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Trade Policy and Public Ownership

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Abstract

This paper discusses the influence of public ownership on trade policy instruments. We demonstrate three important invariance results. First, the degree of public ownership affects neither the level of socially optimal activities nor welfare if the government chooses optimal trade policy instruments. Second, in the case of rivalry between domestic export firms, the optimal export tax is independent of the degree of public ownership. Third, in the case of rivalry in the home market, the optimal import tariff is independent of the degree of public ownership. In this case, the optimal production subsidy decreases with public ownership if the optimal tariff is positive. For the case of Cournot rivalry in a third market, the optimal export subsidy is an increasing function of the public ownership share, while in the case of Bertrand rivalry with differentiated products, the optimal export tax is an increasing function of that parameter.

JEL-Classification: F12, F13.

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1 Introduction

In this paper, we consider the impact of public ownership on trade policies. The main motivation is to explore if and how public ownership changes trade policies because public ownership still plays a crucial role in most countries. Despite an ongoing privatization of state-owned enterprises, in particular in transition countries, the governmental influence by state ownership is still strong. According to a recent OECD indicator of scope of public enterprise sector¹, the governmental influence via state ownership has been reduced but is still prevalent.

The international trade literature has largely ignored the role of public ownership. Obviously, public ownership does not matter under perfect competition, as state-owned firms would also be too small to have any influence on markets. However, the role of state-owned firms is less clear under imperfect competition, because state-owned firms may have different objectives compared to privately owned firms, and both types of firms have market power.² Surprisingly, even the extensive literature on strategic trade policy – genuinely dealing with trade policy under imperfect competition – ignores public ownership but typically proceeds from the assumption that firms are profit-maximizing oligopolists.³

A classic example of strategic trade policy is the duopolistic rivalry between the two jumbo-jet makers, Boeing and Airbus, that allegedly receive overt or covert subsidies from their respective governments. A lesser

¹This indicator has a minimum of zero and a maximum of 6 and measures the pervasiveness of state ownership across business sectors as the proportion of sectors in which the state has an equity stake in at least one firm. For details see <http://www.oecd.org/dataoecd/40/0/35655773.xls>.

²There is a literature on mixed oligopoly, where the typical assumption is that one of the oligopolists is a completely state-owned firm that seeks to maximize social welfare (rather than profit) while its rivals are profit-maximizing private firms. DeFraja and Delbono (1989) show that, in a closed economy without production subsidies to any firm (public or private), privatization of a completely state-owned firm will increase welfare only if the existing number of private firms is large. White (1996) considers privatization in a closed economy where privately owned oligopolists are subsidized.

³For an overview see Brander (1995).

known example is the rivalry between two major producers of regional jets (*i.e.*, aircrafts with less than 100 seats), namely Canada's Bombardier and Brazil's Embraer. These two countries have accused each other of subsidizing their home firm. The theoretical models that explain export subsidies rely on the assumption that the rival firms are profit-maximizing firms. However, not all the firms in the cited examples are entirely owned by private agents. About 15% of Airbus is owned by the French government,⁴ and until recently, a sizeable fraction of Embraer's shares was owned by the Brazilian government. Also many oil companies are state-owned.

It can be argued that firms that are partially state-owned may not maximize profit. A plausible hypothesis is that semi-public firms maximize a weighted average of profit and social welfare, *e.g.* because the board of directors would include representatives of the government, and they may successfully push for an output level that is nearer to the social optimum. The purpose of our paper is to explore the implications of this hypothesis for trade policies.

The paper is organized as follows. Section 2 deals with domestic firms competing in an export market, and one of these firms is partly state-owned. For this model, we prove an "invariance theorem": the optimal export tax is independent of the parameter that represents the degree of state ownership. Section 3 deals with a (semi-public) home firm and a (profit-maximizing) foreign firm competing in the home market. Government policies aim at expanding domestic output and extracting rent from the foreign firm.⁵ We show that the home country's optimal tariff is independent of the parameter representing the extent of public ownership of the home firm, and that the

⁴Airbus is a subsidiary of EADS (European Aeronautic Defence and Space) of which 15% is owned by the French government, and 5% is owned by the Spanish government's holding company SEPI (The Economist, "The Mess at EADS," June 24-30, 2006).

⁵The motive of extracting foreign monopoly rents was explored in Brander and Spencer (1981, 1984) but they did not consider the case where the foreign firm has a rival in the home market. Others (Pal and White, 1998, Chang, 2005) allow domestic production but did not allow the home country to use simultaneously a domestic production subsidy and a tariff.

optimal production subsidy for the home firm is decreasing in this parameter. The maximized welfare level of the home country is however invariant with respect to this parameter. Section 4 deals with the third market model when the home firm is partly state-owned.⁶ We show that, in the case of Cournot rivalry, the optimal export subsidy is an increasing function of the parameter that represents the degree of state ownership, while in the case of Bertrand rivalry with differentiated products, the optimal export tax is an increasing function of that parameter. We also establish an invariance result for this model: the welfare level of the home country is invariant with respect to the degree of state-ownership, provided that the home government uses the optimal tax-subsidy-scheme. Section 5 concludes the paper. For convenience, we have relegated all proofs to the appendix.

Although the international trade literature has basically ignored public ownership, there are a few exemptions, particular on the effects of privatization. Fjell and Pal (1996) extend the closed economy model of DeFraja and Delbono (1989) to include foreign firms exporting to the home country, but they do not consider trade policies. They show that privatization of a public firm will lead to a fall in its output level, and a rise in the output levels of other firms, but industry output will decrease. Pal and White (1998) extend this model by assuming that the home country either subsidizes domestic firms and imposes no tariff, or imposes a tariff and does not subsidize domestic firms (but cannot use both instruments). They show that privatization leads to an increase (decrease) in domestic welfare if optimal subsidies (tariffs) are in place. A second paper by Pal and White (2003) employs a reciprocal dumping mixed-duopoly model, but continues to assume that the home country is allowed to use only one policy instrument.

A few papers have studied the case of oligopoly with a semi-public firm in a closed economy. Fershtman (1990) assumes the semi-public firm's output is a weighted average of Cournot output and socially optimal output. He

⁶This is an extension of the familiar "third market model" (Brander and Spencer, 1985, Eaton and Grossman, 1986).

shows that in a duopoly with linear demand and constant marginal costs, the semi-public firm makes more profit than the private firm. Matsumura (1998) proposes that a semi-public firm's objective function is a weighted average of its profit and "modified" social welfare, which is assumed to give a greater weight to consumer surplus relative to profit. Assuming a duopoly with no taxation, he shows that the optimal degree of public ownership of the semi-public firm is neither zero nor 100 percent. Chang (2005) extends these models to the open economy case, and considers rivalry in the home market between a semi-public domestic firm and a foreign firm that exports to the home market. The semi-public domestic firm maximizes a weighted average of its profit and the home country's welfare. He assumes that the home government either imposes a tariff, or gives a production subsidy to the domestic firm, but not both. By ruling out the simultaneous use of tariff and domestic production subsidy, Chang shows that there is an optimal government ownership share of the domestic firm. In contrast, we show that the share of government ownership is irrelevant for welfare, if the government can use both tariff and domestic production subsidy. None of the above papers considers the home country's simultaneous use of both policy instruments (a production subsidy and a tariff).

