arXiv:hep-th/9506156v4 15 Feb 1996

SOLUTION–GENERATING TRANSFORMATIONS AND THE STRING EFFECTIVE ACTION

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Abstract

We study exhaustively the solution-generating transformations (dualities) that occur in the context of the low-energy effective action of superstring theory.

We first consider target-space duality ("T duality") transformations in absence of vector fields. We find that for one isometry the full duality group is $(SO^{\uparrow}(1,1))^3 \times D_4$, the discrete part ($D_4$) being non-Abelian. We, then, include non-Abelian Yang–Mills fields and find the corresponding generalization of the T duality transformations. We study the α' corrections to these transformations and show that the T duality rules considerably simplify if the gauge group is embedded in the holonomy group.

Next, in the case in which there are Abelian vector fields, we consider the duality group that includes the transformation introduced by Sen that rotates among themselves components of the metric, axion and vector field.

Finally we list the duality symmetries of the Type II theories with one isometry.

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Introduction

In recent years an active field of research has been the study of modified Einstein equations. The modifications that have been considered consist in the addition to pure gravity of extra scalars, antisymmetric tensor fields (called dilatons and axions, respectively) and (Abelian or non-Abelian) Yang–Mills fields. These modified Einstein equations admit special solutions whose consistency crucially depends on the presence of the new fields. For examples of such new solutions, see, for instance, the review articles Refs. [1, 2, 3, 4, 5] and references therein.

One motivation for studying the above-mentioned modifications to General Relativity is that they arise in string theory. In string theory elementary particles are described as the excitations of a string rather than as point-like objects. The size of a string can be characterized by a dimensionful parameter α' (that can also be understood as the inverse of the string tension) in such a way that, in the so-called zero-slope limit $\alpha' \to 0$, an ordinary field theory of point particles is obtained. This zero-slope limit of string theory corresponds to a modified (or extended) Einstein theory of the type discussed above. The complete effective action includes, at higher orders in α', contributions which are of higher order in the Riemann tensor and the Yang–Mills field strength. Since string theory claims to give a consistent description of quantum gravity, solutions of the string effective action are expected to contribute to our understanding of quantum gravity.

Particularly interesting are solitonic and supersymmetric solutions [1, 4, 5] to the low-energy effective field theory since, for different reasons, many of them are expected to be not just exact solutions to the effective action to all orders in α', but exact solutions of string theory.

In general, it appears difficult to find exact solutions to the string equations of motion. One of the reasons for this is that knowledge about the explicit form of the higher order α' corrections to the string effective action have become available only fairly recently [6]. Fortunately, if one considers spacetimes with an isometry, there exist transformations which generate new solutions out of old ones. We will refer to all these symmetries of the equations of motion as "dualities".

The "target-space" ("T") duality transformations of the Type I superstring effective action where first introduced in the bosonic σ–model context

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for general backgrounds with one isometry by Buscher in Refs. [7] (see Ref. [8] for an updated review) as discrete ($\mathbb{Z}_2$) transformations that interchange certain components of the metric and axion fields. Later Roček and Verlinde [9] proved that when the orbit of the isometry is closed, the backgrounds related by Buscher's transformation correspond to the same CFT.

This symmetry is also present in the zero-slope limit of the effective action, and, in this context, (see Ref. [10] for a review with extensive references) the classical T duality group was found to be the continuous $O(1,1)_{\text{Sugra}}$.

On the other hand, using string-field theory arguments, Sen found that in presence of an additional Abelian vector field the duality symmetry was bigger: $O(1,2;\mathbb{Z})$ [11]. At the level of the classical zero-slope effective action, there is a continuous $O(1,2;\mathbb{R})_{\text{Sugra}}$ T duality group. The increase in symmetry is due to the fact that we now can interchange certain components of the metric or axion fields with certain components of the Abelian vector field. We will refer to this kind of transformations as "Sen transformations".

The necessity of isometries strongly suggests the use of techniques of dimensional reduction and a close relationship between the duality symmetries in the original dimensionality and the "hidden symmetries" of the dimensionally reduced theory [12]. For supergravity theories the hidden symmetry groups of most supergravity theories are well known [13] and this has been a fruitful approach in the sense that the duality groups of many dimensionally reduced theories have been found (see, for instance, Ref. [14] were this point of view is advocated). However, the relation with the symmetries of the "original" higher-dimensional theories has not always been carefully studied. It is our purpose to do this here, for the simple case of a single isometry, distinguishing between those dualities which become simple general coordinate transformations or gauge transformations in higher dimensions and those which do not. We will combine this study with a thorough search for all discrete and continuous duality transformations, relating them, when possible, to symmetries of the σ-model or the Type II theories. We will consider three cases: the (Type I) superstring effective action (i) in absence of vector fields, (ii) in presence of non-Abelian Yang–Mills fields and (iii) in presence of a single Abelian vector field.

Our main results are:

1. We find more (both discrete and continuous) duality symmetries. Since

the situation in the literature is unclear, some of them were (perhaps) known in different contexts and sometimes mistaken for each other. We clarify the situation. In particular, we find that the T duality group of the Type I theory with no vector fields is $(SO^\uparrow(1,1))^3 \times D_4$. The appearance of this finite non-Abelian group (D_4) is remarkable.

2. We generalize Buscher's (discrete) transformation to the case in which there are non-Abelian Yang–Mills fields present. Any solution of the zero-slope heterotic string theory effective action with one isometry can now be "T dualized".

3. We present the α' corrections to the generalized (discrete) Buscher's duality transformation and show that it becomes considerably simpler if the gauge group is embedded in the holonomy group.

4. We give the explicit form in terms of the higher-dimensional theory fields of the finite Sen transformation (one Abelian vector field present).

5. We list all the duality symmetries of the Type II theories (including eleven-dimensional supergravity) and relate them with each other and with global coordinate transformations of the higher-dimensional theories, when possible.

6. In this respect, we remark the fact that Buscher's discrete duality transformation is an "unexpected" symmetry in the sense that it is not a global coordinate transformation in higher dimensions[4]. Then, from the higher-dimensional point of view, it is the only interesting solution-generating transformation since all the other transformations are then gauge transformations.

We would like to stress that we are not going to perform full-fledged compactifications, in the sense that in an expansion of the fields in harmonic functions of the compact dimension we will only keep the massless modes, *i.e.* those with no dependence on the coordinate that parametrizes

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[4]If Buscher's discrete duality transformation corresponded to a global coordinate transformation in higher dimensions, it would be a symmetry of all theories which are obtained from a higher dimensional one through dimensional reduction, which is not true. Only theories with the "right" field content have this symmetry.

the compact dimension. The theories that we will obtain in this way will effectively be lower-dimensional theories. We will refer to this procedure as *dimensional reduction*, to distinguish it from (Kaluza-Klein) *compactification*. Dimensional reduction, which has traditionally and successfully been used in supersymmetry and supergravity as a method to obtain new theories and which has been used in many recent works on duality starting from Ref. [12], will be enough for all of purposes.

As a matter of fact, we are ultimately interested in duality symmetries of string theories. While effective actions contain some information about the string massless modes, at least enough to determine their low-energy dynamics, they do not contain much information about the massive modes. It would not make any sense to study the Kaluza-Klein massive modes (whose origin are the higher-dimensional massless modes) without including the original string massive modes at the same time. Dimensional reduction of effective actions is, then, not only the simplest approach, but, in general, the only consistent approach from the low-energy point of view[5].

On the other hand, we expect that all duality symmetries of superstring theories will be duality symmetries of the effective field theories (supergravity theories) [16]. Then, the study of the duality symmetries of effective actions is the easiest way to discover those of the full string theory. In some cases, like the Type II theories, where it is not known how to include the Ramond-Ramond background fields in the σ-model, it is also the only available way [17].

This being said, one should be aware that the effective theory does not always give an adequate representation of the corresponding string theory, particularly where non-perturbative in α' effects occur [18], the results obtained cannot be fully trusted and should be understood as indications but never as proofs of the corresponding results in string theory. This is particularly important in the case of Buscher's T duality transformation. It was shown in Refs. [19, 20] that this transformation seems not to respect unbroken spacetime supersymmetries. This surprising effect has been studied by different authors [21, 22, 18] and the conclusion seems to be that Buscher's

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[5]The only exception to this conclusion might be eleven-dimensional *supergravity*, whose Kaluza-Klein compactification on a circle seems to give the whole spectrum of Type IIA *superstring* theory [15].

duality transformation does *not* break spacetime supersymmetry and that the usual representation of spacetime supersymmetry (and hence the usual effective action) does not describe correctly the dynamics of string theory in this limit.

The intrinsically *stringy* nature of this transformation as different from the rest of the T duality group shows itself here. Since, as we will see, the rest of the conventional $O(1,1)_{\text{Sugra}}$ T duality group[6] corresponds to global coordinate reparametrizations in higher dimensions, it respects automatically supersymmetry.

This article is organized as follows. In Section 1 we review the T duality symmetries of the Type I theory in the absence of vector fields. Here we dimensionally reduce the action in the isometry direction, and we look for symmetries of the lower dimensional theory, as advocated in Refs. [12, 14]. The purpose of this section is to set up our notation and conventions, and to thoroughly review the known results finding some new ones.

In Section 2 we use the technique of dimensional reduction to find the generalization of the discrete (Buscher) T duality rules in the presence of non-Abelian vector fields. We discuss the relation between our results and the σ-model description of T duality.

In Section 3 we discuss how for the special case of an Abelian vector field Sen's solution-generating transformation emerges. In particular, we discuss the connection between the Sen transformation at the one-hand-side and special general coordinate plus gauge transformations at the other-hand-side.

Next, in Section 4 we study the α' corrections to the discrete (Buscher's) T duality transformations. Appendices A and B contain respectively the finite form of Sen's duality transformations in D dimensions and a review of analogous results in the Type II theories and in eleven-dimensional supergravity. We also describe in this last Appendix the relation between the duality transformations studied in the main body of the paper and global coordinate transformations in higher dimensions.

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[6]As we have already said, and we will show in the first section, the T duality group is indeed bigger.

1 T duality Without Vector Fields

In this section we review the T duality symmetries of the bosonic sector of the zero-slope heterotic string effective action (which has the same form as the bosonic string one). Therefore, there are no vector fields present. To keep the discussion general we will work in D dimensions specifying later, where necessary, to the case $D = 10$. Furthermore, for simplicity, we only assume the existence of one isometry direction. The results presented have an obvious generalization to the case of several commuting isometries.

The D–dimensional action we start from is, in the zero-slope limit, given by

$$S_{\text{Sugra}}^{(D)} = \tfrac{1}{2} \int d^D x \sqrt{-\hat{g}} \ e^{-2\hat{\phi}} \left[-\hat{R} + 4(\partial\hat{\phi})^2 - \tfrac{3}{4}\hat{H}^2 \right] , \tag{1}$$

where the fields are the metric, the axion and the dilaton:

$$\left\{ \hat{g}_{\hat{\mu}\hat{\nu}}, \hat{B}_{\hat{\mu}\hat{\nu}}, \hat{\phi} \right\} , \tag{2}$$

and our conventions are those of Ref. [20]. In particular, the axion field-strength $\hat{H}$ is given by

$$\hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}} = \partial_{[\hat{\mu}}\hat{B}_{\hat{\nu}\hat{\rho}]} . \tag{3}$$

We are going to assume that all the backgrounds (solutions of this theory) considered admit one isometry whose orbits can be parametrized by the coordinate x, *i.e.* we assume that there exists a Killing vector $\hat{k}^{\hat{\mu}}$ such that the Lie derivative of all fields with respect to $\hat{k}^{\hat{\mu}}$ is zero and such that

$$\hat{k}^{\hat{\mu}}\partial_{\hat{\mu}} = \partial_{\underline{x}} . \tag{4}$$

It is natural to use *adapted coordinates*[7] (x^μ, x) such that all fields are independent of the redundant coordinate x. Then, the space splits into a $(D-1)$-dimensional space parametrized by the coordinates x^μ and an "internal" space parametrized by the coordinate x. In this internal space, by

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[7]All the D–dimensional entities carry a hat and the $(D-1)$-dimensional ones don't, and $\mu = \underline{0}, \ldots, \underline{D-2}$; $x = x^{\underline{D-1}}$. To distinguish between curved and flat indices, we underline the curved ones ($\xi^{\underline{x}}$, for instance).

assumption, "nothing happens", there is no dynamics, since the fields are independent of x. The theory is effectively $(D-1)$-dimensional, and therefore, following the point of view advocated in Ref. [12], we will reduce dimensionally the action Eq. (1) to find the corresponding effective action.

