectively). The index is expressed as a simple autoregressive model with a constant term. The return of an individual property has two parts. One part depends on the index, and the other depends on the return at the previous period. In our model the degree of the dependence upon these two parts is expressed as a weight. That is to say the weight for the index is a_i, and the weight for the previous return is the rest, i.e. $1-a_i$. The estimates of variance of $\varepsilon_{I,t}$ and $\varepsilon_{i,t}$ are given with the residual of the estimation. Here $\hat{e}$ is the residual and k is the number of samples.

$$I_t = a_I I_{t-1} + c_I + \varepsilon_{I,t} \qquad (3)$$

$$E[\varepsilon_{I,t}] = 0$$

$$V[\varepsilon_{I,t}] = \sum_{j=1}^{k} \hat{e}_{I,j}{}^2 / \left(k-2\right)$$

$$y_{i,t} = a_i I_t + (1 - a_i) y_{i,t-1} + c_i + \varepsilon_{i,t} \tag{4}$$

$$E[\varepsilon_{i,t}] = 0$$

$$V[\varepsilon_{i,t}] = \sum_{j=1}^{k} \hat{e}_{i,j}{}^2 / (k-2)$$

For the LIBOR process we apply the Vasicek model (Vasicek, 1977) as in equation (5) where W_t is a Wiener process and σ determines the volatility of the LIBOR rate. α and β express mean-reversion. $-\alpha / \beta$ is the rate of long term, and $-\beta$ is the strength of mean-reversion. Simplifying the equation (5) we apply the equation (6) in this study.

$$dr = (\alpha + \beta r) dt + \sigma \, dW_t \tag{5}$$

$$r_t = r_{t-1} + (\alpha + \beta r_{t-1}) \Delta t + \varepsilon_{r,t} \tag{6}$$

Where $E[\varepsilon_{r,t}] = 0$

$$V[\varepsilon_{r,t}] = \sigma^2 \Delta t$$

Given the information at $t-1$ we can determine the conditional expected returns of portfolio at t based on the above equations. Here Φ_{t-1} is the information set at $t-1$. Here we suppose that the spread (s) in the hedged portfolio is fixed at $t-1$ and is paid at t.

$$E[R_{N,t} \mid \Phi_{t-1}] = \sum w_i \left(a_i a_I I_{t-1} + (1 - a_i) y_{i,t-1} + a_i c_I + c_i \right) \tag{7}$$

$$E[R_{H,t} \mid \Phi_{t-1}] = \sum w_i \left(a_i a_I I_{t-1} + (1 - a_i) y_{i,t-1} + a_i c_I + c_i \right) + h\left((1 + \beta \Delta t) r_{t-1} + \alpha \Delta t + s - a_I I_{t-1} - c_I \right)$$

(8)

Based on the equations (1),(2),(3),(4) and (6) the conditional variance of return of portfolio are given in (9) and (10). Here we suppose that the covariance between the error terms are negligibly low, i.e. $Cov[I_{t-1}, \varepsilon_{I,t}] = 0$, $Cov[y_{i,t-1}, \varepsilon_{I,t}] = 0$, $Cov[I_{t-1}, \varepsilon_{i,t}] = 0$, $Cov[y_{i,t-1}, \varepsilon_{i,t}] = 0$, $Cov[\varepsilon_{I,t}, \varepsilon_{i,t}] = 0$ and $Cov[\varepsilon_{p,t}, \varepsilon_{q,t}] = 0$ for $p \neq q$. As pointed out above, the spread (s) doesn't have any effect to the variance of return because it isn't a stochastic variable in this study.

$$\begin{aligned} V[R_{N,t} \mid \Phi_{t-1}] &= \sum_{p=1}^{n}\sum_{q=1}^{n} w_p w_q a_p a_q a_I^2 V[I_{t-1}] \\ &+ \sum_{p=1}^{n}\sum_{q=1}^{n} w_p w_q (1-a_p)(1-a_q) Cov[y_{p,t-1}, y_{q,t-1}] \\ &+ 2\sum_{p=1}^{n}\sum_{q=1}^{n} w_p w_q a_p (1-a_q) a_I Cov[I_{t-1}, y_{q,t-1}] \\ &+ \sum_{p=1}^{n} w_p^2 a_p^2 V[\varepsilon_{I,t}] + \sum_{q=1}^{n} w_q^2 V[\varepsilon_{q,t}] \end{aligned} \qquad (9)$$

$$\begin{aligned} V[R_{H,t} \mid \Phi_{t-1}] &= h^2 V[r_t - I_t] + 2hCov[\sum_{i=1}^{n} w_i y_{i,t}, r_t - I_t] + V[\sum_{i=1}^{n} w_i y_{i,t}] \\ &= Ah^2 + 2Bh + C \end{aligned} \qquad (10)$$

Where:

$$\begin{aligned} A &= V[r_t - I_t] \\ &= V[I_t] + V[r_t] - 2Cov[I_t, r_t] \\ &= a_I^2 V[I_{t-1}] + V[\varepsilon_{I,t}] + (1+\beta\Delta t)^2 V[r_{t-1}] + V[\varepsilon_{r,t-1}] - 2a_I(1+\beta\Delta t)Cov[r_{t-1}, I_{t-1}] \end{aligned}$$

$$\begin{aligned} B &= Cov[\sum_{i=1}^{n} w_i y_{i,t}, r_t - I_t] \\ &= Cov[\sum_{i=1}^{n} w_i y_{i,t}, r_t] - Cov[\sum_{i=1}^{n} w_i y_{i,t}, I_t] \\ &= \sum_{i=1}^{n} w_i a_i a_I (1+\beta\Delta t) Cov[I_{t-1}, r_t] + \sum_{i=1}^{n} w_i (1-a_i)(1+\beta\Delta t) Cov[y_{i,t-1}, r_t] \end{aligned}$$

$$-\sum_{i=1}^{n} w_i a_i a_I^2 V[I_{t-1}] - \sum_{i=1}^{n} w_i \left(1-a_i\right) a_I Cov[y_{i,t-1}, I_{t-1}]$$

$$C \;= V[R_{N,t} \mid \Phi_{t-1}]$$

2.3 Hedging Effectiveness

The key issue for the investor is how to decide the hedge ratio, h in the equation (2). Park and Switzer (1995, 1996) apply a mean-variance expected utility function, and they determine the optimal hedge ratio to maximize that utility function. Other various utility functions are introduced in studies, such as Cecchetti, Cumby, and Figlewski (1988). On the other hand, an alternative approach by Howard and D'Antonio (1984) proposes the maximization of the Sharpe ratio. Boveroux and Minguet (1999) indicate that the choice of a hedge ratio that maximizes utility essentially corresponds to adjusting the portfolio's beta.

In this study we apply the minimum-variance hedging strategy derived by Johnson (1960). There is an argument about the appropriateness of the minimum-variance criterion as Alexander and Barbosa (2007) indicate. However our focus is on the demonstration of hedging effectiveness using individual property data, so the simple minimum-variance hedging is appropriate as an extreme case of reducing risk. As shown in equation (10), the variance of return of the hedged portfolio is a function of the hedge ratio h. The hedge ratio to minimize $V[R_{H,t} \mid \Phi_{t-1}]$ is equation (11) and the minimum variance is given as equation (12).

$$h = -\frac{B}{A} \qquad (11)$$

$$\min V[R_{H,t} \mid \Phi_{t-1}] = -\frac{B^2}{A} + C \qquad (12)$$

The hedging effectiveness in this study means how much risk, i.e. variance of return, is reduced by hedging. In this sense the ratio of the variance of return of hedged portfolio to the naked portfolio is appropriate as the indicator of hedging effectiveness. The indicator of Johnson (1960) or Ederington (1979) is based on the same principle. The indicator of Alexander and Sheedy (2008) is the ratio of the standard deviation of returns, and it is consistent with these measures. We adopt the indicator of Alexander and Sheedy (2008) as in equation (13), because it is simple.

$$HE_t = \frac{\sqrt{V[R_{H,t} \mid \Phi_{t-1}]}}{\sqrt{V[R_{N,t}] \mid \Phi_{t-1}}} \tag{13}$$

3. Model Estimation

3.1 Index and Individual Property Data

We used data supplied by the ARES (The Association for Real Estate Securitization in Japan). They provide individual properties data held in J-REITs and the real estate index collected from the reports of the settlement accounts of J-REITs. The index called the ARES J-REIT Property Index is an appraisal-based monthly index, and it is a simple mean of sample properties. The return of each property is calculated on the same basis as the NCREIF Property Index. Their index has been published from April 2006, and their index of office property covers the period from January 2002 until January 2007 at the end of November 2008.

The individual property data provided by ARES is based on the information disclosed in the J-REIT report. As the settlement accounts of J-REIT is made semiannually, normally one record covers 6 months. This includes settlement date of the term, the attributes of the property, appraised value at the end of term, and income return at the term for each property and for each fiscal term. Their data includes 5,722 records for 1,537 properties at the end of September 2007. We used individual office properties that have more than 8 records, i.e. covering more than 4 years, in the Tokyo metropolitan area, excluding the properties that have additional acquisitions or errors in their records. As a result, 746 records for 74 properties were applied in this analysis.

