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Modelling Time Dependent Transient Deformation in New Zealand

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Abstract: Most organisations who maintain regional terrestrial reference frames base their realisations on the global International Terrestrial Reference Frame (ITRS) that uses space based positioning techniques (e.g. GNSS, ITRF, SLR, DORIS) to realize the International Terrestrial Reference Frame (e.g. ITRF2005, ITRF2008). Where ITRF sites are located within stable plates, the coordinate motion is largely of a linear nature. However, sites that are located within plate boundary zones, can exhibit non-linear motion that is the result of other geophysical processes such as earthquake events, volcanic activity, subduction, subsidence and uplift. In such zones, a precise coordinate frame transformation that provides the linkages between global and regional ITRF realisations are not valid. Many of these processes exhibit motion that is time dependent and hence more complex models are required in order to accurately model the deformation. This in turn has implications for national geodetic infrastructure and land surveying applications e.g. Network RTK (NRTK) and GIS.

Although periodic earthquake events that cause co-seismic displacement are a consequence of the tectonic situation in New Zealand, a greater challenge is the time dependent transient deformation caused by post-seismic relaxation and slow slip events (SSE). NRTK systems require accurate, current epoch (or instantaneous) site coordinates, which can be difficult to determine when a site periodically (and unpredictably) undergoes slow slip deformation. In addition, most NRTK systems use simple predictive models (e.g. position and velocity only) and do not have any ability to account for non-linear deformation.

This paper describes some of the methods and results that are being developed to model these complex deformation events. In particular, examples from NRTK sites will be used to illustrate the transient nature of the deformation both in time and space.

Keywords: GNSS, Network RTK, transient deformation

1. Introduction

Global Navigation Satellite Systems (GNSS) provide ubiquitous positioning virtually at anytime and anywhere on Earth. GNSS is capable of, and most surveying applications assume, a positional accuracy at the centimetre or even millimetre level. The actual positional accuracy is dependent upon the accuracy of the reference stations (e.g. continuous GNSS (cGNSS)) used and the ability of a user's position to be accurately transformed to the user's datum.

In regions of the world where the reference stations are largely on stable continental plates, it is possible to assume that the motion of all reference stations is linear and there is no (significant) deformation between marks. Therefore, for most day to day surveying applications the tectonic plate motion can be ignored.

In regions that are not located on stable plates, more care needs to be taken and may require the development and use of more complex deformation models. This is particularly true when operating close to tectonic plate boundaries and is the case in New Zealand. New Zealand's topography is a result of the complex collision of the Australian-Pacific plate boundaries. The boundary includes subduction under the North Island volcanoes, which transitions to oblique strike slip along the Southern Alps and further plate subduction in the southwest of the South Island. The collision causes frequent earthquake events that can create surface displacements in the order of several metres. The effect on the geodetic infrastructure can be massive, but readily dealt with through re-measurement. This is likely to include a combination of GNSS and terrestrial survey measurements, InSAR, seismic and geophysical modelling.

Within the plate boundary zone, the ongoing or long term deformation is uniform [1,2]. Instead of the velocity gradient being constant, the gradient changes gradually across the plate boundary zone and (uniform) deformation can easily be applied to positioning applications.

Earthquakes cause coseismic and post-seismic deformation. This motion can be relatively short term e.g. coseismic deformation that occurs at the time of the earthquake event or after-slip lasting a few weeks to months. However, sufficiently large earthquakes will result in long term viscoelastic post-seismic decay that may last for months to years. It may be relatively small e.g. amplitudes of less than 10mm over decay periods of days to weeks, but cumulatively it will amount to centimetres after a few years.

Unlike instantaneous earthquake events, the motion of plates at most subduction zones may result in periodic creep or slow slip events (SSE) causing non-linear deformation. The surface displacements may be only a few centimetres at a time, but reoccurs periodically. After a few years, the cumulative surface displacement can be similar to a M_L 5-6 earthquake.

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2. Geodetic Position Time Series Modelling

For several decades GNSS positioning has enabled the measurement of tectonic plate motion. Most of these studies, and especially those that use campaign GPS/GNSS, have considered only the uniform or linear deformation. More recently, cGNSS has shown the non-linear and transient nature of deformation, especially in plate boundary zones [3, 4]. It is this type of deformation, typically as a result of earthquake events, that we wish to model.

For this paper, we consider only position time series modelling. Other methods such as geophysical models can also be used. Often these models can be complex and require an understanding of the underlying geological and geophysical processes.

