Square wave solutions in semiconductor lasers with mutual rotated optical coupling

David Sukow

IFISC, Universitat de les Illes Balears, Palma de Mallorca, Spain

Washington and Lee University, Lexington, Virginia USA

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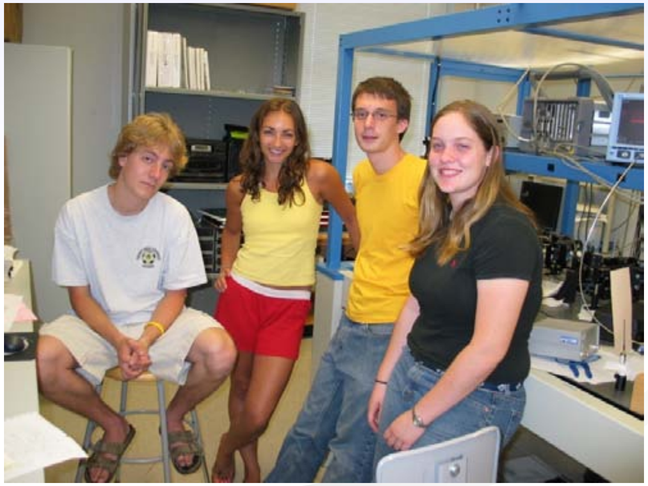
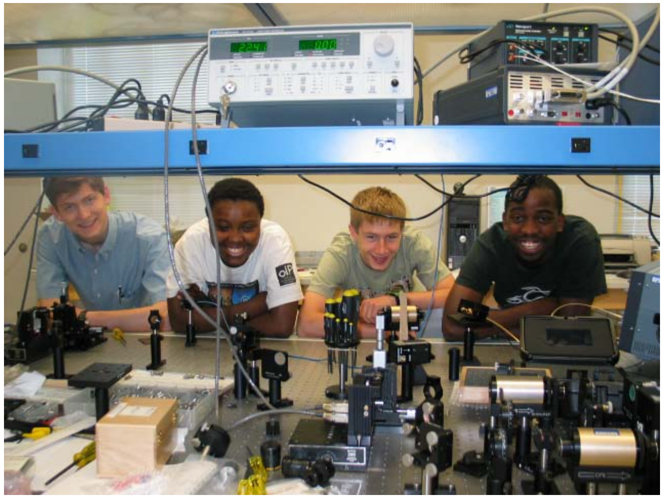
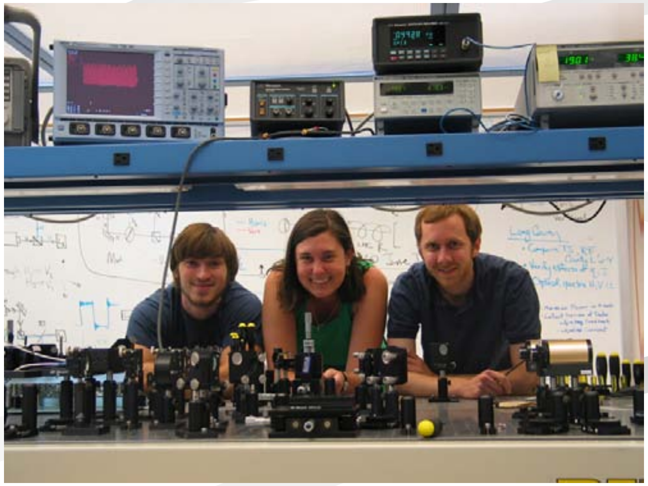






Outline

- EELs with selective orthogonal optical coupling
  - Experimental apparatus
  - Squarewaves and characteristics
- Mathematical model
  - Simulations
  - Noise effects
- Steady states
  - Mixed modes and pure modes
  - Existence and coexistence properties
- Conclusion

Why Rotated Optical Feedback?

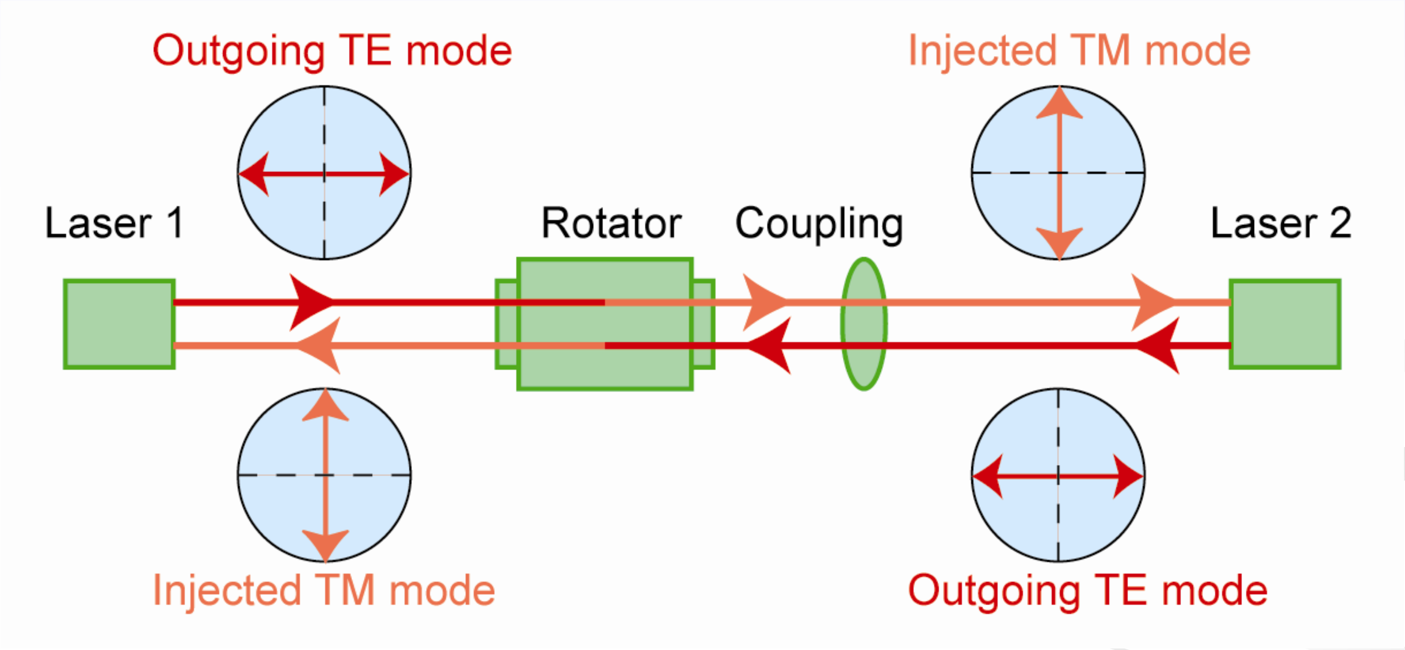
- **Fundamental dynamics**

- Rich delay-dynamical system
- Interesting parallels with optoelectronic systems
- Insight into laser properties