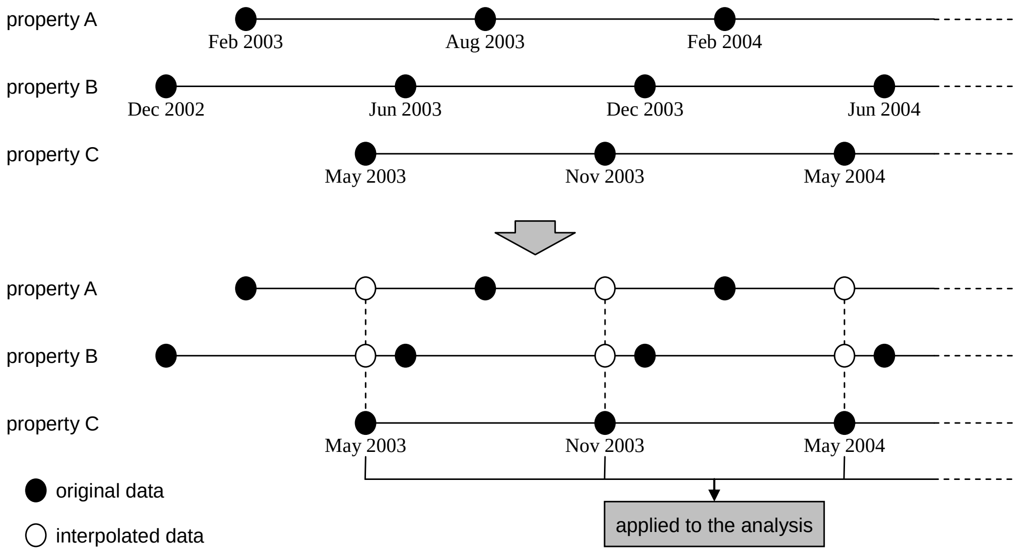


Fig.2 Interpolation for portfolio data

As mentioned above the J-REIT report is published semiannually, the publication month of data may be vary one from other. So if we construct the portfolio with properties whose publication months are the same, we would select only a very restricted sample of properties. We therefore transform the observed return from the different months of data to a common point in time one by cubic spline interpolation like as Fig.2 for each portfolio.

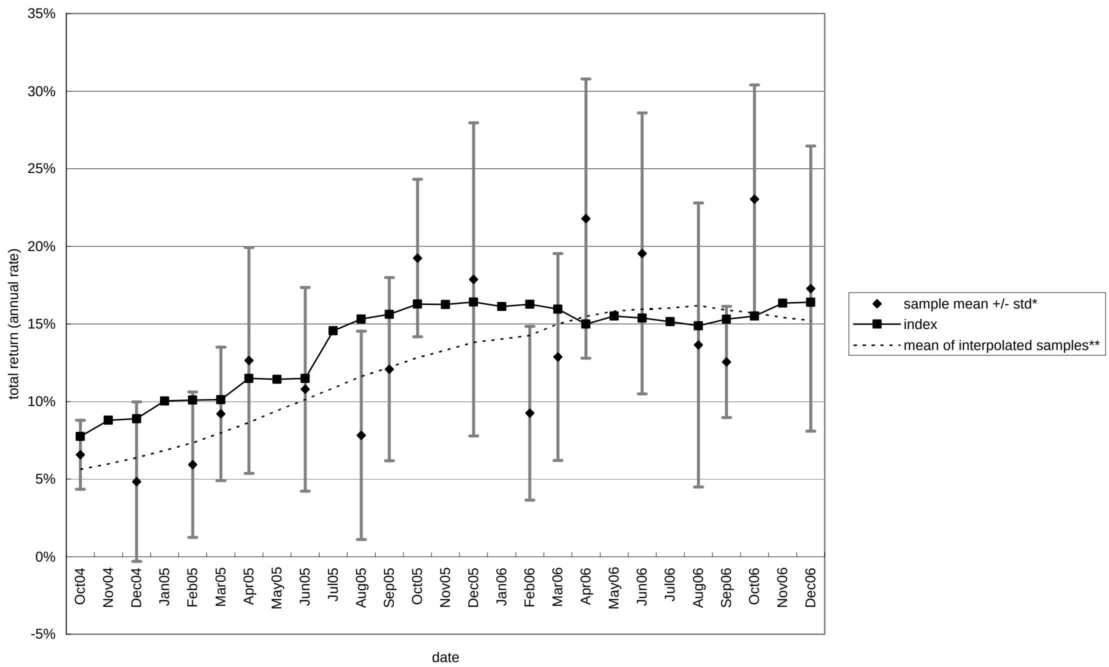


Fig.3 real estate index and return of property

* annualized rate of total return of 6 months investment. This doesn't include interpolated data. The indicated range means plus/minus 1 standard deviation.
** mean value of all samples with interpolated data

Though the individual property data by ARES is available from 2002, sufficient samples are available only from 2004. Fig.3 shows the ARES total return index and the mean value of total return of the individual properties applied in this study. Since the J-REIT report is produced semiannually, only a few data of the individual properties are available for each month and in some periods that have no original data. The mean value of the total return of the individual properties is expressed on Figure 3 as a dot with the confidence interval of 1 standard deviation. Against this the mean values of interpolated data of properties are rather less volatile. The mean of our sample was lower than the index until March 2006 but thereafter was higher than the index. In two periods the difference was statistically significant.

3.2 Estimation of Index Model and Property Model

(1) Procedure

Our analysis is done for two investment horizons, 6 months and 1year. For 6 months investment, we use continuous 6 period observations for each estimation and actual assessment. The first 5 observations are use to estimate the model parameters (a_I, c_I, a_i, c_i) in equations (3)(4) and the variance and covariance in equations (7)(8) ($V[I_{t-1}]$, $Cov[y_{p,t-1}, y_{q,t-1}]$, etc). The prediction of the model from the 5th observation is compared with the final observation for the actual assessment. The date of this 5th observation is called 'period' or 'start date of investment' in the following tables. If there are more than 7 observations for the chosen portfolio, we moved the window of 6 observations in the data coverage.

The number of properties in the portfolio is set 1, 3, 5 and 10. The portfolios with more than one property are constructed randomly. We create 1,000 sample portfolios by the Monte Carlo method with replacement with the weight of each property in the portfolio set equal. The hedge ratio is set to minimize the variance of the return of hedged portfolio as in equation (9), the spread (s) is set zero.

(2) Auto-regressive vs. Wiener Process

As mentioned above, Park and Switzer (1995, 1996) applied a Wiener process for real estate returns as in equation (14). μ_y is the mean drift of the change and σ_y is the volatilities. They assumed that μ_y and σ_y are constant.

$$dy = \mu_y dt + \sigma_y dW_t \quad (14)$$

Here we compare the auto-regressive model that we applied in this study with the Wiener process model using the standard error of estimates. Table.1 shows the mean and standard deviation of the standard error of estimates of both models for each period. The mean value of the standard error of our auto-regressive model is always smaller than the Wiener process, and the standard deviation of the standard error of our model is also smaller than that for the Wiener process in almost all periods. So we conclude that an auto-regressive model is more appropriate for real estate return than a Wiener process model.

Table.1 mean and standard deviation of standard error of estimates

6 month investment

		auto-regressive		Wiener process	
period	properties*	mean	std	mean	std
Feb05	22	0.0377	0.0330	0.0546	0.0546
Mar05	16	0.0405	0.0399	0.0601	0.0579
Jun05	19	0.0291	0.0185	0.0441	0.0283
Aug05	22	0.0284	0.0176	0.0469	0.0323
Sep05	19	0.0308	0.0240	0.0448	0.0372
Oct05	9	0.0269	0.0135	0.0346	0.0107
Dec05	19	0.0314	0.0231	0.0457	0.0233
Feb06	22	0.0296	0.0158	0.0367	0.0178

Mar06	18	0.0230	0.0088	0.0333	0.0197
Apr06	9	0.0265	0.0181	0.0347	0.0124
Jun06	20	0.0363	0.0267	0.0464	0.0295

1 year investment

		auto-regressive		Wiener process	
period	properties*	mean	std	mean	std
Aug05	20	0.0307	0.0251	0.0551	0.0454
Sep05	14	0.0325	0.0259	0.0549	0.0500
Dec05	18	0.0495	0.0398	0.0752	0.0506

* Interpolated samples are NOT included

(3) Summary of the model of the index and properties

The models of the real estate index and properties are estimated by OLS. Table.2 shows the estimates of the index model of the equation (3). The estimates of a_I are highly significant and the standard errors of estimates are quite small until the period of Oct05 for 6 months investment. The estimates of c_I are less significant for almost all periods. Though the significance of a_I after the period of Dec05 is less, the t-statistic is still greater than 2.. For 1 year investment the standard error is a little higher.

For the property model, the significance varies across properties and periods. Of course there are some insignificant cases, but we do not eliminate insignificant cases in this study, because investors cannot select the property in their portfolio based on statistical significance. In practice the investor has to prepare the hedging strategy for the portfolio actually owned. As we assume such a practical situation in order to consider hedging effectiveness, all properties are used for analysis regardless of its significance.

Table.3 shows the mean value of the estimates and statistics of the property model. For 6 months investment the mean of t-statistic of a_i is about or greater than 2, and the mean of t-statistic of c_i is more than 1. The mean of t-statistic of 1-year investment is better. Considering the uniqueness of each individual property we can say the significance of this model is acceptable.

