Dynamical Theory of Superfluidity in One Dimension

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A theory accounting for the dynamical aspects of the superfluid response of one dimensional (1D) quantum fluids is reported. In long 1D systems, the onset of superfluidity is related to the dynamical suppression of quantum phase slips at low temperatures. The effect of this suppression as a function of frequency and temperature is discussed within the framework of the experimentally relevant momentum response function. Applications of these results to the understanding of the superfluid properties of helium confined in 1D pores with nanometer diameter, dislocations in solid ^{4}He, and ultracold atomic gases are also briefly discussed.

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Superfluidity is undoubtedly one of the most important phenomena in quantum physics with fundamental, wide-ranging implications. Although long-range phase coherence is often associated with superfluidity and superconductivity, the former is not a necessary condition for the latter. This was demonstrated by the observation of superfluid response in two dimensions (2D): torsional oscillator (TO) experiments on ^{4}He films have established [1] the existence of superfluidity (observed as a change in the resonance frequency of the TO), despite the lack of long-range order in 2D at finite temperatures. 2D superfluidity without long-range phase coherence can still be understood in terms of the helicity modulus [2], which is a thermodynamic (i.e., static) property. However, superfluidity manifests itself experimentally as a dynamical property, and in 2D, dynamical corrections [3] to the helicity modulus are important in understanding the experimental observations. In one dimension (1D), the helicity modulus vanishes altogether in the thermodynamic limit [4]. Therefore, should superfluidity exist in 1D, dynamics is essential, and not just a correction to the static picture.

Indeed, recent TO experiments have detected superfluidity in long (0.2–0.5 μm) nanometer-wide pores filled with liquid ^{4}He [5,6], where a suppression of the superfluid onset temperature by pressurization and reduction of the pore diameter was observed. In optical lattices, it was found that a Bose-Einstein condensate of ultracold ^{87}Rb atoms exhibits coherent current oscillations [7,8]. However, when confined to 1D, the motion of the same ultracold degenerate gas becomes strongly damped even in the presence of a relatively weak periodic potential [8].

These observations call for a reconsideration of the notion of superfluidity in 1D. Indeed, in this Letter we show that superfluidity in 1D is essentially a dynamical phenomenon, in accord with the fact that experimentally superfluid properties are probed at finite frequencies. Compared to higher dimensions, dynamics in 1D tends to be more constrained by the existence of conserved quantities. Recently, this has been shown to prevent complete thermalization [9] or the total decay of a current [10] in 1D integrable systems. This is, we find, also important for the understanding of superfluidity in 1D.

Furthermore, by analyzing the momentum response of bosons in a periodic potential (which is a relevant model for the ^{4}He systems of Refs. [6,11] and 1D ultracold atomic gases in optical lattices [8,12]), we show that the superfluid onset temperature decreases with decreasing probe frequency (cf. Fig. 1) or decreasing compressibility of the fluid (cf. Fig. 2). The latter can provide an explanation for the pressure-dependent suppression of superfluidity observed in the experiments of Ref. [6].

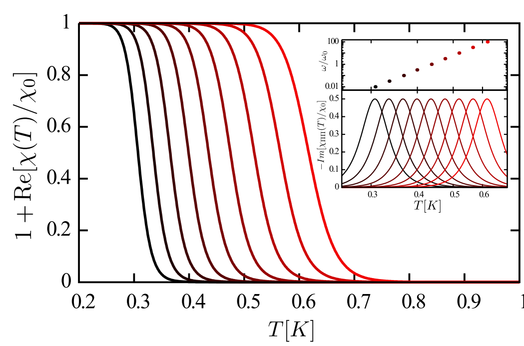

FIG. 1 (color online). Superfluid response for different probe frequencies ω, which ranges from $10^{-2}\omega_0$ (dark) to $10^2\omega_0$ (bright), where $\omega_0 = 2$ kHz [6]. We used two terms for H_{PS} [cf. Eq. (5)] setting $\Delta k_{10} = 0.007a_0^{-1}$, $\Delta k_{11}/\Delta k_{10} = 0.7$, and $g_{10} = g_{11} = 1$; $\chi_0 = M^2 vK/\pi\hbar$. The inset shows $\mathrm{Im}\chi(\omega; T)$ (lower part) and the peak temperature dependence on ω on a log-log scale. Linear density (ρ_0) and cutoff (a_0) are chosen so as to conform to the experimental situation in [6]. The sound velocity, $v = 200$ m/s, and the Luttinger parameter, $K = 8.1$, resulting in onset temperatures comparable to the experimental ones.