We model the resultant surface displacements for the application of surveying positioning. There are two aspects to consider, reference station and non-reference station locations. The modelling of reference station position is typically based on direct position measurements, for example sites that are part of a NRTK operation. For other locations, non-reference station positions, a model needs to be created that enables the interpolation of deformation displacements. Such a model may be created using GNSS/cGNSS position data, but could also include data from other sources, e.g. InSAR and/or geophysical models.

3. Reference Station Position Models

Continuous GNSS sites result in high quality, three dimensional positions for each site on a daily basis. This position data enables the accurate tracking of stations in terms of the ITRF currently being utilized [5]. Over time, the data allows the accurate estimation of the long term (secular) velocity for each site as well as monitoring transient motion. In addition to environmental effects due to annual and semi-annual seasonal motions (periodic terms), in tectonically active regions, transient motion may include instantaneous coseismic deformation, slow release post-seismic relaxation and periodic slow slip events (SSE) that can result in deformation time periods of a few days to weeks to less frequent motion that occurs over months to years.

As well as the long term linear velocity, the time series model needs to include the non-linear components. A number of tools are available to do this at cGNSS reference stations, but tools are not readily available for non-reference station locations. Ideally this requires a sufficiently dense reference station network from which the non-linear terms can be interpolated. The required density of the reference will depend upon the magnitude and degree of non-linearity of the deformation: the greater the non-uniform the deformation, the higher the cGNSS density required in order to accurately model the deformation.

The following sections define the linear and non-linear model for a reference station and the interpolation of non-linear components at non-reference station positions.

3.1 Linear model

For linear site velocities, the model is given as

$$X(t) = X_0 + v_X (t - t_0) \text{ .... (1)}$$

where $X(t)$ is the position (coordinate) (m) at time *t*, X_0 is the reference position (m) at time t_0, v_X is the site velocity (m/yr), t is the time (yr) and t_0 is the reference time (yr).

3.2 Non-linear terms

The linear model is extended to include terms for coseismic, post-seismic decay, transient velocity and slow slip events such that

$$X(t) = X_0 + v_X (t - t_0) + \Delta_{cs} + \Delta_{ps} + \Delta_{vel} + \Delta_{sse} \text{ .... (2)}$$

where Δ_{cs} represents the displacement cause by an earthquake coseismic event (m), Δ_{ps} represents the cumulative post-seismic displacement (m), Δ_{vel} represents the cumulative transient velocity displacement (m) and Δ_{sse} represents the slow slip displacement (m).

Coseismic terms: For n_j coseismic events we have

$$\Delta_{cs} = \sum_{j=1}^{n_j} H_j O_j \text{ .... (3)}$$

where O_j is the coseismic offset for event t_j and H_j is the Heaviside step function with $H_j = 0$ if $t < t_j$ and $H_j = 1$ if $t \geq t_j$.

Post-seismic decay terms: If an earthquake event is sufficiently large, there is likely to be a post-seismic decay signal that may last for years to decades. Post-seismic relaxation is usually a combination of two key physical processes, namely after-slip and viscoelastic relaxation. These processes can be modelled as a decay function e.g. logarithmic, exponential, power law or a combination of the functions. The exact function(s) used is not critical, provided they fit the relaxation adequately.

For n_k post-seismic events we select the appropriate function(s) such that

$$\varDelta_{ps} = \sum_{k=1}^{n_k} H_k \left\{ \begin{array}{c} \left[O_k + A_k \, log\left(1 + \frac{t-t_k}{\tau_k}\right)\right] \\ \left[O_k + A_k \left\{1 - exp\left(-\frac{(t-t_k)}{\tau_k}\right)\right\}\right] \\ \left[O_k + A_k \left(\frac{(t-t_k)}{\tau_k}\right)^{1-p}\right] \end{array} \right. \text{...... (4)}$$

where O_k is the coseismic offset for event t_k, A_k is the amplitude of post-seismic decay (m), τ_k is the decay time scale (yrs), t_k the event time (yrs) and $H_k = 0$ if $t < t_k$ and $H_k = 1$ if $t \geq t_k$. Note that Equation 3 is a special case of Equation 4 if the post-seismic decay is insignificant.

Velocity terms: For large earthquake events, and following the immediate earthquake event, the long term decay may be small and unmeasurable, but a change in the site velocity may occur. In this case a transient velocity may need to be included.