First of all, in this coordinate system, the components of the Killing vector are

$$\hat{k}^{\hat{\mu}} = \delta^{\hat{\mu}\underline{x}}, \quad \hat{k}_{\hat{\mu}} = \hat{g}_{\hat{\mu}\underline{x}}, \quad \hat{k}^{\hat{\mu}}\hat{k}_{\hat{\mu}} = \hat{g}_{\underline{xx}}. \quad (5)$$

The zero-slope limit Buscher's T duality rules were originally derived using the two-dimensional σ-model approach in Refs. [7]. The explicit form of these transformations is:

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\begin{aligned}
\tilde{\hat{g}}_{\mu\nu} &= \hat{g}_{\mu\nu} - (\hat{g}_{\underline{x}\mu}\hat{g}_{\underline{x}\nu} - \hat{B}_{\underline{x}\mu}\hat{B}_{\underline{x}\nu})/\hat{g}_{\underline{xx}}, \\[2mm]
\tilde{\hat{B}}_{\mu\nu} &= \hat{B}_{\mu\nu} + (\hat{g}_{\underline{x}\mu}\hat{B}_{\nu\underline{x}} - \hat{g}_{\underline{x}\nu}\hat{B}_{\mu\underline{x}})/\hat{g}_{\underline{xx}}, \\[2mm]
\tilde{\hat{g}}_{\underline{x}\mu} &= \hat{B}_{\underline{x}\mu}/\hat{g}_{\underline{xx}}, &\qquad \tilde{\hat{B}}_{\underline{x}\mu} &= \hat{g}_{\underline{x}\mu}/\hat{g}_{\underline{xx}}, \\[2mm]
\tilde{\hat{g}}_{\underline{xx}} &= 1/\hat{g}_{\underline{xx}}, &\qquad \tilde{\hat{\phi}} &= \hat{\phi} - \tfrac{1}{2}\log|\hat{g}_{\underline{xx}}|.
\end{aligned}
\quad (6)
$$

The transformations Eqs. (6) also leave the zero-slope limit action $S^{(D)}_{\text{Sugra}}$, given in Eq. (1), invariant in the sense that

$$S^{(D)}_{\text{Sugra}}(\tilde{\hat{g}}, \tilde{\hat{B}}, \tilde{\hat{\phi}}) = S^{(D)}_{\text{Sugra}}(\hat{g}, \hat{B}, \hat{\phi}) + \int d^D x \; A(\hat{g}, \hat{B}, \hat{\phi})\partial_{\underline{x}}B(\hat{g}, \hat{B}, \hat{\phi}), \quad (7)$$

where A and B are some expressions in terms of $\hat{g}, \hat{B}$ and $\hat{\phi}$. This property shows that target-space duality is indeed a symmetry of the equations of motion (to this order in α') and therefore a solution-generating transformation: if a configuration, independent of x, is a solution of the zero-slope limit equations of motion, then the dual configuration is also a solution.

A few remarks are in order

- The duality transformations (6) are only well-defined if $\hat{g}_{\underline{xx}} \neq 0$. This is guaranteed by the condition that the Killing vector $\hat{k}$ is non-null.

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For simplicity we consider from now on only the space-like case. It is not difficult to generalize the formulae to the general case. It is remarkable that it is precisely the restriction of the Killing vector to be non-null that allows us to perform dimensional reduction in that direction. Things are radically different in the null case, and we do not know which kind of "null duality symmetries", if any, exist. Recently, the dimensional reduction of the Einstein theory in a null direction has been studied in Ref. [23], but it is not clear yet whether their results can be applied to our problem since in our case the existence of a null Killing vector is not enough to prove that the integrability condition $R_{vv} = 0$ (v is the corresponding null coordinate), on which their results are based, always holds.

- For the special case in which the configuration is given by the product of a $(D-1)$-dimensional Minkowski space times a circle we have that $\hat{g}_{\underline{xx}} \sim R$, where R is the radius of the torus, and the duality transformation corresponds to the well-known transformation $R \to 1/R$ [24].

- The dual of the dual gives back the original configuration. Therefore, this duality transformation, that we will call from now on B, generates a $\mathbb{Z}_2^{(B)}$ symmetry group.

- We are after symmetries of the equations of motion[8], and, therefore, we will consider as good symmetries transformations that, instead of leaving invariant the action, as B, scale it.

To show that Equation (7) holds, it is convenient to use a supergravity interpretation of duality via dimensional reduction [12]. We use the standard techniques of Scherk and Schwarz in Refs. [25]. Thus, the D–dimensional fields decompose as follows:

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[8]From the point of view of string theory, the only meaning of the effective action (when it exists) is that the conditions for the vanishing of the beta functionals can be derived from its minimization. From the supergravity point of view, the action is meaningful and a good symmetry of the theory will always leave the action invariant.

$$\hat{g}_{\mu\nu} \; = \; g_{\mu\nu} - k^2 A_\mu A_\nu \,, \qquad\qquad \hat{B}_{\mu\nu} \; = \; B_{\mu\nu} + A_{[\mu} B_{\nu]} \,,$$

$$\hat{g}_{\underline{x}\mu} \; = \; -k^2 A_\mu \,, \qquad\qquad\qquad \hat{B}_{\underline{x}\mu} \; = \; B_\mu \,, \qquad\qquad (8)$$

$$\hat{g}_{\underline{x}\underline{x}} \; = \; -k^2 \,, \qquad\qquad\qquad\quad \hat{\phi} \; = \; \phi + \tfrac{1}{2} \log k \,,$$

where

$$\{ g_{\mu\nu}, B_{\mu\nu}, \phi, A_\mu, B_\mu, k \} \,, \qquad\qquad (9)$$

are the $(D-1)$-dimensional fields and we have used the notation

$$k = |\hat{k}_{\hat\mu} \hat{k}^{\hat\mu}|^{\frac{1}{2}} \,. \qquad\qquad (10)$$

Observe that $\hat{k}_{\hat\mu} \hat{k}^{\hat\mu} = \hat{g}_{\underline{x}\underline{x}} = -k^2$. Similarly, the $(D-1)$-dimensional fields are given in terms of the D–dimensional fields by

$$g_{\mu\nu} \; = \; \hat{g}_{\mu\nu} - \hat{g}_{\underline{x}\mu} \hat{g}_{\underline{x}\nu} / \hat{g}_{\underline{x}\underline{x}} \,, \qquad\qquad B_\mu \; = \; \hat{B}_{\underline{x}\mu} \,,$$

$$B_{\mu\nu} \; = \; \hat{B}_{\mu\nu} + \hat{g}_{\underline{x}[\mu} \hat{B}_{\nu]\underline{x}} / \hat{g}_{\underline{x}\underline{x}} \,, \qquad\qquad \phi \; = \; \hat{\phi} - \tfrac{1}{4} \log |\hat{g}_{\underline{x}\underline{x}}| \,, \qquad (11)$$

$$A_\mu \; = \; \hat{g}_{\underline{x}\mu} / \hat{g}_{\underline{x}\underline{x}} \,, \qquad\qquad\qquad k \; = \; |\hat{g}_{\underline{x}\underline{x}}|^{\frac{1}{2}} \,.$$

Ignoring the integral over x, the D–dimensional action Eq. (1) is identically equal to[9]

$$S^{(D-1)}_{\text{Sugra(red)}} \; = \; \tfrac{1}{2} \int d^{(D-1)}x \; \sqrt{-g} \; e^{-2\phi} \Big[-R + 4(\partial\phi)^2 - \tfrac{3}{4} H^2$$

$$-(\partial \log k)^2 + \tfrac{1}{4} k^2 F^2(A) + \tfrac{1}{4} k^{-2} F^2(B) \Big] \,, \qquad (12)$$

where

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[9]Dropping the integral over x is consistent with our dimensional reduction philosophy. However, when global transformations of this coordinate are involved, we will find that the lower-dimensional action scales while the higher-dimensional action is invariant. In both dimensions the equations of motion are invariant.

$$F_{\mu\nu}(A) \;\; = \;\; 2\partial_{[\mu}A_{\nu]} \,, \qquad\qquad F_{\mu\nu}(B) = 2\partial_{[\mu}B_{\nu]} \,,$$

$$H_{\mu\nu\rho} \;\; = \;\; \partial_{[\mu}B_{\nu\rho]} + \tfrac{1}{2}A_{[\mu}F_{\nu\rho]}(B) + \tfrac{1}{2}B_{[\mu}F_{\nu\rho]}(A) \,. \tag{13}$$

The action Eq. (12) can be interpreted as a $(D-1)$-dimensional action for the above $(D-1)$-dimensional fields. It is known since the old supergravity days [13, 26] that this action is invariant under the rigid non-compact ("supergravity") symmetry group

$$O(1,1)_{\text{Sugra}} = SO^{\uparrow}(1,1)_{\text{Sugra}} \;\times\; \mathbb{Z}_2^{(B)} \times \; \mathbb{Z}_2^{(S)} \,. \tag{14}$$

$SO^{\uparrow}(1,1)$ is the part of $O(1,1)$ connected with the identity, and $\mathbb{Z}_2^{(B)} \times \mathbb{Z}_2^{(S)}$ is its *mapping class group*. The first of the discrete symmetries, $\mathbb{Z}_2^{(B)}$, is generated by the transformation that we denote by B:

$$\tilde{A}_\mu = B_\mu \,, \qquad \tilde{B}_\mu = A_\mu \,, \qquad \tilde{k}^2 = k^{-2} \,, \tag{15}$$

while the other fields are invariant. In D dimensions these rules correspond to Buscher's duality rules Eqs. (6) (hence the name). The second $\mathbb{Z}_2$ symmetry is generated by the transformation

$$A'_\mu = -A_\mu \,, \qquad B'_\mu = -B_\mu \,, \tag{16}$$

that we denote by S. On the other hand, the $SO^{\uparrow}(1,1)_{\text{Sugra}}$ transformation (with continuous rigid parameter α)[10] is a scaling of the fields with diferent powers ("weights") of the factor e^α. The weights in nine dimensions (*i.e.* when we take $D = 10$) are given in Table 1.

As it is explained in Appendix B, and following the classical reasoning of Ref. [13], since the action above was obtained by dimensional reduction of the coordinate x, one would generally expect an $SO(1,1)$ T duality group of rescalings and reflections of the compact coordinate $x' = e^\alpha x$, $x' = -x$. This is exactly the origin of $SO^{\uparrow}(1,1)_{Sugra} \times \mathbb{Z}_2^{(S)}$, and an analogous global symmetry is expected in any theory which can be obtained from (one)-dimensional

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[10]The parameter α takes values in $\mathbb{R}$. Therefore, $SO^{\uparrow}(1,1)$ is isomorphic to the multiplicative group $\mathbb{R}^+$ or to the additive group $\mathbb{R}$.