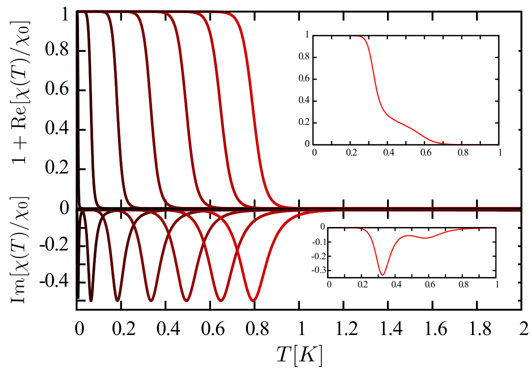

FIG. 2 (color online). Superfluid response vs T for different values of the Luttinger parameter K which ranges in unit steps from 3.2 (dark) to 9.2 (bright), ($g_{10} = g_{11} = 1$, $\Delta k_{10} = 0.001 a_0^{-1}$, and $\Delta k_{11}/\Delta k_{10} = 0.7$). The insets ($K = 6.2$) show the effect of a larger value of the PS momenta ($g_{01} = g_{11} = 1$, $\Delta k_{10} = 0.5 a_0^{-1}$, and $\Delta k_{11}/\Delta k_{10} = 0.7$), resulting in two dissipation peaks.

First, let us discuss the concept of superfluidity in general. As emphasized above, some of the most striking experimental manifestations of superfluidity are dynamical in nature. In particular, we can consider the following simple gedanken experiment: in the initial state a fluid is prepared in equilibrium with a pipe-shaped container that is moving at small velocity v along the direction of the pipe axis. If the container is suddenly stopped, a normal fluid will eventually come to rest by virtue of its interaction with the container, which makes it thermalize. On the other hand, in a superfluid, a fraction $\rho_s(T)/\rho_0$ of the total density ρ_0 will continue to move indefinitely with velocity v because the superfluid velocity is topologically constrained by the quantization of the condensate phase [13]. Indeed, the superfluid component behaves as a zero viscosity fluid, which does not interact with the container. Nevertheless, it is clear that the interaction between the fluid and the container, which breaks translational invariance, is crucial in defining superfluidity.

The superfluid density ρ_s is also related to the helicity modulus, Υ, which is a static quantity. The latter is defined as response of the system to a change of the boundary conditions of the many body wave function. To understand this definition physically, imagine that we give the fluid a boost to a state moving with constant velocity $\mathbf{v}$ by means of the transformation $\mathcal{U}_{\mathbf{v}} = e^{iM\mathbf{v}\cdot\mathbf{R}/\hbar}$, where $\mathbf{R} = \sum_j \mathbf{r}_j$ is the sum of the coordinates of the particles and M their mass. Taking, e.g., $\mathbf{v} = v\hat{\mathbf{x}}$, the transformed boson field $\Psi_{\mathbf{v}}(\mathbf{r}) = \mathcal{U}_{\mathbf{v}}^{-1}\Psi(\mathbf{r})\mathcal{U}_{\mathbf{v}}$ obeys twisted boundary conditions: $\Psi_{\mathbf{v}}(x + L, \ldots) = e^{iMvL/\hbar}\Psi_{\mathbf{v}}(x, \ldots)$, where L is the system length. Hence, the helicity modulus [2] is

$$\Upsilon(T) = \frac{\hbar^2 \rho_s(T)}{M} = \frac{\hbar^2}{M^2} \frac{\partial^2 f(T, \mathbf{v} = v\hat{\mathbf{x}})}{\partial v^2}\bigg|_{v=0}, \qquad (1)$$

where $f(T, \mathbf{v})$ is the system free energy per unit volume computed in the reference frame moving with velocity $\mathbf{v}$.

The discussion above implicitly assumes that the container acts as a boundary condition that thermalizes a part of the fluid, i.e., the normal component that is dragged along with velocity $\mathbf{v}$ [13,14]. However, for 1D fluids lacking a condensate, there is no obvious separation between a normal and a superfluid component. Furthermore, the latter cannot be defined from the helicity modulus, as $\Upsilon(T)$ vanishes in 1D in the thermodynamic limit at all temperatures T [4]. Therefore, the problem of defining superfluidity in 1D is essentially a dynamical one. Indeed, the dynamics of a fluid in 1D is more constrained by (quasi-) conserved quantities [9,10] than in higher dimensions, implying that no fraction of it can easily come to thermal equilibrium with the container. And even if the interaction with the container eventually makes the fluid thermalize, an anomalously long thermalization time would make the system appear superfluid in realistic experiments. These observations are incorporated into the theory of 1D superfluids which we describe below.