For n_l transient velocity terms we have

$$\Delta_{vel} = \sum_{l=1}^{n_l}\left[v_{X_l}\left(t_{1,l} - t_0\right)\right] \quad t_{1,l} < t < t_{2,l} \text{ ....}(5)$$

where v_{X_i} is the velocity (m/yr), $t_{1,l}$ is the start time and $t_{2,l}$ is the end time of the velocity event.

Slow slip event terms: A characteristic of subduction zones are SSE that are dependent upon the degree of coupling on the fault zone. Fault plane slipping can translate to surface motion at the centimetre level that can occur at periods of days to weeks, months or even years. A function that can conveniently model this motion is the error function.

For n_p SSE we have

$$\Delta_{sse} = \sum_{p=1}^{n_p}\left[0.5A_p \operatorname{erf}\left(\frac{t - t_p}{\tau_p}\right)\right] \text{ ....}(6)$$

where A_p is the amplitude of SSE (m), τ_p is the duration of the event and t_p is the mid-point time of the event.

3. Non-reference Station Position Models

For non-reference station positions, accounting for surface deformation is more difficult since there may few or no direct measurements. Other methods need to be considered e.g. InSAR data or geophysical models. When the density of the cGNSS stations is sufficient, then an interpolation technique may be used.

Two common methods of representing deformation include a rectangular grid with regularly spaced nodes or a triangulated grid (irregular) with explicitly defined triangles and nodes [6]. The type of grid is not critical rather, that the surface deformation can be accurately modelled for the application. Assuming that the deformation is modelled (Equation 2) incorporating appropriate terms, then the non-linear deformation is gridded at a suitable grid interval over the spatial extent of the deformation. The grid interval will depend upon the magnitude of the deformation and extent of the deformation. It may also be modelled as nested grids with variable grid intervals to accommodate variations in the magnitude of the deformation, see for example, the Christchurch 2010-11 earthquake sequence in [7].

Coseismic decay terms: Modelling the coseismic deformation (Equation 3), is straight forward. Two grids corresponding to the offset east and offset north are created. Bi-linear interpolation is used to estimate the offset for a given location and the offset applied if the time is after the earthquake event (i.e. $t \geq t_j$) and zero otherwise (i.e. $t < t_j$). To reflect the spatial dependency of the coseismic deformation interpolation, we modify Equation 3 such that

$$\Delta_{cs}(\phi,\lambda) = \sum_{j=1}^{n_j} H_j O_j(\phi,\lambda) \text{ ....}(7)$$

for location latitude ϕ and longitude λ.

Post-seismic decay terms: Both the post-seismic decay (either logarithmic, exponential or power law) and SSE include a variable, τ, for the time scale of the deformation. In the case of the post-seismic decay (Equation 4), the decay time scale, τ_k, is difficult to estimate using least squares. For decay functions, the time scale parameter is insensitive, especially if the time series is short following the event. To help overcome this, we stack the position time series for multiple sites and use non-linear least squares to estimate one time scale parameter for all sites plus an amplitude parameter per site. We stack all three position components, but another option is to estimate a separate decay time scale for each component. Overall, there appears to be little improvement in doing so.

Both the coseismic offset O_k and amplitude of the post-seismic decay, A_k, vary are spatially and can represented as grid files. Equation 4 (for the logarithmic function only), becomes

$$\Delta_{ps}(\varphi,\lambda) = \sum_{k=1}^{n_k} H_k \left[\begin{matrix} O_k(\varphi,\lambda) + \\ A_k(\varphi,\lambda)\log\left(1 + \frac{t-t_k}{\tau_k}\right)\end{matrix}\right] \text{........ }(8)$$

Velocity terms: Any post-seismic transient velocity terms will also be spatial dependent. Hence Equation 6 becomes

$$\Delta_{vel}(\phi,\lambda) = \sum_{l=1}^{n_l}\left[v_{X_l}(\phi,\lambda)\left(t_{1,l} - t_0\right)\right] \text{ .... }(9)$$

Slow slip event terms: Modelling SSE includes variables for the duration of the event variable, τ_p and amplitude, A_p. The amplitude is well defined, but τ_p is ill-defined since the start and end times of an event can be difficult to pick. Similar to the post-seismic decay, stacking multiple SSE time series enables a representative event duration to be estimated for a region along with separate amplitudes for each site and position component.