2 Rivalry between exporting firms

Let us begin with a very simple model. Two domestic firms, 1 and 2, produce a homogeneous good and export their entire outputs, q_1 and q_2 , to a foreign country that does not produce the good. They have identical marginal cost, c .⁷ Let $Q = q_1 + q_2$. The foreign country's imports demand function is $P = P(Q)$, where $P(0) > c$ and $P'(Q) < 0$. It is well known that the rivalry of the two domestic firms implies that the home country as a whole fails to exploit its potential monopoly power in the foreign market. In other words, the two

⁷As long as international markets are segmented and marginal costs are constant, it does not matter whether a domestic demand exists or not because the optimal export tax does not depend on domestic demand. See Rodrik (1989).

firms over-produce. When both firms maximize profits non-cooperatively, the socially optimal policy is to impose an export tax, at the same rate for both firms.

Consider now the case where one firm is semi-public and has the objective function of maximizing a weighted average of its own profit and the home country's welfare, with weights $1 - \theta$ and θ , respectively. In this context, we ask the following questions: should the export tax rates be different for the two firms, and should they depend on θ ? Our answer for each of these questions is "no." We now proceed to prove this "invariance theorem."

Let us note that if the home country can directly control the outputs of the domestic firms, it will choose Q to maximize social welfare, which is the export revenue minus the production cost:

$$W = P(Q)Q - cQ. \quad (1)$$

The socially optimal industry output, denoted by Q^* , must therefore equate the marginal export revenue with the marginal production cost:

$$P'(Q^*)Q^* + P(Q^*) = c. \quad (2)$$

As usual, we assume that the second order condition is satisfied. Let us start with the standard case of two domestic firms which are privately owned. Suppose that direct control is not possible. The home government can choose an export tax t to influence the quantity exported. Assume the firms behave as Cournot rivals, and that each wants to maximize its profit. Firm i 's profit function (net of tax) is

$$\tilde{\pi}_i = P(q_i + q_{-i})q_i - cq_i - tq_i. \quad (3)$$

For this simple case, we obtain

Lemma 1 *The socially optimal tax is equal to*

$$t^* = -(Q^*/2)P'(Q^*) > 0. \quad (4)$$

Proof: See Appendix A.1.1.⁸

Consider now the case where firm 1 is partly state-owned. We assume its objective is to maximize a weighted average of its profit $\tilde{\pi}_1$, as given by (3), and social welfare W , as given by (1). Let $\theta \in (0, 1)$ be the weight attached to social welfare. Since the good is not consumed in the home country, social welfare is just the export revenue minus production cost. Thus firm 1's problem is:

$$\max_{q_1} (1 - \theta) [P(q_1 + q_2)q_1 - cq_1 - tq_1] + \theta [P(q_1 + q_2)(q_1 + q_2) - c(q_1 + q_2)].$$

We assume that firm 1 takes t and q_2 as given and may derive the following invariance result:

Proposition 1 *The social optimum can be achieved as a Nash equilibrium by applying an optimal export tax. The optimal export tax is independent of the parameter θ that represents the degree of public ownership. At the optimal tax, the Nash equilibrium output of each firm is independent of θ .*

Proof: see Appendix A.1.2.

Remark 1 *Proposition 1 can be generalized to the case of n domestic firms that are Cournot rivals, of which $m < n$ firms are semi-public.*

Proposition 1 is a surprising and important result. Its intuition can be best explained by considering the case of a completely state-owned firm. This firm would maximize welfare, and any tax, including the optimal export tax, would not influence its behavior because government tax revenues and the firm's tax bills cancel out for welfare. Hence, given that the privately owned firm produces half of the socially optimal output, a completely state-owned firms would also produce half of the socially optimal output. Proposition 1 demonstrates that the optimal tax guarantees also that any partially state-owned firm will keep this level as it has no incentive to produce less or more. Obviously, these results extend easily to the case of more than two firms.

⁸Lemma 1 coincides with Rodrik's (1989) optimal export tax for the case of only two firms.

3 Rivalry in home market

We now turn to the case where the rivalry takes place in the home market which is assumed to be segregated from the foreign market. We assume constant marginal costs, so that output decisions for the home market are unrelated to output decisions for the foreign market. This allows us to focus on the home market. The home market is served by a domestic firm, firm 1, and a foreign firm, firm 2, located in country 2. The quantities they supply to the home market are denoted by y (domestic production) and x (export by firm 2), respectively.

There is a literature that deals with optimal tariffs to extract rent from a foreign monopolist (*e.g.* Katrak, 1977, Brander and Spencer, 1981 and 1984). These authors assume that there are no domestic firms that can produce or compete with the foreign monopolist. Assuming the home country's choice variable is a tariff rate t , Brander and Spencer (1984) show that the optimal tariff is positive if the domestic demand curve is not too convex and the foreign firm's marginal cost is constant or increasing. On the other hand, if an increase in tariff causes a larger increase in the consumer's price ($dp/dt > 1$) then an import subsidy is optimal.⁹ The scenario where a domestic firm can compete with the foreign firm is taken up by Pal and White (1998), but they assume that the home government may use only one policy instrument: either a production subsidy for the domestic firm, or an import tariff.¹⁰

In this section, we allow the home government to have recourse to both production subsidy s and import tariff t , and study how the optimal pair (s, t) changes with the degree of public ownership of the domestic firm. To facilitate understanding of our results, it is useful to consider first the benchmark case where the government can control the domestic output (by control and command).

⁹For example, under constant marginal cost, an import subsidy is optimal if $-2 < R < -1$ where $R \equiv Qp''/p'$.

¹⁰Haufler, Schjelderup and Stähler (2005) consider the role of commodity taxes in a trade model under different rules of taxation.

Let $Q = x + y$ where y is produced by a domestic firm and x is the quantity exported by the foreign firm to the home country. To make the problem non-trivial, we assume the marginal cost of the home firm is higher than that of the foreign firm, *i.e.*, $c_1 > c_2$. Otherwise the optimal quantity imported would be zero. Let $\Delta = c_1 - c_2$ denote the cost disadvantage of the domestic firm. The domestic inverse demand function is $p = p(Q)$, where $p(0) > c_1$ and $p'(Q) < 0$. As usual, we assume

$$p'(Q) + xp''(Q) < 0 \quad (5)$$

for all $Q \geq 0$ and all $x \in [0, Q]$. Suppose the home government can precommit the domestic output y and the tariff rate t . Appendix A.2.1 proves

Lemma 2 *Domestic welfare is equal to*

$$W = \int_0^{x+y} p(Q)dQ - c_1y - \underbrace{[c_2x - x^2p'(x+y)]}_{\equiv \Gamma}.$$

Expression Γ is the cost (to the home country) of obtaining x . Since the foreign production cost is c_2x , we can interpret $-x^2p'(x+y) \equiv \rho$ as the rent accrued to the foreign firm. For a given x , if domestic output y increases, the rent accrued to the foreign firm will change by $\rho_y = -x^2p''$. Hence, an increase in domestic production will increase (decrease) the foreign firm's rent if the inverse demand curve is concave (convex). Armed with these prerequisites, we are now able to explain our next proposition.