Name	$g_{\mu\nu}$	$B_{\mu\nu}$	A_μ	B_μ	e^ϕ	k	$S^{(9)}_{\text{Sugra(red)}}$
$SO^\uparrow(1,1)_{\text{Sugra}}$	0	0	1	-1	0	-1	0
$SO^\uparrow(1,1)_{x-y}$	1	1	1	0	$\frac{7}{4}$	$-\frac{1}{2}$	0
$SO^\uparrow(1,1)_{\text{string}}$	0	0	0	0	1	0	-2

Table 1: *Weights of the heterotic fields under the two $SO^\uparrow(1,1)$ symmetries of the action $S^{(9)}_{\text{Sugra(red)}}$ and the third $SO^\uparrow(1,1)$ which scales it in nine dimensions (so taking $D = 10$).*

reduction of another theory. It will come as no surprise that in all the cases considered in the following sections this duality group is always present and we will not mention its origin again. On the other hand, the presence of the second $\mathbb{Z}_2^{(B)}$ is completely unexpected (for instance, it is not present in Type II actions as explained in Ref. [17]) and ultimately related to the stringy origin of the action we started with.

In addition to these well-known symmetries, there is another $SO^\uparrow(1,1)_{x-y}$ scaling transformation and a discrete $\mathbb{Z}_2^{(A)}$ transformation that leave the action invariant. The weights of the fields under the $SO^\uparrow(1,1)_{x-y}$ scaling in nine dimensions are given in the second row in Table 1.

The $\mathbb{Z}_2^{(A)}$ symmetry group is generated by the transformation that we call A

$$B_{\mu\nu} \rightarrow -B_{\mu\nu}, \qquad B_\mu \rightarrow -B_\mu. \qquad (17)$$

One may naively think that the total symmetry group is just $SO^\uparrow(1,1)_{\text{Sugra}} \times SO^\uparrow(1,1)_{x-y} \times \mathbb{Z}_2^{(B)} \times \mathbb{Z}_2^{(S)} \times \mathbb{Z}_2^{(A)}$. However, a careful analysis shows that the actual symmetry group is

$$SO^\uparrow(1,1)_{\text{Sugra}} \times SO^\uparrow(1,1)_{x-y} \times D_4. \qquad (18)$$

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where D_4 is the symmetry group of rotations of a square with undirected sides, which has two generators b, c that obey [11] $c^4 = b^2 = (bc)^2 = 1$. In our case the generators are $b = B$ (Buscher's duality transformation Eq. (6)) and the order four element $c = AB$ (the A is given by Eq. (17)). Note that A and B do not commute.

Finally we note that there is an additional $SO^\uparrow(1,1)_{\text{string}}$ scaling transformation ("string") which is a symmetry of the equations of motion but not of the action, which scales under it. The non-zero scaling weights in nine dimensions are given in the last line of Table 1.

We conclude that the full group of global symmetries of the equations of motion is

$$SO^\uparrow(1,1)_{\text{Sugra}} \times \ SO^\uparrow(1,1)_{x-y} \times \ SO^\uparrow(1,1)_{\text{string}} \times \ D_4 \,. \qquad (19)$$

The whole symmetry group (except for the B transformation) can be understood from a higher dimensional point of view and from the Type II theories point of view. This is discussed in Appendix B. Also, all the transformations in the discrete part of the symmetry group D_4 can be understood from the σ-model with a D-dimensional target-space point of view. In particular, the transformation A consists in the change of sign of the D-dimensional axion $(\hat{B}_{\hat{\mu}\hat{\nu}} \rightarrow -\hat{B}_{\hat{\mu}\hat{\nu}})$ plus the interchange between right-movers and left-movers $z \rightleftharpoons \overline{z}$.

2 Duality In Presence Of Non-Abelian Vector Fields

Since our ultimate goal is to study the duality symmetries of the full heterotic string effective action, it is natural to study, as an intermediate step, the effect of the addition of non-Abelian vector fields on the duality symmetries found in the previous section. Then, our starting point is the so-called "Sugra+YM" action:

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[11]See for instance Ref. [27] where a representation in terms of two-dimensional matrices is given in page 25.

$$S^{(D)}_{\text{Sugra+YM}} = \tfrac{1}{2} \int d^D x \sqrt{-\hat{g}} \; e^{-2\hat{\phi}} \left[-\hat{R} + 4(\partial\hat{\phi})^2 - \tfrac{3}{4}\hat{H}^2 + \tfrac{1}{4g^2}\hat{F}^I_{\hat{\mu}\hat{\nu}}\hat{F}^{\hat{\mu}\hat{\nu}}_I \right] , \quad (20)$$

in terms of the fields

$$\left\{ \hat{g}_{\hat{\mu}\hat{\nu}}, \hat{B}_{\hat{\mu}\hat{\nu}}, \hat{V}^I_{\hat{\mu}}, \hat{\phi} \right\} , \qquad (21)$$

which, in the case $D = 10$ corresponds to the bosonic sector of $N = 1, D = 10$ supergravity coupled to Yang–Mills and is interesting by itself. Here $\hat{F}^I$ is the curvature of the Yang–Mills vector field $\hat{V}^I$, I is a Yang–Mills index (which we raise and lower with δ_{IJ}) and g is the coupling constant. $\hat{H}$ contains now a Yang–Mills Chern–Simons term:

$$\hat{F}^I_{\hat{\mu}\hat{\nu}}(\hat{V}) = 2\partial_{[\hat{\mu}}\hat{V}^I_{\hat{\nu}]} - f_{KL}{}^I\hat{V}^K_{\hat{\mu}}\hat{V}^L_{\hat{\nu}} ,$$

$$\hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}} = \partial_{[\hat{\mu}}\hat{B}_{\hat{\nu}\hat{\rho}]} - \tfrac{1}{2g^2}\left[\hat{V}^I_{[\hat{\mu}}\hat{F}_{\hat{\nu}\hat{\rho}]I}(\hat{V}) + \tfrac{1}{3}f_{IJK}\hat{V}^I_{[\hat{\mu}}\hat{V}^J_{\hat{\nu}}\hat{V}^K_{\hat{\rho}]} \right] . \qquad (22)$$

In principle, there is an ambiguity in the relative sign between $\partial\hat{B}$ and the Yang–Mills Chern–Simons term. In fact, there are two theories whose only difference is this relative sign and which are related by the change of sign of $\hat{B}_{\hat{\mu}\hat{\nu}}$ ($\mathbb{Z}_2^{(A)}$) which is no longer a symmetry of each separate theory. We, therefore, anticipate that the group D_4 is broken to $\mathbb{Z}_2^{(B)} \times \mathbb{Z}_2^{(S)}$ in each theory. In fact $\mathbb{Z}_2^{(A)}$ is a duality transformation that brings us from one theory to the other, exactly as happens in the Type II duality of Ref. [17]. From the σ-model point of view, these theories are related by a change of the sign of $\hat{B}_{\hat{\mu}\hat{\nu}}$ and the simultaneous interchange of left- and right-movers. For the sake of definiteness, we will work with the above choice of relative sign.

Following the standard procedure for dimensional reduction [25] (see also Section 1) we get in the $(D-1)$-dimensional theory one graviton, one axion, two Abelian vector fields, one non-Abelian vector field, the dilaton, a scalar and a set of additional scalars that transform in the adjoint representation of the Yang–Mills group:

$$\left\{ g_{\mu\nu}, B_{\mu\nu}, A_\mu, B_\mu, V^I_\mu, \phi, k, \ell^I \right\} . \qquad (23)$$

These fields are related to the D–dimensional fields by

$$g_{\mu\nu} = \hat{g}_{\mu\nu} - \hat{g}_{\underline{x}\mu}\hat{g}_{\underline{x}\nu}/\hat{g}_{\underline{xx}},$$

$$B_{\mu\nu} = \hat{B}_{\mu\nu} + \hat{g}_{\underline{x}[\mu}\hat{B}_{\nu]\underline{x}}/\hat{g}_{\underline{xx}} - \tfrac{1}{2g^2}\hat{V}^I_{\underline{x}}\hat{g}_{\underline{x}[\mu}\hat{V}_{\nu]I}/\hat{g}_{\underline{xx}},$$

$$V^I_\mu = \hat{V}^I_\mu - \hat{V}^I_{\underline{x}}\hat{g}_{\underline{x}\mu}/\hat{g}_{\underline{xx}}, \qquad A_\mu = \hat{g}_{\underline{x}\mu}/\hat{g}_{\underline{xx}},$$

$$B_\mu = \hat{B}_{\underline{x}\mu} - \tfrac{1}{2g^2}\hat{V}^I_{\underline{x}}\hat{V}_{\mu I} + \tfrac{1}{2g^2}\hat{V}^I_{\underline{x}}\hat{V}_{\underline{x}I}\hat{g}_{\underline{x}\mu}/\hat{g}_{\underline{xx}},$$

$$\phi = \hat{\phi} - \tfrac{1}{4}\log|\hat{g}_{\underline{xx}}|, \qquad k = |\hat{g}_{\underline{xx}}|^{\frac{1}{2}}, \qquad \ell^I = \tfrac{1}{g}\hat{V}^I_{\underline{x}}. \qquad (24)$$

The D–dimensional fields decompose into the $(D-1)$-dimensional ones as follows:

$$\hat{g}_{\mu\nu} = g_{\mu\nu} - k^2 A_\mu A_\nu, \qquad \hat{B}_{\mu\nu} = B_{\mu\nu} + A_{[\mu}B_{\nu]} + \tfrac{1}{g}\ell^I A_{[\mu}V_{\nu]I},$$

$$\hat{g}_{\underline{x}\mu} = -k^2 A_\mu, \qquad \hat{B}_{\underline{x}\mu} = B_\mu + \tfrac{1}{2g}\ell^I V_{\mu I},$$

$$\hat{g}_{\underline{xx}} = -k^2, \qquad \hat{\phi} = \phi + \tfrac{1}{2}\log k, \qquad (25)$$

$$\hat{V}^I_{\underline{x}} = g\,\ell^I, \qquad \hat{V}^I_\mu = V^I_\mu + g\,\ell^I A_\mu.$$

The $(D-1)$-dimensional vector fields and two-form are defined in such a way that they transform in a standard way specified below under the (infinitesimal) gauge symmetries that they inherit from the D–dimensional fields:

1. x-independent reparametrizations $x' = x - \xi(x^\mu)$:

$$\delta\hat{g}_{\mu\nu} = 2\hat{g}_{\underline{x}(\mu}\partial_{\nu)}\xi, \qquad \delta\hat{B}_{\mu\nu} = -2\hat{B}_{\underline{x}[\mu}\partial_{\nu]}\xi,$$

$$\delta\hat{g}_{\mu\underline{x}} = \hat{g}_{\underline{xx}}\partial_\mu\xi, \qquad \delta\hat{V}^I_\mu = \hat{V}^I_{\underline{x}}\partial_\mu\xi, \qquad (26)$$

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2. x–independent gauge transformations of the axion field:

$$\delta\hat{B}_{\hat{\mu}\hat{\nu}} = 2\partial_{[\hat{\mu}}\hat{\Sigma}_{\hat{\nu}]}\,, \tag{27}$$

3. x–independent gauge transformations of the Yang–Mills field (accompanied by a Nicolai–Townsend (N–T) transformation of the axion two-form)

$$
\begin{aligned}
\delta\hat{V}_{\hat{\mu}}^{I} &= \partial_{\hat{\mu}}\Lambda^{I} + f_{JK}{}^{I}\Lambda^{J}\hat{V}_{\hat{\mu}}^{K}\,, \\[2mm]
\delta\hat{B}_{\hat{\mu}\hat{\nu}} &= \tfrac{1}{g^{2}}\hat{V}_{[\hat{\mu}}^{I}\partial_{\hat{\nu}]}\Lambda_{I}\,.
\end{aligned}
\tag{28}
$$

These three gauge symmetries correspond to the following four gauge symmetries of the $(D-1)$-dimensional fields:

1. Gauge transformations of the vector field A_{μ} (plus N–T transformation of the axion two-form):

$$
\begin{aligned}
\delta A_{\mu} &= \partial_{\mu}\xi\,, \\[2mm]
\delta B_{\mu\nu} &= -B_{[\mu}\partial_{\nu]}\xi\,,
\end{aligned}
\tag{29}
$$

2. Gauge transformations of the axion field

$$\delta B_{\mu\nu} = 2\partial_{[\mu}\Sigma_{\nu]} \tag{30}$$

where $\Sigma_{\mu} = \hat{\Sigma}_{\mu}$,

3. Gauge transformations of the vector field B_{μ} (plus a N–T transformation)

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$$\delta B_\mu \;=\; \partial_\mu \Sigma \,,$$

$$\delta B_{\mu\nu} \;=\; -A_{[\mu}\partial_{\nu]}\Sigma \,, \tag{31}$$

where $\Sigma = -\hat{\Sigma}_{\underline{x}}$,

4. Gauge transformation of the vector field V_μ (plus a N–T transformation)

$$\delta V_\mu^I \;=\; \partial_\mu \Lambda^I + f_{JK}{}^I \Lambda^J V_\mu^K \,,$$

$$\delta B_{\mu\nu} \;=\; \tfrac{1}{g^2} V_{[\mu}^I \partial_{\nu]}\Lambda_I \,. \tag{32}$$

The gauge–invariant $(D-1)$-dimensional vector and axion field strengths are, accordingly

$$F_{\mu\nu}(A) \;=\; 2\partial_{[\mu}A_{\nu]} \,, \qquad\qquad F_{\mu\nu}(B) = 2\partial_{[\mu}B_{\nu]} \,,$$

$$F_{\mu\nu}^I(V) \;=\; 2\partial_{[\mu}V_{\nu]}^I - f_{JK}{}^I V_\mu^J V_\nu^K \,,$$

$$H_{\mu\nu\rho} \;=\; \partial_{[\mu}B_{\nu\rho]} + \tfrac{1}{2}A_{[\mu}F_{\nu\rho]}(B) + \tfrac{1}{2}B_{[\mu}F_{\nu\rho]}(A)$$

$$-\tfrac{1}{2g^2}\left[V_{[\mu}^I F_{\nu\rho]I}(V) + \tfrac{1}{3}f_{IJK}V_{[\mu}^I V_\nu^J V_{\rho]}^K \right] \,. \tag{33}$$

The dimensionally reduced action is given by

$$S_{\text{Sugra+YM(red)}}^{(D-1)} \;=\; \tfrac{1}{2}\int d^{(D-1)}x \,\sqrt{-g}\, e^{-2\phi}\left\{ -R + 4(\partial\phi)^2 - \tfrac{3}{4}H^2 \right.$$

$$\left. +\tfrac{1}{4g^2}\left(\frac{k^2+\ell^2}{k^2}\right)\text{Tr }F^2(V) - \left[(\partial\log k)^2 + \frac{1}{2k^2}(\mathcal{D}\ell)^2\right] \right.$$

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Name	$g_{\mu\nu}$	$B_{\mu\nu}$	A_μ	B_μ	e^ϕ	k	ℓ	V_μ	$1/g^2$	$S^{(9)}$
$SO^\uparrow(1,1)_{\text{Sugra}}$	0	0	1	-1	0	-1	-1	0	0	0
$SO^\uparrow(1,1)_{x-y}$	1	1	1	0	$\frac{7}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	1	0
$SO^\uparrow(1,1)_{\text{string}}$	0	0	0	0	1	0	0	0	0	-2

Table 2: *Weights of the* Sugra+YM *fields under the two* $SO^\uparrow(1,1)$ *(pseudo-) duality symmetries of the action* $S^{(9)}_{\text{Sugra+YM(red)}}$ *and the third* $SO^\uparrow(1,1)$ *which scales it in* $D = 9$.

$$+\tfrac{1}{4}\left[\frac{(2k^2+\ell^2)^2}{4k^2}F^2(A) + k^{-2}F^2(B) + \frac{\ell^2}{k^2}F(A)F(B)\right]$$

$$+\tfrac{1}{g}F^I(V)\left[\ell_I\left(\frac{2k^2+\ell^2}{4k^2}\right)F(A) + \frac{\ell_I}{2k^2}F(B)\right]\right\}, \qquad (34)$$

where $\ell^2 \equiv \ell^I\ell_I$, $(\mathcal{D}\ell)^2 \equiv \mathcal{D}_\mu\ell^I\mathcal{D}^\mu\ell_I$ and the covariant derivative $\mathcal{D}_\mu\ell^I$ is defined by

$$\mathcal{D}_\mu\ell^I = \partial_\mu\ell^I + f_{JK}{}^I\ell^J V_\mu^K. \qquad (35)$$

As happened in the previous section, the action is invariant under a rigid $SO^\uparrow(1,1)_{\text{Sugra}} \times \mathbb{Z}_2^{(B)} \times \mathbb{Z}_2^{(S)}$ symmetry. The continuous $SO^\uparrow(1,1)_{\text{Sugra}}$ transformations are scalings and the weights of the fields in nine dimensions are given in Table 2.

The action of the discrete B transformation that generates the first $\mathbb{Z}_2^{(B)}$ is

$$\tilde{A}_\mu = B_\mu, \qquad\qquad \tilde{B}_\mu = A_\mu,$$

$$\tilde{k}^2 = \frac{4k^2}{(\ell^2+2k^2)^2}, \qquad\qquad \tilde{\ell}^I = \frac{2\ell^I}{\ell^2+2k^2} \qquad (36)$$

and the action of the transformation S which generates $\mathbb{Z}_2^{(S)}$ is

$$
\begin{aligned}
A'_\mu &= -A_\mu\,, & B'_\mu &= -B_\mu\,, \\[2mm]
\ell^{I\prime} &= -\ell^I\,.
\end{aligned}
\tag{37}
$$

If one allows for transformations of the coupling constant g (*pseudo-dualities* [28]) the $SO^\uparrow(1,1)_{x-y}$ symmetry of the action found in the previous section can be promoted to a symmetry in the presence of Yang–Mills fields. The weights are given in the second row of Table 2 for nine dimensions. Finally, the $SO^\uparrow(1,1)_{\text{string}}$ trivially extends to this case. In consequence the full symmetry group of the equations of motion is

$$
SO^\uparrow(1,1)_{\text{Sugra}} \times\ SO^\uparrow(1,1)_{x-y} \times\ SO^\uparrow(1,1)_{\text{string}} \times\ \mathbb{Z}_2^{(B)} \times \mathbb{Z}_2^{(S)}.
\tag{38}
$$

We see that the non-Abelian discrete group D_4 from the previous section indeed breaks into the Abelian group $\mathbb{Z}_2^{(B)} \times \mathbb{Z}_2^{(S)}$, as mentioned above, due to the presence of the Yang–Mills Chern–Simons term in the axion field strength.

In D dimensions the transformation B corresponds to the following generalization of Buscher's T duality rules[12]:

$$
\begin{aligned}
\tilde{\hat{g}}_{\mu\nu} &= \hat{g}_{\mu\nu} + \left[\hat{g}_{\underline{x}\underline{x}}\hat{G}_{\underline{x}\mu}\hat{G}_{\underline{x}\nu} - 2\hat{G}_{\underline{x}\underline{x}}\hat{G}_{\underline{x}(\mu}\hat{g}_{\nu)\underline{x}}\right]/\hat{G}_{\underline{x}\underline{x}}^2\,, \\[3mm]
\tilde{\hat{B}}_{\mu\nu} &= \hat{B}_{\mu\nu} - \hat{G}_{\underline{x}[\mu}\hat{G}_{\nu]\underline{x}}/\hat{G}_{\underline{x}\underline{x}}\,, \\[3mm]
\tilde{\hat{g}}_{\underline{x}\mu} &= -\hat{g}_{\underline{x}\mu}/\hat{G}_{\underline{x}\underline{x}} + \hat{g}_{\underline{x}\underline{x}}\hat{G}_{\underline{x}\mu}/\hat{G}_{\underline{x}\underline{x}}^2\,, & \tilde{\hat{B}}_{\underline{x}\mu} &= -\hat{B}_{\underline{x}\mu}/\hat{G}_{\underline{x}\underline{x}} + \hat{G}_{\underline{x}\mu}/\hat{G}_{\underline{x}\underline{x}}\,, \\[3mm]
\tilde{\hat{g}}_{\underline{x}\underline{x}} &= \hat{g}_{\underline{x}\underline{x}}/\hat{G}_{\underline{x}\underline{x}}^2\,, & \tilde{\hat{\phi}} &= \hat{\phi} - \tfrac{1}{2}\log|\hat{G}_{\underline{x}\underline{x}}|\,, \\[3mm]
\tilde{\hat{V}}_{\underline{x}}^I &= -\hat{V}_{\underline{x}}^I/\hat{G}_{\underline{x}\underline{x}}\,, & \tilde{\hat{V}}_\mu^I &= \hat{V}_\mu^I - \hat{V}_{\underline{x}}^I\hat{G}_{\underline{x}\mu}/\hat{G}_{\underline{x}\underline{x}}\,,
\end{aligned}
\tag{39}
$$

with

---

[12]The analogous result for Abelian vector fields was first given in Refs. [29, 30]

$$\hat{G}_{\hat\mu\hat\nu} = \hat{g}_{\hat\mu\hat\nu} + \hat{B}_{\hat\mu\hat\nu} - \frac{1}{2g^2}\hat{V}_{\hat\mu}^I \hat{V}_{\hat\nu I}\,. \tag{40}$$

These are the duality rules of the theory corresponding to the choice of sign in Eq. (22). Since the theory corresponding to the other choice can be obtained by performing an A transformation ($\hat{B}_{\hat\mu\hat\nu} \to -\hat{B}_{\hat\mu\hat\nu}$), its duality rules can be also obtained by performing an A transformation in the above rules.

Note that the factor $\sqrt{-\hat{g}}\,e^{-2\hat\phi}$ which occurs as an overall factor in the D–dimensional Lagrangian is invariant under this set of transformations, since the determinant of the metric transforms as follows:

$$\sqrt{-\tilde{\hat{g}}} \;=\; \hat{G}_{\underline{xx}}^{-1}\,\sqrt{-\hat{g}}\,. \tag{41}$$

Finally, it is remarkable that the duality rules of $\hat{G}_{\hat\mu\hat\nu}$ are of the following particular simple form:

$$
\begin{aligned}
\tilde{\hat{G}}_{\mu\nu} &= \hat{G}_{\mu\nu} - \hat{G}_{\underline{x}\mu}\hat{G}_{\nu\underline{x}}/\hat{G}_{\underline{xx}}, &\qquad \tilde{\hat{G}}_{\underline{xx}} &= 1/\hat{G}_{\underline{xx}}, \\
\tilde{\hat{G}}_{\underline{x}\mu} &= +\hat{G}_{\underline{x}\mu}/\hat{G}_{\underline{xx}}, &\qquad \tilde{\hat{G}}_{\mu\underline{x}} &= -\hat{G}_{\mu\underline{x}}/\hat{G}_{\underline{xx}}\,.
\end{aligned}
\tag{42}
$$

3   Duality In Presence Of One Abelian Vector Field

A particularly interesting case of the action considered in the previous section is the one in which the gauge group is $(U(1))^n$, because new duality transformations that interchange components of the metric or axion with components of the Abelian vector fields are now possible[13]. It is known that, as a consequence, the $O(1,1)_{\text{Sugra}}$ duality group of Section 2 becomes $O(1,n+1)_{\text{Sugra}}$ [30, 29, 26]. The symmetries $SO^\uparrow(1,1)_{x-y} \times SO^\uparrow(1,1)_{\text{string}}$ do not extend to larger symmetry groups and remain as in the previous section.