- **Applications**

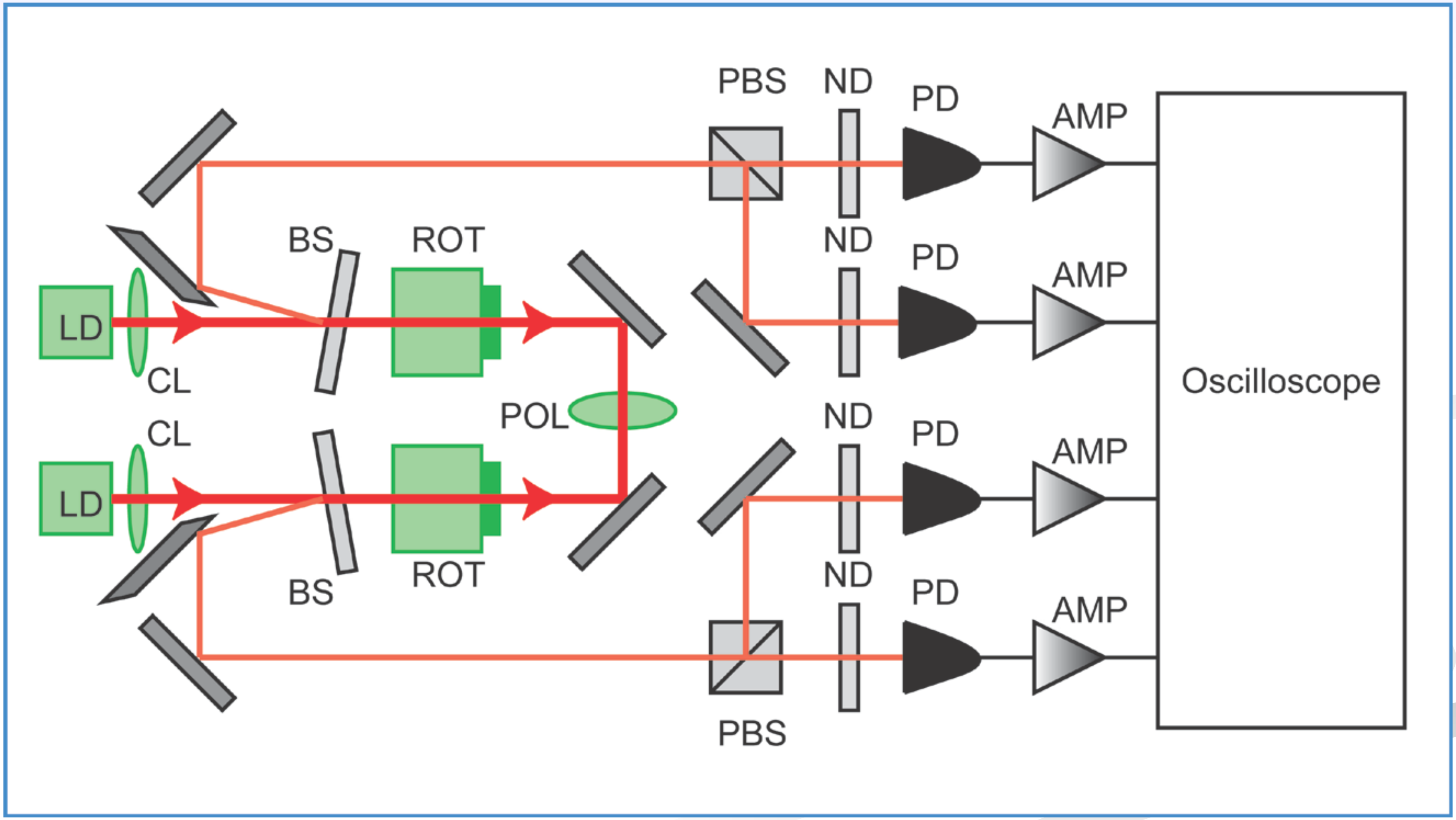
- Chaos communication
- Atomic clocks
- Optical digital logic
- Telecommunications and optical data storage
- Random number generation

Mutual Coupling via Orthogonal Optical Injection

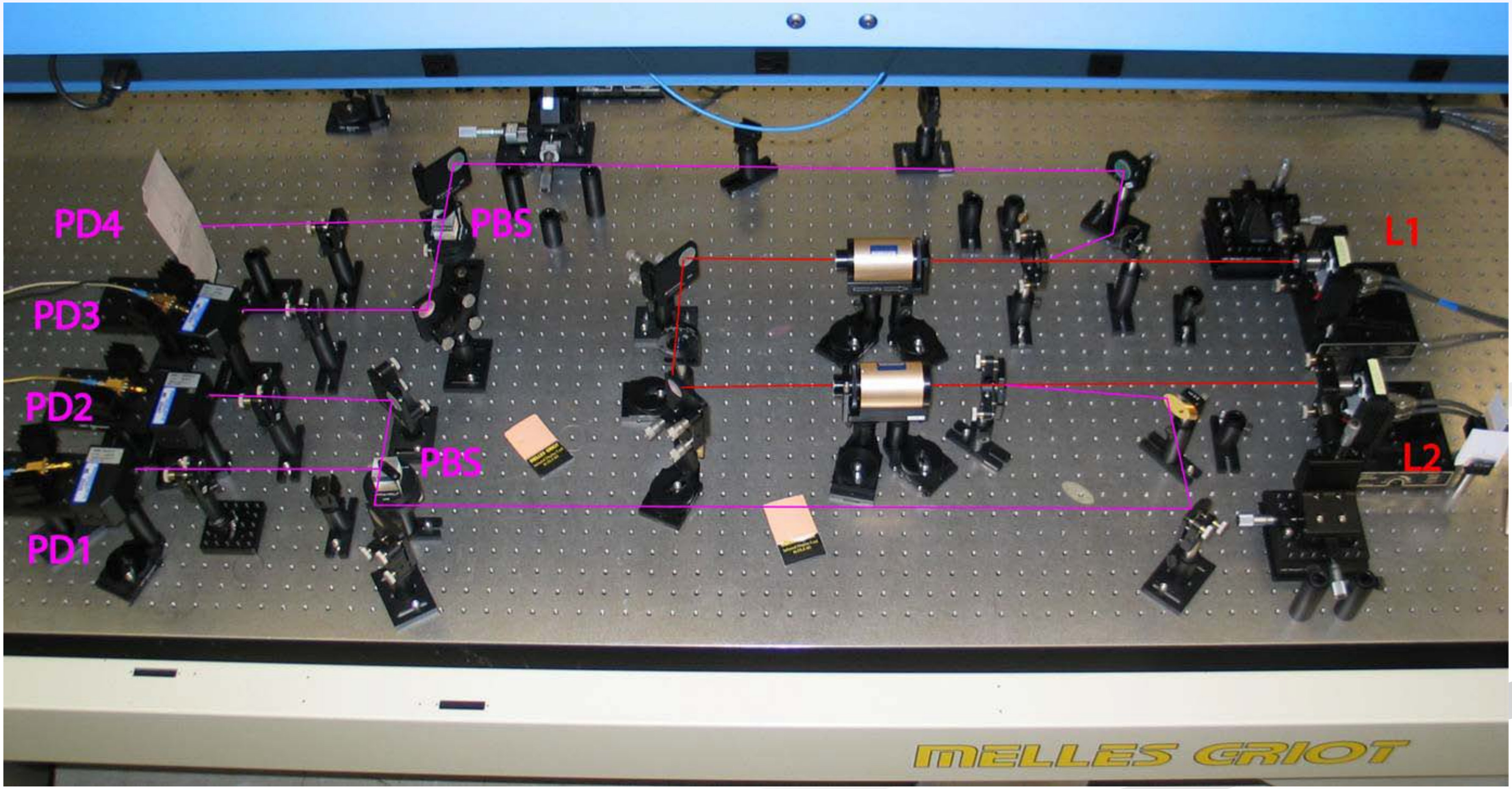


- Both lasers naturally operate in TE mode (horizontal polarization)
- TE mode of each laser rotated to TM before injection
- **Selective** mutual coupling via TE to TM modes **only**