Proposition 2 *1. It is optimal to import if and only if the domestic firm has a cost disadvantage, *i.e.*, $\Delta > 0$.*

*2. Assume an interior solution (*i.e.* $x^* > 0$ and $y^* > 0$). At the optimal consumption level Q^* , given that $\Delta > 0$, consumer price exceeds (respectively, falls short of) marginal cost of the domestic firm if and only if $p''(Q^*) < 0$ (respectively, $p''(Q^*) > 0$.)*

Proof: see Appendix A.2.1.

The intuition for part 1 of Proposition 2 is straightforward. As for part 2, the government has to balance two conflicting incentives, *i.e.*, to increase domestic consumption and to extract rents from the foreign firm. If the demand function is linear, both effects cancel each other such that the consumer price must be equal to the marginal cost of the domestic firm. This equality does not hold if $p(Q)$ is strictly concave or convex. If the inverse demand curve is strictly concave ($p'' < 0$), an increase in domestic production will increase the rent accrued to the foreign firm at any given level of x . In such a case, the home country refrains from expanding domestic output toward the level that would equate price to marginal cost. It lowers domestic output so as to extract more rent from the foreign firm. On the contrary, if the inverse demand function is strictly convex ($p'' > 0$), more domestic output will extract more rent from the foreign firm. Our next lemma gives the tariff which manages trade optimally.

Lemma 3 *Assume the government can directly control the output of the domestic firm. The optimal tariff is*

$$t^* = \frac{\Delta}{2} \left[1 - \frac{\Delta p''}{2(p')^2} \right] \equiv \frac{\Delta}{2} \left[1 + \frac{x^*}{Q^*} R \right] \quad (6)$$

where R is the elasticity of the slope of the inverse demand curve,

$$R \equiv \frac{Q^* p''}{p'}$$

Proof: see Appendix A.2.1.

Note that $R > 0$ if $p(Q)$ is a concave function. It follows from Lemma 3 that the optimal tariff is positive if and only if $\infty < (x^*/Q^*)R < -1$, *i.e.*, if the demand curve is not too convex.¹¹ This is a less demanding condition than part 2 of Proposition 2. The reason is that a moderately positive tariff may still result in a consumer price above the domestic marginal cost (recall that $c_1 > c_2$).

¹¹It follows that when there is no domestic production, the condition for a positive optimal tariff is $\infty > R > -1$, which is consistent with Brander and Spencer (1984, p. 231.)

The optimal output levels of both firms can be achieved by decentralized measures. First, consider the case where both the domestic firm and the foreign firm maximize their profits and assume the optimal domestic output y^* is strictly positive. Instead of dictating y^* to the domestic firm, the home country can give the domestic firm a subsidy s^* while maintaining the same tariff t^* as in (6):

Lemma 4 *Assume the domestic firm's objective is profit-maximization. The optimal subsidy is*

$$s^* = -y^*p' - \frac{\Delta}{2} + t^* = -y^*p' + \frac{\Delta}{2} \left[\frac{x^*}{Q^*} R \right]. \quad (7)$$

Proof: see Appendix A.2.2.

The optimal subsidy is positive if R is positive. In any case

$$s^* > (-y^*p') - \Delta, \quad (8)$$

where the inequality follows from the second order condition (see (A.4) in Appedix A.2.1). Thus, under the assumption that optimal domestic production is positive, the optimal production subsidy is s^* is positive provided the demand curve is not too convex.

We now turn to the case where the domestic firm 1 is semi-public. Its objective is to maximize a weighted average of its profit and social welfare. Firm 1's profit function, exclusive of the subsidies, is

$$\pi^{(1)} = p(y+x)y - c_1y,$$

and its profit inclusive of the subsidies is

$$\tilde{\pi}^{(1)} = \pi^{(1)} + sy.$$

Unlike the “benchmark case” of direct control of domestic output, where the foreign firm reacts to the precommitted y , in the present scenario, the two firms choose their outputs simultaneously. The welfare function of country 1

is the sum of the domestic firm's profit, the domestic consumer surplus, and the government revenue:

$$W = \tilde{\pi}^{(1)} + \left[\int_0^{x+y} p(Q)dQ - p(x+y)(x+y) \right] + [tx - sy]. \quad (9)$$

Upon simplification,

$$W = p(y+x)y - c_1y + \left[\int_0^{x+y} p(Q)dQ - p(x+y)(x+y) \right] + tx. \quad (10)$$

As firm 1 is partly state-owned, its objective function is

$$V^{(1)} = (1 - \theta)\tilde{\pi}^{(1)} + \theta W, \quad (11)$$

and we arrive at

Proposition 3 *Assume the domestic firm is semi-public, and θ is the weight it places on social welfare.*

1. *The optimal tariff is independent of the degree of public ownership and equal to*

$$\hat{t} = t^* = \frac{\Delta}{2} \left[1 - \frac{\Delta p''}{2(p')^2} \right].$$

This is positive if and only if $\infty < R < -1$.

2. *The optimal domestic production subsidy is equal to*

$$\hat{s} = -y^*p' + \frac{\Delta}{2(1-\theta)} \left[\frac{x^*}{Q^*} R \right] + \frac{\theta x^* p'(q^*)}{1-\theta}.$$

3. *The optimal subsidy is negatively related to the degree of public ownership if and only if the optimal tariff is positive.*

Proof: see Appendix A.2.3.

Proposition 3 shows that the tariff stays constant whereas the subsidy goes down with public ownership if the tariff is positive. The reason is that the government can commit to a tariff and a subsidy whereas a partially state-owned firm cannot precommit to domestic production. Hence, only the

government is able to fully extract rents from the foreign firm. A partially state-owned firm, however, competes against a foreign rival on a level playing field and will only take into account the effects of its output on welfare for the fixed tariff rate and the equilibrium import level. Hence, the change in welfare as perceived by the semi-public firm will only include the change in consumer surplus and the change in the foreign rent via price effects as described by Lemma 2. It will ignore the decline in tariff revenues due to a decline in imports because imports and domestic production are determined simultaneously. Consequently, the marginal change in welfare due to increasing domestic production will be larger (and positive) as perceived by the semi-public firm compared to the government, leading to output expansion beyond y^* if the subsidy is not adjusted. Therefore, the government has to reduce the subsidy with public ownership as it correctly anticipates that a semi-public firm would produce too much under a subsidy designed for a private firm.

4 Rivalry between a domestic and a foreign firm in a third market

In this section, we re-examine the third market model under the complication that the home firm is partly state-owned. There are two firms, denoted by 1 and 2, located respectively in the home country and the foreign country. They export all their outputs to a third market. We start with the case in which firms are Cournot rivals. For simplicity, assume the goods are perfect substitutes. The inverse demand function is $P = P(Q)$, where $Q = q_1 + q_2$. The subsidy-inclusive profit of firm i is denoted by $\tilde{\pi}^{(i)}$:

$$\tilde{\pi}^{(i)} = P(q_1 + q_2)q_i - C^{(i)}(q_i) + s_i q_i \equiv \pi^{(i)}(q_i, q_j) + s_i q_i,$$

where s_i is the export subsidy rate set by country i . By assumption, the goods are not consumed in the home country, and therefore there is no domestic consumer surplus. The social welfare of country 1 is taken to be equal to the subsidy-inclusive profit of firm 1 minus the cost of the subsidy:

$$W^{(1)} = \tilde{\pi}^{(1)} - s_1 q_1 = \pi^{(1)}(q_1, q_2).$$

As before, we assume that firm 1's objective is to maximize a weighted average of $\tilde{\pi}^{(1)}$ and $W^{(1)}$, *i.e.*,

$$V^{(1)} \equiv (1 - \theta)\tilde{\pi}^{(1)} + \theta W^{(1)} = \pi^{(1)}(q_1, q_2) + (1 - \theta)s_1 q_1.$$

Firm 2 (the foreign firm) seeks to maximize $\tilde{\pi}^{(2)}$.