In New Zealand at least, a SSE can manifest over a large region and may migrate across the region. That is, it starts at a particular time and location and then propagates in time and space. For an event the amplitude, mid-point time and duration of the event exhibit a spatially variable component. Including location parameters in Equation 6 gives

$$\Delta_{sse}(\phi,\lambda) = \sum_{p=1}^{n_p}\left[0.5A_p(\phi,\lambda)\operatorname{erf}\left(\frac{t - t_p(\phi,\lambda)}{\tau_p(\phi,\lambda)}\right)\right] \text{ .... }(10)$$

4. Time-dependent modelling in New Zealand

Active deformation in New Zealand results in frequent non-linear and transient land motion. We will describe the time-dependent modelling of two events, namely; the Dusky Sound 2009 earthquake, located in southwest of the South Island, and the Kapiti Coast slow slip event located on the bottom of the North Island.

4.1 Dusky Sound 2009 Earthquake

The M_w 7.8 Dusky Sound earthquake event occurred in Fiordland, New Zealand, on 15th July 2009 with an epicentre located 12km below Dusky Sound. It was the largest recorded earthquake event since the Hawke’s Bay 1931 events and the Canterbury 2010-2011 events [8]. It is also the only event of its size to have occurred on a subduction interface in New Zealand for which high quality geodetic data has been recorded [9].

As a consequence, the event has resulted in post-seismic relaxation that extends for over 500 km from the epicentre as well as a significant change in velocity following the event. For the purposes of this paper, we ignore the coseismic deformation as it has been reported elsewhere [9].

As described in Section 3, we use non-linear least squares to estimate the post-seismic decay time scale, τ_k, by stacking multiple position time series. To do this we used 13 cGNSS sites from the lower South Island. The coseismic offsets and post-seismic decay amplitudes for each cGNSS site are estimated by assuming that the decay time scale is a constant for the event. the decay time scale for the Dusky Sound event was $\tau_k = 0.019$ years (~7 days). The maximum decay amplitudes are over 10 mm in the western Fiordland region, but rapidly decrease to less than 2 mm at approximately 250-300 km from the epicentre (Figure 1). For example, the cumulated surface displacement for a point if the southwest corner of the grid (dark red) is over 100mm after 5 years. For a point in the green portion of the grid, the cumulative surface displacement is over 20 mm after 5 years.

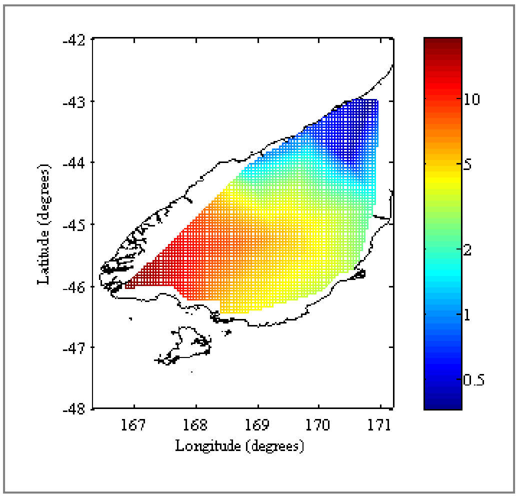

Fig. 1: Post-seismic decay amplitude for the Dusky Sound 2009 earthquake.

The post-seismic amplitude delay can only be measured where cGNSS sites exit. Figure 1 clearly shows the lack of sites in the west of the South Island since there is no grid data for these locations.

In addition to the post-seismic decay, a significant change in velocity occurred compared to the pre-earthquake velocity. This has resulted in velocity changes of over 3 mm/yr, which equates to a surface displacement of over 15 mm after 5 years.

4.2 Kapiti Coast 2013 Slow Slip Event

The Hikurangi Trench subduction zone results in frequent SSE along the East Coast of the North Island and north South Island. A large SSE, which started in late 2013, has resulted in surface displacements of over 50 mm along the Kapiti Coast (west of Wellington). The nature of these events is that they occur every 5-7 years (based on 20 years of time series data) and have a duration of several months to over a year. During this time, the velocities of sites affected by the SSE will change resulting in substantial changes in direction.

