

Scissors modes: The first overtoneKeisuke Hatada,^{1,2} Kuniko Hayakawa,^{2,3} and Fabrizio Palumbo²¹*Instituto de Ciencia de Materiales de Aragón, CSIC-Universidad de Zaragoza, E-50009 Zaragoza, Spain*²*INFN Laboratori Nazionali di Frascati, c.p. 13, I-00044 Frascati, Italy*³*Centro Fermi, Compendio Viminale, Roma I-00184, Italy*

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Scissors modes were predicted in the framework of the two-rotor model. This model has an intrinsic harmonic spectrum, so that the level above the scissors mode, the first overtone, has excitation energy twice that of the scissors mode. Because the latter is of the order of 3 MeV in the rare-earth region, the energy of the overtone is below threshold for nucleon emission, and its width should remain small enough for the overtone to be observable. We find that $B(E2) \uparrow_{\text{overtone}} = \frac{1}{64\theta_0^2} B(E2) \uparrow_{\text{scissors}}$, where θ_0 is the zero-point oscillation amplitude, which in the rare-earth region is of order 10^{-1} .

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The scissors mode has an excitation energy of the order of 3 MeV in the rare-earth region [1,2]. The model that led to its prediction, the two-rotor model [3], has the spectrum of a planar harmonic oscillator, with some constraints on the states to be discussed below. The first overtone should then have an energy of the order of 6 MeV, below threshold for nucleon emission. As a consequence, its width should remain of purely electromagnetic nature and be small enough for this mode to be observable, even though scissors modes have a modest collectivity and are substantially fragmented [1,2].

The possible occurrence of the first overtone has been considered in Ref. [4], but to our knowledge has not been thoroughly investigated so far. The main reason is perhaps that its excitation amplitude is expected to be too small. Indeed, excitation amplitudes in the two-rotor model are proportional to powers of θ_0 , the amplitude of the zero point oscillation, which in the rare-earth region is of order 10^{-1} . Now $B(M1) \uparrow_{\text{scissors}} \sim 1/\theta_0^2$, but $B(E2) \uparrow_{\text{scissors}} \sim \theta_0^2$, and the first overtone needs to be excited by an $E2$ multipole. All other methods used in the study of scissors modes—the schematic random-phase approximation [5], the interacting boson model [6], the sum rule method [7], and a geometrical model [8]—give similar results.

The proportionality $B(M1) \uparrow_{\text{scissors}} \sim B(E2) \uparrow_{\text{scissors}}$ has been observed experimentally [9] [even though $B(M1) \uparrow_{\text{scissors}}$ in reality scales as δ^2 instead of $\delta^{\frac{3}{2}}$ as predicted by the two-rotor model if the dependence of the moment of inertia on the deformation parameter of Ref. [10] is adopted]. This is regarded as significant evidence of the scissors nature of the low-lying magnetic transitions and their collectivity. We expect that such a proportionality should hold for the $E2$ strength of excitation of the first overtone as well. Even if this strength were small, its study might contribute to a deeper assessment of the nature and collectivity of the scissors modes. We then decided to start an investigation of the overtone within the two-rotor model, because this model allows a first insight into the relevant dynamics in a simple framework. We thus came to the surprising result that $B(E2) \uparrow_{\text{overtone}}$ is of zero order in

the expansion with respect to θ_0 , and, precisely,

$$B(E2) \uparrow_{\text{overtone}} = \frac{1}{64\theta_0^2} B(E2) \uparrow_{\text{scissors}}. \quad (1)$$

In view of the smallness of θ_0 , this amplitude is quite substantial. The above result might be relevant to some of the other electrically charged systems for which scissors modes have been predicted: metal clusters [11], quantum dots [12], and, in particular, crystals [13], for which an expansion in powers of θ_0 holds. In all these systems one of the blades of the scissors must be identified with a moving cloud of particles (electrons in metal clusters and quantum dots, an atom in a cell in crystals) and the other one with a structure at rest, the lattice.