We consider a two-stage game. In the first stage, the government of country 1 sets the export subsidy rate s_1 . (We assume s_2 is exogenous, which we set at zero for simplicity.) In stage 2, the firms simultaneously choose their output levels to maximize their objective function. Furthermore, we assume that the outputs are strategic substitutes, in the sense that an increase in the output of one firm will reduce the marginal profit of the other firm:¹²

Assumption 1 *Outputs are strategic substitutes:*

$$\tilde{\pi}_{ij}^{(i)} \equiv \frac{\partial^2 \tilde{\pi}^{(i)}}{\partial q_i \partial q_j} = q_i P''(Q) + P'(Q) < 0$$

for all $q_i \in [0, Q]$ and for all $Q > 0$.

Given Assumption 1, we can now derive

Proposition 4 *In the case of international Cournot rivalry in a third market, the optimal rate of export subsidy is an increasing function of the degree of state ownership θ . The outcomes (in terms of export quantity and social welfare level) are however independent of the degree of state ownership.*

Proof: see Appendix A.3.1.

We can demonstrate Proposition 4 by Figure 1. From the first-order condition, we can derive that $dq_1/d\theta < 0$ holds for firm 1's reaction curve (see (A.30) in Appendix A.3.1), *i.e.*, an increase in public ownership shifts firm 1's

¹²For the notion of strategic substitutes and complements, see Bulow, Geanakoplos and Klemperer (1985).

reaction curve to the left. In Figure 1, RR denotes the reaction curve without public ownership and VV denotes the reaction curve with semi-public ownership.¹³ The optimal policy makes the domestic firm behave as if it were a Stackelberg leader. SS denotes the optimal after-subsidy reaction curve which is determined such that the output in equilibrium maximizes domestic profits and hence the domestic iso-profit curve is tangential to the foreign firm's reaction curve R^*R^* . Since public ownership shifts the reaction curve to the left, a higher subsidy is needed to put the semi-public firm in a position such that it behaves in the Nash equilibrium as if it were a Stackelberg leader.

This result seems to be surprising. Why does VV lie to the left of RR ? The semi-public firm acknowledges at least partially that the subsidy is not part of welfare. At the same time, the semi-public firm competes against its rival on a level playing field and cannot commit to a high output level. Hence, public ownership weakens the influence of the subsidy as it cancels out for the welfare component of firm 1's objective function. Consequently, the subsidy must be larger in order to achieve the same output levels as with two private firms.

A similar analysis applies to the case of Bertrand rivalry (in the third market) between a semi-public home firm and a profit-maximizing foreign firm. As firms compete by prices, we now assume that the two goods are imperfect substitutes. Let $x^{(1)}$ and $x^{(2)}$ denote the demands for outputs of firm 1 and firm 2 respectively. The demand functions are assumed to be

$$x^{(1)} = x^{(1)}(p_1, p_2) \text{ and } x^{(2)} = x^{(2)}(p_1, p_2)$$

where p_i is the price the consumers in the third market have to pay for one unit of good i . We use the following notation:

$$x_i^{(i)} = \frac{\partial x^{(i)}}{\partial p_i}, x_j^{(i)} = \frac{\partial x^{(i)}}{\partial p_j}, x_{ij}^{(i)} = \frac{\partial^2 x^{(i)}}{\partial p_i \partial p_j}, x_{ii}^{(i)} = \frac{\partial^2 x^{(i)}}{\partial p_i^2} \text{ etc.}$$

and make

¹³Reactions curves do not have to be linear or parallel to each other.

Assumption 2 Demand for good i decreases with p_i . The goods are substitutes: an increase in p_j will increase the demand for good i , where $i \neq j$:

$$x_i^{(i)} < 0 \text{ and } x_j^{(i)} > 0.$$

The cost functions are denoted by $C^{(i)}(x^{(i)})$, and the government of country 1 levies an export tax t on $x^{(1)}$. (As before, the foreign tax is exogenous and set to zero for simplicity.) We denote by $\Pi^{(i)}$ firm i 's gross revenue minus production cost:

$$\Pi^{(i)}(p_1, p_2) = p_i x^{(i)}(p_1, p_2) - C^{(i)}[x^{(i)}(p_1, p_2)].$$

We use the upper-case letter $\Pi^{(i)}$ to distinguish this function from the function $\pi^{(i)}(q_1, q_2)$ introduced in the Cournot rivalry case. Let $\tilde{\Pi}^{(1)}$ denote the after-tax profit function of firm 1:

$$\tilde{\Pi}^{(1)}(p_1, p_2) = (p_1 - t)x^{(1)}(p_1, p_2) - C^{(1)}[x^{(1)}(p_1, p_2)].$$

Again, since there is no domestic consumer surplus, the social welfare of country 1 is the sum of $\tilde{\Pi}^{(1)}$ and the government's tax revenue:

$$W^{(1)}(p_1, p_2) = \tilde{\Pi}^{(1)}(p_1, p_2) + tx^{(1)}(p_1, p_2) = \Pi^{(1)}(p_1, p_2).$$

We assume that firm 1 is partly state-owned, and its objective is to maximize $V^{(1)}$, which is a weighted average of $\tilde{\Pi}^{(1)}$ and $W^{(1)}$:

$$V^{(1)} = (1 - \theta)\tilde{\Pi}^{(1)}(p_1, p_2) + \theta W^{(1)}(p_1, p_2) = \Pi^{(1)}(p_1, p_2) - (1 - \theta)tx^{(1)}(p_1, p_2)$$

Firm 2, on the other hand, is privately owned, and seeks to maximize its profit.

We consider a two-stage game. In stage 1, the government of country 1 sets t . In stage 2, given t , the firms simultaneously choose the consumer prices p_1 and p_2 . Furthermore, we assume that prices are strategic complements in the sense that an increase in the price of good j will raise the marginal contribution of p_i to the profit of firm i :

Assumption 3 *Prices are strategic complements:*

$$\Pi_{ij}^{(i)}(p_1, p_2) > 0 \text{ for } j \neq i.$$

Given Assumptions 2 and 3, we can now derive

Proposition 5 *In the case of international Bertrand rivalry in the third market, the optimal rate of export tax is an increasing function of the degree of state ownership θ . The outcomes (in terms of export quantity and social welfare level) are however independent of the degree of state ownership.*

Proof: see Appendix A.3.2.

We can also demonstrate Proposition 5 by Figure 2. From the first-order condition, we can derive that $dp_1/d\theta > 0$ holds for firm 1's reaction curve (see (A.34) in Appendix A.3.2), *i.e.*, an increase in public ownership shifts firm 1's reaction curve downwards. In Figure 2, RR denotes the reaction curve without public ownership and VV denotes the reaction curve with semi-public ownership. The optimal policy makes the domestic firm behave as if it were a Stackelberg leader. SS denotes the optimal after-tax reaction curve which is determined such that the prices in equilibrium maximizes domestic profits and hence the domestic iso-profit curve is tangential to the foreign firm's reaction curve. Since public ownership shifts the reaction curve downwards, a higher tax is needed to put the semi-public firm in a position such that it behaves in the Nash equilibrium as if it were a Stackelberg leader.