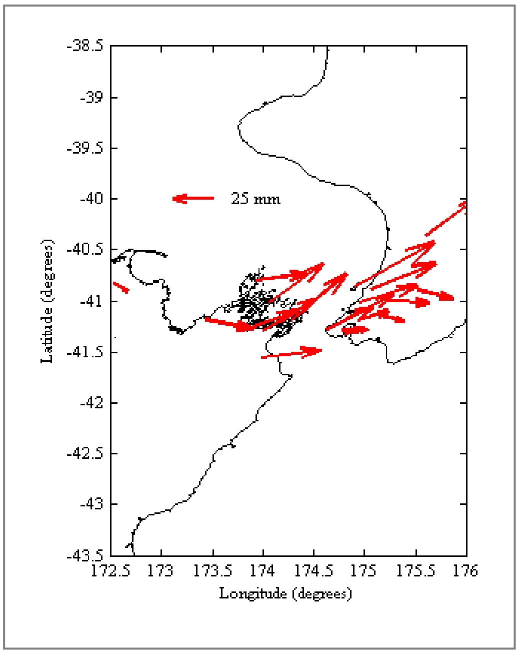

Fig. 2: Slow Slip Event deformation vectors for the Kapiti Coast Slow Slip Event.

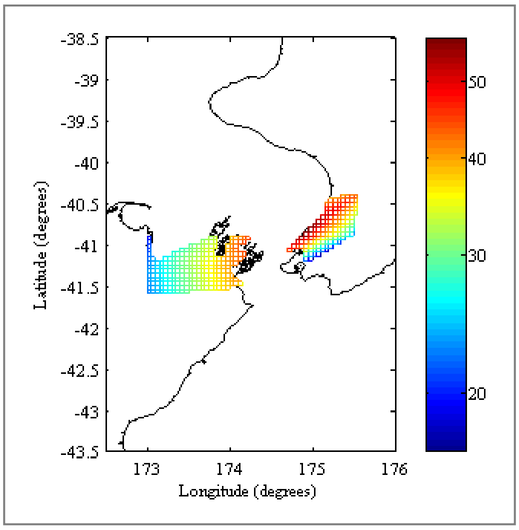

Fig. 3: Slow Slip Event deformation grid for the Kapiti Coast Slow Slip Event.

Using Equation 10, we assume that the mid-point time and the duration of the event, $t_p(\phi,\lambda)$ and $\tau_p(\phi,\lambda)$ respectively, are approximately constant over this region. Figures 2 and 3 show the total surface displacement of the SSE that occurred over a period of 1.3 years.

The direction of the long term secular velocity in this region is ~325° (approximately northwest). During the SSE, the direction of motion is between east and northeast (Figure 2), or approximately perpendicular to the long term motion. The maximum total displacement is over 50 mm along the Kapiti coastline.

5. Discussion

Post-seismic relaxation and SSE can cause significant surface displacements over short and long time periods. Transient motions affect surveying and positioning applications and it may not be possible to ignore these effects if accurate positions are required. There are two key areas that need to be consider: 1) the accuracy of the reference stations e.g. cGNSS and 2) the accuracy of the transformation from the epoch of observation to reference epoch of the local datum.

For example, a GNSS observed static baseline network. Typically, baselines will be observed relative to (cGNSS) reference stations. The coordinates of the reference station may have changed due to the ongoing deformation and need to be correct at the time of observation

GNSS processing engines, both PPP and baseline applications, determine coordinates at the time of observation in the current ITRF realization. Often it is necessary to transform the computed coordinate to the local datum. But applying a secular velocity that accounts for the motion between time of observation and the datum reference epoch may not be sufficiently accurate. The motion may have to include transient motion for post seismic decay and slow slip events.

Accurate coordinates, at the time of observation, are required for reference stations for a NRTK. Currently most NRTK application only account for linear motions as given by Equation 1. The positions need to account for both non-linear and transient motion. In addition, a rover GNSS position e.g. computed using kinematic or stop and go techniques, must be transformed to the local datum, applying transient motion based on an interpolation grid.

6. Summary

Current day positioning applications that require survey level accuracy, i.e. centimetre and millimetre; simply applying a linear model that accounts for uniform site velocities may not be sufficient. Where significant deformation is occurring, such as in a plate boundary zone, a model of the non-linear and transient deformation needs to be applied. The model may have to include the effect of post-seismic decay, transient velocities and SSE deformation.

This is required for reference stations (e.g. cGNSS) in order to maintain an accurate geodetic infrastructure that may be used for NRTK applications.

Today the current trend is towards survey application that generates current epoch (instantaneous) coordinates, for example, GNSS post processing engines and NRTK applications. For these applications, a user requirement may be to transform the position to the local coordinate system in term of the reference epoch of the datum. When non-linear deformation occurs, a more complex model is required to do this.