Table.2 estimates of the index model

6 month investment

period	estimates		t-statistic		Probability		s.e.*
	a_I	c_I	a_I	c_I	a_I	c_I	
Feb05	1.6916	-0.0080	12.5776	-2.3951	0.0063	0.1389	0.0018
Mar05	1.5612	-0.0051	8.6414	-1.0965	0.0131	0.3873	0.0025
Jun05	1.1511	0.0070	9.7406	1.9917	0.0104	0.1846	0.0027
Aug05	1.6761	-0.0078	21.1440	-2.8488	0.0022	0.1043	0.0018
Sep05	1.7590	-0.0109	20.4919	-3.6342	0.0024	0.0681	0.0019
Oct05	1.5278	-0.0050	14.2148	-1.1759	0.0049	0.3607	0.0028
Dec05	1.4028	-0.0006	9.5563	-0.0994	0.0108	0.9299	0.0040
Feb06	0.9098	0.0180	3.0637	1.2001	0.0921	0.3530	0.0115
Mar06	0.8516	0.0202	2.5354	1.1713	0.1267	0.3621	0.0131
Apr06	0.6769	0.0282	2.0193	1.5217	0.1809	0.2675	0.0135
Jun06	0.6540	0.0302	2.0850	1.6944	0.1724	0.2323	0.0120

1 year investment

period	estimates		t-statistic		Probability		s.e.*
	a_I	c_I	a_I	c_I	a_I	c_I	
Aug05	1.5152	0.0225	2.9228	0.6288	0.0998	0.5937	0.0238
Sep05	1.4998	0.0217	2.5231	0.5210	0.1277	0.6543	0.0264
Dec05	0.9949	0.0536	2.7282	1.8000	0.1122	0.2137	0.0196

* s.e.: standard error of estimates

Table.3 summary of estimates of the property model

6 month investment

period	properties*	mean of estimates		mean of t-statistic**		mean of probability		mean of s.e.***
		a_i	c_i	a_i	c_i	a_i	c_i	
Feb05	52	0.9504	-0.0120	1.9262	0.9969	0.3076	0.5410	0.0363
Mar05	55	0.9639	-0.0112	1.9614	1.0349	0.2971	0.5251	0.0353
Jun05	57	0.9053	-0.0090	2.3979	1.2834	0.2691	0.5080	0.0298
Aug05	51	0.9290	-0.0149	2.3604	1.0295	0.2642	0.5499	0.0301
Sep05	57	0.9519	-0.0126	2.5741	1.0951	0.2684	0.5440	0.0277
Oct05	64	0.9492	-0.0134	2.5890	1.2604	0.2620	0.4992	0.0250
Dec05	65	0.7415	-0.0045	2.1258	1.1147	0.3319	0.5214	0.0260
Feb06	67	0.8389	-0.0065	2.5124	1.5305	0.3383	0.4824	0.0266
Mar06	67	0.7754	-0.0051	1.9746	1.1638	0.3352	0.5068	0.0270
Apr06	62	0.7397	-0.0026	1.9722	1.2754	0.3580	0.4795	0.0271
Jun06	67	0.738	-0.0001	1.9325	1.2395	0.3305	0.4825	0.0264

1

1 year investment

period	properties*	mean of estimates a_i	mean of estimates c_i	mean of t-statistic** a_i	mean of t-statistic** c_i	mean of probability a_i	mean of probability c_i	mean of s.e.***
Aug05	47	0.8661	-0.0445	3.1205	2.0543	0.2391	0.3751	0.0344
Sep05	49	0.8311	-0.0399	3.2011	2.2963	0.2688	0.3800	0.0342
Dec05	50	0.6959	-0.0201	4.9213	2.8498	0.3165	0.3938	0.0371

* Interpolated samples are included. ** mean of absolute value of t-statistic *** s.e.: standard error of estimates

3.3 Estimation of LIBOR Model

We estimated the parameters α, β and σ in the equation (6) by GMM (Generalized Method of Moments, Hansen, 1982) under the following orthogonal conditions.

$$
\begin{aligned}
&E[\varepsilon_{r,t}] = 0 \\
&E[\varepsilon_{r,t}\, r_t] = 0 \\
&E[\varepsilon_{r,t}^2 - \sigma^2 \Delta t] = 0 \\
&E[\left(\varepsilon_{r,t}^2 - \sigma^2 \Delta t\right) r_t] = 0
\end{aligned}
\tag{15}
$$

We applied 10 years (from Oct 1997 to Oct 2007) monthly data for annualized 6 months LIBOR and 1 year LIBOR. The results of the estimation are as Table.4, and both estimations are significant for all parameters. Calculating $-\alpha/\beta$ the long term rate of 6 months LIBOR is 4.895%, and the 1-year LIBOR is 4.887%. The mean values of sample data are 5.222% and 5.335% respectively. Though mean-reverting rates are slightly smaller than the arithmetic mean values of sample data, both of them are very close.

Table.4 Estimation of LIBOR model

		α	β	σ
6mth LIBOR	Estimate	0.011709	-0.239205	0.002789
	Error	0.003546	0.064254	0.000160
	t-statistic	3.302220	-3.722800	17.403700
1y LIBOR	Estimate	0.018557	-0.379727	0.003705
	Error	0.004898	0.085942	0.000204
	t-statistic	3.788600	-4.418400	18.198100

4. Model Based Hedging Effectiveness

4.1 Variance of Return

The variances of return of the portfolio, $V[R_{N,t} | \Phi_{t-1}]$ and $V[R_{H,t} | \Phi_{t-1}]$, are calculated for each period by the equations (9) and (10) respectively. Table.5 shows the mean and the standard deviation of the square root of the variance of return of each portfolio for each period, and Fig.4 shows the probability distribution of square root of the variance of return. As n, the number of properties in the portfolio, becomes greater, the mean of the variance of return becomes smaller for any period. The standard deviation of the variance of return also becomes to be smaller when n becomes greater. And the distribution of the variance of return of $n=1$ fluctuates heavily depending on the period, but the increase of n make the fluctuation moderate as shown in Fig.4. Especially for the hedged portfolio of $n=10$, there is little fluctuation through the whole period. The diversification effects of the portfolio are obvious.

Comparing the naked portfolio and the hedged portfolio in Table.5, the mean of the variance of return of the hedged portfolio is always smaller than the naked portfolio for any n. On the other hand the standard deviation of the variance of return of the hedged portfolio isn't always smaller than the naked portfolio especially for $n=1$ or $n=3$. But there is a tendency for the standard deviation of the hedged portfolio to be smaller than the naked portfolio when n increases. As shown in Fig.4, the peak of distribution of the hedged portfolio is located slightly lower than the naked portfolio.

Comparing the 6 months investment and the 1 year investment in Table.5, the mean of the variance of return of the 1 year investment is smaller than the 6 months investment except December 2005 and the hedged portfolio of $n=10$. Since there are a few results for the 1 year investment, it is difficult to find the consistent tendency about the mean of the variance of return between the 6 months investment and the 1 year investment. But with a few exceptions, the standard deviation of the variance of return of the 1 year investment is generally greater than the 6 months investment.

Table.5 mean and standard deviation of variance of return*

6 month investment

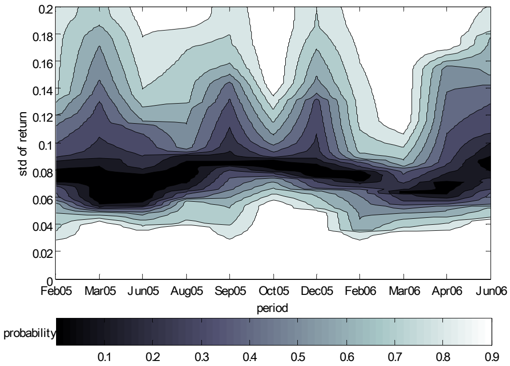
period	n=1 naked	n=1 hedge	n=3 naked	n=3 hedge	n=5 naked	n=5 hedge	n=10 naked	n=10 hedge
Feb05	0.0918	0.0792	0.0660	0.0525	0.0526	0.0384	0.0451	0.0299
	(0.0665)	(0.0692)	(0.0253)	(0.0283)	(0.0157)	(0.0166)	(0.0102)	(0.0101)
Mar05	0.0989	0.0848	0.0658	0.0532	0.0532	0.0404	0.0460	0.0308
	(0.0722)	(0.0789)	(0.0244)	(0.0281)	(0.0138)	(0.0146)	(0.0110)	(0.0122)
Jun05	0.0787	0.0651	0.0553	0.0407	0.0467	0.0329	0.0406	0.0246
	(0.0388)	(0.0344)	(0.0210)	(0.0191)	(0.0152)	(0.0139)	(0.0071)	(0.0068)
Aug05	0.0863	0.0608	0.0740	0.0428	0.0660	0.0326	0.0621	0.0237
	(0.0384)	(0.0368)	(0.0213)	(0.0176)	(0.0155)	(0.0110)	(0.0119)	(0.0051)
Sep05	0.1013	0.0686	0.0757	0.0416	0.0669	0.0329	0.0627	0.0245
	(0.0515)	(0.0518)	(0.0203)	(0.0181)	(0.0150)	(0.0103)	(0.0099)	(0.0060)
Oct05	0.0865	0.0561	0.0655	0.0378	0.0583	0.0300	0.0517	0.0229
	(0.0172)	(0.0260)	(0.0130)	(0.0154)	(0.0107)	(0.0099)	(0.0088)	(0.0059)
Dec05	0.1049	0.0748	0.0727	0.0504	0.0586	0.0377	0.0514	0.0254
	(0.0635)	(0.0586)	(0.0188)	(0.0221)	(0.0153)	(0.0151)	(0.0088)	(0.0065)
Feb06	0.0746	0.0681	0.0545	0.0437	0.0495	0.0368	0.0448	0.0297
	(0.0293)	(0.0301)	(0.0129)	(0.0119)	(0.0086)	(0.0072)	(0.0070)	(0.0043)
Mar06	0.0654	0.0594	0.0496	0.0421	0.0440	0.0356	0.0393	0.0291
	(0.0186)	(0.0189)	(0.0104)	(0.0095)	(0.0072)	(0.0062)	(0.0057)	(0.0038)
Apr06	0.0843	0.0761	0.0551	0.0486	0.0473	0.0408	0.0400	0.0332
	(0.0442)	(0.0397)	(0.0160)	(0.0145)	(0.0113)	(0.0097)	(0.0070)	(0.0056)
Jun06	0.0991	0.0943	0.0586	0.0541	0.0484	0.0432	0.0407	0.0341
	(0.0487)	(0.0481)	(0.0183)	(0.0173)	(0.0140)	(0.0124)	(0.0106)	(0.0085)

