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The human development index as a criterion for optimal planning*

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Abstract

Planning strategies that maximize the Human Development Index (HDI) imply equitable outcomes – even though inequality aversion is not in the index itself. Moreover, the weight on income in the HDI plays only an indirect role in determining optimal allocations.

Keywords: Human development index; Planning

JEL classification: O21; O15

1. Introduction

The Human Development Index (HDI) is a composite index published annually by the UN Human Development Report Office, since 1990, which is designed to measure "human well being" in different countries.[1] The index combines measures of life expectancy, school enrolment, literacy, and income to provide a broader-based measure of well-being and development than income alone. Since its publication, this index has become widely cited and is commonly used as a way of ranking the quality of life in different countries. The impact of the HDI ranking on policy is reflected by the fact that some national governments have taken to announcing their HDI ranking and their aspirations for improving it.[2]

In this paper, we consider the implications of using the HDI as a criterion for economic development plans. In particular, we examine the consequences of pursuing plans that maximize the HDI score for a given country. To do this, we construct a simple economic model where a planner chooses optimal education, health, and consumption expenditures to maximize a well-defined objective function that includes the HDI index as a special case. To keep the analysis as simple as possible, we restrict our attention to economies with incomes high enough so that consumption levels themselves do not affect educational attainment or life expectancy. We show that despite the fact that the HDI includes an income index as one of its components, the optimal plan involves maximizing expenditure on the other two components of the index: education and health. This optimal plan tends to imply equitable outcomes. We also identify circumstances under which income's weight in the index is irrelevant to optimal plans. At most, income plays an indirect role in determining optimal plans.[3]

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[1] For a detailed description see http://hdr.undp.org/statistics/indices/.

[2] For example, in a recent speech, the President of India, Dr. Abdul Kalam, exalts Indians to work together to improve India's current HDI rank of 127 to achieve a rank of 20; see Kalam (2005). The HDI is discussed in recent Indian budgets; see Budget of India (2005). In announcing Canada's number one ranking in 1998, Prime Minister Jean Chrétien stated: "While the HDI tracks Canada's impressive achievements, it also tells us where we can improve."; see Chrétien (1998).

[3] This is an interesting result because the "capabilities approach", or equivalently "human development approach", de-emphasizes valuing income *per se* [e.g. Sen (1985), Anand and Ravallion (1993)]. Anand and Ravallion (1993, p.136-37) note that while the philosophy of the Human Development Report has been heavily influenced by the capabilities approach, the inclusion of income in the HDI is problematic because "…it is not a direct indicator of any achievement or functioning, …".

2. The model

We consider a static closed economy model, where a planner acts to maximize the following objective function, which nests the HDI:

$$I(w,W) = wI^{y}(y) + (1-w)[WI^{e}(e) + (1-W)I^{l}(l)] \quad (1)$$

Here, $I^{y}(y)$, $I^{e}(e)$, and $I^{l}(l)$ represent indexes of per capita income (*y*), educational attainment (*e*), and life expectancy (*l*) respectively. These are assumed to be differentiable, increasing and concave in their respective arguments. The parameters *w* and *W* are weights used when constructing the composite index, given in equation (1).[4]

Educational attainment is assumed to be a differentiable increasing function of expenditures on both education (*E*) and health (*H*). Thus:

$$e = e(E,H), \quad e_{E} > 0, \quad e_{H} \geq 0 \quad (2)$$

Similarly, life expectancy is differentiable and increasing in both of these arguments:

$$l = l(E,H), \quad l_{E} \geq 0, \quad l_{H} > 0 \quad (3)$$

To simplify the analysis, we are assuming that the economy in question has a level of per capita income high enough so that neither income nor consumption substantially affect life expectancy and educational attainment as measured in the HDI. This is formalized by the following assumption about per capita consumption *c*:

$$c \geq c_{\min} \quad (4)$$

[4] The HDI is a special case of this index, where *w* = 1/3 and *W* = 1/2, so that each of the three component indexes are equally weighted.

where $c_{\min}$ is a parameter which identifies the level of consumption beyond which no further increments in consumption will increase educational attainment or life expectancy.[5] If this constraint is relaxed the results we derive below are even stronger.