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References

[1] Beavan, J., M. Moore, C. Pearson, M. Henderson, B. Parsons, S. Bourne, P. England, D. Walcott, G. Blick, D. Darby and K. Hodgkinson (1999). Crustal deformation during 1994-1998 due to oblique continental collision in the central Southern Alps, New Zealand, and implications for seismic potential of the Alpine fault. *Journal of Geophysical Research-Solid Earth* **104**(B11): 25233-25255.

[2] Wallace, L. M., J. Beavan, R. McCaffrey, K. Berryman and P. Denys (2007). Balancing the plate motion budget in the South Island, New Zealand using GPS, geological and seismological data. *Geophysics Journal International* **168**(1): 332-352.

[3] Douglas, A., J. Beavan, L. Wallace and J. Townend (2005). Slow slip on the northern Hikurangi subduction interface, New Zealand. *Geophysical Research Letters* **32**(16).

[4] Wallace, L. M. and J. Beavan (2010) Diverse slow slip behavior at the Hikurangi subduction margin, New Zealand. *Journal of Geophysical Research* **115**, B12, B12402 doi: 10.1029/2010jb007717.

[5] Hackl, M., R. Malservisi, U. Hugentobler and R. Wonnacott (2011) Estimation of velocity uncertainties from GPS time series: Examples from the analysis of the South African TrigNet network. *Journal of Geophysical Research: Solid Earth* **116**, B11, B11404 doi: 10.1029/2010jb008142.

[6] Jordan, A., P. Denys and G. Blick (2007). Implementing localised deformation models into a semi-dynamic datum. Dynamic Planet - Monitoring and Understanding a Dynamic Planet with Geodetic and Oceanographic Tools. P. Tregoning and C. Rizos. Cairns, Australia, IAG Symposium, 22-26 August 2005. **130:** 631-637.

[7] Crook, C. and N. Donnelly (2013). Updating the NZGD2000 deformation model. Joint Proceedings of the NZIS conference: Celebrating the Past, Redefining the Future and SIRC NZ 2013 Conference. P. Denys, M. Strack, A. B. Moore and P. Whigham. Dunedin, New Zealand, New Zealand Institute of Surveyors**:** 40-46.

[8] Kaiser, A., C. Holden, J. Beavan, D. Beetham, R. Benites, A. Celentano, D. Collet, J. Cousins, M. Cubrinovski, G. Dellow, P. Denys, E. Fielding, B. Fry, M. Gerstenberger, R. Langridge, C. Massey, M. Motagh, N. Pondard, G. McVerry, J. Ristau, M. Stirling, J. Thomas, S. R. Uma and J. Zhao (2012) "The Mw 6.2 Christchurch earthquake of February 2011: preliminary report." New Zealand Journal of Geology and Geophysics **55**, 67-90 DOI: 10.1080/00288306.2011.641182.

[9] Beavan, J., S. Samsonov, P. Denys, R. Sutherland, N. Palmer and M. Denham (2010) "Oblique slip on the Puysegur subduction interface in the 2009 July M_W 7.8 Dusky Sound earthquake from GPS and InSAR observations: implications for the tectonics of southwestern New Zealand." Geophysical Journal International **183**, 1265-1286 DOI: 10.1111/j.1365-246X.2010.04798.x.

Biography

Paul Denys: I have been an academic staff member at the School of Surveying, Otago University since 1995. I teach papers in Survey Methods and Survey Mathematics. My primary interest is GNSS positioning and geodetic data analysis with a focus on active deformation. New Zealand offers an excellent opportunity to study and understand the broad scale deformation of the Australian-Pacific plate boundary as well as focusing on specific problems: Central Otago and Cascade deformation, Southern Alps uplift and sea level rise. I have also been involved with the geodetic analysis of the Christchurch earthquake sequence and its application to the maintenance of the geodetic infrastructure.

Chris Pearson: Since 2011 Chris has been a lecturer/research fellow at School of Surveying, Otago University where he has been active in measuring earth deformation and has collaborated with LINZ to develop tools such as PositioNZ-PP and made contributions to the NZGD2000 datum. Prior to this he worked for the US National geodetic Survey where he was project lead for maintaining the US National Deformation Model. Chris is currently acting as an advisor to the Government of Nepal on modernizing their national datum.