Experimental Schematic



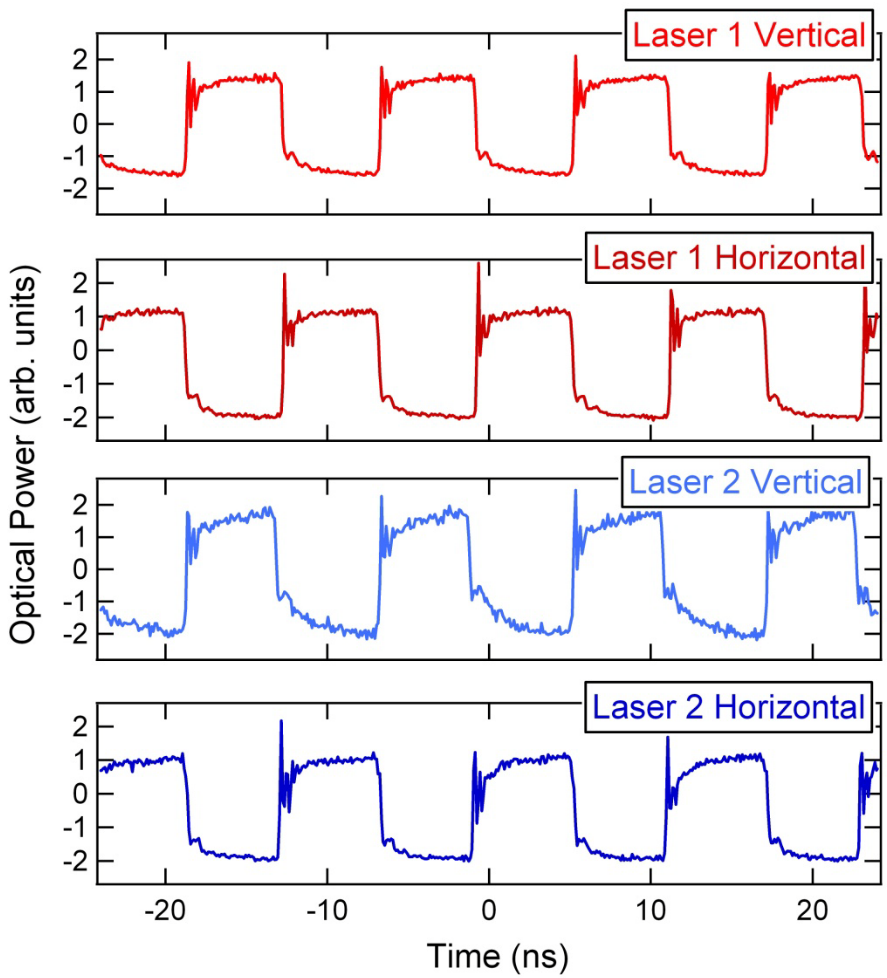
Physical Experiment



Mutually Coupled Lasers Experimental Configuration

- Both lasers are SDL-5401, temperature stabilized.
- Edge-emitting laser with dominant TE mode (horizontal polarization).
- Both nominal wavelengths λ=818 $\pm$ 0.1 nm.
- Both current thresholds I_{th} = 18.5 mA.
- U-shaped cavity for ease of alignment and similar detection path lengths.
- Cavity length L = 1.67 m
- One-way photon time of flight τ = 5.57 ns
- TE $\rightarrow$ TM mutual coupling only, no secondary reflections

Experimental Results: Squarewaves

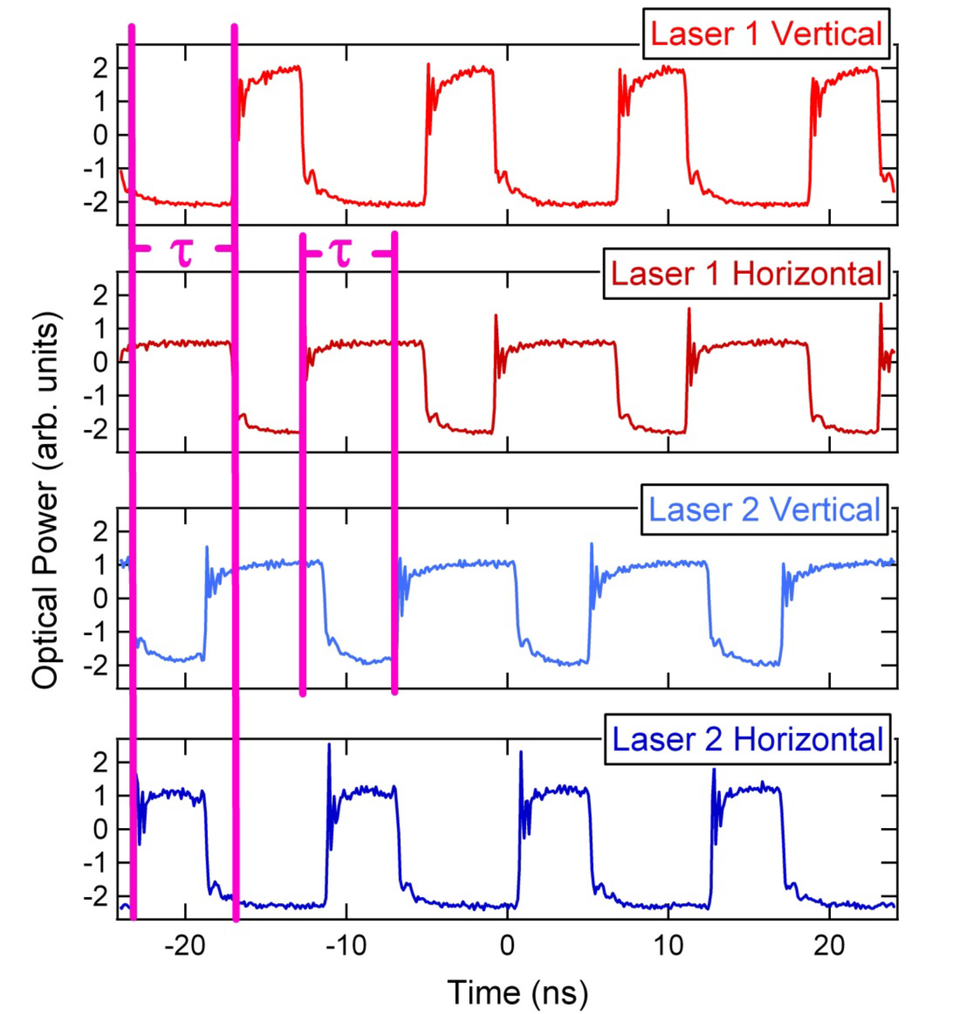


Sukow *et al.*, Phys. Rev. E **81** 025206 (2010).

- Both lasers exhibit squarewaves in both modes.
- For each individual laser, modes are in antiphase.
- Makes sense physically.
- Damped oscillations appear at onset of pulse.
- Period is twice the one-way time-of-flight between the lasers, 2τ, also the cavity roundtrip time.

- Strong coupling (48.7%).
- I = 38.88 mA for both.