The intuition for the necessity of a higher tax is similar to the case of strategic substitutes. The semi-public firm cannot commit to a higher price but will acknowledge that taxes do not count for welfare. Hence, the influence of the tax is weakened, and a larger tax is needed in order to achieve the optimal price levels.

5 Concluding remarks

This paper has demonstrated the role of public ownership for trade policies. We have shown that public ownership does neither change the level of socially

optimal activities nor the welfare level itself. It may, however, change the level of optimal trade taxes. If it does, we have shown that a semi-public firm cannot achieve the same results as the government on its own because it competes with rivals on a level playing field. This lack of commitment disqualifies public ownership as a tool for extracting rents from foreign firms.

Given our strong invariance results, the reasons why public ownership is so prevalent seem to be obscure or at least beyond our model setup. The assumption that the use of trade policy instruments is restricted does not help in general. In particular, if subsidies for supporting your national champion in a third market were restricted, public ownership would be harmful because it would make the domestic firm less aggressive. It seems that trade policy cannot explain why firms are partially state-owned.

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Appendix

A.1 Rivalry between domestic exporting firms

A.1.1 The benchmark case: profit-maximizing duopolists

The first-order condition for firm i is

$$P'(q_i + q_{-i})q_i + P(q_i + q_{-i}) = c + t \text{ for } i = 1, 2. \quad (\text{A.1})$$

Assume that the second-order condition is satisfied, *i.e.*, for all $q_i \in [0, Q]$ and for all $Q > 0$, $P''(Q)q_i + 2P'(Q) < 0$. The Cournot equilibrium outputs are denoted by $q_i^C(t)$. Let

$$Q^C(t) = \sum_{i=1}^2 q_i^C(t).$$

Then, adding the two equations (A.1) for $i = 1, 2$, we get

$$P'(Q^C(t))Q^C(t) + 2P(Q^C(t)) = 2(c + t)$$

To ensure that $Q^C(t)$ coincides with the socially optimal output Q^* , the government must set the export tax rate at t^* according to (4). We now verify that this tax rate makes each firm i produce the quantity $q_i = Q^*/2$. Firm i takes as given the tax rate t^* and the output of the other firm, which is $q_{-i} = Q^*/2$ (this turns out to be true in equilibrium). So its first-order condition is

$$P'(q_i + \frac{Q^*}{2})q_i + P(q_i + \frac{Q^*}{2}) = c + t^* \equiv c - (Q^*/2)P'(Q^*).$$

Clearly, by choosing $q_i = Q^*/2$, the firm satisfies this condition. This argument also applies to the other firm. It follows that, in a Cournot equilibrium with the export tax rate t^* , the equilibrium industry output is identical to the socially optimal output. \square

A.1.2 The mixed-duopoly case

Suppose that the government sets the same t^* as in the standard duopoly case (see eq. (4)), and suppose that firm 2 chooses $q_2 = Q^*/2$ as before (we will verify that this is in fact the optimal choice for firm 2). Then the first-order condition for firm 1 is

$$(1 - \theta) \left[P'(q_1 + \frac{Q^*}{2})q_1 + P(q_1 + \frac{Q^*}{2}) - c - t^* \right] + \theta \left\{ P'(q_1 + \frac{Q^*}{2})(q_1 + \frac{Q^*}{2}) + P(q_1 + \frac{Q^*}{2}) - c \right\} = 0.$$

Now, clearly, if firm 1 chooses $q_1 = Q^*/2$, then the expression inside the squared brackets is zero, because t^* satisfies (4), and the expression inside the curly brackets $\{\dots\}$ is also zero, because Q^* satisfies (2). Therefore the first-order condition for firm 1 is satisfied at $q_1 = Q^*/2$. The second-order condition is

$$(1 - \theta) [q_1 P'' + 2P'] + \theta \{QP'' + 2P'\} < 0$$

which is also satisfied. It remains to check that firm 2, by choosing $q_2 = Q^*/2$, also satisfies its own first- and second-order condition. This is easily verified. \square

A.2 Rivalry in the home market

A.2.1 The benchmark case: direct control of domestic output

The foreign firm takes y and t as given, and chooses the export quantity $x \geq 0$ to maximize its profit:

$$\max_{x \geq 0} \pi_2 = p(x+y)x - (c_2 + t)x.$$

Assume an interior maximum for this problem. Then the first-order condition is

$$\frac{d\pi_2}{dx} = xp'(x+y) + p(x+y) - c_2 - t = 0. \quad (\text{A.2})$$

The second-order condition is

$$\frac{d^2\pi_2}{dx^2} = 2p' + xp'' < 0 \quad (\text{A.3})$$

which is satisfied because of (5). The second-order condition can be expressed as

$$\frac{x}{Q} \left[\frac{Qp''}{p'} \right] > -2. \quad (\text{A.4})$$

From the first-order condition we obtain the foreign firm's reaction function: x is expressed as a function of y and t . The function $x = x(y, t)$ is implicitly defined by

$$xp'(x+y) + p(x+y) - c_2 - t = 0.$$

The partial derivatives are

$$\begin{aligned} \frac{\partial x}{\partial t} &= \frac{1}{2p' + xp''} < 0, \\ \frac{\partial x}{\partial y} &= -\frac{(p' + xp'')}{2p' + xp''} = [-(p' + xp'')] \frac{\partial x}{\partial t} < 0. \end{aligned}$$

We can invert the function $x = x(y, t)$ to get the function $t = t(x, y)$. This function tells us that, given y , if the home country wants the foreign firm to supply x , the required tariff rate is

$$t(x, y) = xp'(x + y) + p(x + y) - c_2. \quad (\text{A.5})$$

Social welfare of the home country is the utility of consuming $x + y$ minus the cost of obtaining $x + y$, *i.e.*,

$$W = \int_0^{x+y} p(Q)dQ - c_1y - [p(x + y) - t]x. \quad (\text{A.6})$$

Substitute for t , using (A.5), we get

$$W = \int_0^{x+y} p(Q)dQ - c_1y - p(x + y)x + [xp'(x + y) + p(x + y) - c_2]x.$$

which upon simplification leads to Lemma 2. \square

The home country chooses x and y to maximize W . This yields two first order conditions:

$$W_y \equiv \frac{\partial W}{\partial y} = p(x + y) - c_1 + x^2p''(x + y) \leq 0 \quad (= 0 \text{ if } y > 0), \quad (\text{A.7})$$

$$W_x \equiv \frac{\partial W}{\partial x} = p(x + y) - c_2 + x^2p''(x + y) + 2xp' \leq 0 \quad (= 0 \text{ if } x > 0). \quad (\text{A.8})$$

Note that sufficiency is ensured if (i) $x^2p''(x + y)$ is concave in (x, y) , *e.g.* if $p' = -b < 0$, and (ii) the integral is concave in (x, y) , *e.g.* $p(q)$ is linear. Suppose we have an interior maximum ($x^* > 0$ and $y^* > 0$). Then the first-order conditions become

$$W_y \equiv p(x + y) - c_1 + x^2p''(x + y) = 0, \quad (\text{A.9})$$

$$W_x \equiv p(x + y) - c_2 + x^2p''(x + y) + 2xp' = 0. \quad (\text{A.10})$$

The second-order conditions are

$$W_{yy} = p' + x^2p''' < 0, \quad (\text{A.11})$$

$$W_{xx} = 2(p' + 2xp'') + (p' + x^2p''') < 0 \quad (\text{A.12})$$

and

$$J \equiv W_{xx}W_{yy} - (W_{xy})^2 = 2(p')^2 + 2x^2 [(p')(p''') - 2(p'')^2] > 0. \quad (\text{A.13})$$

Condition (A.13) can be expressed as

$$Z \equiv p' + x^2 p''' - \frac{2x^2 (p'')^2}{p'} < 0. \quad (\text{A.14})$$

Example 1: Assume

$$p(Q) = e - e^Q \text{ for } Q \in [0, 1]$$

Then $p' = p'' = p''' = -e^Q$, and $J = 2e^{2Q}(1 - x^2) > 0$ for $x < Q \leq 1$.