We are only going to consider the case $n = 1$ since it is the simplest and shows all the interesting features.

_____

[13]The Abelian case is obtained by first rescaling $V \to gV$ and then putting all structure constants equal to zero. Observe that after this rescaling, the weight of V under $SO^\uparrow(1,1)_{x-y}$ becomes $\frac{1}{2}$ and the "pseudo-duality" becomes a duality.

The $(D-1)$-dimensional action is again given by Eq. (34), specified to the Abelian case. This action can be rewritten into the following form, presented in Refs. [12, 26], which makes the $O(1,2)_{\text{Sugra}}$–invariance manifest :

$$S^{(D-1)}_{\text{Sugra}+U(1)} = \frac{1}{2} \int d^{(D-1)}x \, \sqrt{-g} \, e^{-2\phi} \left\{ -R + 4(\partial\phi)^2 - \frac{3}{4}H^2 \right.$$

$$\left. + \frac{1}{8}\text{Tr}\left(\partial_\mu M^{-1}\partial^\mu M\right) - \frac{1}{4}\mathcal{F}^i_{\mu\nu}(\mathcal{A})\mathcal{F}^{\mu\nu}_i(\mathcal{A}) \right\}, \qquad (43)$$

where

$$\mathcal{A}^i_\mu = \begin{pmatrix} A_\mu \\ B_\mu \\ V_\mu \end{pmatrix}, \qquad \mathcal{F}^i_{\mu\nu}(\mathcal{A}) = \partial_\mu \mathcal{A}^i_\nu - \partial_\nu \mathcal{A}^i_\mu ,$$

$$\qquad (44)$$

$$\mathcal{F}_{\mu\nu i} = M^{-1}_{ij}\mathcal{F}^j_{\mu\nu}, \quad i = 1,2,3$$

and

$$M^{-1}_{ij} = \begin{pmatrix} -(2k^2+\ell^2)^2/4k^2 & -\ell^2/2k^2 & -(2k^2\ell+\ell^3)/2k^2 \\[2mm] -\ell^2/2k^2 & -1/k^2 & -\ell/k^2 \\[2mm] -(2k^2\ell+\ell^3)/2k^2 & -\ell/k^2 & -(k^2+\ell^2)/k^2 \end{pmatrix}. \qquad (45)$$

The explictly $O(1,2)_{\text{Sugra}}$–invariant axion field-strength can be written as

$$H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]} + \frac{1}{2}\mathcal{A}^i_{[\mu}\mathcal{F}^j_{\nu\rho]}(\mathcal{A})\eta_{ij}, \qquad (46)$$

where η is the $O(1,2)$ metric in an off-diagonal basis:

$$\eta_{ij} = \eta^{ij} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \qquad (47)$$

Note that the matrix M^{-1}_{ij} itself is an $O(1,2)$ matrix since it leaves invariant the metric η

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$$(M^{-1})^T \eta M^{-1} = \eta \,. \tag{48}$$

Under an $O(1,2)_{\mathrm{Sugra}}$ transformation Ω the vectors and scalars transform in this way [12]:

$$\mathcal{A}'_\mu = \Omega \mathcal{A}_\mu \,, \qquad (M^{-1})' = \Omega M^{-1} \Omega^T \,. \tag{49}$$

Let us now consider the explicit form of the $O(1,2)_{\mathrm{Sugra}}$ transformations in more detail. First of all, $O(1,2)_{\mathrm{Sugra}} = SO^\uparrow(1,2)_{\mathrm{Sugra}} \times \mathbb{Z}_2^{(B)} \times \mathbb{Z}_2^{(S)}$. The $\mathbb{Z}_2^{(B)} \times \mathbb{Z}_2^{(S)}$ transformations are again given by Eqs. (36,37), specified to the Abelian case. The $\mathbb{Z}_2^{(B)}$ leads to the generalized Buscher's rules given in Eqs. (39). We next consider the continuous $SO^\uparrow(1,2)_{\mathrm{Sugra}}$ transformations. It is convenient to first consider the $so(1,2)$ Lie algebra with generators J_1, J_2 and J_3:

$$[J_1, J_2] = J_3 \,, \quad [J_2, J_3] = -J_1 \,, \quad [J_3, J_1] = J_2 \,. \tag{50}$$

In a 3×3-matrix representation they satisfy the symmetry property:

$$(J_i \eta)^T = -(J_i \eta) \,, \qquad i = 1, 2, 3 \,. \tag{51}$$

In the off-diagonal basis Eq. (47) the generators J_i are represented by the following 3×3 matrices[14]:

$$J_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \,, \qquad J_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ -1 & -1 & 0 \end{pmatrix} \,, \tag{52}$$

$$J_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \,.$$

In terms of

$$J_+ = (J_2 + J_1)/\sqrt{2} \,, \qquad J_- = (J_2 - J_1)/\sqrt{2} \,, \tag{53}$$

---

[14] Note that J_1, J_3 generate $so(1,1)$ subalgebras while J_2 generates an $so(2)$ subalgebra.

we have the following commutation relations:

$$[J_3, J_+] = J_+ , \quad [J_3, J_-] = -J_- , \quad [J_+, J_-] = J_3 . \tag{54}$$

The exponentiation of J_3, J_+ and J_- leads to the following $SO^\uparrow(1,2)$ group elements:

$$\exp \alpha J_3 = \begin{pmatrix} e^\alpha & 0 & 0 \\ 0 & e^{-\alpha} & 0 \\ 0 & 0 & 1 \end{pmatrix} ,$$

$$\exp \beta J_- = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2}\beta^2 & 1 & -\beta \\ -\beta & 0 & 1 \end{pmatrix} , \tag{55}$$

$$\exp \gamma J_+ = \begin{pmatrix} 1 & \frac{1}{2}\gamma^2 & -\gamma \\ 0 & 1 & 0 \\ 0 & -\gamma & 1 \end{pmatrix} .$$

An arbitrary $SO^\uparrow(1,2)$ group element can be written as the product of these basis elements. Using Eqs. (49) one can verify that the transformations in the basis above induce the following transformations in $D-1$ dimensions. First of all, the transformation generated by J_3 in $D-1$ dimensions is just the $SO^\uparrow(1,1)_{\mathrm{Sugra}}$ transformation of previous sections. We next consider the transformation generated by J_-. The $(D-1)$-dimensional rules are given by

$$A'_\mu = A_\mu , \qquad\qquad (k^2)' = k^2 ,$$

$$B'_\mu = B_\mu - \beta V_\mu + \tfrac{1}{2}\beta^2 A_\mu , \qquad\qquad \ell' = \ell + \beta , \tag{56}$$

$$V'_\mu = V_\mu - \beta A_\mu .$$

The corresponding transformation of the D–dimensional fields is

$$\hat{V}'_{\underline{x}} = \hat{V}_{\underline{x}} + \beta ,$$

$$\hat{B}'_{\underline{x}\mu} = \hat{B}_{\underline{x}\mu} - \tfrac{1}{2}\beta\hat{V}_\mu . \tag{57}$$

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All other fields are invariant. It turns out that this transformation is a special finite $U(1)$ gauge transformation accompanied of a N–T transformation

$$\hat{V}'_{\hat{\mu}} = \hat{V}_{\hat{\mu}} + \partial_{\hat{\mu}}\Lambda \,,$$

$$\hat{B}'_{\hat{\mu}\hat{\nu}} = \hat{B}_{\hat{\mu}\hat{\nu}} + \hat{V}_{[\hat{\mu}}\partial_{\hat{\nu}]}\Lambda \,, \tag{58}$$

with the parameter Λ given by $\Lambda = \beta x$ [11].

Finally, we consider the transformation generated by J_+ . This is the solution-generating transformation which was first introduced by Sen [11]. The rules in $D-1$ dimensions are given by

$$A'_\mu = A_\mu + \tfrac{1}{2}\gamma^2 B_\mu - \gamma V_\mu \,, \qquad (k^2)' = \left(\frac{4k}{4+4\gamma\ell+(\ell^2+2k^2)\gamma^2}\right)^2 ,$$

$$B'_\mu = B_\mu \,, \qquad\qquad\qquad \ell' = \frac{4\ell+2(\ell^2+2k^2)\gamma}{4+4\gamma\ell+(\ell^2+2k^2)\gamma^2} \,, \tag{59}$$

$$V'_\mu = -\gamma B_\mu + V_\mu \,.$$

This transformation does not correspond to any gauge transformation in D dimensions. Indeed, as we have seen, from the entire group $O(1,2)_{\text{Sugra}}$, only this transformation and Buscher's (B) are non-trivial solution generating transformations. They correspond to the subgroup $O(1) \times O(2)$, while the other transformations belong to the coset $O(1,2)_{\text{Sugra}}/(O(1) \times O(2))$ of pure gauge transformations [11].

It is instructive to also consider the infinitesimal form of the $SO^\uparrow(1,2)$ transformations in $D-1$ dimensions:

$$\delta A_\mu = \alpha A_\mu - \gamma V_\mu \,,$$

$$\delta B_\mu = -\alpha B_\mu - \beta V_\mu \,,$$

$$\delta V_\mu = -\beta A_\mu - \gamma B_\mu \,,$$

$$\delta k^2 = -2\alpha k^2 - 2\gamma \ell k^2 \,,$$

$$\delta \ell = -\alpha \ell + \beta - \tfrac{1}{2}\gamma(\ell^2 - 2k^2) \,. \tag{60}$$

These infinitesimal rules lead to the commutation relations of the $so(1,2)$ algebra:

$$[\delta_\alpha, \delta_\beta] = \delta_{\beta'} \qquad \text{with} \quad \beta' = \alpha\beta \,,$$

$$[\delta_\alpha, \delta_\gamma] = \delta_{\gamma'} \qquad \text{with} \quad \gamma' = -\alpha\gamma \,, \tag{61}$$

$$[\delta_\beta, \delta_\gamma] = \delta_{\alpha'} \qquad \text{with} \quad \alpha' = \gamma\beta \,.$$

One may verify that the action (34) is indeed left invariant by the infinitesimal transformations given above.

The rules of transformation of the scalars k^2 and ℓ are complicated and they lead to even more complicated rules for the D–dimensional fields that are given in Appendix A. They considerably simplify if we assume that $\hat{V}_\mu = \hat{V}_{\underline{x}} = 0$[15]. In that case they are given by

---

[15]The J_+–transformation is often used to construct a charged solution ($\hat{V}' \neq 0$) out of an uncharged one ($\hat{V} = 0$). Therefore, most times it is enough to know the J_+–transformation for the case that $\hat{V}_\mu = \hat{V}_{\underline{x}} = 0$.

$$\hat{g}'_{\mu\nu} = \hat{g}_{\mu\nu} - \frac{\hat{g}_{\underline{x}\mu}\hat{g}_{\underline{x}\nu}}{\hat{g}_{\underline{xx}}} + \frac{4\hat{g}_{\underline{xx}}}{(2-\gamma^2\hat{g}_{\underline{xx}})^2}\left(\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}} + \tfrac{1}{2}\gamma^2\hat{B}_{\underline{x}\mu}\right)\left(\frac{\hat{g}_{\underline{x}\nu}}{\hat{g}_{\underline{xx}}} + \tfrac{1}{2}\gamma^2\hat{B}_{\underline{x}\nu}\right),$$

$$\hat{B}'_{\mu\nu} = \hat{B}_{\mu\nu} + 2\gamma^2\frac{\hat{g}_{\underline{x}[\mu}\hat{B}_{\underline{x}\nu]}}{2-\gamma^2\hat{g}_{\underline{xx}}}, \qquad\qquad \hat{B}'_{\underline{x}\mu} = \frac{2\hat{B}_{\underline{x}\mu}}{2-\gamma^2\hat{g}_{\underline{xx}}},$$

$$\hat{g}'_{\underline{x}\mu} = \frac{4\hat{g}_{\underline{xx}}}{(2-\gamma^2\hat{g}_{\underline{xx}})^2}\left(\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}} + \tfrac{1}{2}\gamma^2\hat{B}_{\underline{x}\mu}\right), \quad \hat{g}'_{\underline{xx}} = \frac{4\hat{g}_{\underline{xx}}}{(2-\gamma^2\hat{g}_{\underline{xx}})^2}, \tag{62}$$

$$\hat{V}'_\mu = -\gamma\hat{B}_{\underline{x}\mu} - 2\gamma\frac{\hat{g}_{\underline{xx}}}{2-\gamma^2\hat{g}_{\underline{xx}}}\left(\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}} + \tfrac{1}{2}\gamma^2\hat{B}_{\underline{x}\mu}\right),$$

$$\hat{V}'_{\underline{x}} = -2\gamma\frac{\hat{g}_{\underline{xx}}}{2-\gamma^2\hat{g}_{\underline{xx}}}, \qquad\qquad \hat{\phi}' = \hat{\phi} + \tfrac{1}{2}\log\frac{2}{2-\gamma^2\hat{g}_{\underline{xx}}}.$$