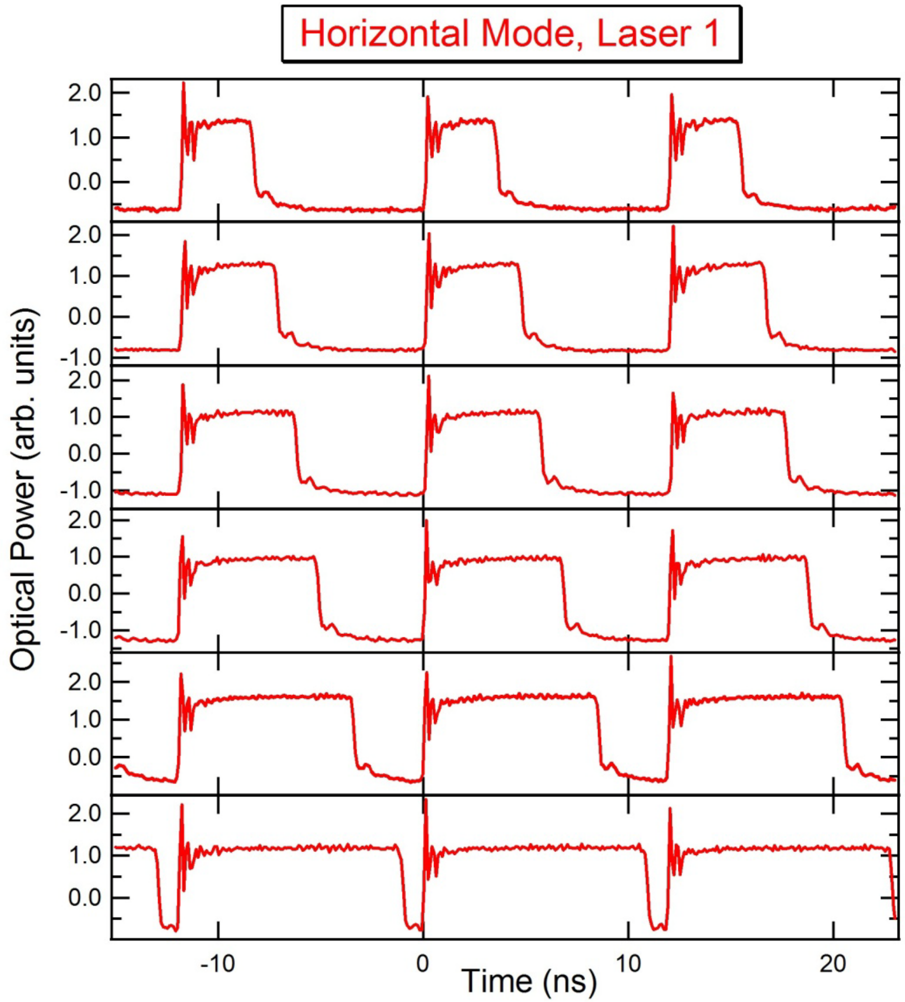
Experimental Results II: Asymmetry



- Squarewaves still appear in both modes, but with plateaus of different durations
- Modes remain in antiphase *within* each individual laser
- Horizontal mode of each laser leads the vertical mode of the other by τ
- Total period remains 2τ

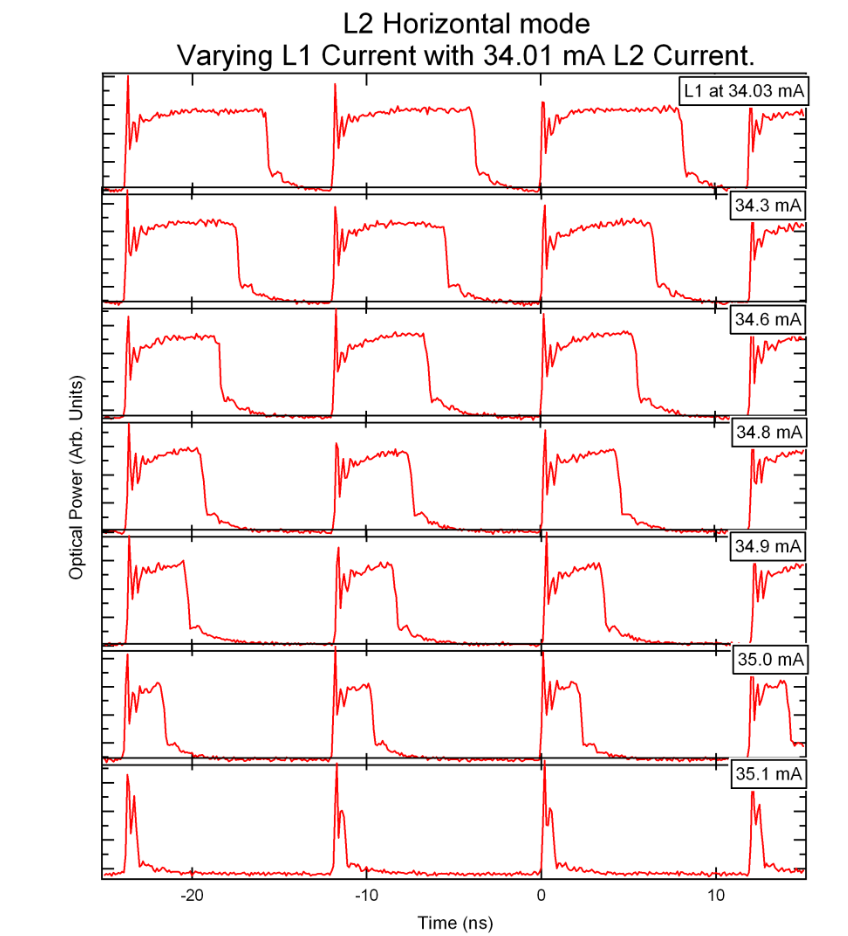
- I = 38.88 mA for both
- Coupling is weaker.

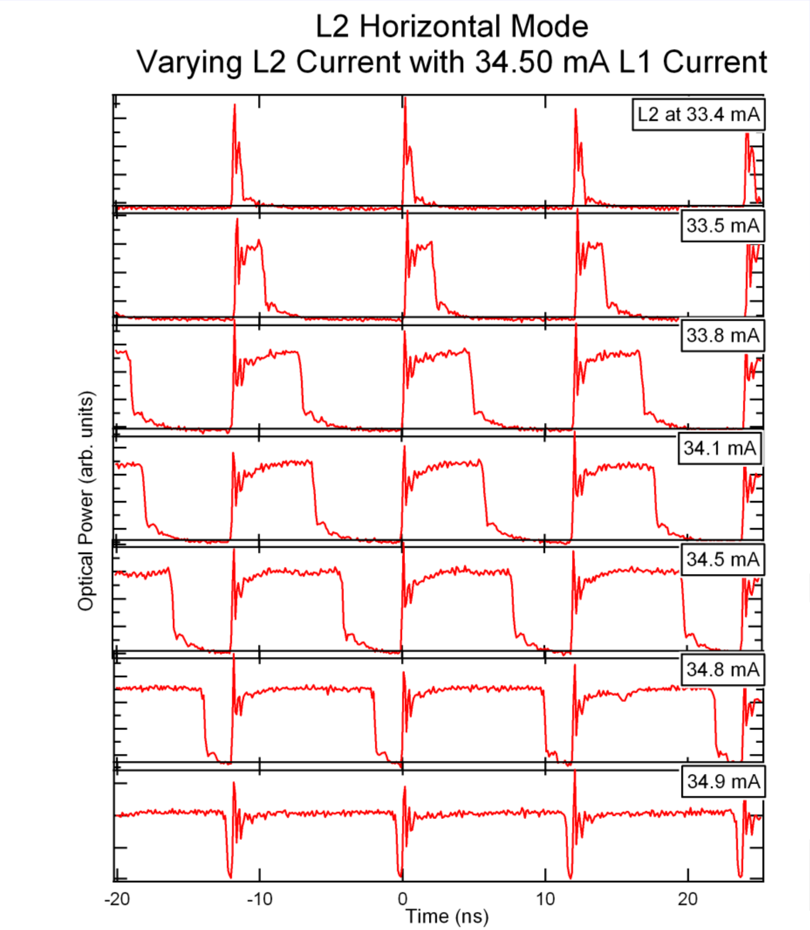
Asymmetry is Smoothly Tunable



- **What governs the duty cycle?**
- Plateaus change duration smoothly as a function of several experimentally accessible parameters: coupling strength, alignment, and laser pump currents.
- Diagram at right shows variation with coupling.
- Implies squarewaves will be lost if coupling is too weak.
- Do initial conditions matter?

Asymmetry and Pump Current





- Higher pumps produce longer plateaus in TE mode of that laser.
- Squarewaves are lost if pump currents are too dissimilar.