Focusing on the dynamics of the superflow brings about the notion of phase slips (PS). Semiclassicaly, a PS is a topological excitation in 1D that "unwinds" the phase difference imposed upon the fluid. The creation of PS induces the decay of the superflow, which is the 1D counterpart of quantized vortices moving perpendicular to the flow in higher dimensions. The importance of PS for superfluidity in 1D has been pointed out by several authors in the past. A calculation of the thermal production rate of PS was first reported in Ref. [15], and has been extended later to the quantum regime [16,17]. In homogeneous systems, the PS production rate is exponentially small at low temperatures, implying that the lifetime of the superflow in 1D can be astronomically long; this is a manifestation of the anomalously slow thermalization in 1D discussed above. However, understanding the suppression of superfluidity in the experiments mentioned above [6,8] would require a finite PS rate even at low temperatures. Moreover, the connection of the PS production rate to TO measurements [6] or to other quantities that are measurable in 1D ultracold atomic gases [12] remains obscure.

Therefore, we need a systematic and dynamical formulation of superfluidity in 1D. Since superfluidity is defined by the (non-)response of the fluid against the moving container, it can be defined in terms of the momentum response function $\chi_{\mu\nu}(\boldsymbol{r}, t) = -i\hbar^{-1}\vartheta(t) \times \langle[\pi_\mu(\boldsymbol{r}, t), \pi_\nu(\mathbf{0}, 0)]\rangle$ (μ, $\nu = x, y, z$), where

$$\boldsymbol{\pi}(\boldsymbol{r}) = \frac{\hbar}{2i}[\Psi^\dagger(\boldsymbol{r})\nabla\Psi(\boldsymbol{r}) - \nabla\Psi^\dagger(\boldsymbol{r})\Psi(\boldsymbol{r})] \qquad (2)$$

is the momentum current operator. The Fourier transform of the momentum response $\chi_{\mu\mu}$ is a rank-2 tensor, and for an isotropic fluid $\chi_{\mu\nu}(\boldsymbol{q}, \omega) = (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})\chi_T(q, \omega) + \frac{q_\mu q_\nu}{q^2}\chi_L(q, \omega)$, where $\chi_{T(L)}(q, \omega)$ is the transverse (longitudinal) momentum current response. In dimensions larger than 1, the effects of the interaction potential can be accounted for in the boundary conditions of the imposed velocity field [14]. As a consequence, the density of the normal fluid, which is dragged along by the container, is given by [14,18]

$$\rho_n = -\frac{1}{M}\lim_{q\to 0}\lim_{\omega\to 0}\chi_T(q,\omega), \tag{3}$$

where M is the particle mass. The superfluid density is then $\rho_s = \rho_0 - \rho_n$, ρ_0 being the total density of the fluid.

In contrast, in 1D, $\chi(q,\omega)$ is a scalar and does not split into transverse and longitudinal parts. Furthermore, in 1D the interaction with the container affects the entire fluid and therefore its effect cannot be replaced by suitable boundary conditions. It has to be explicitly accounted for in the calculation of the momentum response.

Since we are interested in the low-temperature transport properties, we shall rely upon the Tomonaga-Luttinger liquid (TLL) description of 1D fluids [20–24], where the low temperature or frequency degrees of freedom of the system are described by two collective (canonically conjugate) fields, $\theta(x,t)$ and $\partial_x\phi(x,t)/\pi$, which account for phase and density fluctuations, respectively. The effective Hamiltonian takes the form $H = H_0 + H_{\rm PS} + H_{\rm irr}$, where

$$H_0 = \frac{\hbar v}{2\pi}\int dx[K(\partial_x\theta(x))^2 + K^{-1}(\partial_x\phi(x))^2]. \tag{4}$$

H_0 describes the system in the low-temperature limit as a 1D fluid where sound waves (phonons) propagate with velocity v and whose compressibility is $K/(\hbar\pi v\rho_0^2)$ [21,22]. $H_{\rm PS}$ and $H_{\rm irr}$ respectively, represent the PS and "irrelevant" interactions in the renormalization group sense; their explicit forms will be presented later.

Using the TLL Hamiltonian (4), we find that $\chi(\omega,T) = \lim_{q\to 0}\chi(q,\omega,T) = 0$, where T is the temperature, implying there is no normal component and the system behaves as a perfect superfluid at any ω, T. This rather unrealistic result is a consequence of the infinite number of conservation laws in the TLL (4). In reality, the additional interactions in $H_{\rm PS}$ and $H_{\rm irr}$ cause decay of superflow. With an eye on the experiments [6,8,11,25], we shall take a periodic potential to describe the interaction with the container. In the experiments of Ref. [6], the walls of the pore are covered by an inert layer of solid helium, which may be regarded as periodic (allowing for a disorder potential is straightforward [20] and will not alter our conclusions substantially).