The above formulae have been given in Eq. (2.12) of Ref. [2] for the special case that $\hat{g}_{\underline{x}\mu} = \hat{B}_{\underline{x}\mu} = 0$. In that case the fields $\hat{g}_{\underline{x}\mu}, \hat{B}_{\underline{x}\mu}$ and $\hat{V}_\mu$ remain zero after the Sen transformation; $\hat{g}_{\mu\nu}$ and $\hat{B}_{\mu\nu}$ remain invariant while the other fields transform as follows:

$$\hat{g}'_{\underline{xx}} = \frac{4\hat{g}_{\underline{xx}}}{(2-\gamma^2\hat{g}_{\underline{xx}})^2},$$

$$\hat{V}'_{\underline{x}} = -2\gamma\frac{\hat{g}_{\underline{xx}}}{2-\gamma^2\hat{g}_{\underline{xx}}}, \tag{63}$$

$$\hat{\phi}' = \hat{\phi} + \tfrac{1}{2}\log\frac{2}{2-\gamma^2\hat{g}_{\underline{xx}}}.$$

These formulae differ from Eq. (2.12) of [2]. To obtain the formula of Ref. [2] (with parameter α_H) one should perform successively an α, β and γ transformation with parameters[16]

$$\alpha = 2\log\cosh\alpha_H \qquad \beta = -i\sqrt{2}\frac{\sinh\alpha_H}{\cosh^3\alpha_H} \qquad \gamma = i\sqrt{2}\tanh\alpha_H. \tag{64}$$

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[16]Note that the normalization of the vector field V_μ in [2] differs from ours with a factor of $2i$.

Finally, the infinitesimal form of the $SO^{\uparrow}(1,2)$ tranformations in D dimensions is given by:

$$\delta\hat{g}_{\underline{xx}} = -2\alpha\hat{g}_{\underline{xx}} - 2\gamma\hat{V}_{\underline{x}}\hat{g}_{\underline{xx}},$$

$$\delta\hat{g}_{\underline{x}\mu} = -\alpha\hat{g}_{\underline{x}\mu} - \gamma\left(\hat{V}_{\mu}\hat{g}_{\underline{xx}} + \hat{V}_{\underline{x}}\hat{g}_{\underline{x}\mu}\right),$$

$$\delta\hat{g}_{\mu\nu} = -2\gamma\hat{V}_{(\mu}\hat{g}_{\underline{x}\nu)},$$

$$\delta\hat{B}_{\underline{x}\mu} = -\alpha\hat{B}_{\underline{x}\mu} - \tfrac{1}{2}\beta\hat{V}_{\mu} - \tfrac{1}{2}\gamma\left(\hat{V}_{\underline{x}}\hat{B}_{\underline{x}\mu} - \hat{g}_{\underline{x}\mu}\hat{V}_{\underline{x}} + \hat{g}_{\underline{xx}}\hat{V}_{\mu}\right),$$

$$\delta\hat{B}_{\mu\nu} = -\gamma\left(\hat{V}_{[\mu}\hat{B}_{\underline{x}\nu]} + \hat{g}_{\underline{x}[\mu}\hat{V}_{\nu]}\right), \qquad (65)$$

$$\delta\hat{V}_{\underline{x}} = -\alpha\hat{V}_{\underline{x}} + \beta - \gamma\left(g_{\underline{xx}} + \tfrac{1}{2}\hat{V}_{\underline{x}}^{2}\right),$$

$$\delta\hat{V}_{\mu} = -\gamma\left(\hat{g}_{\underline{x}\mu} + \hat{B}_{\underline{x}\mu} + \tfrac{1}{2}\hat{V}_{\underline{x}}\hat{V}_{\mu}\right),$$

$$\delta\hat{\phi} = -\tfrac{1}{2}\alpha - \tfrac{1}{2}\gamma\hat{V}_{\underline{x}}.$$

As in the $(D-1)$-dimensional case (see above) these infinitesimal rules lead to the commutation relations of the $so(1,2)$ algebra given in (62).
To conclude we give the full symmetry group of the equations of motion:

$$SO^{\uparrow}(1,2)_{\text{Sugra}} \times\ SO^{\uparrow}(1,1)_{x-y} \times\ SO{\uparrow}(1,1)_{\text{string}} \times\ \mathbb{Z}_2^{(B)} \times\ \mathbb{Z}_2^{(S)}. \qquad (66)$$

4 α' Corrections

In the first section we considered the zero-slope limit of the bosonic sector heterotic string effective action and in the following sections we have added Yang–Mills fields to it. This is consistent from the supergravity point of view. However, from the heterotic string theory point of view, the Yang–Mills term

is already first order in α' and, strictly speaking, one has to add to the action all the other terms linear in α'. Therefore, to this order, the action that we have to consider is [6]

$$S_{\text{Sugra}+\alpha'}^{(D)} = \tfrac{1}{2} \int d^D x \ \sqrt{-\hat{g}} \ e^{-2\hat{\phi}} \Big\{ -\hat{R} + 4(\partial\hat{\phi})^2 - \tfrac{3}{4}(\hat{H}^{(1)})^2$$

$$+ \tfrac{1}{4} \Big[\beta \text{Tr} \ \hat{F}^2 \left(\hat{V} \right) + \alpha \text{Tr} \ \hat{R}^2 \left(\hat{\Omega}^{(0)} \right) \Big] \Big\} , \qquad (67)$$

Here $\hat{H}^{(1)}$ is the axion field strength up to linear order in α'

$$\hat{H}^{(1)} = \hat{H}^{(0)} - \left(\beta \hat{\omega}^{YM} + \alpha \hat{\omega}^{(0)L} \right) , \qquad (68)$$

$\hat{H}^{(0)}$ is the zero order in α' axion field strength, $\hat{\omega}^{YM}$ is the Yang–Mills Chern–Simons form and $\hat{\omega}^{(0)L}$ is the (zero order in α') Lorentz Chern–Simons form. They are respectively given by[17]

$$\hat{H}^{(0)} = \partial\hat{B} ,$$

$$\hat{\omega}^{YM} = \tfrac{1}{2}\hat{V}^I \hat{F}_I \left(\hat{V} \right) + \tfrac{1}{6} f_{IJK} \hat{V}^I \hat{V}^J \hat{V}^K , \qquad (69)$$

$$\hat{\omega}^{(0)L} = \tfrac{1}{2}\hat{\Omega}^{(0)\hat{a}\hat{b}} \hat{R}^{\hat{a}\hat{b}} \left(\hat{\Omega}^{(0)} \right) + \tfrac{1}{3}\hat{\Omega}^{(0)\hat{a}\hat{b}} \ \hat{\Omega}^{(0)\hat{a}\hat{c}} \ \hat{\Omega}^{(0)\hat{c}\hat{b}} .$$

where the (zero order in α') torsionful spin connection $\hat{\Omega}^{(0)}$ is defined by

$$\hat{\Omega}_{\hat{\mu}}{}^{(0)\hat{a}\hat{b}} = \hat{\omega}_{\hat{\mu}}{}^{\hat{a}\hat{b}} + \tfrac{3}{2}\hat{H}_{\hat{\mu}}{}^{(0)\hat{a}\hat{b}} . \qquad (70)$$

Finally, α and β are constants which are related to α' as follows

$$\alpha = 2\alpha' , \qquad \beta = \tfrac{1}{15}\alpha' . \qquad (71)$$

Note that the action used in the previous sections can be obtained by setting $\alpha = 0$ and $\beta = 1/g^2$.

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[17]We use a short-hand notation in which the antisymmetrized world indices are not indicated.

Once we assume that all the solutions to the equations of motion derived from the above action have an isometry, we expect the usual duality group $SO(1,1)_{\text{Sugra}}$. The question we really need to address now is whether a generalization of Buscher's duality transformation exists.

This transformation should coincide with the B transformations found in previous sections in the appropriate limits (Buscher's original transformations Eqs. (6) in the limit $\alpha' \to 0$ and the generalization Eqs. (39,40)). Then, we are actually looking for the complete form of the α' corrections to Eqs. (6) and we know the contribution of the vector fields to them.

We are now going to argue, using a simple observation, that the corrections to the T duality rules can be obtained in a straightforward manner. We first note that the torsionful spin connection $\hat{\Omega}^{(0)}$ is a dependent field given in terms of the D–bein and the axion. This fixes the zero-th order duality rules of $\hat{\Omega}^{(0)}$. To calculate these we first have to give the duality rules of the D–bein. For this purpose we parametrize the D–bein as follows:

$$(\hat{e}_{\hat{\mu}}{}^{\hat{a}}) = \begin{pmatrix} e_{\mu}{}^{a} & kA_{\mu} \\ 0 & k \end{pmatrix}, \qquad (\hat{e}_{\hat{a}}{}^{\hat{\mu}}) = \begin{pmatrix} e_{a}{}^{\mu} & -A_{a} \\ 0 & k^{-1} \end{pmatrix}, \qquad (72)$$

where $A_a = e_a{}^{\mu}A_{\mu}$. Note that this is the first time that we are forced to use k instead of k^2. To lowest order in α' the duality rule of k^2 is given by $\tilde{k}^2 = 1/k^2$. This means that for k we have

$$\tilde{k} = \mp\frac{1}{k}. \qquad (73)$$

These two signs are not really different since the two possibilities are related to each other by a discrete Lorentz transformation (in tangent space) $\tilde{\hat{e}}_{\hat{\mu}}{}^{x} = -\hat{e}_{\hat{\mu}}{}^{x}$. In D dimensions this leads to the following lowest-order rule of the D-bein, (here we choose the upper sign in Eq. (73)):

$$\begin{aligned} \tilde{\hat{e}}_{\underline{x}}{}^{\hat{a}} &= \frac{1}{\hat{g}_{\underline{xx}}}\hat{e}_{\underline{x}}{}^{\hat{a}}, \\ \tilde{\hat{e}}_{\mu}{}^{\hat{a}} &= \hat{e}_{\mu}{}^{\hat{a}} - \frac{1}{\hat{g}_{\underline{xx}}}\left(\hat{g}_{\underline{x}\mu} - \hat{B}_{\underline{x}\mu}\right)\hat{e}_{\underline{x}}{}^{\hat{a}}, \end{aligned} \qquad (74)$$

so the dual of $\hat{\Omega}^{(0)}$ is

$$\tilde{\hat{\Omega}}_{\underline{x}}^{(0)\hat{a}\hat{b}} = -\hat{\Omega}_{\underline{x}}^{(0)\hat{a}\hat{b}}/\hat{G}_{\underline{xx}}^{(0)}, \quad \tilde{\hat{\Omega}}_{\mu}^{(0)\hat{a}\hat{b}} = \hat{\Omega}_{\mu}^{(0)\hat{a}\hat{b}} - \hat{\Omega}_{\underline{x}}^{(0)\hat{a}\hat{b}}\hat{G}_{\underline{x}\mu}^{(0)}/\hat{G}_{\underline{xx}}^{(0)}, \qquad (75)$$

where $\hat{G}_{\hat{\mu}\hat{\nu}}^{(0)} = \hat{g}_{\hat{\mu}\hat{\nu}} + \hat{B}_{\hat{\mu}\hat{\nu}}$ is the zero-slope limit of $\hat{G}_{\hat{\mu}\hat{\nu}}$. We now observe that this duality rule is identical to that of a non-Abelian vector field $\hat{V}_{\hat{\mu}}^I$ (see Eq. (39)), to lowest order in α', when we consider the pair of Lorentz indices $\hat{a}\hat{b}$ as a Yang–Mills index.