To make this Rapid Communication self-contained we report the main features of the two-rotor model. Its classical Hamiltonian is

$$H = \frac{1}{2\mathcal{I}_n} \vec{I}_n^2 + \frac{1}{2\mathcal{I}_p} \vec{I}_p^2 + V, \quad (2)$$

where \vec{I}_n , \vec{I}_p , \mathcal{I}_n , and \mathcal{I}_p are the angular momenta and the moments of inertia of the neutron and proton bodies assumed to have ellipsoidal shape and V their interaction potential. Introducing the total angular momentum \vec{I} and the vector \vec{S} ,

$$\vec{I} = \vec{I}_n + \vec{I}_p, \quad \vec{S} = \vec{I}_n - \vec{I}_p, \quad (3)$$

the Hamiltonian (2) can be rewritten as the sum of the rotational Hamiltonian of the nucleus as a whole plus the Hamiltonian of the intrinsic motion

$$H = \frac{\vec{I}^2}{2\mathcal{I}} + H_{\text{intr}}, \quad (4)$$

where

$$\mathcal{I} = \frac{4\mathcal{I}_p\mathcal{I}_n}{\mathcal{I}_p + \mathcal{I}_n} \quad (5)$$

$$H_{\text{intr}} = \frac{1}{2\mathcal{I}} \vec{S}^2 + \frac{\mathcal{I}_n - \mathcal{I}_p}{4\mathcal{I}_n\mathcal{I}_p} \vec{I} \cdot \vec{S} + V. \quad (6)$$

We assume the potential to depend only on the angle 2θ between the symmetry axes $\hat{\zeta}_n, \hat{\zeta}_p$ of the rotors

$$\cos(2\theta) = \hat{\zeta}_n \cdot \hat{\zeta}_p. \quad (7)$$

It is therefore convenient to introduce this variable together with a set of other variables that identify the axes $\hat{\zeta}_n, \hat{\zeta}_p$. We chose the Euler angles α, β, γ of the intrinsic frame defined by

$$\hat{\xi} = \frac{\hat{\zeta}_n \times \hat{\zeta}_p}{\sin(2\theta)}, \quad \hat{\eta} = \frac{\hat{\zeta}_n - \hat{\zeta}_p}{2 \sin \theta}, \quad \hat{\zeta} = \frac{\hat{\zeta}_n + \hat{\zeta}_p}{2 \cos \theta}. \quad (8)$$

The correspondence $(\hat{\zeta}_n, \hat{\zeta}_p) \leftrightarrow (\alpha, \beta, \gamma, \theta)$ is one-to-one and regular for $0 < \theta < \pi/2$. These variables are not sufficient to describe all the configurations of the classical system, but describe uniquely the quantum system owing to the constraint

$$\vec{I}_n \cdot \hat{\zeta}_n = \vec{I}_p \cdot \hat{\zeta}_p = 0, \quad (9)$$

appropriate to quantum bodies with axial symmetry. These constraints are automatically satisfied if the wave functions depend on $\hat{\zeta}_n, \hat{\zeta}_p$ only. Quantization can be obtained assuming the standard representation for the total angular momentum \vec{I} , and for \vec{S} the realization

$$S_\xi = i \frac{\partial}{\partial \theta}, \quad S_\eta = -\cot \theta I_\zeta, \quad S_\zeta = -\sin \theta I_\eta, \quad (10)$$

where I_ξ, I_η , and I_ζ are the components of the total angular momentum on the intrinsic axes.

The two-rotor model was reformulated [14], adopting a more appropriate quantization procedure and including the realistic case in which the neutron rotor is bigger than that of protons. The resulting intrinsic Hamiltonian is

$$\begin{aligned} H_{\text{intr}} &= \frac{1}{2\mathcal{I}} \left[-\frac{d^2}{d\theta^2} - 2 \cot(2\theta) \frac{d}{d\theta} + \cot^2 \theta I_\zeta^2 + \tan^2 \theta I_\eta^2 \right] \\ &+ \frac{\mathcal{I}_n - \mathcal{I}_p}{4 \mathcal{I}_p \mathcal{I}_n} \left[-\tan \theta I_\zeta I_\eta - \cot \theta I_\eta I_\zeta + i I_\xi \frac{d}{d\theta} \right] + V(\theta). \end{aligned} \quad (11)$$

We separate the range of θ in two regions:

$$s_I = s(\theta) s\left(\frac{\pi}{4} - \theta\right), \quad s_{II} = s\left(\frac{\pi}{2} - \theta\right) s\left(\theta - \frac{\pi}{4}\right), \quad (12)$$

where $s(x)$ is the step function: $s(x) = 1, x > 0$ and zero otherwise. They are obtained from each other by the reflection of θ with respect to $\pi/4$. It is convenient to introduce the notation

$$R_\theta f(\theta) = \overset{\circ}{f}(\theta), \quad (13)$$

where

$$\overset{\circ}{f}(\theta) = f\left(\frac{\pi}{2} - \theta\right), \quad (14)$$

so that $\overset{\circ}{s}_I = s_{II}$. We assume $\overset{\circ}{V} = V$, as appropriate to the geometry of the system. Because we know that the angle between the neutron-proton axes is very small we can assume

for the potential a quadratic approximation

$$V = \frac{1}{2} C \theta_0^2 x^2 s_I + \frac{1}{2} C \theta_0^2 y^2 s_{II}, \quad (15)$$

where

$$\theta_0 = (\mathcal{I}C)^{-\frac{1}{4}}, \quad x = \frac{\theta}{\theta_0}, \quad y = \frac{\frac{\pi}{2} - \theta}{\theta_0}. \quad (16)$$