We now prove part 1 of Proposition 2. First, we establish the why it is optimal to import *only if* the domestic firm has a cost disadvantage. Subtracting equation (A.9) from (A.10), we get

$$x^* = \frac{c_1 - c_2}{[-2p'(x^* + y^*)]} \equiv \frac{\Delta}{[-2p'(x^* + y^*)]}. \quad (\text{A.15})$$

It follows that $x^* > 0$ only if $\Delta > 0 \Leftrightarrow c_1 > c_2$.

Next, we prove why it is optimal to import *if* the domestic firm has a cost disadvantage. (If $\Delta > 0$, then it is optimal to import a positive amount from the foreign firm.) Suppose the contrary, *i.e.* $\Delta > 0$ and yet $x^* = 0$ and $y^* > 0$. Then equations (A.7) and (A.8) become, respectively,

$$p(y^*) - c_1 = 0$$

$$p(y^*) - c_2 \leq 0$$

which implies $c_2 \geq c_1$, a contradiction. As for part 2, substitute (A.15) into (A.9):

$$p(x^* + y^*) - c_1 + \frac{(c_1 - c_2)^2 p''(x^* + y^*)}{4 [p'(x^* + y^*)]^2} = 0. \quad (\text{A.16})$$

This equation determines the optimal total consumption, $Q^* \equiv x^* + y^*$. We assume that the equation

$$p(Q^*) - c_1 + \frac{(c_1 - c_2)^2 p''(Q^*)}{4 [p'(Q^*)]^2} = 0 \quad (\text{A.17})$$

has a unique solution $Q^* > 0$ (*e.g.* the linear demand case, or see example 1 below). Eq. (A.17) gives Q^* as a function of c_1 and Δ , and it clearly shows that $p(Q^*) > (<)c_1$ if $p''(Q^*) < (>)0$. \square

Example 1 (continued): Assume $c_1 < e$. Then equation (A.17) yields $Q^* = \ln [4(e - c_1)/(4 + \Delta^2)] < 1$. Using this, we obtain $x^* \in (0, Q^*)$ and $y^* \in (0, Q^*)$ provided that c_1 is sufficiently small relative to Δ . In this case, $p(Q^*) > c_1$. See the discussion of Lemma 2 above for an explanation.

Given the optimal consumption Q^* as determined by (A.17), we can compute the derivative of Q^* with respect to c_1 and Δ . Re-write (A.17) as

$$[p'(Q^*)]^2 [p(Q^*) - c_1] + \frac{\Delta^2}{4} p''(Q^*) = 0.$$

Differentiate totally to get

$$\left\{ [2(p - c_1)p'p''] + [p']^3 + \frac{\Delta^2}{4} p''' \right\} dQ^* - [p']^2 dc_1 + \frac{\Delta}{2} p'' d\Delta = 0.$$

Using (A.16), (A.15) and (A.14), this equation becomes

$$(p')^2 Z dQ^* - (p')^2 dc_1 + \frac{\Delta}{2} p'' d\Delta = 0,$$

for which we get

$$\frac{\partial Q^*}{\partial \Delta} = -\frac{(\Delta/2)p''}{(p')^2 Z} < 0 \text{ iff } p'' < 0, \quad (\text{A.18})$$

i.e., given c_1 , a larger Δ will lead to greater consumption iff the inverse demand curve is concave; and

$$\frac{\partial Q^*}{\partial c_1} = \frac{1}{Z} < 0. \quad (\text{A.19})$$

Having computed $Q^* = Q^*(c_1, \Delta)$, we can express optimal import as

$$x^* = \frac{\Delta}{-2p'(Q^*(c_1, \Delta))} \quad (\text{A.20})$$

and optimal domestic output as

$$y^* = Q^* - x^* = Q^*(c_1, \Delta) + \frac{\Delta}{2p'(Q^*(c_1, \Delta))}, \quad (\text{A.21})$$

respectively. We must verify that $y^* > 0$. This is satisfied iff

$$2Q^*(c_1, \Delta)p'(Q^*(c_1, \Delta)) + \Delta < 0,$$

i.e., if the domestic cost disadvantage is not too large, *i.e.*,

$$\Delta < -2Q^*p'(Q^*).$$

The optimal tariff is

$$t^* = t(x^*, y^*) = x^*p'(Q^*) + p(Q^*) - c_2. \quad (\text{A.22})$$

From (A.22), we find that

$$t^* = (p - c_2) - \frac{\Delta}{2} = (p - c_1) + \frac{\Delta}{2}. \quad (\text{A.23})$$

Substituting (A.17) into (A.23), using (A.20), we obtain

$$\begin{aligned} t^* &= \frac{\Delta}{2} \left[1 + \frac{\Delta}{2(-p')} \frac{p''}{(-p')} \right] = \frac{\Delta}{2} \left[1 + \frac{x^*Q}{Q} \frac{p''}{(p')} \right] \\ &= \frac{\Delta}{2} \left[1 + \frac{x^*R}{Q} \right] \end{aligned}$$

which is Lemma 3. \square

A.2.2 The optimal subsidy

The optimal production subsidy must satisfy

$$p(Q^*) + y^*p'(Q^*) - c_1 + s^* = 0,$$

i.e.,

$$s^* = -y^*p' - (p - c_1). \quad (\text{A.24})$$

Using (A.23), we can express equation (A.24) as (7). \square

A.2.3 Optimal subsidy and optimal tariff in a mixed duopoly

Using (9) and (11), we get

$$V^{(1)} = \tilde{\pi}^{(1)} + \theta \left[\int_0^{x+y} p(Q) dQ - p(x+y)(x+y) \right] + \theta [tx - sy]$$

i.e.,

$$V^{(1)} = \pi^{(1)} + \theta \left[\int_0^{x+y} p(Q) dQ - p(x+y)(x+y) \right] + \theta tx + (1 - \theta) sy$$

The first-order condition of firm 1 is

$$\frac{\partial V^{(1)}}{\partial y} = p + yp' - c_1 + (1 - \theta)s - \theta(y + x)p' = 0 \quad (\text{A.25})$$