1 year investment

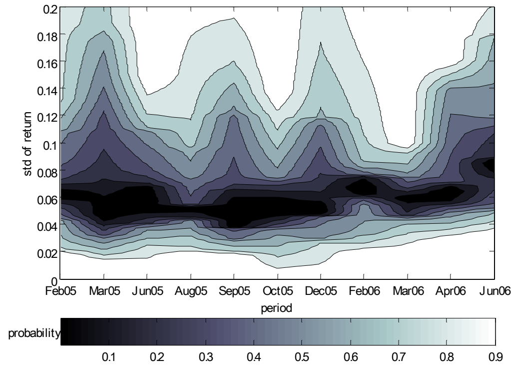
Period	n=1 naked	n=1 hedge	n=3 naked	n=3 hedge	n=5 naked	n=5 hedge	n=10 naked	n=10 hedge
Aug05	0.0768	0.0508	0.0652	0.0369	0.0637	0.0320	0.0617	0.0269
	(0.0776)	(0.0485)	(0.0309)	(0.0162)	(0.0248)	(0.0117)	(0.0176)	(0.0076)
Sep05	0.0708	0.0520	0.0646	0.0390	0.0628	0.0336	0.0587	0.0279
	(0.0344)	(0.0300)	(0.0251)	(0.0154)	(0.0227)	(0.0118)	(0.0179)	(0.0081)
Dec05	0.0949	0.0732	0.0868	0.0589	0.0725	0.0459	0.0583	0.0336
	(0.0838)	(0.0653)	(0.0514)	(0.0343)	(0.0332)	(0.0206)	(0.0203)	(0.0118)

* These numbers are the mean and the standard deviation of square root of the variance of return. The return is annualized for each investment horizon.

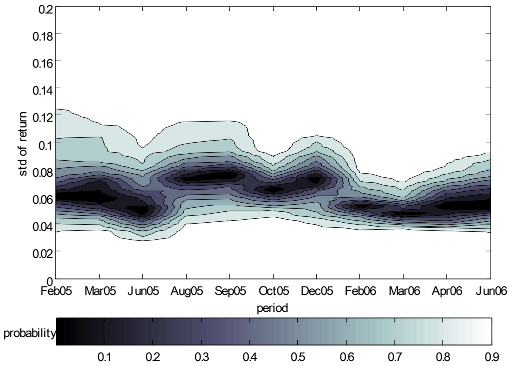
** Upper cell is mean and bracketed lower cell is standard deviation n=1, Naked



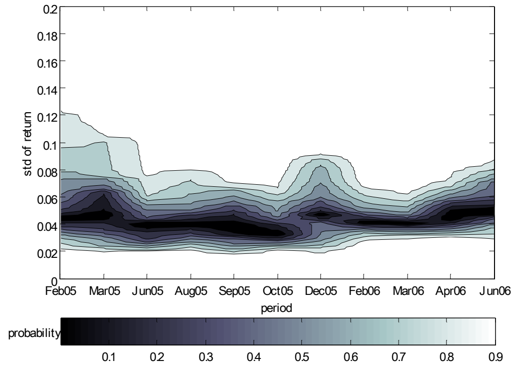
n=1, Hedge



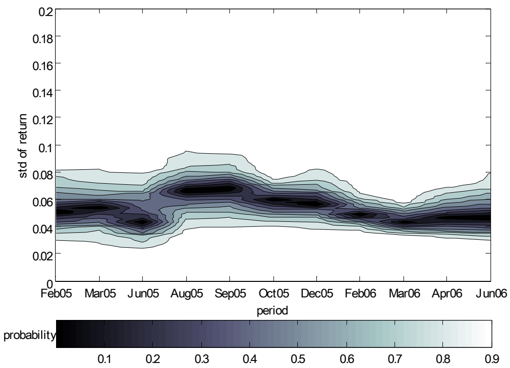
n=3, Naked



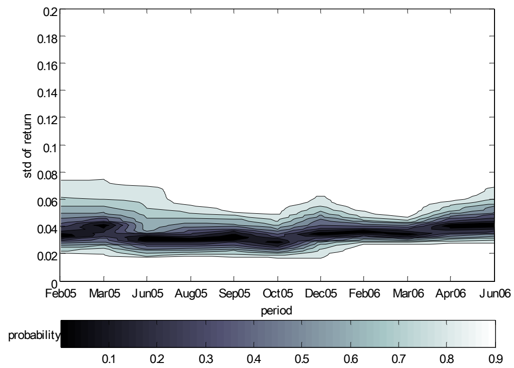
n=3, Hedge



n=5, Naked



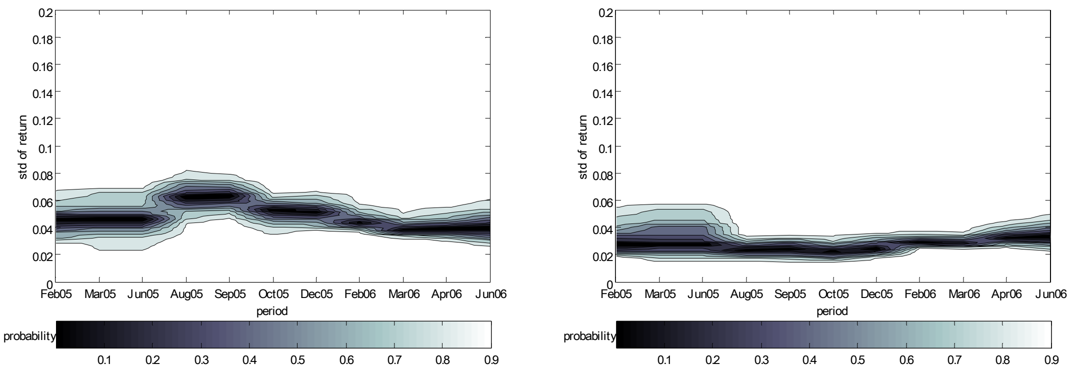
n=5, Hedge



n=10, Naked

n=10, Hedge

Fig.4 Probability distribution of variance of return (6 months investment)



4.2 Hedging Effectiveness

The hedging effectiveness in this study is defined as the ratio of square root of the variance of return of the hedged portfolio to the naked portfolio as the equation (13). So the smaller number indicates the greater hedging effectiveness. Table.6 shows the mean and the standard deviation of hedging effectiveness. The hedging effectiveness differs considerably between the periods. For $n=1$, the best is 0.6276 on October 2005, and the worst is 0.9463 on June 2006. For $n=10$, the best is 0.3906 on August 2005, and the worst is 0.8419 on June 2006. This latter best case is the best of all, and the standard deviation of return of the hedged portfolio is 39% of the naked portfolio in this case.

Table.6 mean and standard deviation of hedging effectiveness

6 month investment

period	n=1	n=3	n=5	n=10
Feb05	0.8211	0.7693	0.7204	0.6595
	(0.1285)	(0.1474)	(0.1436)	(0.1328)
Mar05	0.7577	0.7704	0.7495	0.6541
	(0.2189)	(0.1561)	(0.1200)	(0.1236)
Jun05	0.8246	0.7375	0.7022	0.6000
	(0.1956)	(0.1573)	(0.1315)	(0.0927)
Aug05	0.6887	0.5815	0.4998	0.3906
	(0.1661)	(0.1658)	(0.1377)	(0.0978)
Sep05	0.6361	0.5455	0.4997	0.3939
	(0.2203)	(0.1765)	(0.1369)	(0.0945)
Oct05	0.6276	0.5694	0.5244	0.4593
	(0.1939)	(0.1674)	(0.1754)	(0.1607)
Dec05	0.6654	0.6749	0.6473	0.5083
	(0.1936)	(0.1786)	(0.1902)	(0.1701)

Feb06	0.8996 (0.0782)	0.7991 (0.0826)	0.7436 (0.0749)	0.6656 (0.0534)
Mar06	0.9012 (0.0680)	0.8480 (0.0686)	0.8089 (0.0628)	0.7441 (0.0499)
Apr06	0.9086 (0.0752)	0.8815 (0.0540)	0.8638 (0.0489)	0.8324 (0.0338)
Jun06	0.9463 (0.0552)	0.9236 (0.0529)	0.8959 (0.0460)	0.8419 (0.0411)

1 year investment

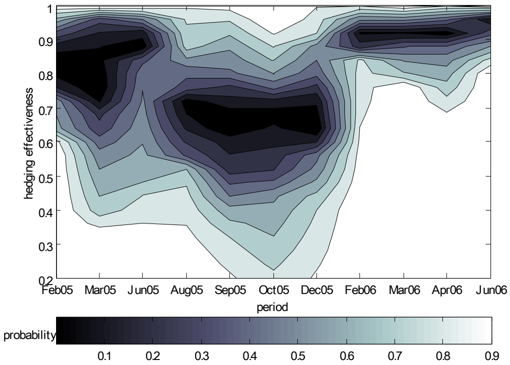
period	n=1	n=3	n=5	n=10
Aug05	0.6908 (0.1816)	0.5902 (0.1418)	0.5186 (0.1098)	0.4386 (0.0418)
Sep05	0.7303 (0.1595)	0.6120 (0.1288)	0.5452 (0.0967)	0.4819 (0.0601)
Dec05	0.7367 (0.1477)	0.6813 (0.1000)	0.6390 (0.0842)	0.5795 (0.0605)

As shown in Table.6, when n becomes greater, the mean of hedging effectiveness becomes higher for any period. And though there are a few exceptions, the standard deviation of returns is smaller for the portfolio with greater number of properties. So we can find the diversification effect also impacts the hedging effectiveness

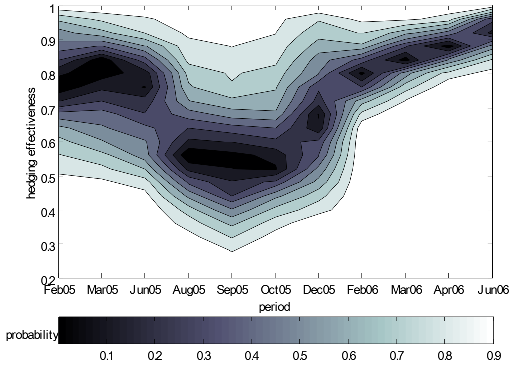
Comparing the 6 months investment and the 1 year investment, the mean of hedging effectiveness of the 1 year investment is lower than the 1year investment. The standard deviation of the 1 year investment is smaller than the 6 months investment with only one exception ($n = 1$ on August 2005).