In this simple static economy, we abstract away from capital, and assume full employment of labour. Total labour in the economy is normalized to one unit. Given this, output per capita is determined by the following differentiable production technology:

$$y = f(e,l), \quad f_e \geq 0, \quad f_l \geq 0 \tag{5}$$

Here, education levels affect output through human capital in the usual way. Also, increments in life expectancy increase the effective size of the labour force and thereby increase production.

Once produced, the single good in the economy can be allocated to three possible uses: aggregate consumption ($l \cdot c$), education expenditure (E), and health expenditure (H). Therefore, the economy must respect the aggregate constraint:

$$lc + E + H \leq y \tag{6}$$

Observe that consumption, c, is on items other than health and education and that we allow total consumption to be proportional to life expectancy.

2. The Planner's Problem

Given the concavity of the objective function (1) and the convexity of the constraints (4) and (6), using equations 1-6, the planner's problem can be formulated as the concave programming problem (P1):

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[5] This assumption is consistent with Anand and Ravallion's (1993) "capability expansion through social services". According to this explanation (also see Sen, 1981), the public provision of essential goods and services leads to improved social outcomes and income matters if it is used to finance suitable public services and alleviate poverty. For example, Anand and Ravallion find in a sample of 22 developing countries that after controlling for health expenditures and poverty (as measured by the proportion of population consuming less one dollar a day in 1985 at PPP), life expectancy is not affected by consumption. Even the unconditional plot of income against life expectancy displays an income threshold (achieved by all developed countries) beyond which there is no discernable relationship (e.g. Deaton, 2003)). Anand and Ravallion contrast schools of thought on the importance of social services versus private consumption for human development.

$$\underset{\{c,E,H\}}{Max}\, I(w,W) = wI^{y}(f(l(E,H),e(E,H))) + (1-w)\big(WI^{e}(e(E,H)) + (1-W)I^{l}(l(E,H))\big)$$

subject to: i) $l(E,H)c + E + H - f\big(l(E,H),e(E,H)\big) \leq 0$

ii) $-c \leq -c_{\min}$

The Lagrangian for the problem is:

$$\begin{aligned}\mathcal{L} &= wI^{y}(f(l(E,H),e(E,H))) + (1-w)\big(WI^{e}(e(E,H)) + (1-W)I^{l}(l(E,H))\big)\\ &\quad + \lambda_1\big(f(l(E,H),e(E,H)) - E - H - l(E,H)c\big) + \lambda_2(c - c_{\min}) \qquad (7)\end{aligned}$$

Proposition 1. *Maximizing the HDI requires setting consumption at the minimum level:* $c^* = c_{\min}$.

Proof. The Kuhn-Tucker maximum conditions are necessary and sufficient for a global maximum. Among these conditions are the following:

$$c\big(\lambda_2 - \lambda_1 l(E,H)\big) = 0 \qquad (8)$$

$$\lambda_2(c - c_{\min}) = 0 \qquad (9)$$

$$\lambda_1 \geq 0, \quad \lambda_2 \geq 0 \qquad (10)$$

We now show that $\lambda_2 > 0$. Suppose not. Then, by (10), $\lambda_2 = 0$. By (8), since $l(E,H) > 0$ and $c > 0$, this implies that $\lambda_1 = 0$. Since the objective function is strictly increasing in *E* and *H*, the resource constraint (6) binds, and so $\lambda_1 > 0$. This is a contradiction. Thus, $\lambda_2 > 0$. By (9), this then implies that $c = c_{\min}$. ■

The intuition behind this result is quite straightforward. Consumption does not enter the index (the objective function) or the production technology, but costs the planner through the resource constraint, so the optimal plan will set consumption to its minimal allowed value.[6]