The effect of the periodic potential characterized by a minimum wave number G can be represented by [20–22],

$$H_{\rm PS} = \sum_{n>0,m}\frac{\hbar v g_{nm}}{\pi a_0^2}\int dx\cos(2n\phi(x) + \Delta k_{nm}x). \tag{5}$$

This term is often ignored because of its oscillatory nature due to Δk_{nm}; for example, it does not open the gap even if the operator is relevant in the renormalization group sense. Nevertheless, for the discussion of superfluidity, $H_{\rm PS}$ is essential as it represents the effect of PS. The g_{mn} are dimensionless couplings related to the strength of the periodic potential and the interatomic interactions, and $a_0 \sim \rho_0^{-1}$ is a short-distance cutoff (ρ_0 being the fluid's linear density); $\hbar\Delta k_{mn} = (2n\pi\rho_0 - 2mG)\hbar$ are the set of all possible (lattice) momenta carried by the PS. The smallest $|\Delta k_{mn}|$ is a measure of the incommensurability between the 1D fluid density and the container potential. In the absence of the container potential, $G = 0$ and $g_{n,m\neq 0} = 0$ and $vK = v_F = \hbar\pi\rho_0/M$.

In addition, $H_{\rm irr}$ contains operators like $H_{\rm irr}^{n,m} = f_{mn}\int dx(\partial_x\phi)^n(\partial_x\theta)^m$ $(m+n>2)$, accounting, e.g., for the curvature of the phonon dispersion [22,24]. The $H_{\rm irr}^{12}$ term, for which $f_{12} = \frac{\hbar v K}{2\pi^2\rho_0}$ [24], is particularly important for momentum conservation. The momentum current is given by $\pi(x,t) = Mj(x,t)$, where $j(x,t) \simeq -\frac{1}{\pi}\partial_t\phi(x,t)$ [20] from the continuity equation. Heisenberg's equation of motion using $H_0 + H_{\rm irr}^{12}$, yields

$$\begin{aligned}\Pi(t) &= \int dx\pi(x,t) = J(t) + \frac{vK}{v_F}P(t)\\ &= \frac{MvK}{\pi}\int dx\left[1 + \frac{1}{\pi\rho_0}\frac{\partial\phi}{\partial x}(x,t)\right]\frac{\partial\theta}{\partial x}(x,t)\end{aligned} \tag{6}$$

where $J = \frac{MvK}{\pi}\int dx\partial_x\theta(x)$ is the particle (mass) current and $P = \frac{\hbar}{\pi}\int dx\partial_x\phi(x)\partial_x\theta(x)$ is proportional to the energy current. J describes the contribution from the motion of the center of mass of the fluid to Π, whereas vKP/v_F is the phonon contribution. The latter contribution to the total momentum arises due to the curvature term $H_{\rm irr}^{12}$.

Thus the total momentum Π is written in terms of two conserved currents J and P. Both of the currents are conserved in the pure TLL (4) (and in fact even with $H_{\rm irr}$), implying perfect superfluidity. However, neither J nor P (and therefore Π) commute with $H_{\rm PS}$. Thus, when the PS are properly accounted for, the currents J and P are expected to acquire finite (and *a priori* different) decay rates. Nevertheless, the decay of the currents could be anomalously slow thanks to the constrained dynamics in 1D. In order to study the fate of the superflow, we need to compute the momentum response in the presence of $H_{\rm PS}$, taking into account the two approximately conserved currents. This can be achieved within the memory matrix formalism [19,26,27], which has been successfully employed to compute the ac conductivity of charged 1D systems [26,27]. In terms of the memory matrix $M(\omega;T)$, the momentum response can be written as

$$\chi(\omega,T) = {\rm Tr}\{VC(\omega,T)iM(\omega,T)\boldsymbol{\chi}(T)\}, \tag{7}$$

where $C(\omega,T) = [\omega\mathbf{1} + iM(\omega,T)]^{-1}$, $V_{\alpha\beta} = (vK/v_F)^{\alpha+\beta-2}$ $(\alpha,\ \beta = 1, 2)$, and $\boldsymbol{\chi}(T)$ is the matrix of static susceptibilities [4]. The real part of the momentum response $\chi(\omega,T)$ is related to the normal fluid density ρ_n, similar to the higher-dimensional expression (3). On the other hand, the imaginary part of $\chi(\omega, T)$ represents the dissipation, as is the case in general linear response theory. These meanings of $\chi(\omega, T)$ in the context of TO experiments can be confirmed by considering the equation of motion for the TO coupled to the 1D fluid [4].