Mathematical Model

Mutual Coupling Model

$$\frac{dE_1^h}{ds} = (1+i\alpha)Z_1 E_1^h + \xi_1^h$$

$$\frac{dE_1^v}{ds} = (1+i\alpha)k(Z_1-\beta)E_1^v + \eta E_2^h(s-\tau) + \xi_1^v$$

$$T\frac{dZ_1}{ds} = P_1 - Z_1 - (1+2Z_1)\left(\left|E_1^h\right|^2 + \left|E_1^v\right|^2\right)$$

$$\frac{dE_2^h}{ds} = (1+i\alpha)Z_2 E_2^h + \xi_2^h$$

$$\frac{dE_2^v}{ds} = (1+i\alpha)k(Z_2-\beta)E_2^v + \eta E_1^h(s-\tau) + \xi_2^v$$

$$T\frac{dZ_2}{ds} = P_2 - Z_2 - (1+2Z_2)\left(\left|E_2^h\right|^2 + \left|E_2^v\right|^2\right)$$

- Rate equations for ***E^h***, ***E^v***, and ***Z*** for both lasers
- Mutual coupling *only* via injection of ***E^h*** from each laser into ***E^v*** of the other
- ***E^h*** is fundamental lasing mode if $\eta = 0$.
- Noise terms ξ added in field equations.
- Material parameters assumed to be the same.

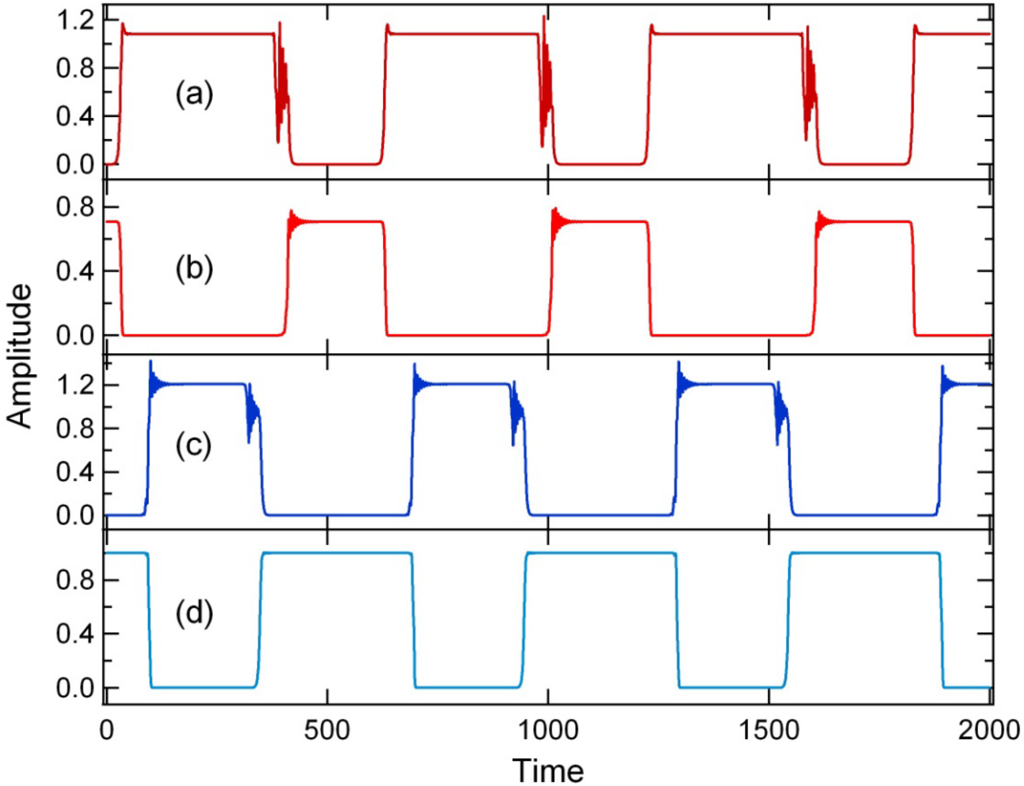
$$k = g_1/g_2, \qquad \eta = kr\tau_1$$

$$r = \frac{\eta^2}{\beta^2 k^2 (1+\alpha^2)}, \quad \beta = \frac{1}{2}\left(\frac{g_1\tau_1}{g_2\tau_2} - 1\right) > 0$$

Chen and Liu, Appl. Phys. Lett. **50** 1406 (1987)
Heil, Uchida *et al.*, Phys Rev. A **68**, 033811 (2003)
Gavrielides *et al.*, Proc SPIE **6115** (2006)

Simulations

- Numerical simulations reproduce squarewaves, with all the expected timing relationships, and tunable asymmetry.



- From the model, it can be shown that

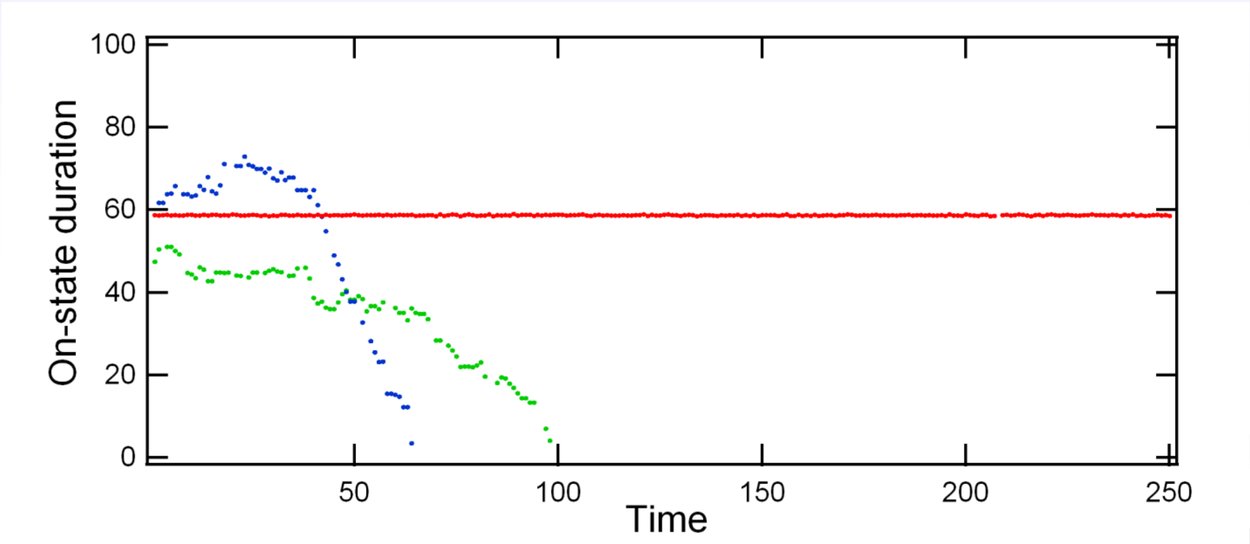
$$E_1^v(s) \approx \frac{\eta}{k\beta(1+i\alpha)} E_2^h(s-\tau)$$

$$E_2^v(s) \approx \frac{\eta}{k\beta(1+i\alpha)} E_1^h(s-\tau)$$

$T = 100,\ \tau = 3000, \alpha = 2,\ \beta = 0.03,$
$P_1 = P_2 = 1.0,\ \ \eta = 0.36$

- **The Problem: simulated square waves decay to steady states.**

Noise and Squarewave Stability



- Blue, green, and red are in order of increasing noise
- Blue and green decay into pure mode
- Red appears to be stable
- Suggests square waves may be supported by noise

Williams, Garcia-Ojalvo, and Roy, Phys Rev. A **55** 2376 (1997).
Kuske, Cordillo, and Greenwood, J. Theor. Biol. **245**, 459 (2007).