Notice that, if we assume $c_{\min}$ is the same for each person in the economy, then Proposition 1 implies that consumption for each individual would be set equal to the same value under the optimal plan. That is, the optimal plan is egalitarian, at least with respect to consumption, even though no inequality aversion appears explicitly in the HDI itself. If education and health facilities are also equally accessible to everyone in the economy, (as would be the case, for example, if agents are homogeneous and given diminishing returns to individual expenditures on education and health) then *maximizing the HDI implies equality of treatment*. This is important because some critics of the HDI have argued that some sort of inequality aversion should be built into the index explicitly (e.g. Foster et al., 2005). Proposition 1 indicates that, if governments use the existing HDI as an objective function to devise their plans, then this leads to equitable outcomes – through the implied emphasis on maximizing funding to education and health.

4. The Role of Income

This emphasis on education and health expenditures naturally leads us to consider the question of the importance of the income index $I^y(y)$ in the HDI. In the optimal plan, given that $c = c_{\min}$, the remaining problem of how to allocate resources to E and H is affected by $I^y(y)$ only because of the effects of E and H on production, *indirectly* through life expectancy $l(E,H)$ and education $e(E,H)$. By way of contrast, both $l(E,H)$ and $e(E,H)$ have *direct* impacts on the indexes $I^e(e)$, and $I^l(l)$ respectively. Particularly in developed economies, it seems reasonable to argue that the marginal effects of E and H on production may be quite small. This reasoning is formalized in the following proposition.

[6] Observe that if $c_{\min} = 0$, then $c = 0$. This unrealistic corner solution arises only because we have excluded consumption from the education and health functions, e and l, on the grounds that $c_{\min} > 0$ is sufficiently high not to affect those functions. If we resolve the planner's problem without the minimum consumption constraint (ii), we would have to specify the e and l functions as positively related to c (over the range up to $c_{\min}$) to get an internal solution. This internal solution would have a smaller value of c and a larger value of public expenditures than (P1). Consumption is even more de-emphasized without the minimum consumption constraint.

Proposition 2.

a) *If* $f_l = f_e = 0$ *then the weight w on the income index* $I^y(y)$ *in the HDI plays no role in determining optimal plans.*

b) *If* $f_l > 0$ *or* $f_e > 0$ *then the weight w on the income index* $I^y(y)$ *in the HDI affects only the trade-off between expenditures on education E and health H.*

Proof.

a) Consider an alternative objective function where only the indexes $I^e(e)$ and $I^l(l)$ have weight: $I(\hat{W}) = \hat{W}I^e(e(E,H)) + (1-\hat{W})I^l(l(E,H))$. When $f_l = f_e = 0$ then $I(w,W)$ is simply an affine transformation of $I(\hat{W})$.

b) By Proposition 1, $c = c_{\min}$. Problem P1 is therefore equivalent to the following problem, P2, in which determines the choices of *E* and *H*:

$$\underset{\{E,H\}}{Max}\, I(w,W,c_{\min}) = wI^y(f(l(E,H),e(E,H))) + (1-w)\left(WI^e(e(E,H)) + (1-W)I^l(l(E,H))\right)$$

subject to: $l(E,H)c_{\min} + E + H - f\left(l(E,H),e(E,H)\right) \le 0$ ■

5. Conclusion

The HDI is a widely cited statistic that is commonly used as a measure of well-being in different countries. Here, we have examined some of the implications that follow if government planners decide to use maximization of the HDI as a criterion for optimal plans. We have found that, if they do so, planners will tend to heavy emphasize expenditures on education and health over consumption on other items. This leads the economy towards a more egalitarian allocation – even though inequality aversion does not appear explicitly in the HDI itself. Our simple static normative analysis ignores capital accumulation and growth issues and does not look at the positive issues around incentive and participation constraints. We leave these issues for future research.

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