Fig.5 is the probability distribution of the 6 months investment's hedging effectiveness. There is a break in the hedging effectiveness between June and August 2005. The hedging effectiveness becomes lower after that gradually. This tendency becomes obvious when n increases. We consider the cause of this movement in the next section.

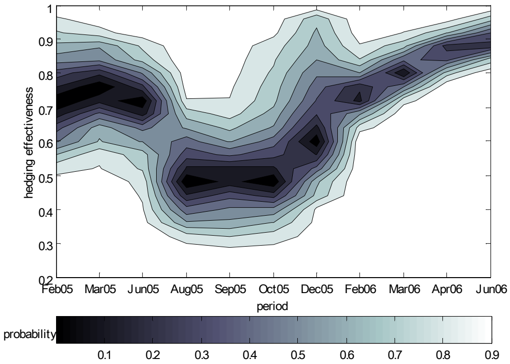
n=1 n=3



n=5



n=10



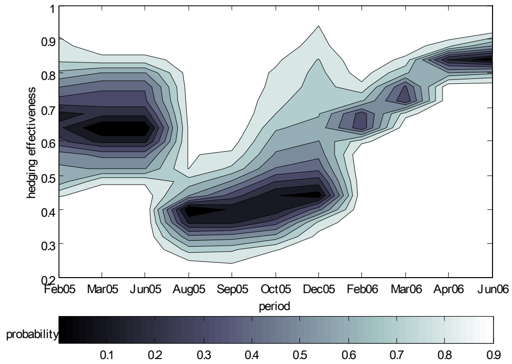


Fig.5 Probability distribution of hedging effectiveness (6 months investment)

4.3 Determinant of Hedging Effectiveness

The variance of return of the hedged portfolio, a minimum-variance portfolio, is given in equation (12). That is to say the reduction in variance of return depends on the parameters A and B given in equation (10). The parameter A depends on $V[I_t]$ (variance of index), $V[r_t]$ (variance of LIBOR) and $Cov[I_t, r_t]$ (covariance of index and LIBOR). The parameter B depends on $Cov[\sum w_i y_{i,t}, r_t]$ (covariance of property returns and LIBOR) and $Cov[\sum w_i y_{i,t}, I_t]$ (covariance of property returns and index). Fig.6 shows the fluctuation of these factors and the hedging effectiveness for each number of properties in the portfolio.

n=1

n=3

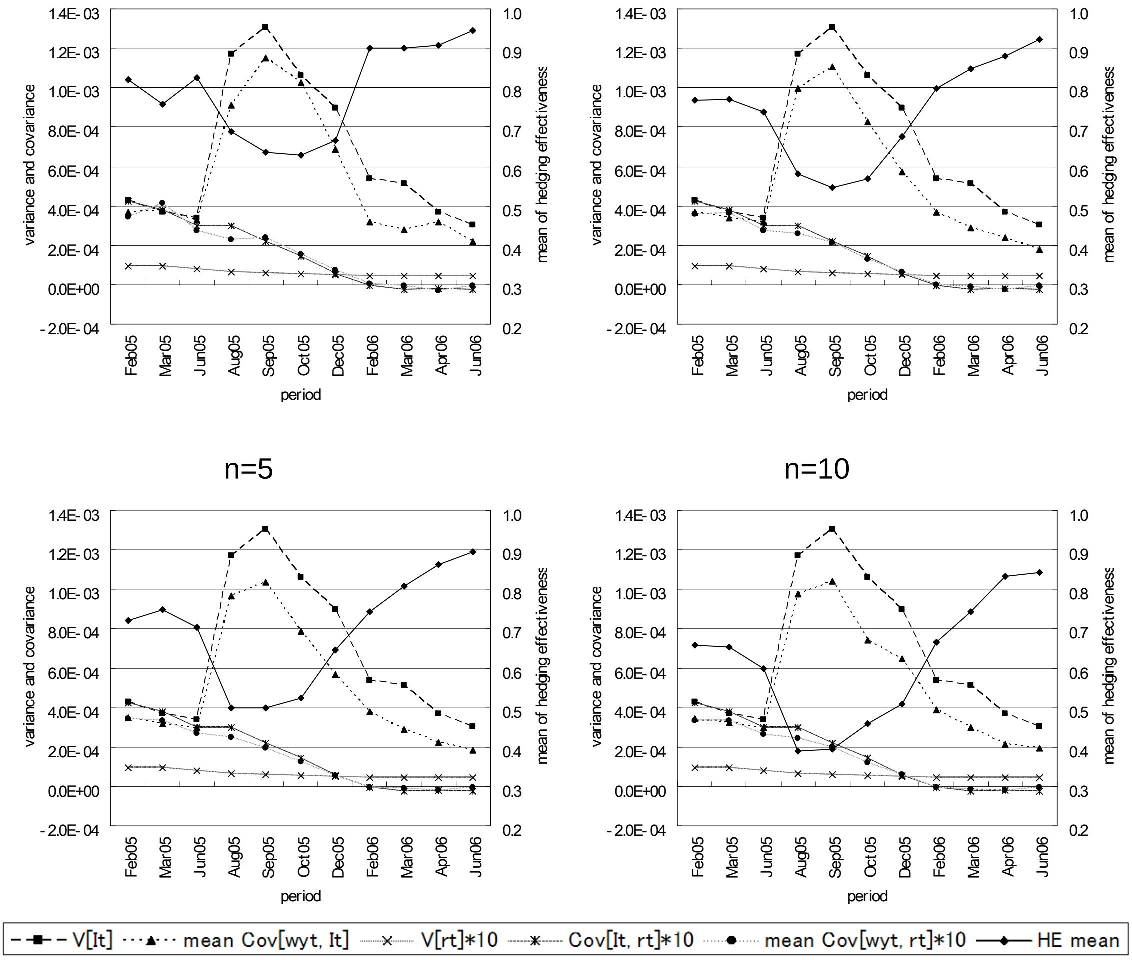


Fig.6 Hedging Effectiveness Factors

* Since the values of $V[r_t]$, $Cov[I_t, r_t]$ and $Cov[\sum w_i y_{i,t}, r_t]$ are considerably smaller than the others, their values are multiplied by a factor of 10 when presented in this figure.

Table.7 regressions for hedging effectiveness

6 months investment

	n=1	n=3	n=5	n=10
R square	0.3960	0.611634	0.7357	0.8461
s.e.	0.1433	0.1151	0.0970	0.0752
F-statistics	29.9672**	680.3517**	1035.2900**	1575.6210**
samples	234	2165	1865	1438
constant	1.1087	1.3391	1.4660	1.8045
	(8.4576)**	(21.4794)**	(21.8457)**	(25.5834)**
$V[r_t]$	-20034.1162	-73351.3526	-102868.7962	-179288.1786
	(-0.8756)	(-6.1859)**	(-7.8745)**	(-12.8591)**
$V[I_t]$	-202.1544	-149.8692	-119.8846	-116.5907

	(-3.8210)**	(-7.4822)**	(-6.1557)**	(-5.8988)**
$Cov[I_t, r_t]$	581.2318	6611.0867	8547.6393	13081.2486
	(0.2265)	(5.0011)**	(5.8429)**	(8.0137)**
$Cov[\sum w_i y_{i,t}, r_t]$	-290.3318	601.5796	1889.1773	6285.1488
	(-0.4508)	(1.7879)*	(4.3946)**	(9.1533)**
$Cov[\sum w_i y_{i,t}, I_t]$	-108.8612	-325.7864	-464.9742	-642.8434
	(-4.2984)**	(-26.0386)**	(-33.8721)**	(-40.0975)**

1 year investment

	n=1	n=3	n=5	n=10
R square	0.340012	0.493274	0.469299	0.6100
s.e.	0.1772	0.1172	0.0902	0.0504
F-statistics	8.7581	219.0266	182.3430	312.2680
samples	90	1130	1036	1002
constant	2.3082	2.9817	3.1178	2.8656
	(2.7523)**	(15.2103)**	(14.1333)**	(7.0952)**
$V[r_t]$	-97076.4518	-146973.3900	-161591.1377	-150814.8913
	(-1.8583)*	(-11.7811)**	(-11.0017)**	(-5.5142)**
$V[I_t]$	-21.9104	-3.5337	1.5756	-4.1332
	(-1.4507)	(-0.9510)	(0.4504)	(-1.5556)
$Cov[I_t, r_t]$	6171.4083	7694.2281	7839.7982	6905.9100
	(1.7759)*	(9.6275)**	(9.2574)**	(4.2585)**
$Cov[\sum w_i y_{i,t}, r_t]$	-367.1686	-202.9320	-102.4349	-17.5641
	(-1.0294)	(-1.1917)	(-0.4895)	(-0.0821)
$Cov[\sum w_i y_{i,t}, I_t]$	-19.1259	-33.2805	-30.2912	-16.6237
	(-3.7217)**	(-20.5320)**	(-17.8646)**	(-10.9040)**

lower bracketed cell is t-statistics of estimate
** 1% significant
* 10% significant

As shown Fig.6 the fluctuations of $V[I_t]$ and $Cov[\sum w_i y_{i,t}, I_t]$ are relatively larger than the others, the hedging effectiveness moves in inverse proportion to them. In order to determine the relative importance of these variables we estimate the regression model with these 5 factors as independent values. This result is shown in Table.7.