Steady States

Steady States

$$0=(1+i\alpha)Z_1E_1^h$$
$$0=(1+i\alpha)k(Z_1-\beta)E_1^v+\eta E_2^h$$
$$0=P_1-Z_1-(1+2Z_1)\left(\left|E_1^h\right|^2+\left|E_1^v\right|^2\right)$$

$$0=(1+i\alpha)Z_2E_2^h$$
$$0=(1+i\alpha)k(Z_2-\beta)E_2^v+\eta E_1^h$$
$$0=P_2-Z_2-(1+2Z_2)\left(\left|E_2^h\right|^2+\left|E_2^v\right|^2\right)$$

- Let $E_i^{v,h}=A_i^{v,h}e^{i\Phi_i^{v,h}}$
  then

$$Z_i=0$$
$$\tan\left(\Phi_2^h-\Phi_1^v\right)=\tan\left(\Phi_1^h-\Phi_2^v\right)=\alpha$$
$$A_1^{v\,2}=rA_2^{h\,2},\qquad A_2^{v\,2}=rA_1^{h\,2}$$
$$A_1^{h\,2}=\frac{P_1-rP_2}{1-r^2},\qquad A_2^{h\,2}=\frac{P_2-rP_1}{1-r^2}$$

$$r=\frac{\eta^2}{\beta^2k^2\left(1+\alpha^2\right)}$$

Steady States: Two Varieties

- Pure-Mode Solutions
- One laser dominates completely.
- For dominant laser, TE is on and TM is off. Opposite case for the other laser.
- Two such solutions.

- Mixed-Mode Solutions
- Neither laser dominates completely.
- All four optical fields are nonzero simultaneously.

- The steady states depend on the coupling relative to the pumping ratio.

Mixed-Mode Steady States

- Neither laser dominates completely. All four fields contribute.

$$E_1^h \neq 0,\ E_1^v \neq 0,\ E_2^h \neq 0,\ E_2^v \neq 0$$

- Decomposing with $E_i^{v,h} = A_i^{v,h} e^{i\phi_i^{v,h}}$,

$$A_1^{v\,2} = rA_2^{h\,2}, \qquad A_2^{v\,2} = rA_1^{h\,2}$$

$$A_1^{h\,2} = \frac{P_1 - rP_2}{1-r^2}, \qquad A_2^{h\,2} = \frac{P_2 - rP_1}{1-r^2}$$

$$r = \frac{\eta^2}{k^2\beta^2\left(1+\alpha^2\right)}$$

- These mixed-mode steady states are possible for

  $r < P_1/P_2$ (ratio less than 1), and

  $r > P_2/P_1$ (ratio greater than 1)

Sukow *et al.*, Proc. SPIE. **6997** (2008).

Pure Mode Steady States

- If Laser 1 is dominant,

$$E_1^v = E_2^h = 0$$

$$Z_1 = 0$$

- Then with $E_i^{v,h} = A_i^{v,h} e^{i\phi_i^{v,h}}$,

$$\left|E_1^h\right| = \sqrt{P_1}$$

$$\tan\left(\Phi_1^h - \Phi_2^v\right) = \alpha$$

$$k^2 (Z_2 - \beta)^2 \left(1 + \alpha^2\right) A_2^{v\,2} = \eta^2 P_1$$

$$A_2^{v\,2} = \frac{P_2 - Z_2}{1 + 2Z_2} \geq 0$$

- If Laser 2 is dominant,

$$E_2^v = E_1^h = 0$$

$$Z_2 = 0$$

- Then

$$\left|E_2^h\right| = \sqrt{P_2}$$

$$\tan\left(\Phi_2^h - \Phi_1^v\right) = \alpha$$

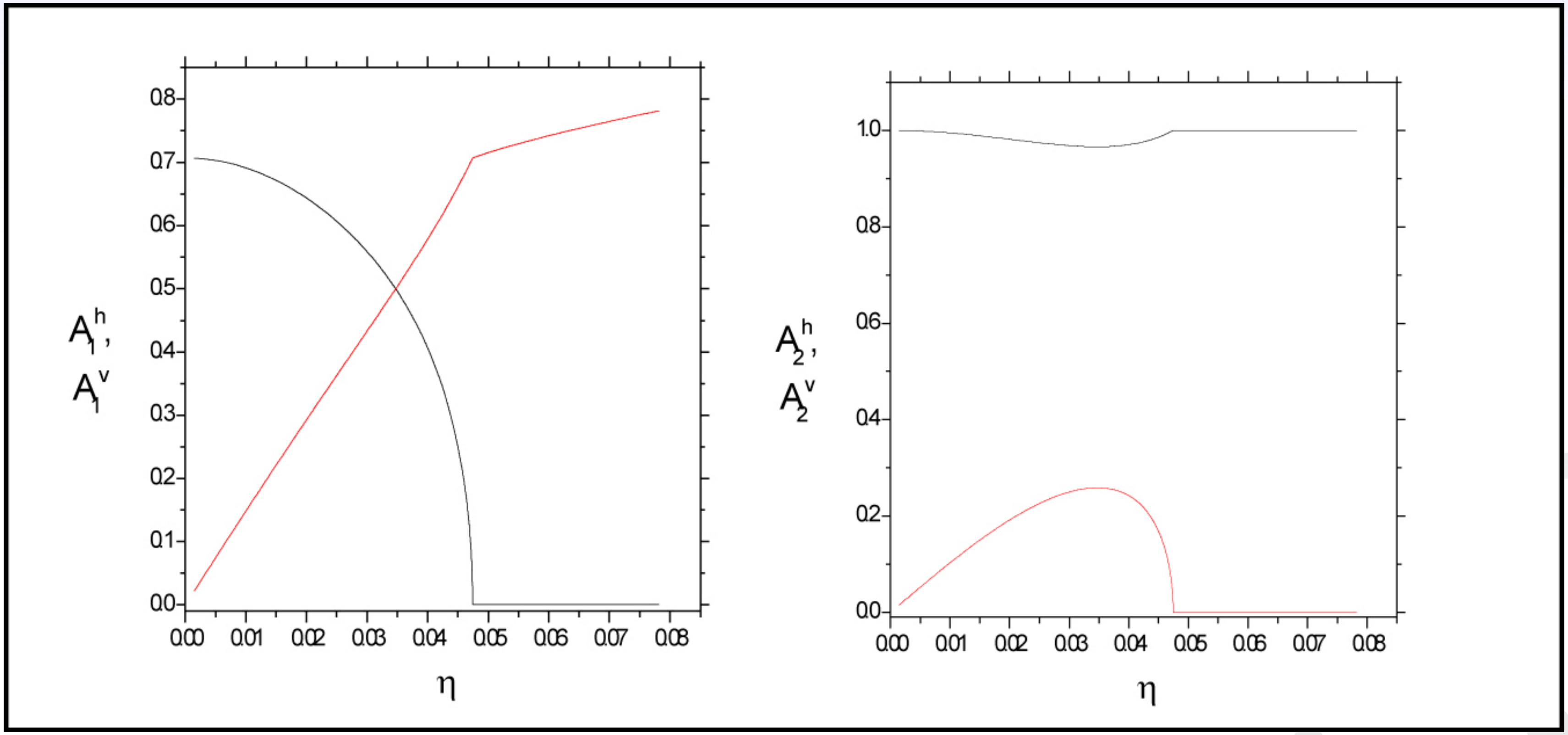
$$k^2 (Z_1 - \beta)^2 \left(1 + \alpha^2\right) A_1^{v\,2} = \eta^2 P_2$$