The first-order condition of firm 2 is

$$p + xp' - c_2 - t = 0. \quad (\text{A.26})$$

Clearly, the home country can achieve the same output pair (x^*, y^*) as in the benchmark case, by setting the following tariff and subsidy vector (\hat{t}, \hat{s}) , where \hat{t} is identical to t^* and \hat{s} is a modification of s^* (where s^* is given by equation (8)):

$$\hat{t} = t^* = \frac{\Delta}{2} \left[1 - \frac{\Delta p''}{2(p')^2} \right], \quad (\text{A.27})$$

$$(1 - \theta)\hat{s} - \theta Q^* p'(Q^*) = s^* \equiv -y^* p' + \frac{\Delta}{2} \left[\frac{x^*}{Q^*} R \right],$$

That is,

$$\hat{s} = \frac{s^* + \theta Q^* p'}{1 - \theta} = \frac{-y^* p'}{1 - \theta} + \frac{\Delta}{2(1 - \theta)} \frac{x^*}{Q^*} R + \frac{\theta Q^* p'}{1 - \theta},$$

or

$$\hat{s} = -y^* p' + \frac{\Delta}{2(1 - \theta)} \left[\frac{x^*}{Q^*} R \right] + \frac{\theta x^* p'(q^*)}{1 - \theta}. \quad (\text{A.28})$$

It is easy to verify that given \hat{s} , firm 1 will choose y^* , if it expects firm 2 to choose x^* . Similarly, given \hat{t} , firm 2 will choose x^* , if it expects firm 1 to choose y^* . It follows that the pair (x^*, y^*) is achievable as a Nash equilibrium, by setting (\hat{t}, \hat{s}) as in (A.27) and (A.28).

The effect of an increase in θ on \widehat{s} can easily be computed. From equation (A.28), we derive

$$\begin{aligned}
\frac{d\widehat{s}}{d\theta} &= -\frac{\Delta}{2(1-\theta)^2} \left[\frac{x^*}{Q^*} R \right] + \frac{x^* p'}{(1-\theta)^2} \\
&= \left[\frac{1}{(1-\theta)^2} \right] \left\{ x^* p' - \frac{\Delta}{2} \left(\frac{x^* R}{Q^*} \right) \right\} \\
&= \left[\frac{1}{(1-\theta)^2} \right] x^* p' \left\{ 1 - \frac{\Delta}{2} \left(\frac{R}{Q^* p'} \right) \right\} \\
&= \left[\frac{1}{(1-\theta)^2} \right] x^* p' \left\{ 1 - \frac{\Delta}{2} \left(\frac{p''}{(p')^2} \right) \right\} = t^* \left[\frac{1}{(1-\theta)^2} \right] x^* p'
\end{aligned}$$

and it follows that an increase in θ will decrease \widehat{s} if and only if $t^* > 0$. \square

A.3 Rivalry in the third market

A.3.1 Cournot rivalry in the third market

In what follows, we use the following notations:

$$\begin{aligned}
\pi_i^{(i)}(q_i, q_j) &\equiv \frac{\partial \pi^{(i)}(q_i, q_j)}{\partial q_i}, \\
\pi_{ij}^{(i)}(q_i, q_j) &\equiv \frac{\partial^2 \pi^{(i)}(q_i, q_j)}{\partial q_i \partial q_j}.
\end{aligned}$$

Solving for the output game in the second stage, we find that, given s_1 , the first-order condition of firm 1 is

$$\frac{\partial V^{(1)}}{\partial q_1} \equiv V_1^{(1)} \equiv \pi_1^{(1)}(q_1, q_2) + (1-\theta)s_1 = 0,$$

i.e.,

$$q_1 P'(Q) + P(Q) - C_q^{(1)} = -(1-\theta)s_1. \tag{A.29}$$

The second-order condition is $\pi_{11}^{(1)} < 0$. Clearly, for any given q_2 , the greater is θ , the smaller is the response of firm 1's output to an increase in the subsidy rate s_1 because

$$\left. \frac{dq_1}{d\theta} \right|_{dq_2=0} = \frac{s_1}{\pi_{11}^{(1)}} < 0. \tag{A.30}$$

For firm 2, the first-order condition is

$$\pi_2^{(2)}(q_1, q_2) = 0$$

and the second-order condition $\pi_{22}^{(2)} < 0$. The two first-order conditions yield the Cournot equilibrium outputs q_1 and q_2 as functions of s_1 . We now determine the sign of dq_1/ds_1 and dq_2/ds_1 . This is done by differentiating the system of two first-order conditions totally:

$$\begin{bmatrix} \pi_{11}^{(1)} & \pi_{12}^{(1)} \\ \pi_{12}^{(2)} & \pi_{22}^{(2)} \end{bmatrix} \begin{bmatrix} \frac{dq_1}{ds_1} \\ \frac{dq_2}{ds_1} \end{bmatrix} = \begin{bmatrix} -(1-\theta) \\ 0 \end{bmatrix}.$$

The determinant of the matrix is

$$J = \pi_{11}^{(1)}\pi_{22}^{(2)} - \pi_{12}^{(2)}\pi_{12}^{(1)},$$

for which we assume $J > 0$, using the usual stability argument. Then the effects of an increase in s_1 on the equilibrium outputs of the mixed oligopoly are as follows. The semi-public firm 1 will increase its output if $\theta < 1$:

$$\frac{dq_1}{ds_1} = -\frac{1}{J} \left(\pi_{22}^{(2)} \right) (1-\theta) > 0 \text{ if } \theta < 1.$$

Using Assumption 1 above, we can see that an increase in s_1 will reduce foreign firm's equilibrium output if $\theta < 1$:

$$\frac{dq_2}{ds_1} = \frac{1}{J} \left(\pi_{12}^{(2)} \right) (1-\theta) < 0 \text{ if } \theta < 1$$

Thus the ratio of these two equilibrium responses is equal to the slope of firm 2's reaction function:

$$\frac{\frac{dq_2}{ds_1}}{\frac{dq_1}{ds_1}} = -\frac{\pi_{12}^{(2)}}{\pi_{22}^{(2)}} < 0 \tag{A.31}$$

In Stage 1, the government of country 1 chooses the export subsidy rate s_1 to maximize its welfare

$$W^{(1)} = P[q_1(s_1), q_2(s_1)] q_1(s_1) - C^{(1)}[q_1(s_1)],$$

which yields the first-order condition:

$$\frac{dW^{(1)}}{ds_1} = q_1 P'(Q) \left[\frac{dq_1}{ds_1} + \frac{dq_2}{ds_1} \right] + P(Q) \frac{dq_1}{ds_1} - C_q^{(1)} \frac{dq_1}{ds_1} = 0.$$

Re-arrange terms to get

$$[q_1 P'(Q) + P(Q) - C_q^{(1)}] \frac{dq_1}{ds_1} = -q_1 P'(Q) \frac{dq_2}{ds_1}. \quad (\text{A.32})$$