The intrinsic Hamiltonian is then invariant with respect to the transformation

$$R = R_\xi \left(\frac{\pi}{2} \right) R_\theta, \quad (17)$$

where R_ξ is the rotation operator around the ξ axis, so that we can study the eigenvalue equation separately in the regions I and II . The linear derivative in the first term of H_{intr} can be eliminated by the transformation

$$(U\Phi)(\theta) = \frac{1}{\sqrt{2 \sin(2\theta)}} \Phi'(\theta). \quad (18)$$

Neglecting terms of order θ_0 we get

$$\begin{aligned} H'_{\text{intr}} &= U H_{\text{intr}} U^{-1} \\ &= \frac{1}{2\mathcal{I}} \left\{ -\frac{d^2}{d\theta^2} - [2 + \cot^2(2\theta)] + \cot^2 \theta I_\zeta^2 + \tan^2 \theta I_\eta^2 \right\} + V(\theta). \end{aligned} \quad (19)$$

We then write accordingly

$$H'_{\text{intr}} \approx H_I s_I + H_{II} s_{II}, \quad (20)$$

where, setting $\hbar = 1$,

$$\begin{aligned} H_I &= \frac{1}{2} \omega \left[-\frac{d^2}{dx^2} + \frac{1}{x^2} \left(I_\zeta^2 - \frac{1}{4} \right) + x^2 \right], \\ H_{II} &= \frac{1}{2} \omega \left[-\frac{d^2}{dy^2} + \frac{1}{y^2} \left(I_\eta^2 - \frac{1}{4} \right) + y^2 \right], \end{aligned} \quad (21)$$

with

$$\omega = \sqrt{\frac{C}{\mathcal{I}}}. \quad (22)$$

The eigenfunctions and eigenvalues of H_I are [14]

$$\varphi_{Kn}(x) = \sqrt{\frac{n!}{(n+K)! \theta_0}} x^{K+\frac{1}{2}} L_n^K(x^2) e^{-\frac{1}{2}x^2}, \quad (23)$$

$$\epsilon_{nK} = \omega(2n + K + 1). \quad (24)$$

L_n^K are Laguerre polynomials and the wave functions φ are normalized according to

$$\int_0^\infty dx [\varphi_{Kn}(x)]^2 = \frac{1}{2}. \quad (25)$$

Even if the nucleus in its ground state has axial symmetry, this symmetry is, in general, lost in excited states, so that the component of angular momentum along any intrinsic axis is not conserved, resulting in a superposition of intrinsic states with different K -quantum number. The only states that have been theoretically analyzed so far are not affected by K mixing.

They are the ground state, $I = K = n = 0$; the scissors modes, $I = 1, 2$, $K = 1$, $n = 0$, and the state $I = K = 0$, $n = 1$, which cannot be excited by electromagnetic radiation.

The total wave functions must respect the r symmetry with respect to both the neutron and the proton axes: Configurations of the nucleus differing by independent rotations through π around the ξ axis of the ζ_n , ζ_p vectors are indistinguishable. Enforcing this symmetry, one finds two conditions [14]. The first has been solved in general, and requires that the wave functions have the form

$$\Psi_{IM\sigma} = \sum_{K \geq 0} \mathcal{F}_{MK}^I(\alpha, \beta, \gamma) \Phi_{IK\sigma}(\theta), \quad (26)$$

where

$$\mathcal{F}_{MK}^I = \sqrt{\frac{2I+1}{16(1+\delta_{K0})\pi^2}} [\mathcal{D}_{MK}^I + (-1)^I \mathcal{D}_{M-K}^I]. \quad (27)$$

I, M are the nucleus angular momentum and its component on the z axis of the laboratory system, and σ labels the states. We impose the normalization

$$\int_0^{2\pi} d\alpha \int_0^\pi d\beta \int_0^{2\pi} d\gamma \int_0^{\frac{\pi}{2}} d\theta |\Psi_{IM\sigma}|^2 = 1. \quad (28)$$

Notice that the normalization of the Φ in Eq. (26) is different from that in Ref. [14]. The second condition coming from r invariance constrains the parity of the intrinsic functions with respect to R_θ and must be worked out state by state.