$$A_1^{v\,2} = \frac{P_1 - Z_1}{1 + 2Z_1} \geq 0$$

- **Pure mode steady states are possible for r > P_1/P_2 (ratio < 1)**

Steady States

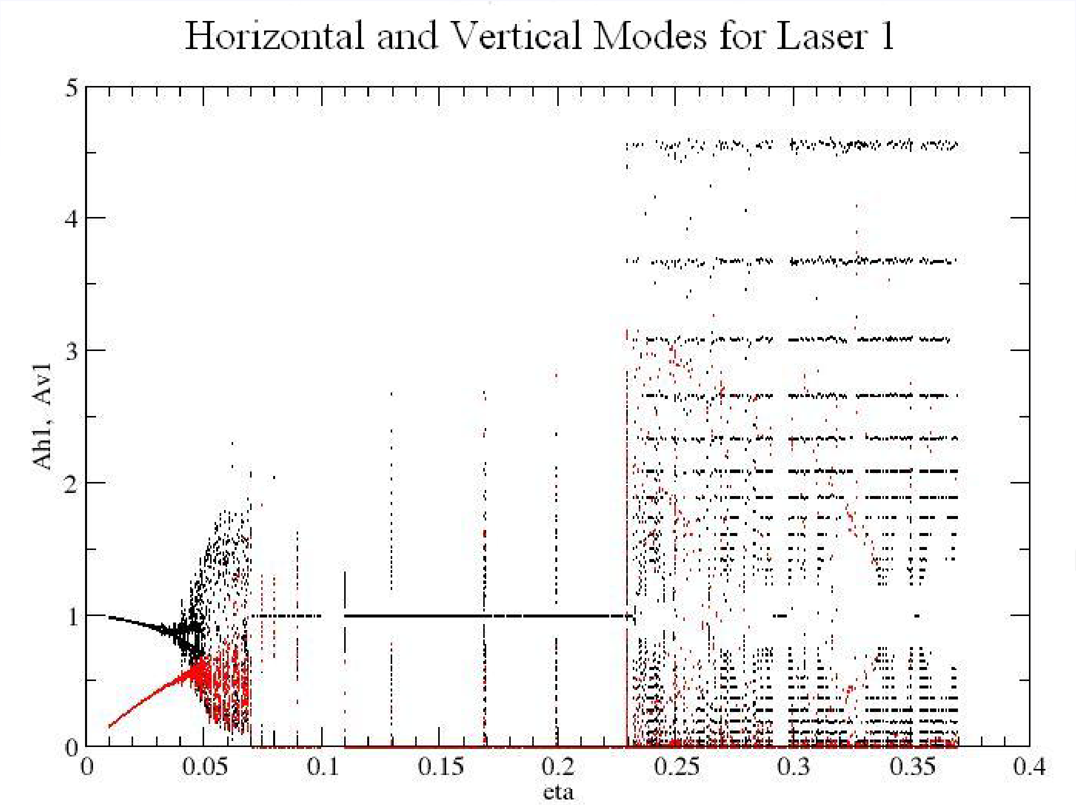
Laser 1 Laser 2



Black = TE mode, Red = TM mode

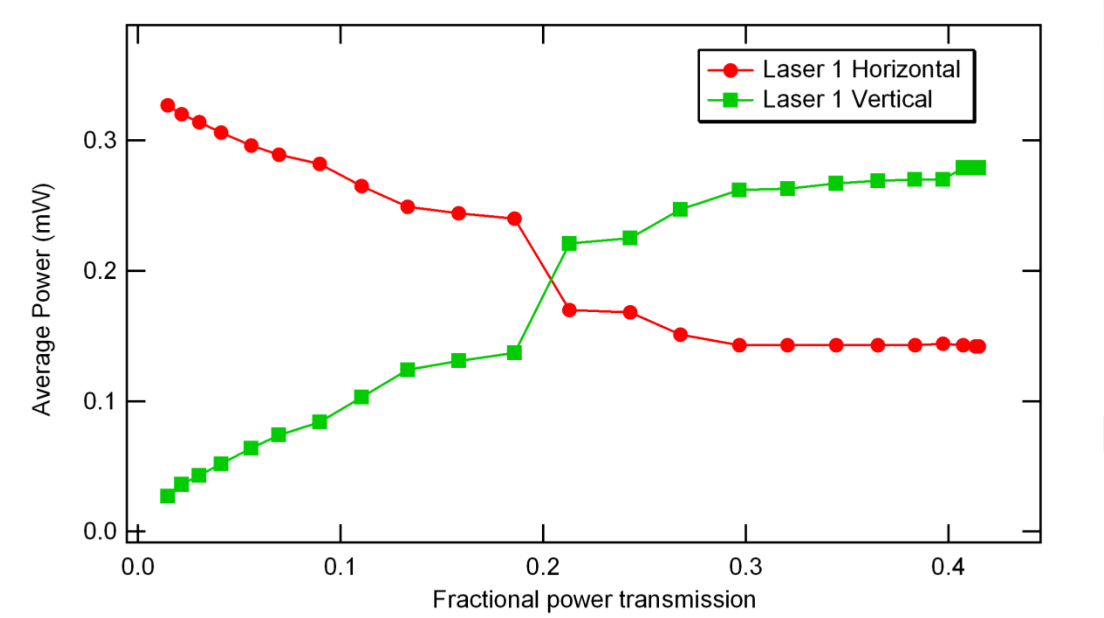
$$T = 150\,,\ \tau = 3500,\ \alpha = 2,\ \beta = 0.03,\ P_1 = 0.5,\ P_2 = 1.0$$

Bifurcation Diagram: One Laser



- Mixed mode emerges from solitary state, bifurcates to limit cycle, then strange attractor.
- Pure polarization mode solution appears near $\eta = 0.07$.

Experimental Results
Average Modal Powers vs. Coupling



$I_1 = 37\text{ mA}$
$(P_1 = 1.0)$

$I_2 = 37\text{ mA}$
$(P_2 = 1.0)$

- Mixed-mode steady state up to 0.09 roundtrip power transmission
- Oscillatory or pulsating up to 0.186
- Sudden jump to squarewaves at 0.213
- Complex dynamics above 0.267

Steady States: Both Lasers

- Pure mode solution 1

$$A_1^h = 0 \text{ and } A_2^h = \sqrt{P_2}$$

- Pure mode solution 2

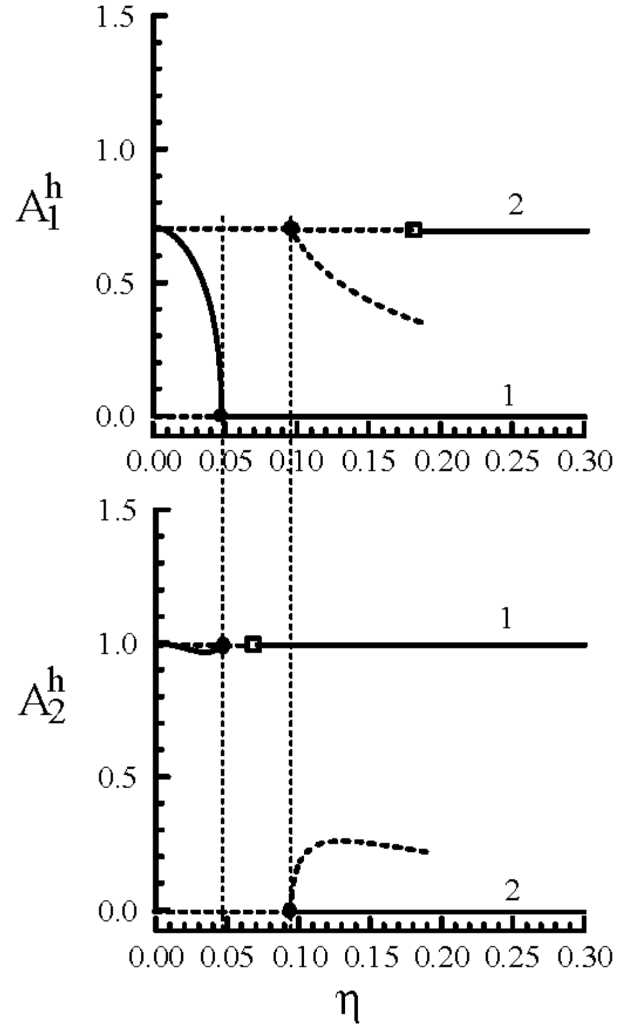
$$A_1^h = \sqrt{P_1} \text{ and } A_2^h = 0$$

- Mixed mode bifurcation points are solid dots at:

  η =0.047 (r = ½)

  η =0.095 (r =2)

- Pure modes coexist if η is sufficiently large.