In Fig. 1 we have plotted them against the absolute temperature, for different values of the probe frequency. (We have used $g_{10} = g_{11} = 1$ as a representative set of coupling constants; we verified that the result depends only weakly on the choice of their values.) In the inset we show the dissipation peak positions as a function of the probe frequency. The parameters of the system are chosen so as to reproduce onset temperatures comparable to those experimentally observed in the liquid ^{4}He filled nanopores of Ref. [6] where the probe frequency equals 2 kHz. As the probe frequency is decreased (corresponding to darker colored curves), the onset temperature decreases. Indeed, this behavior can be anticipated by taking the limit of $\omega \to 0^+$ in (7), which yields $\chi(\omega \to 0, T) = \mathrm{Tr}[V\boldsymbol{\chi}(T)] = -\frac{M^2 vK}{\hbar\pi} - (\frac{vK}{v_F})^2 \frac{\pi(k_B T^2)}{6\hbar v^3}$. This is in stark contrast with the vanishing of $\chi(\omega, T)$ obtained from (4) by neglecting the PS. The limiting behavior at $\omega \to 0$ is also consistent with the behavior of the helicity modulus: $\Upsilon(T) \to 0$ in an infinitely long TLL (4), at any finite T. This signals the absence of superfluidity in the thermodynamic (i.e., static) sense. On the other hand, constrained dynamics in 1D leads to an anomalously long lifetime of currents, leading to a dynamical superfluid response that is observable even at very low probing frequencies like the 2 kHz employed in Ref. [6]. Note, however, that a helicity modulus can be defined for finite systems as $\Upsilon(T, L)$ [4], the thermodynamic helicity modulus being $\Upsilon(T) = \lim_{L\to\infty} \Upsilon(T, L)$. By contrast, letting $T \to 0$ first in $\Upsilon(T, L)$, followed by $L \to \infty$, yields the Drude weight or charge stiffness at $T = 0$, which is not a measure of superfluidity [28].

In Fig. 2 we show the real and imaginary part of the momentum response for several values of the parameter K, which determines the compressibility of the fluid. The superfluidity onset temperature is suppressed as the compressibility decreases, as expected. In the experiment of Ref. [6], the value of K is expected to decrease as pressure is applied to the system. Thus, the results displayed in Fig. 2 are consistent with the experimental observation that the onset temperature is suppressed by pressurizing the sample. Furthermore, since the dynamical coupling between J and P is taken into account in (7), for certain values of the parameters, we predict the appearance of a double superfluid onset (cf. inset of Fig. 2), which seems to be consistent with preliminary experimental results [29].

Let us briefly mention another example of ^{4}He system, where application of the present analysis may be a possibility. It has been suggested [11,25] that, in single ^{4}He crystals, an explanation of the observed superfluid response [30] is related to the behavior of dislocations [25], which, by quantum Monte Carlo simulations, are shown to be well described as TLL with $K \simeq 5$ [11]. The results reported here are applicable to such systems and improve on earlier theoretical treatments [11,17]. Experimentally, it may also be interesting to investigate the similarities in the TO response of the 1D nanopore systems [6] and ^{4}He single crystals.

One of the most important predictions of our theory is the dependence of the superfluid response on the probe frequency as shown in Fig. 1. In principle, this can be verified in TO experiments on liquid ^{4}He by changing the oscillator frequency. However, in practice, it is difficult to change the TO frequency over several orders of magnitude. On the other hand, our general analysis of superfluidity in 1D is not limited to liquid ^{4}He. 1D superfluidity can be realized in ultracold atomic systems. There, the imaginary part of momentum response can be probed by using a phase modulated optical lattice [12] and measuring the rate of energy absorption. The latter can be inferred from time of flight measurements [31]. Indeed, ultracold atom systems offer the interesting possibility to probe the frequency dependence of $\chi(\omega, T)$ over a much wider range of ω than what is accessible through the TO in helium systems.

To sum up, we have reported a theory of superfluidity for 1D quantum fluids, showing that superflow is essentially a dynamical phenomenon related to the suppression of quantum PS at low temperatures. Our calculations go beyond previous theoretical treatments by computing the experimentally accessible dynamical momentum response using the memory matrix formalism. The results agree well with the recent torsional oscillator experiments on liquid ^{4}He in 1D nanopores, and several predictions are made including the frequency dependence of the superfluid response. Furthermore, the present theory can be applied to a host of different physical systems.

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technical details of the calculations of the helicity modulus, the memory function, and the torsional oscillator response.
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