Concerning the level of the independent values and coefficients, the major determinants of hedging effectiveness are $V[I_t]$ and $Cov[\sum w_i y_{i,t}, I_t]$. Observing the equation (12) the increase of A operates to reduce the hedging effectiveness, and the increase of B improves the hedging effectiveness. Though the increase of $Cov[\sum w_i y_{i,t}, I_t]$ decreases B, since B normally takes negative value, the absolute value of B becomes large with the increase of $Cov[\sum w_i y_{i,t}, I_t]$. It means that the increase of $Cov[\sum w_i y_{i,t}, I_t]$ improves the hedging effectiveness. So it is appropriate that the coefficient of $Cov[\sum w_i y_{i,t}, I_t]$ takes negative value. On the other hand it should be strange that the coefficient of $V[I_t]$ takes negative value. Because the increase of $V[I_t]$ increases A, the increase of A lowers the hedging effectiveness. But in the case of this study, as the changes in $V[I_t]$ and $Cov[\sum w_i y_{i,t}, I_t]$ are almost the same shown in Fig.6, it is inevitable that those two coefficients take same direction as a result of regression. Nevertheless the weight of $Cov[\sum w_i y_{i,t}, I_t]$ is much larger than $V[I_t]$ except the case of $n=1$, so we can say that the hedging effectiveness is mainly defined by $Cov[\sum w_i y_{i,t}, I_t]$. The break in the hedging effectiveness between June 2005 and August 2005 is brought about by the rapid increases in the covariance of property returns and index. The decline of the covariance after that reduces the hedging effectiveness.

Thus the covariance of property returns and index is the main factor, and it is highly volatile as shown in Fig.6. This means that the basis risk of hedging with TRS is very high, and the hedging effectiveness varies depending on the period. The mean of the covariance of property returns and index does not change as much even if n increases. But the weight of the coefficient in the regression becomes heavier with the increase of n as shown in Table.7. This is because the covariance of property returns and index tends to concentrate around the mean value with the increase of n. Fig.7 shows the dispersion of the covariance of property returns and index and the hedging effectiveness on September 2005. In the case of $n=1$ or $n=3$, there are some portfolios that take extreme covariance. But in the case of $n=10$, there is no such portfolio, and the covariance and the hedging effectiveness concentrate on the center. This means that the reason of improvement of hedging effectiveness with the increase of the number of properties in portfolio is not the rise of the average covariance but the convergence to the average covariance.

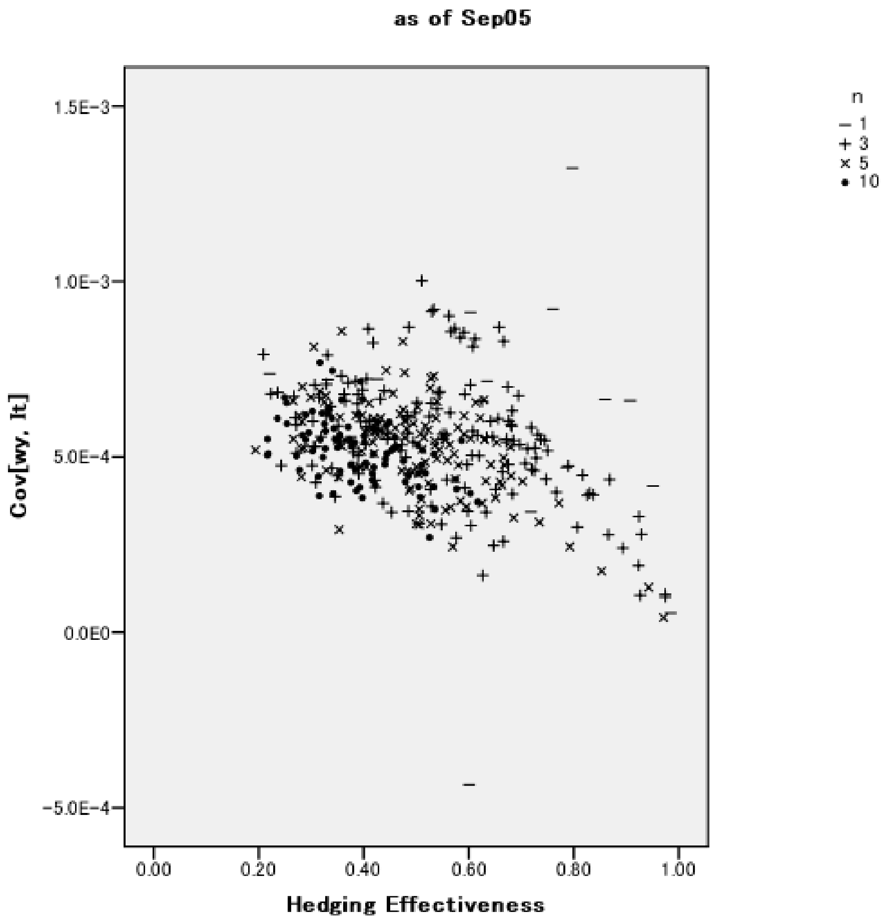


Fig.7 Covariance and Hedging Effectiveness

5. Actual Assessment of Hedging Effectiveness

5.1 Actual Performance Score

Though the model is estimated with historical data in the past, investors want to discover whether they can actually get the expected hedging effectiveness based on the estimated model. Since the term length covered by the data of this study is relatively short long, we do not have enough data to calculate the variance of returns ex post.. Then we devise the following indicator to assess the actual hedging effectiveness with one period data.

As mentioned before we use 6 periods data for each calculation. The first 5 periods are used to estimate the model, and the last period is used for the actual assessment. Our indicator expressed in the equations (13) and (14) is similar to the Sharpe ratio. This indicator is the ratio of the difference between the actual return on the last period and the expected return predicted by the model to the square root of the variance of return predicted by the model. This indicator represents the actual excess return for the risk the investor expect to take. We call this indicator the actual performance score.

$$S_{N,t} = \frac{\hat{R}_{N,t+1} - E[R_{N,t+1}]}{\sqrt{V[R_{N,t}]}} \tag{13}$$

$$S_{H,t} = \frac{\hat{R}_{H,t+1} - E[R_{H,t+1}]}{\sqrt{V[R_{H,t}]}} \tag{14}$$

Where:

$\hat{R}_{N,t+1}$: actual return of the naked portfolio from t to $t+1$

$\hat{R}_{H,t+1}$: actual return of the hedged portfolio from t to $t+1$

$E[R_{N,t+1}]$: expected return of the naked portfolio at $t+1$ predicted at t

$E[R_{H,t+1}]$: expected return of the hedged portfolio at $t+1$ predicted at t

5.2 Summary of Score

Table.8 shows the mean and the standard deviation of the actual performance score for each portfolio and for each period. It shows the probability distribution of the actual performance score of the 6 months investment. Though the mean score of the hedged portfolio is better than the naked from August 2005 until April 2006 (except April 2006 of n=10) as shown in the shadowed cell in Table.8, the standard deviation of the hedged portfolio is always larger than the naked portfolio, and the range of distribution of the hedged portfolio is always wider than the naked portfolio as shown in Fig.8. This means that the difference between actual return and expected return of the hedged portfolio doesn't always correspond to the risk the investor expects to take compared with the naked portfolio.