We can combine this with the observation that we already know how to construct duality-invariant actions for the Yang–Mills fields. In fact we can extend our results for the Yang–Mills fields to a more general action formula: given a vector field which, to lowest order in α', transforms as given in Eqs. (39), an action can be constructed which is duality invariant up to linear order in α'. The action is given by Eq. (20) (with the identification $1/g^2 = \beta = \alpha'/15$ and the Yang–Mills field being replaced by the vector field in question) and the corresponding duality rules are given by Eqs. (39).

We now apply the above action formula to the case that the gauge group of the vector field is given by the direct product of the Yang–Mills group times the D-dimensional Lorentz group. This leads to the action given in Eq. (67). The corresponding duality rules, to linear order in α' are given by Eqs. (39), where

$$\hat{G}_{\hat{\mu}\hat{\nu}} = \hat{g}_{\hat{\mu}\hat{\nu}} + \hat{B}_{\hat{\mu}\hat{\nu}} - \tfrac{1}{2}\left\{\alpha\hat{\Omega}_{\hat{\mu}}^{(0)\hat{a}\hat{b}}\hat{\Omega}_{\hat{\nu}}^{(0)\hat{a}\hat{b}} + \beta\hat{V}_{\hat{\mu}}^I\hat{V}_{I\hat{\nu}}\right\}. \qquad (76)$$

instead of Eq. (40).

We would like to stress the following points:

- The duality rules that we just have obtained considerably simplify if the gauge group is embedded into the holonomy group since in that case the last two terms in Eq. (76) cancel against each other. We note that for $\alpha = 0$ and Abelian vector fields, the duality rules of the gauge fields are those of Refs. [30, 29]. These rules can be derived using the σ-model approach if the gauge fields couple to the string via bosonic group coordinates. The same rules can also be derived for the case that the gauge fields couple via heterotic fermions to the string. However, in that case, to obtain the same answer, one has to take into account

the Yang–Mills anomaly. In the general case with $\alpha \neq 0$ one also should consider the Lorentz anomaly. In case the embedding is made, there is an anomaly cancellation which leads to the simplified duality rules mentioned above. In particular, the simplified duality rules of the vector fields are now the ones given in Ref. [31].

- Since the structure of the higher order in α' corrections seems to be such that the torsionful spin connection $\hat{\Omega}$ enters always in the same way as the Yang-Mills vector field $\hat{V}^I$ (apart from the fact that $\hat{\Omega}$ has to be redefined at each order but $\hat{V}^I$ does not), one may expect that the structure of higher order in α' corrections to the duality rules will be such that Eqs. (39) can still be used but the effective metric (76) will get higher order corrections in which $\hat{\Omega}$ and $\hat{V}^I$ will appear in the same way. If this was true, the embedding of the gauge group into the holonomy group would produce a cancellation of all the corrections and Buscher's original transformations would not get corrections. This is also consistent with the results in Ref. [31].

- It is interesting to note that a combination similar to the effective metric given in Eq. (76) also appears in Ref. [32]. There it was observed that a manifestly supersymmetric way of cancelling the Green-Schwarz anomaly in the heterotic string effective action is to make a redefinition of the metric. The new metric is essentially our effective metric. This suggests that this effective metric could play an important role in the heterotic string effective action and that it could be the right object in terms of which many terms could be expressed. A geometrical or physical interpretation is still lacking.

We thus conclude that Buscher's duality transformations have a straight-forward generalization to first order in α'. Are the other zero-slope duality symmetries also preserved? The answer is yes (except for the A transformation). The duality symmetries are then those of the Sugra+YM action.

Finally, we can also ask what happens to the $SO(1,2)_{\text{Sugra}}$ duality group if we take just a single Abelian field. Do these transformations receive α' corrections as well or are they already exact up to this order? The situation is not entirely clear: it is true that both the α as the β transformations (the ones that are gauge transformations and g.c.t.'s) do not receive corrections

and we suppose that the $so(1,2)$ algebra holds. However this does not mean that the non-trivial solution generating transformation γ cannot have corrections because of the algebra structure. It would be entirely consistent with the absence of α' corrections in the α, β transformations and in the $so(1,2)$ algebra to assume that the γ transformations have α' corrections that commute with the α and β transformations. More work is necessary to answer these questions and we hope to present our results elsewhere soon [33].

Acknowledgements

A   The Sen Transformations

In this appendix we give the explicit form of the solution generating transformation introduced by Sen in D dimensions. A special case plus the infinitesimal form of these formulae are given in Eqs. (62) and Eqs. (65), respectively. The general and finite Sen rules are:

$$\hat{g}'_{\underline{xx}} = \frac{16}{N^2}\hat{g}_{\underline{xx}},$$

$$\hat{g}'_{\underline{x}\mu} = \frac{16}{N^2}\hat{g}_{\underline{xx}}\left[\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}} - \gamma\left(\hat{V}_\mu - \hat{V}_{\underline{x}}\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}\right)\right.$$

$$+\tfrac{1}{2}\gamma^2\left(\hat{B}_{\underline{x}\mu}-\tfrac{1}{2}\hat{V}_{\underline{x}}\hat{V}_\mu+\tfrac{1}{2}\hat{V}_{\underline{x}}^2\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}\right)\Bigg]\ ,$$

$$\hat{g}'_{\mu\nu}\ =\ \hat{g}_{\mu\nu}-\frac{\hat{g}_{\underline{x}\mu}\hat{g}_{\underline{x}\nu}}{\hat{g}_{\underline{xx}}}$$

$$+\frac{16}{N^2}\hat{g}_{\underline{xx}}\left[\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}-\gamma\left(\hat{V}_\mu-\hat{V}_{\underline{x}}\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}\right)+\tfrac{1}{2}\gamma^2\left(\hat{B}_{\underline{x}\mu}-\tfrac{1}{2}\hat{V}_{\underline{x}}\hat{V}_\mu+\tfrac{1}{2}\hat{V}_{\underline{x}}^2\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}\right)\right]$$

$$\times\left[\frac{\hat{g}_{\underline{x}\nu}}{\hat{g}_{\underline{xx}}}-\gamma\left(\hat{V}_\nu-\hat{V}_{\underline{x}}\frac{\hat{g}_{\underline{x}\nu}}{\hat{g}_{\underline{xx}}}\right)+\tfrac{1}{2}\gamma^2\left(\hat{B}_{\underline{x}\nu}-\tfrac{1}{2}\hat{V}_{\underline{x}}\hat{V}_\nu+\tfrac{1}{2}\hat{V}_{\underline{x}}^2\frac{\hat{g}_{\underline{x}\nu}}{\hat{g}_{\underline{xx}}}\right)\right]\ ,$$

$$\hat{B}'_{\underline{x}\mu}\ =\ \hat{B}_{\underline{x}\mu}-\tfrac{1}{2}\hat{V}_{\underline{x}}\hat{V}_\mu+\tfrac{1}{2}\hat{V}_{\underline{x}}^2\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}+\tfrac{1}{N}\left[2\hat{V}_{\underline{x}}+\gamma\left(\hat{V}_{\underline{x}}^2-2\hat{g}_{\underline{xx}}\right)\right]$$

$$\times\left[\hat{V}_\mu-\hat{V}_{\underline{x}}\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}-\gamma\left(\hat{B}_{\underline{x}\mu}-\tfrac{1}{2}\hat{V}_{\underline{x}}\hat{V}_\mu+\tfrac{1}{2}\hat{V}_{\underline{x}}^2\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}\right)\right]\ ,$$

$$\hat{B}'_{\mu\nu}\ =\ \hat{B}_{\mu\nu}-\hat{V}_{\underline{x}}\frac{\hat{g}_{\underline{x}[\mu}\hat{V}_{\nu]}}{\hat{g}_{\underline{xx}}}-\gamma\left(\hat{V}_{[\mu}-\hat{V}_{\underline{x}}\frac{\hat{g}_{\underline{x}[\mu}}{\hat{g}_{\underline{xx}}}\right)\hat{B}_{\underline{x}\nu]}$$

$$+\tfrac{2}{N}\left[2\hat{V}_{\underline{x}}+\gamma\left(\hat{V}_{\underline{x}}^2-2\hat{g}_{\underline{xx}}\right)\right]$$

$$\times\left[\frac{\hat{g}_{\underline{x}[\mu}\hat{V}_{\nu]}}{\hat{g}_{\underline{xx}}}-\gamma\frac{\hat{g}_{\underline{x}[\mu}}{\hat{g}_{\underline{xx}}}\left(\hat{B}_{\underline{x}\nu]}-\tfrac{1}{2}\hat{V}_{\underline{x}}\hat{V}_{\nu]}\right)+\tfrac{1}{2}\gamma^2\left(\hat{V}_{[\mu}-\hat{V}_{\underline{x}}\frac{\hat{g}_{\underline{x}[\mu}}{\hat{g}_{\underline{xx}}}\right)\hat{B}_{\underline{x}\nu]}\right]\ ,$$

$$\hat{V}'_{\underline{x}}\ =\ \tfrac{2}{N}\left[2\hat{V}_{\underline{x}}+\gamma\left(\hat{V}_{\underline{x}}^2-2\hat{g}_{\underline{xx}}\right)\right]\ ,$$

$$\hat{V}'_\mu\ =\ \hat{V}_\mu-\hat{V}_{\underline{x}}\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}-\gamma\left(\hat{B}_{\underline{x}\mu}-\tfrac{1}{2}\hat{V}_{\underline{x}}\hat{V}_\mu+\tfrac{1}{2}\hat{V}_{\underline{x}}^2\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}\right)$$

$$+\tfrac{2}{N}\left[2\hat{V}_{\underline{x}}+\gamma\left(\hat{V}_{\underline{x}}^2-2\hat{g}_{\underline{xx}}\right)\right]$$

$$\times\left[\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}-\gamma\left(\hat{V}_\mu-\hat{V}_{\underline{x}}\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}\right)+\tfrac{1}{2}\gamma^2\left(\hat{B}_{\underline{x}\mu}-\tfrac{1}{2}\hat{V}_{\underline{x}}\hat{V}_\mu+\tfrac{1}{2}\hat{V}_{\underline{x}}^2\frac{\hat{g}_{\underline{x}\mu}}{\hat{g}_{\underline{xx}}}\right)\right]\ ,$$

$$\hat{\phi}' = \hat{\phi} + \tfrac{1}{2}\log\tfrac{4}{N} .$$

where

$$N = 4 + 4\gamma\hat{V}_{\underline{x}} + \gamma^2\left(\hat{V}_{\underline{x}}^2 - 2\hat{g}_{\underline{xx}}\right) . \tag{77}$$

Note that, unlike the case of the Buscher transformations, the effective metric $\hat{G}_{\hat{\mu}\hat{\nu}}$ defined in (40) does not seem to play any special role in the above transformations. We have verified that under $SO(1,2)$ the effective metric does not transform into itself. This is in contradistinction with the $\mathbb{Z}_2$ transformations (see Eq. (42)).