Using firm 1's first-order condition, eq. (A.29), we can re-write the social optimal condition (A.32) as

$$-(1 - \theta) s_1 \frac{dq_1}{ds_1} = -q_1 P'(Q) \frac{dq_2}{ds_1}.$$

This equation gives the optimal rate of subsidy s_1 to the semi-public firm:

$$s_1^* = \frac{1}{(1 - \theta)} q_1 P'(Q) \left[\frac{\frac{dq_2}{ds_1}}{\frac{dq_1}{ds_1}} \right] = \frac{1}{(1 - \theta)} \left\{ [-P'(Q) q_1] \left[\frac{\pi_{12}^{(2)}}{\pi_{22}^{(2)}} \right] \right\} > 0, \quad (\text{A.33})$$

where the second equality follows from (A.31). It follows that an increase in θ will increase s_1^* *provided the value of the expression inside the curly brackets remains constant* when θ changes. We now show that it in fact does not change, if the optimal subsidy is imposed. To do this, it suffices to show that the optimal subsidy always ensures that the stage-two equilibrium output pair (q_1, q_2) is identical to the pair (q_1^L, q_2^F) which would be obtained if firm 1 were the quantity-setting leader, and firm 2 were the quantity-setting follower.¹⁴ Re-write the optimal subsidy condition above as

$$(1 - \theta) s_1^* = -q_1 P'(Q) \left(\frac{\pi_{12}^{(2)}}{\pi_{22}^{(2)}} \right) \equiv -\pi_2^{(1)} \left(\frac{\pi_{12}^{(2)}}{\pi_{22}^{(2)}} \right) > 0$$

Using firm 1's first-order condition, this gives

$$\pi_1^{(1)} = \pi_2^{(1)} \left(\frac{\pi_{12}^{(2)}}{\pi_{22}^{(2)}} \right).$$

But this is precisely the condition that determines the pair (q_1^L, q_2^F) , which would be obtained if firm 1 were the quantity-setting leader, and firm 2 is the quantity-setting follower. \square

¹⁴It is well known that "the optimal export subsidy would move the industry equilibrium to what would be the Stackelberg leader-follower position with the domestic firm as the leader." (Brander and Spencer, 1985, Proposition 3).

A.3.2 Bertrand rivalry in the third market

The cost functions' derivatives are denoted by

$$C_x^{(i)} \equiv \frac{dC^{(i)}}{dx^{(i)}}, \quad C_{xx}^{(i)} \equiv \frac{d^2C^{(i)}}{(dx^{(i)})^2}$$

We assume $C_x^{(i)} > 0$. Given t , the first-order condition of firm 1 is

$$V_1^{(1)} = \Pi_1^{(1)} - (1 - \theta)tx_1^{(1)} = \left[x_1^{(1)} + (p_1 - C_x^{(1)})x_1^{(1)} \right] - (1 - \theta)tx_1^{(1)} = 0$$

and the second-order condition is

$$\Pi_{11}^{(1)} - (1 - \theta)tx_{11}^{(1)} < 0.$$

From the first-order condition, we may derive that

$$\left. \frac{dp_1}{d\theta} \right|_{dp_2=0} = -\frac{tx_1^{(1)}}{\Pi_{11}^{(1)} - (1 - \theta)tx_{11}^{(1)}} < 0 \quad (\text{A.34})$$

For firm 2, the first-order conditions is

$$\Pi_2^{(2)} = 0$$

and the second-order condition is

$$\Pi_{22}^{(2)} < 0.$$

The two first-order conditions yield the Bertrand equilibrium prices p_1 and p_2 as functions of t . We now determine the sign of dp_1/dt and dp_2/dt . This is done by differentiating the system of two first-order conditions totally:

$$\begin{bmatrix} \Pi_{11}^{(1)} - (1 - \theta)tx_{11}^{(1)} & \Pi_{12}^{(1)} - (1 - \theta)tx_{12}^{(1)} \\ \Pi_{12}^{(2)} & \Pi_{22}^{(2)} \end{bmatrix} \begin{bmatrix} \frac{dp_1}{dt} \\ \frac{dp_2}{dt} \end{bmatrix} = \begin{bmatrix} (1 - \theta)x_1^{(1)} \\ 0 \end{bmatrix}.$$

The determinant of the matrix is

$$J = \Pi_{11}^{(1)}\Pi_{22}^{(2)} - \Pi_{12}^{(2)}\Pi_{12}^{(1)} - \Pi_{22}^{(2)}(1 - \theta)tx_{11}^{(1)} + \Pi_{12}^{(2)}(1 - \theta)tx_{12}^{(1)},$$

for which we assume $J > 0$, using the usual stability argument. Then the responses of the Bertrand equilibrium prices to the tax rate t are

$$\frac{dp_1}{dt} = \frac{1}{J}\Pi_{22}^{(2)}(1 - \theta)x_1^{(1)} > 0 \text{ if } \theta < 1,$$

$$\frac{dp_2}{dt} = -\frac{1}{J}\Pi_{12}^{(2)}(1-\theta)x_1^{(1)} > 0 \text{ if } \theta < 1.$$

The ratio of these two responses is equal to the slope of firm 2's reaction function in the space (p_1, p_2) :

$$\frac{\frac{dp_2}{dt}}{\frac{dp_1}{dt}} = -\frac{\Pi_{12}^{(2)}}{\Pi_{22}^{(2)}} > 0$$

In stage 1, the government of country 1 chooses t so as to maximize its welfare

$$W^{(1)}(t) = p_1(t)x^{(1)}(p_1(t), p_2(t)) - C^{(1)}[x(p_1(t), p_2(t))].$$

The first-order condition is

$$\frac{dW^{(1)}}{dt} = x^{(1)}\frac{dp_1}{dt} + (p_1(t) - C_x^{(1)}) \left[x_1^{(1)}\frac{dp_1}{dt} + x_2^{(1)}\frac{dp_2}{dt} \right] = 0.$$

Re-arranging terms yields

$$\left[x^{(1)} + (p_1 - C_x^{(1)}) x_1^{(1)} \right] \frac{dp_1}{dt} = (p_1 - C_x^{(1)}) x_2^{(1)} \frac{dp_2}{dt},$$

and hence

$$(1-\theta)t x_1^{(1)} = -\frac{(p_1 - C_x^{(1)}) x_2^{(1)} \frac{dp_2}{dt}}{\frac{dp_1}{dt}}$$

and

$$t^* = \frac{1}{(1-\theta)} \left\{ \left(\frac{\Pi_{12}^{(2)}}{\Pi_{22}^{(2)}} \right) \frac{(p_1 - C_x^{(1)}) x_2^{(1)}}{x_1^{(1)}} \right\} > 0$$

hold. It follows that an increase in θ will increase t^* *provided the term inside the curly brackets does not change* when θ changes. We now show that this term in fact does not change, if the optimal tax is imposed. To do this, it suffices to show that the optimal tax always ensures that the stage-two equilibrium prices (p_1, p_2) are identical to the prices (p_1^L, p_2^F) , which would be obtained if firm 1 were the price-setting leader, and firm 2 were the price-setting follower. Re-write the optimal tax condition above as

$$(1-\theta)t^* x_1^{(1)} = (p_1 - C_x^{(1)}) x_2^{(1)} \left(\frac{\Pi_{12}^{(2)}}{\Pi_{22}^{(2)}} \right) \equiv \Pi_2^{(1)} \left(\frac{\Pi_{12}^{(2)}}{\Pi_{22}^{(2)}} \right).$$

Using firm 1's first-order condition, this gives

$$\Pi_1^{(1)} = \Pi_2^{(1)} \left(\frac{\Pi_{12}^{(2)}}{\Pi_{22}^{(2)}} \right).$$

But this precisely the condition that determines the prices (p_1^F, p_2^F) , which are obtained if firm 1 is the price-setting leader, and firm 2 is the price-setting follower. \square

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