Table.8 mean and standard deviation of actual performance score

6 month investment

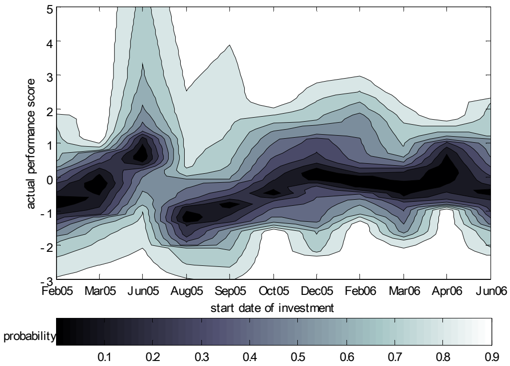
start date	n=1		n=3		n=5		n=10	
	1-naked	1-hedge	3-naked	3-hedge	5-naked	5-hedge	10-naked	10-hedge
Feb05	-0.6864 (1.2122)	-1.0395 (1.6250)	-0.4412 (0.8999)	-0.8717 (1.3291)	-0.4117 (0.8348)	-0.9125 (1.3137)	-0.4572 (0.6300)	-1.0832 (1.2251)
Mar05	-0.3366 (1.0090)	-1.1418 (2.2930)	-0.0514 (0.8600)	-0.5590 (1.1822)	-0.1735 (0.7505)	-0.7540 (1.1222)	-0.2932 (0.6182)	-1.1372 (1.1218)
Jun05	1.1830 (2.3420)	0.8061 (3.4105)	1.0365 (1.6412)	0.6183 (2.6447)	0.8892 (1.3977)	0.4249 (2.2744)	0.6221 (0.9671)	0.0007 (1.8064)
Aug05	-1.0909 (1.2316)	-0.8055 (1.6307)	-0.7256 (0.9772)	0.0600 (1.3448)	-0.6365 (0.7838)	0.4612 (1.4086)	-0.9408 (0.6144)	-0.0717 (1.3447)
Sep05	-0.7315 (1.4471)	-0.0720 (2.1840)	-0.9357 (0.7955)	-0.0838 (1.4214)	-0.8572 (0.7462)	0.2097 (1.2657)	-0.8073 (0.4738)	0.7225 (1.1294)
Oct05	-0.1856 (1.0858)	1.3283 (1.9985)	-0.2265 (0.9821)	1.3574 (1.4894)	-0.1837 (0.9353)	1.6111 (1.2998)	-0.1431 (0.8448)	2.0604 (1.1968)
Dec05	0.0882 (1.4338)	1.0334 (2.3145)	0.1767 (1.1606)	1.2518 (1.1832)	0.1567 (1.1245)	1.3369 (1.1069)	-0.0224 (0.9118)	1.8130 (1.1667)
Feb06	0.2166 (1.2728)	0.5966 (1.4878)	0.1171 (0.9175)	0.6577 (1.1242)	0.0152 (0.7840)	0.6477 (1.0378)	-0.0107 (0.6248)	0.7803 (0.9165)
Mar06	-0.3156 (1.1293)	-0.1722 (1.2419)	-0.1563 (0.9591)	0.0816 (1.0973)	-0.0488 (0.8535)	0.2689 (1.0307)	0.0703 (0.7058)	0.5070 (0.9049)
Apr06	0.1925 (0.9217)	0.2061 (1.0396)	-0.0567 (0.7323)	-0.0462 (0.8358)	-0.1160 (0.6636)	-0.1084 (0.7668)	-0.2175 (0.5204)	-0.2244 (0.6236)
Jun06	-0.3156 (1.2869)	-0.4030 (1.3709)	0.2953 (1.3337)	0.2467 (1.4437)	0.2632 (1.1172)	0.2329 (1.2272)	-0.0451 (0.8147)	-0.1194 (0.9567)

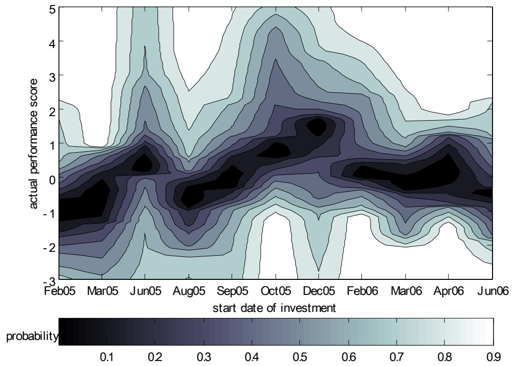
1 year investment

start date	n=1		n=3		n=5		n=10	
	naked	hedge	naked	hedge	naked	hedge	naked	hedge
Aug05	-0.3060	0.6622	-0.4545	1.0837	-0.5724	1.2178	-0.6976	1.3778
	(1.2794)	(1.3254)	(0.8201)	(0.9929)	(0.6487)	(0.9617)	(0.4180)	(0.8952)
Sep05	-0.4538	0.4062	-0.5944	0.7860	-0.7114	0.8473	-0.7593	1.0298
	(1.4850)	(1.5383)	(0.7425)	(0.9313)	(0.6409)	(0.9460)	(0.4870)	(0.8924)
Dec05	-0.4547	0.4217	-0.7159	0.2483	-0.7188	0.3530	-0.6833	0.5811
	(0.8231)	(0.9820)	(0.6806)	(0.8275)	(0.6692)	(0.8761)	(0.5788)	(0.8581)

n=1, Naked

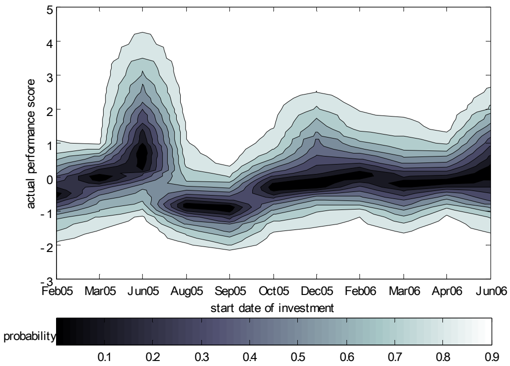
n=1, Hedge

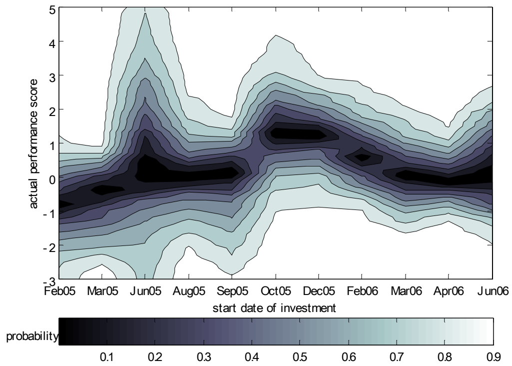




n=3, Naked

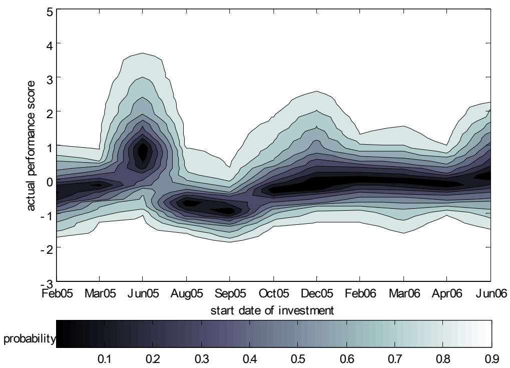
n=3, Hedge

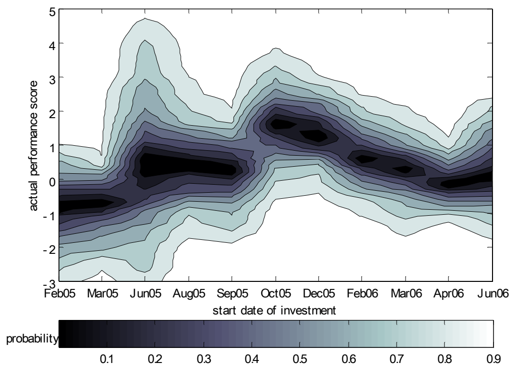




n=5, Naked

n=5, Hedge





n=10, Naked

n=10, Hedge

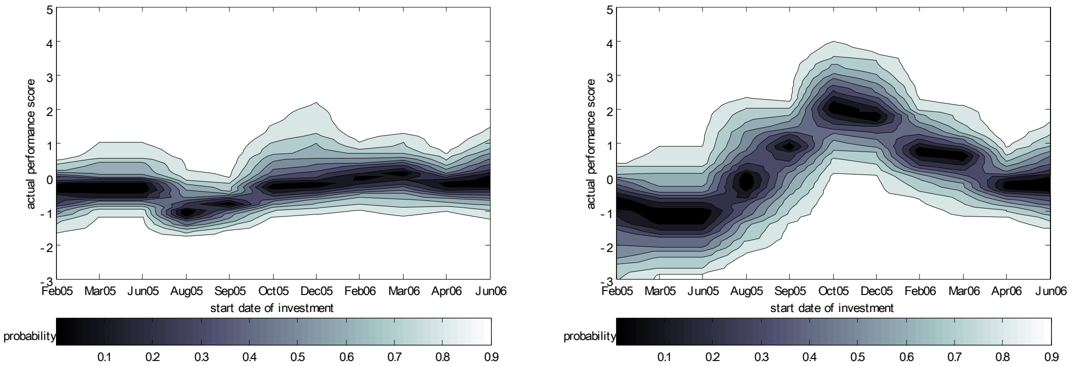


Fig.8 Probability distribution of actual performance score (6 months investment)

5.3 Prediction Error and Actual Performance

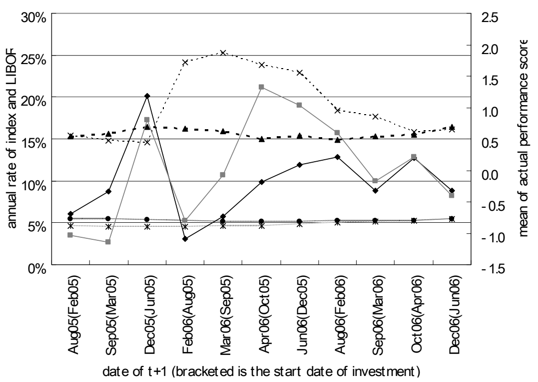
As shown the equations (7) and (8) the expected return depend on $a_I I_{t-1}$. That is to say the difference between actual return and expected return depends on the prediction of I_t. Fig.3 in Section 3.1 shows that there is a break in the index process between June and July 2005. This should affect the estimated model of index and the accuracy of the prediction.

Fig.9 shows the index prediction and the actual performance score. The index is overestimated after August 2005 as the start date of investment because the break is included to the terms of model estimation. For LIBOR there is no such difference between the actual value and the prediction. The difference between the actual index and the predicted index is largest at August 2005 as the start date of investment, and it becomes smaller gradually to June 2006. As the overestimation of the index means that the actual return would be smaller than the prediction, the actual performance score becomes smaller by the overestimation. Actually the score of naked portfolio sharply falls at August 2005, and afterwards recovers. The score of the hedged portfolio also falls in August, but its degree is smaller than the naked portfolio and the recovery after is quite large. Since the hedged portfolio pays the index, the overestimation of index means that the actual payment of the index should be smaller than the expectation. This is the reason why the mean score of the hedged portfolio is better than the naked from August 2005 until April 2006.