$k = 1$

$\alpha = 2$

$\beta = 0.03$

$P_1 = 0.5$

$P_2 = 1.0$

Pure Modes
Existence and Coexistence

- Pure modes exist if

$$\eta^2 \geq k^2 \beta^2 \left(1+\alpha^2\right)$$

- If this condition is met, it is possible for either pure mode to exist individually, or both may coexist simultaneously, depending on operating conditions.

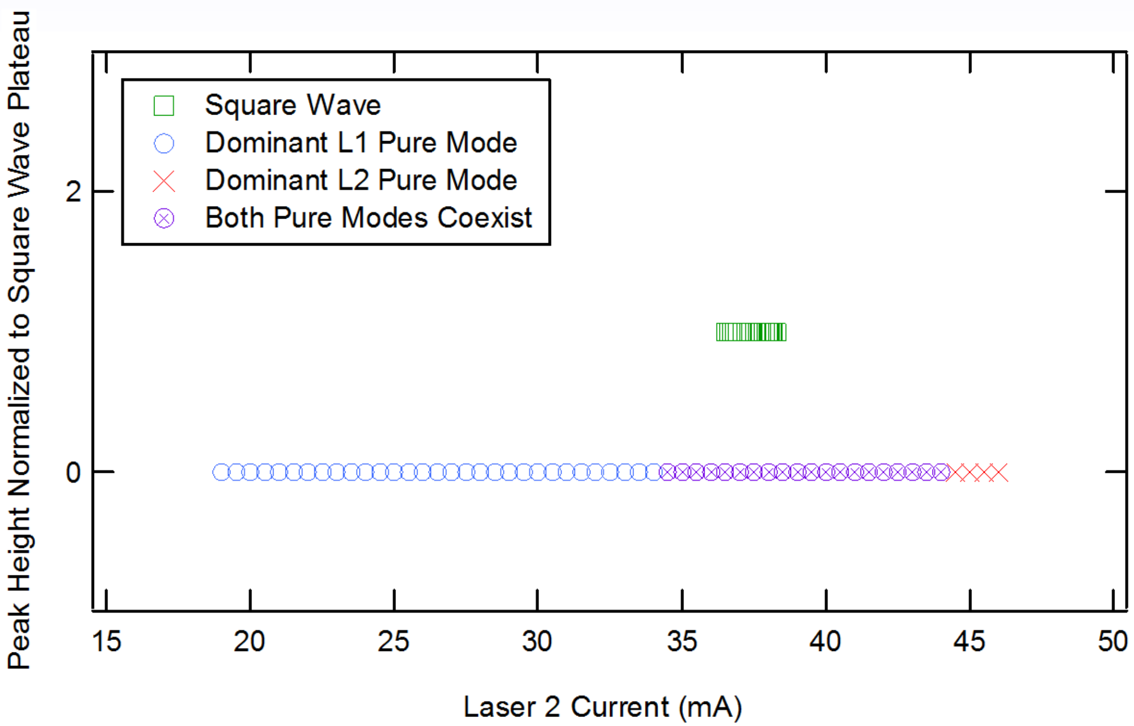
- Suppose P_1 is fixed. Then the pure modes coexist over a range of P_2 :

$$P_{2\,\text{upper}} = \frac{\eta^2}{k^2 \beta^2 \left(1+\alpha^2\right)} P_1$$

$$P_{2\,\text{lower}} = \frac{1}{P_{2\,\text{upper}}}$$

Pure Mode Coexistence: Experimental Results

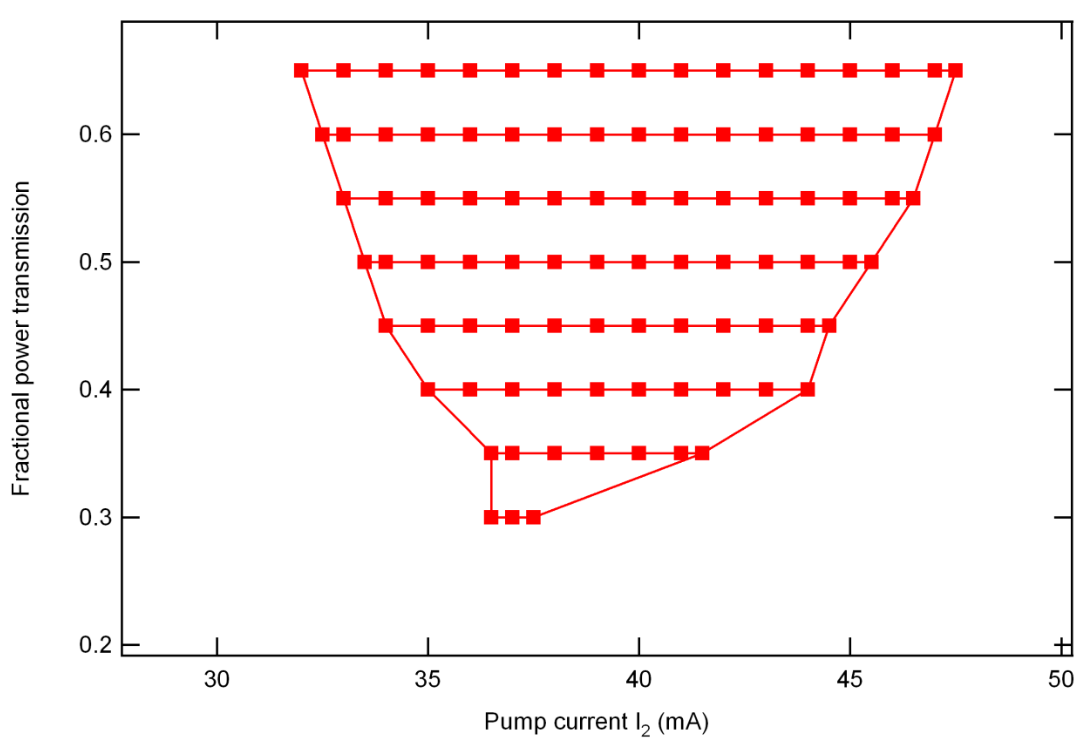
Bifurcation Diagram for Short-Cavity System for Varying L2 Current (L1 Current: 38.00 mA



- Power transmission 57.6%

- Laser 1 current fixed: 38 mA
- Laser 2 current varies
- Pure modes have region of coexistence where I_2 is similar to I_1
- *Square wave solutions only observed where pure modes coexist.*
- Many other complex solutions also coexist

Pure Mode Coexistence: Experimental Feedback Dependence



- I_1 fixed at 38 mA
- Plot shows currents I_2 where pure modes coexist, depending on coupling strength.
- Weaker coupling leads to a smaller coexistence region of pure modes, to a nonzero minimum.

$$P_{2\,\mathrm{upper}} = \frac{\eta^2}{k^2\beta^2\left(1+\alpha^2\right)}P_1, \quad P_{2\,\mathrm{lower}} = \frac{1}{P_{2\,\mathrm{upper}}}$$

Summary

- EELs with mutual, selective, rotated optical coupling display tunable, asymmetric square waves, with antiphase relations between waves of the same laser, and fixed timing relations with the other laser.

- Simulations reproduce experimentally observed features, but appear to be unstable. Noise may play a role in determining the stability properties of square waves.

- This system possesses pure-mode and mixed-mode steady states, which are observed experimentally and theoretically. Square wave solutions appear only where both pure modes coexist.

- Further study is required to fully understand the square wave solutions. Is noise is a requirement? Can simpler analytic models offer insight and reproduce the same effects?

- More square waves in a future talk this morning!