The ground state and the scissors mode are labeled by $\sigma = 0, 1$, respectively, and their intrinsic wave functions are

$$\Phi_{000} = \varphi_{00} s_I + \overset{\circ}{\varphi}_{00} s_{II}, \quad \Phi_{111} = \varphi_{10} s_I - \overset{\circ}{\varphi}_{10} s_{II}. \quad (29)$$

Let us now study the first overtone, labeled by $\sigma = 2$. The Hamiltonian H_{II} couples states with $K = 0, 2$. It is then easy to see that the total eigenfunction must involve a superposition of φ_{01} and φ_{20} . These states are degenerate and decoupled in region I . To find which superposition is an eigenfunction of the total Hamiltonian we diagonalize I_η^2

$$\begin{aligned} I_\eta^2 \mathcal{G}_{M0} &= 0, \quad \mathcal{G}_{M0} = \frac{1}{2} (\mathcal{F}_{M0}^2 + \sqrt{3} \mathcal{F}_{M2}^2), \\ I_\eta^2 \mathcal{G}_{M2} &= 4 \mathcal{G}_{M2}, \quad \mathcal{G}_{M2} = \frac{1}{2} (\sqrt{3} \mathcal{F}_{M0}^2 - \mathcal{F}_{M2}^2). \end{aligned} \quad (30)$$

The total eigenfunction must then be an appropriate superposition of $\mathcal{G}_{M0} \overset{\circ}{\varphi}_{01}$, $\mathcal{G}_{M2} \overset{\circ}{\varphi}_{20}$ in region II and of $\mathcal{F}_{M0}^2 \varphi_{01}$, $\mathcal{F}_{M2}^2 \varphi_{20}$ in region I . It is easy to verify that the intrinsic wave functions that satisfy the constraints [14] of r invariance are

$$\begin{aligned} \Phi_{202} &= \frac{1}{\sqrt{2}} \left[\varphi_{01} s_I - \frac{1}{2} (\sqrt{3} \overset{\circ}{\varphi}_{20} + \overset{\circ}{\varphi}_{01}) s_{II} \right], \\ \Phi_{222} &= \frac{1}{\sqrt{2}} \left[\varphi_{20} s_I + \frac{1}{2} (\overset{\circ}{\varphi}_{20} - \sqrt{3} \overset{\circ}{\varphi}_{01}) s_{II} \right]. \end{aligned} \quad (31)$$

The different normalization of the Φ should be kept in mind in a comparison with Ref. [14].

The collective motion of the first overtone has a simple geometrical description in the intrinsic frame: In region I it is

a superposition of the state φ_{01} , which is a kind of breathing mode, and of the state φ_{20} , which is a relative rotation of the neutron-proton axes as in the scissors mode but with angular momentum $K = 2$. We already mentioned that the spectrum of the two-rotor model is identical to that of the planar harmonic oscillator. We remark, however, that the first and second excited states of the planar harmonic oscillator have degeneracy 2 and 3, respectively, while all the intrinsic states of the two-rotor model discussed so far are nondegenerate because of the r symmetry.

The surprising feature of our result is that the $E2$ excitation amplitude of the first overtone gets a nonvanishing contribution to zero order in the expansion with respect to θ_0 . Indeed, the quadrupole operator to this approximation is [14]

$$\begin{aligned} M(E2, \mu) &= e Q_{20} \left[\mathcal{D}_{\mu 0}^2 \left(s_I - \frac{1}{2} s_{II} \right) + \frac{1}{2} \sqrt{\frac{3}{2}} (\mathcal{D}_{\mu 2}^2 + \mathcal{D}_{\mu -2}^2) s_{II} \right], \end{aligned} \quad (32)$$

where $e Q_{20}$ is quadrupole moment in the intrinsic frame. We immediately see that to zero order in θ_0 we cannot excite the scissors modes from the ground state, but we can excite the first overtone, with the amplitude

$$\begin{aligned} \langle \Psi_{2M2} | M(E2, \mu) | \Psi_{000} \rangle &= \frac{1}{4\sqrt{2}} e Q_{20} \langle \varphi_{20} | \varphi_{00} \rangle \left\{ -\sqrt{3} \mathcal{F}_{M2,0}^2 + \mathcal{F}_{M2,2}^2 \right\} \\ &\quad \times \sqrt{\frac{3}{2}} (\mathcal{D}_{\mu 2}^2 + \mathcal{D}_{\mu -2}^2) - \mathcal{D}_{\mu 0}^2 | \mathcal{F}_{M0}^0 \rangle. \end{aligned}$$

Notice that this amplitude is entirely attributable to the $K = 2$ component of the wave function, because $\langle \varphi_{01} | \varphi_{00} \rangle = 0$. Finally,

$$\langle \Psi_{2M2} | M(E2, \mu) | \Psi_{000} \rangle = \frac{1}{8} \sqrt{\frac{3}{5}} e Q_{20} C_{002\mu}^{2M}, \quad (33)$$

where $C_{002\mu}^{2M}$ is a Clebsch-Gordan coefficient. We thus get Eq. (1).