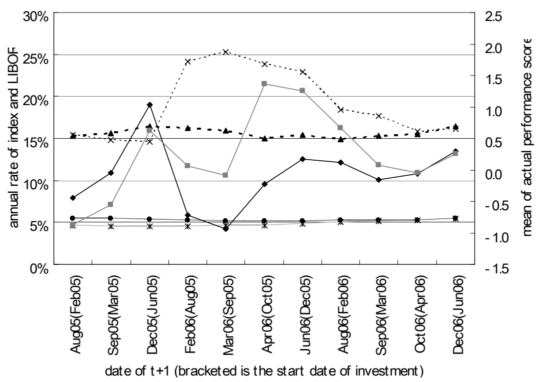
This effect of the overestimation of index seems to fade away from around December 2005 for the naked portfolio. This is because the correlation of the individual property to the index, i.e. a_i, declines after December 2005 as shown in Table.3 in Section 3.2. For the hedged portfolio the overestimation effect lasts until around March 2006. As the hedged portfolio has the index payment, the effect is greater for it than for the naked portfolio.

Needless to say, the appropriateness of the model also affects the actual performance. As the return of portfolio critically depends on the prediction of the index, the prediction error includes the difference between the actual return and the expectation compared with the expected risk that investor take. One extreme change tends to make an enormous impact on the estimation of the model, because the model is estimated with a few observations in this study. But from the other point of view, the quality of the index is also a critical factor to affect to the actual performance. As mentioned in Section 3.1, the index applied in this study is a simple mean of about 200 properties. The break between June and July 2005 would certainly reflect the real change but there might be some doubt whether that sudden change typifies the whole market change.

n=1



n=3



n=5

n=10

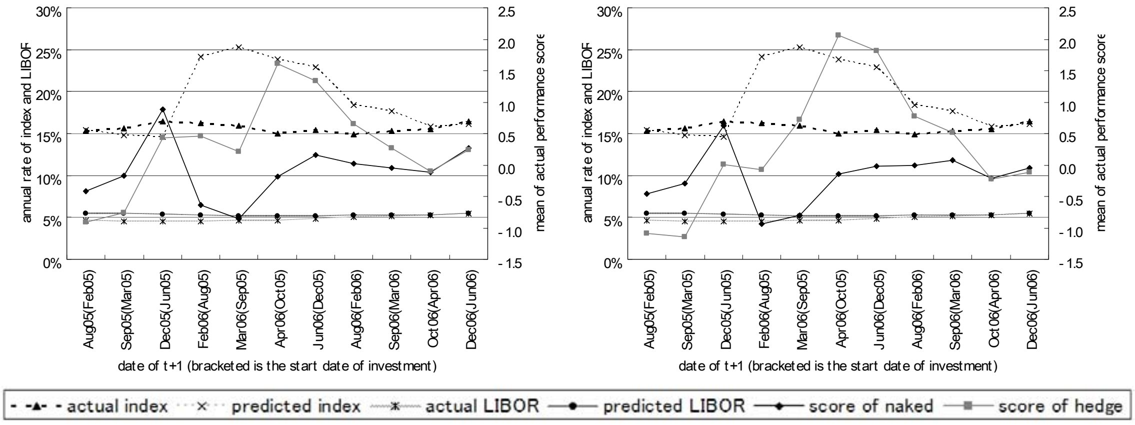


Fig.9 index prediction and performance score

6. Conclusions

Applying the real estate index and the individual property data in Japanese market, we develop a model for the real estate portfolio hedged with TRS, and assess the hedging effectiveness. We assess the actual performance if investors hedge the portfolio based on the model prediction. As a result, the variance of return is certainly reduced by the hedge with TRS, and the standard deviation of return of the hedged portfolio is 39% of the naked portfolio in the best case. We also confirm that in general the more diversified the portfolio, the more effective the hedge.

But the hedging effectiveness differs substantially between periods, because the covariance of property returns and index is highly volatile. This means that the basis risk of hedging with TRS is high, and the hedging effectiveness subsequently varies depending on the period. We find the diversification effect in hedging effectiveness is brought by not the rise of the average covariance but the convergence to the average covariance.

For actual performance assessment, we find that the difference between actual return and expected return of the hedged portfolio doesn’t always correspond to the risk the investor expects to take compared with the naked portfolio. This is because the return of portfolio depends critically on the prediction of the index and the prediction error of the index model is too large. As the model is estimated with relatively few observations in this study because of the restriction of data, one extreme change tends to make an enormous impact on the estimation. The sudden break included in the actual index process makes the model estimation problematic, and this produces a large prediction error.

Acknowledgment

This work was supported by KAKENHI* (19510155).

* Grant-in-Aid for Scientific Research (C) in Japan

References

Alexander, Carol and Andreza Barbosa (2007) “Effectiveness of Minimum-Variance Hedging”, Journal of Portfolio Management, Vol. 33 No. 2, 46-59.

Alexander, Carol and Elizabeth Sheedy Eds. (2008) The Professional Risk Managers' Guide to Finance Theory and Application (PRMIA Professional Risk Management Series), McGraw-Hill, 400pp.

Amihud, Yakov and Haim Mendelson (1989) “The Effects of Beta, Bid-Ask Spread, Residual Risk, and Size on Stock Returns”, The Journal of Finance, Vol. 44, No. 2, 479-486.

Baillie, Richard T. and Robert J. Myers (1991) “Bivariate Garch Estimation of the Optimal Commodity Futures Hedge”, Journal of Applied Econometrics, Vol. 6, No. 2, 109-124.

Benveniste, Lawrence, Dennis R. Capozza and Paul J. Seguin (2001) “The Value of Liquidity”, Real Estate Economics, Vol. 29, No. 4, 633-660

Bond, Shaun A., Soosung Hwang, Zhenguo Lin and Kerry D. Vandell (2007) “Marketing Period Risk in a Portfolio Context: Theory and Empirical Estimates from the UK Commercial Real Estate Market”, Journal of Real Estate Finance and Economics, Vol. 34, 447-461.

Boveroux, Philippe and Albert Minguet (1999) “Selecting Hedge Ratio Maximizing Utility or Adjusting Portfolio’s Beta”, Applied Financial Economics, Vol. 9, 423–432

Bjork, Tomas and Eric Clapham (2002) “On the pricing of real estate index linked swaps”, Journal of Housing Economics, Vol. 11, 418-432

Buttimer, Richard J, James B. Kau and V. Carlos Slawson (1997) “A model for Pricing Securities Dependent upon a Real Estate Index”, Journal of

Housing Economics, Vol.6, 16-30

Case, Karl E. and Robert J. Shiller (1996) "Mortgage Default Risk and Real Estate Prices: The Use of Index-Based Futures and Options in Real Estate", Journal of Housing Research, Vol. 7, No. 2, 243-258

Cecchetti, Stephen G., Robert E. Cumby and Stephen Figlewski (1988) "Estimation of the Optimal Futures Hedge", The Review of Economics and Statistics, Vol. 70, No. 4, 623-630.

Collet, David, Colin Lizieri and Charles Ward (2003) "Timing and the Holding Periods of Institutional Real Estate", Real Estate Economics, Vol. 31, No. 2, 205-222

Ederington, Louis H. (1979) "The Hedging Performance of the New Futures Markets", The Journal of Finance, Vol. 34, No. 1, 157-170.

Englund, Peter, Min H. Wang and John M. Quigley (2002) "Hedging Housing Risk", Journal of Real Estate Finance and Economics, Vol. 24, No. 2, 167-200

Hansen, Lars P. (1982) "Large Sample Properties of the Generalized Methods of Moments", Econometrica, Vol. 50, 1029-10.

Harris, Richard D. F. and Evarist Stoja (2008) "The Limits to Minimum-Variance Hedging" presented at the academic research seminars of the ICMA center in the University of Reading on 30th April 2008.

Howard, Charles T. and Louis J. D'Antonio (1984) "A Risk-Return Measure of Hedging Effectiveness", Journal of Financial and Quantitative Analysis, Vol. 19, No. 1, 101-112

Iacoviello, Matteo and Francois Ortalo-Magne (2003) "Hedging Housing Risk in London", Journal of Real Estate Finance and Economics, Vol. 27, No. 2, 191-209

Johnson, Leland L. (1960) "The Theory of Hedging and Speculation in Commodity Futures", The Review of Economic Studies, Vol. 27, No. 3, 139-151.

Moridaira, Soichiro (2006) "Pricing Model of Real Estate Index Derivatives" in

Takeaki Kariya and Masahisa Fujita Eds. 'JAREFE Journal 2006: Evolution of Real Estate Financial Engineering', Toyokeizai, 254pp. (Japanese Literature)

Park, Tae H., and Lorne N. Switzer (1996) "An Economic Analysis of Real Estate Swaps", Canadian Journal of Economics, Vol. 29, Special Issue: Part 2, S527-S533.

Park, Tae H., and Lorne N. Switzer (1995) "Risk Management of Real Estate: The Case of Real Estate Swaps", Journal of Real Estate Finance and Economics, Vol. 11, 219-233.

Patel, Kanak and Ricardo Pereira (2008) "Pricing Property Index Linked Swaps with Counterparty Default Risk", Journal of Real Estate Finance and Economics, Vol. 36, 5-21

Shiller, Robert J. and Allan N. Weiss (1999) "Home Equity Insurance", Journal of Real Estate Finance and Economics, Vol. 19, No. 1, 21-47

Syz, Juerg, Paolo Vanini and Marco Salvi (2007) "Property Derivatives and Index-Linked Mortgages", Journal of Real Estate Finance and Economics, Online First.

Vasicek, Oldrich (1977). "An Equilibrium Characterisation of the Term Structure". Journal of Financial Economics Vol. 5, 177-188.