B Duality Symmetries In $D = 11, 10$ And 9 Type II Theories

In this Appendix we will discuss duality symmetries in eleven, ten and nine dimensions for Type II theories. We will use the results and conventions of Ref. [17].

This Appendix is organized in subsections with increasing number of isometries and decreasing number of dimensions for each case.

B.1 No isometries

D=11 There is a single $SO^{\uparrow}(1,1)_{\mathrm{brane}}$ symmetry whose weights are given in Table 3. This symmetry essentially counts the mass dimension of the different fields. We also stress that $\hat{\hat{C}}$ is a pseudo-tensor that changes sign under improper g.c.t. 's.

B.2 One isometry

D=11 In addition to the symmetries of the previous section, we have to consider the subgroup of g.c.t. 's that preserve the condition that the fields do not depend on the coordinate y. This group is

$$GL(1,\mathbb{R}) = SO^{\uparrow}(1,1) \times \mathbb{Z}_2^{(y)} . \tag{78}$$

Name	$\hat{\hat{C}}$	$\hat{\hat{g}}$	$S^{(11)}$
$SO^{\uparrow}(1,1)_{\text{brane}}$	$\frac{3}{2}$	1	$\frac{9}{2}$

Table 3: *Weights of the $D = 11, N = 1$ supergravity fields and action under $SO^{\uparrow}(1,1)_{\text{brane}}$.*

Name	$\hat{C}$	$\hat{g}$	$\hat{B}^{(1)}$	$\hat{A^{(1)}}$	$e^{\hat{\phi}}$	$S_{IIA}^{(10)}$
$SO^{\uparrow}(1,1)_{\text{brane}}$	1	1	1	0	$\frac{1}{2}$	$\frac{3}{2}$
$SO^{\uparrow}(1,1)_y$	0	1	1	-1	$\frac{3}{2}$	1
$\mathbb{Z}_2^{(y)}$	$-$	$+$	$+$	$-$	$+$	$+$

Table 4: *Weights of the $D = 10$ Type IIA supergravity fields and action under $SO^{\uparrow}(1,1)_{\text{brane}} \times SO^{\uparrow}(1,1)_y \times \mathbb{Z}_2^{(y)}$.*

D=10, Type IIA Taking into account that $\hat{\hat{C}}$ changes sign under the (now) internal $\mathbb{Z}_2^{(y)}$, the eleven-dimensional transformations become the group

$$SO^{\uparrow}(1,1)_{\text{brane}} \times SO^{\uparrow}(1,1)_y \times \mathbb{Z}_2^{(y)}, \tag{79}$$

of global symmetries of the equations of motion. The $SO^{\uparrow}(1,1)$'s act as scalings and the weights and sign changes under $\mathbb{Z}_2^{(y)}$ are summarized in Table 4.

D=10, Type IIB This theory has a manifest $SL(2,\mathbb{R})$ duality which in the

string frame acts of the fields as follows

$$\hat{\jmath}'_{\hat{\mu}\hat{\nu}} \;=\; |c\hat{\lambda} + d|\,\hat{\jmath}_{\hat{\mu}\hat{\nu}}\,, \qquad \hat{\lambda}' = \frac{a\hat{\lambda} + b}{c\hat{\lambda} + d}\,,$$

$$\begin{pmatrix} \tilde{\mathcal{B}}^{(2)\prime}_{\hat{\mu}\hat{\nu}} \\ \tilde{\mathcal{B}}^{(1)\prime}_{\hat{\mu}\hat{\nu}} \end{pmatrix} \;=\; \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \tilde{\mathcal{B}}^{(2)}_{\hat{\mu}\hat{\nu}} \\ \tilde{\mathcal{B}}^{(1)}_{\hat{\mu}\hat{\nu}} \end{pmatrix}, \tag{80}$$

where $ad - bc = 1$ and $\hat{\lambda} = \hat{\ell} + ie^{-\varphi}$. There are several specially interesting subgroups of $SL(2,\mathbb{R})$. One is a $\mathbb{Z}_2$ generated by

$$\hat{\lambda}' = -1/\lambda\,, \qquad \hat{\jmath}'_{\hat{\mu}\hat{\nu}} = |\lambda|\,\hat{\jmath}_{\hat{\mu}\hat{\nu}}\,, \qquad \hat{\mathcal{B}}^{(2)\prime}_{\hat{\mu}\hat{\nu}} = \hat{\mathcal{B}}^{(1)}_{\hat{\mu}\hat{\nu}}\,, \qquad \hat{\mathcal{B}}^{(2)\prime}_{\hat{\mu}\hat{\nu}} = -\hat{\mathcal{B}}^{(1)}_{\hat{\mu}\hat{\nu}}\,. \tag{81}$$

This transformation inverts the string coupling constant (for $\hat{\ell} = 0$) and that is why it makes sense to identify $SL(2,\mathbb{R})$ with the S-duality group. On the other hand, $\hat{\mathcal{B}}^{(1)}$ is a Type I field, whose origin is the elementary excitations of the string, but $\hat{\mathcal{B}}^{(2)}$ is a Ramond-Ramond-type field, whose origin is in solitonic modes on the worldsheet. Therefore, this transformation has also an "electric-magnetic" side from the worldsheet point of view.

Another subgroup is a scaling $\widetilde{SO}^{\uparrow}(1,1)_y$ given in Table 5. It can be obtained from the $SO^{\uparrow}(1,1)_y$ of Type IIA using Type II Buscher duality [20]. Using it, we have also translated $SO^{\uparrow}(1,1)_{\mathrm{brane}} \times \mathbb{Z}_y$ to the Type IIB language. the results are given also in Table 5.

The total group on global symmetries of the equations of motion is, then

$$GL(2,\mathbb{R}) = SL(2,\mathbb{R}) \times \widetilde{SO}^{\uparrow}(1,1)_{\mathrm{brane}} \times \tilde{\mathbb{Z}}_2^{(y)}\,. \tag{82}$$

We would like to remark that this is exactly the global symmetry group that one would expect in a ten-dimensional theory that has been obtained by dimensional reduction from a twelve-dimensional theory with no global symmetries whatsoever.

Name	$\hat{D}$	$\hat{j}$	$\hat{\mathcal{B}}^{(1)}$	$\hat{\mathcal{B}}^{(2)}$	$\hat{\ell}$	$e^{\hat{\varphi}}$	$S_{IIB}^{(10)}$
$\widetilde{SO}^{\uparrow}(1,1)_{\text{brane}}$	1	1	1	0	-1	1	2
$\widetilde{SO}^{\uparrow}(1,1)_{y}$	0	1	1	-1	-2	2	0
$\tilde{\mathbb{Z}}_2^{(y)}$	$-$	$+$	$+$	$-$	$-$	$+$	$+$

Table 5: *Weights of the $D = 10$ Type IIB supergravity fields and $(\hat{F}(\hat{D}) = 0$ truncated) action under $\widetilde{SO}^{\uparrow}(1,1)_{\text{brane}} \times \widetilde{SO}^{\uparrow}(1,1)_{y} \times \tilde{\mathbb{Z}}_2^{(y)}$.*

D=10, Type I Truncating any of the Type II theories by setting the Ramond-Ramond fields to zero we obtain the symmetries of the Type I theory. These are

$$SO^{\uparrow}(1,1)_{\text{brane}} \times SO^{\uparrow}(1,1)_{y} \times \mathbb{Z}_2^{(A)}. \tag{83}$$

The $SO^{\uparrow}(1,1)_y$ group is the same as in Type IIA and the same as $\widetilde{SO}^{\uparrow}(1,1)_y$ in Type IIB, and is the only subgroup that remains of $SL(2,\mathbb{R})$.

B.3 Two isometries

D=11 Upon dimensional reduction to nine dimensions, the global symmetry group that we expect is

$$GL(2,\mathbb{R}) \times SO^{\uparrow}(1,1)_{\text{brane}} = SL(2,\mathbb{R}) \times SO^{\uparrow}(1,1) \times SO^{\uparrow}(1,1)_{\text{brane}} \times \mathbb{Z}_2. \tag{84}$$

D=10, Type IIA and B In presence of an isometry (in ten dimensions), the Type IIA and Type IIB theories are related by Type II duality

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[17]. There are other global symmetries which are not covariant from the ten-dimensional point of view. They are become covariant when we rewrite the theories in nine-dimensional language and so we will discuss them below.

D=10, Type I Upon truncation of the Ramond-Ramond-type fields both Type II theories become the Type I theory and the Type II duality that related them becomes the $\mathbb{Z}_2^{(B)}$ Buscher duality that we also discuss below in nine dimensions.

D=9, Type II In nine dimensions there is a single Type II theory whose global symmetry group is the one we expected:

$$SL(2,\mathbb{R}) \times\ SO^{\uparrow}(1,1)_{x+y} \times\ SO^{\uparrow}(1,1)_{\text{brane}} \times\ \mathbb{Z}_2^{(x)}. \qquad (85)$$

This $SL(2,\mathbb{R})$ group is a symmetry of the action. From the Type IIB point of view it is the manifest $SL(2,\mathbb{R})$ symmetry of the original theory and from the point of view of the Type IIA is part of the symmetry predicted in eleven dimensions. It contains one particular subgroup of scalings: $SO^{\uparrow}(1,1)_{x-y}$ corresponding to the eleven-dimensional g.c.t. $x \to e^{\alpha}x$, $y \to e^{-\alpha}y$. $SO^{\uparrow}(1,1)_{x+y}$ scales the fields and the action and corresponds to the eleven-dimensional g.c.t. $x \to e^{\alpha}x$, $y \to e^{\alpha}y$. Combining it with $SO^{\uparrow}(1,1)_{\text{brane}}$, a second scaling symmetry of the action can be obtained. Finally, $\mathbb{Z}_2^{(x)}$ corresponds to improper g.c.t.s in the internal space, for instance $x \to -x$ (up to $SL(2,\mathbb{R})$ rotations). The weights of the different nine-dimensional fields are summarized in Table 6.

D=9, Type I After truncation of the Type II theory, two interesting and opposite phenomena take place: the breaking of the Type I $SL(2,\mathbb{R})$ to just $SO^{\uparrow}(1,1)_{x-y}$, and a discrete symmetry enhancement from $\mathbb{Z}_2^{(x)}$ to D_4, due to the appearance of two new $\mathbb{Z}_2$'s: $\mathbb{Z}_2^{(A)}$ and $\mathbb{Z}_2^{(B)}$. The appearance of $\mathbb{Z}_2^{(A)}$ is related to the disappearance of the topological term in the action. The appearance of $\mathbb{Z}_2^{(B)}$ is more subtle and was discussed in Ref. [17]. In Table 1 the weights of the nine-dimensional Type I fields under certain combination of these symmetries are given.

Name	C	g	$B^{(1)}$	$B^{(2)}$	$A^{(1)}$	$A^{(2)}$	B	k	ℓ	e^ϕ	$S^{(9)}_{\text{II}-\text{red}}$
$SO^{\uparrow}(1,1)_{\text{brane}}$	1	1	1	1	0	0	1	$\frac{1}{2}$	0	$\frac{1}{4}$	3
$SO^{\uparrow}(1,1)_{x-y}$	0	1	1	-1	-1	1	0	$-\frac{1}{2}$	-2	$\frac{7}{4}$	0
$SO^{\uparrow}(1,1)_{x+y}$	0	1	1	1	-1	-1	2	$\frac{3}{2}$	0	$\frac{3}{4}$	2
$\mathbb{Z}_2^{(x)}$	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$-$	$+$	$+$

Table 6: *Weights of the $D = 9$ Type II supergravity fields and action under $SL(2,\mathbb{R}) \times SO^{\uparrow}(1,1)_{x+y} \times SO^{\uparrow}(1,1)_{\text{brane}} \times \mathbb{Z}_2^{(x)}$.*

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