While the first overtone cannot be excited by a $M1$ transition, it can decay to the scissors mode through such a transition to order θ_0^{-2} . The $E2$ decay to the scissors mode can instead only occur to order θ_0^2 and will be neglected. After the excitation of the first overtone one should then observe photons of energy 2ω and ω as well. Let us then evaluate the $M(M1)$ transition amplitude. The magnetic dipole moment in the intrinsic frame is

$$M(M1, \nu) = \frac{e}{2m} \sqrt{\frac{3}{4\pi}} S_\nu. \quad (34)$$

Then in the laboratory frame we have

$$M(M1, \nu) = -\frac{1}{\sqrt{2}} (\mathcal{D}_{\nu 1}^1 - \mathcal{D}_{\nu -1}^1) \mathcal{M} (s_I - s_{II}) \frac{d}{d\theta}, \quad (35)$$

where

$$\mathcal{M} = i \sqrt{\frac{3}{16\pi}} (g_p - g_n) \frac{e}{2m_p}, \quad (36)$$

g_n and g_p being the orbital gyromagnetic factors of neutrons and protons, respectively, and m_p is the proton mass. We need an approximate expression of the derivative operator in regions *I* and *II*:

$$\begin{aligned}\theta_0 U \frac{d}{d\theta} U^{-1} &\approx \frac{d}{dx} - \frac{1}{2x}, & \text{in region I,} \\ \theta_0 U \frac{d}{d\theta} U^{-1} &\approx -\frac{d}{dy} + \frac{1}{2y}, & \text{in region II.}\end{aligned}\quad (37)$$

Setting

$$\nabla_\theta = \frac{d}{d\theta} - \frac{1}{2\theta} \quad (38)$$

by a straightforward calculation we get

$$\langle \Psi_{IM_22} | M(M1, \nu) | \Psi_{IM_11} \rangle = -\frac{\pi}{\sqrt{3}} \mathcal{M} \mathcal{T}, \quad (39)$$

where

$$\begin{aligned}\mathcal{T} &= \langle \mathcal{F}_{M_20}^2 | \mathcal{F}_{\nu 1}^1 | \mathcal{F}_{M_11}^1 \rangle \langle \varphi_{01} - \sqrt{3} \varphi_{20} | \nabla_\theta | \varphi_{01} \rangle \\ &+ \langle \mathcal{F}_{M_22}^2 | \mathcal{F}_{\nu 1}^1 | \mathcal{F}_{M_11}^1 \rangle \sqrt{3} \langle \varphi_{20} - \varphi_{01} | \nabla_\theta | \varphi_{01} \rangle.\end{aligned}\quad (40)$$

At variance with the *E2* excitation of the first overtone both intrinsic components $K = 0, n = 1$ and $K = 2, n = 0$

contribute to this decay amplitude. We thus find

$$\langle \Psi_{IM_22} | M(M1, \nu) | \Psi_{IM_11} \rangle = \frac{\sqrt{2} + \sqrt{3}}{2\sqrt{10}\theta_0} C_{1M_11\nu}^{2M_2} \mathcal{M}, \quad (41)$$

and we can finally relate the *M1* decay strength of the overtone to the scissors mode with the *M1* strength of the scissors mode excitation:

$$B(M1; \text{overtone} \rightarrow \text{scissors}) \approx \frac{1}{4} B(M1) \uparrow_{\text{scissors}}. \quad (42)$$

A distinctive feature of the overtone in the two-rotor model is the mixing of intrinsic states with different *K*-quantum number, which is necessary to respect the *r* symmetry. This mixing is determined by the different form that the intrinsic hamiltonian takes in regions *I* and *II*. It would be very interesting to investigate if the approaches in Refs. [5–8] will confirm the structure of the overtone we found in the two-rotor model. Indeed, so far microscopic calculations, while reproducing some experimental features as fragmentation, which are outside the possibility of a semiclassical model, agree with the two-rotor model about the nature of the scissors modes. In particular, the interacting boson model, in the semiclassical approximation obtained using coherent states, exactly reproduces [15] the Hamiltonian of the two-rotor model in region *I*. We would be very surprised if the agreement would not extend to region *II*, and we think it would be very interesting to know the exact results for the overtone in the interacting boson model.

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