Kaluza-Klein Monopoles and Gauged Sigma-Models

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Abstract

We propose an effective action for the eleven-dimensional (bosonic) Kaluza-Klein monopole solution. The construction of the action requires that the background fields admit an Abelian isometry group. The corresponding sigma-model is gauged with respect to this isometry. The gauged sigma-model is the source for the monopole solution. A direct (double) dimensional reduction of the action leads to the effective action of a 10-dimensional D-6-brane (IIA Kaluza-Klein monopole). We also show that the effective action of the 10-dimensional heterotic Kaluza-Klein monopole (which is a truncation of the IIA monopole action) is T-dual to the effective action of the solitonic 5-brane. We briefly discuss the kappa-symmetric extension of our proposal and the possible role of gauged sigma-models in connection with the conjectured M-theory 9-brane.

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Introduction

Eleven-dimensional supergravity [1] is believed to describe the low-energy behaviour of (uncompactified) M-theory [2], which may be a theory of supermembranes [3]. To gain a better understanding of M-theory it is therefore of interest to study the different kinds of solutions to the supergravity equations of motion. Of particular interest are those solutions that, upon reduction to ten dimensions, lead to the D-p-brane solutions of IIA supergravity [4]. By now, the 11-dimensional origin of the D-p-brane solutions (with p even) are well understood with the exception of the D-8-brane whose 11-dimensional interpretation is still a mystery (see, however, the discussion).

It turns out that 11-dimensional supergravity admits two "brane" solutions, the M-2-brane and M-5-brane. Their reduction leads to the D-2-brane and D-4-brane solution, respectively. There is no such brane interpretation for the D-0-brane and D-6-brane. These solutions are related to a purely gravitational Brinkmann wave (W11) and a Kaluza-Klein monopole (KK11) in eleven dimensions [5]. The metric corresponding to these solutions does not split up into isotropic worldvolume and transverse directions and therefore does not describe a standard brane. Nevertheless, due to their 10-dimensional D-brane interpretation, these solutions are expected to play an important role in M-theory. It is therefore of interest to get a better understanding of these solutions and the role they play in M-theory.

A natural question to ask is, what is the effective action corresponding to the Brinkmann wave and Kaluza-Klein monopole? Such effective actions occur as source terms in the supergravity equations of motion. The effective action of the Brinkmann wave is a massless particle (or any other kind of massless p-brane [6]) moving at the speed of light. The zero modes of the 11-dimensional Kaluza-Klein monopole have been recently discussed in [7]. In this letter we will make a concrete proposal for the KK11 effective action that involves those zero modes. To motivate our proposal we first show that the purely gravitational part[5] of the action is the source of the KK11 solution. Next, we show that, up to the yet undetermined Wess-Zumino term, a direct dimensional reduction of the KK11 action leads to the effective action of a 10-dimensional D-6-brane, while a double dimensional reduction leads to the effective action for a IIA Kaluza-Klein monopole (KK10A). In the latter reduction we only consider the purely gravitational part of the action. Finally, we will show that the effective action of the 10-dimensional heterotic Kaluza-Klein monopole (KKh), which is a truncation of the KK10A action, is T-dual to the effective action of the heterotic solitonic 5-brane (P5h), again

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[5]A precise definition will be given in the next section.

up to the WZ term [6].

1   The KK11 Effective Action

Our starting point is the 11-dimensional KK11 solution [8][6]

$$ds^2 = \eta_{ij}dy^i dy^j - H^{-1}\left(dz + A_m dx^m\right)^2 - H(dx^m)^2\,, \tag{1.1}$$

where $i,j = 0,\ldots,6$, $m,n = 7,8,9$, and $z = x^{10}$ and where

$$F_{mn} = 2\partial_{[m}A_{n]} = \epsilon_{mnp}\partial_p H\,, \qquad \partial_m \partial_m H = 0\,. \tag{1.2}$$

The solution (1.1) has 8 isometries and therefore it represents an extended object. At first sight one might think that the solution represents a 7-brane (with non-isotropic worldvolume directions) but it turns out that the isometry in the direction z is special and cannot be interpreted as a worldvolume direction [7]. We are therefore dealing with a 6-brane, with a 7-dimensional worldvolume, that has an additional isometry in one of the 4 transverse directions.

The KK11 solution preserves half of the supersymmetry and must correspond, after gauge fixing, to a 7-dimensional supersymmetric field theory. The natural candidate for such a field theory involves a vector multiplet with 3 scalars and one vector [7]. We are now faced with a dilemma. Since the KK11-monopole moves in 11 dimensions, we have 11 embedding coordinates. Fixing the diffeomorphisms of the 7-dimensional worldvolume we are left with 4 instead of 3 scalars. At this point one might argue that, to eliminate the extra scalar d.o.f., we need an extra diffeomorphism, i.e. an 8-dimensional worldvolume, but this would upset the counting of the worldvolume vector components. We therefore need a new mechanism to eliminate the unphysical scalar degree of freedom. As we will see below, this can be done by gauging an Abelian isometry in the effective sigma-model, i.e. we propose to work with a *gauged sigma-model*.

A second characteristic feature of our proposal is that the "KK11-brane" couples to a scalar k constructed from the Killing vector k^μ that generates the isometry we are gauging. The coupling manifests itself as a factor k^2 in front of the kinetic term in the effective action. Since, in coordinates adapted to the isometry, $g_{zz} = -k^2$ and the length of the z-dimension is

$$2\pi R_z = \int dz |g_{zz}|^{1/2} = \int dz\,k\,, \tag{1.3}$$

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[6]The discussion below can easily be extended to arbitrary dimensions d. For simplicity we restrict ourselves to d=11.

the tension of the KK11-brane (and of the KK brane in any dimension) is proportional to R_z^2.

To be concrete, we propose the following expression for the kinetic term of the KK11 effective action (a proper WZ term needs to be added [6]):

$$S_{\mathrm{KK11}} = -T_{\mathrm{KK11}} \int d^7\xi \; k^2 \sqrt{|\det\left(\partial_i X^\mu \partial_j X^\nu \Pi_{\mu\nu} + k^{-1}\mathcal{F}_{ij}\right)|}\,, \qquad (1.4)$$

where k^μ is the Killing vector associated to the isometry direction z and

$$k^2 = -k^\mu k^\nu g_{\mu\nu}\,. \qquad (1.5)$$

Furthermore,

$$\begin{cases} \Pi_{\mu\nu} & = \; g_{\mu\nu} + k^{-2}k_\mu k_\nu\,, \\[2mm] \mathcal{F}_{ij} & = \; \partial_i V_j - \partial_j V_i - k^\mu \partial_i X^\nu \partial_j X^\rho C^{(3)}_{\mu\nu\rho}\,. \end{cases} \qquad (1.6)$$

The field $C^{(3)}$ is the 3-form potential of 11-dimensional supergravity.

Observe that the components of the "metric" $\Pi_{\mu\nu}$ in the directions of k^μ vanish:

$$k^\mu \Pi_{\mu\nu} = 0\,. \qquad (1.7)$$

$\Pi_{\mu\nu}$ is effectively a 10-dimensional metric and, in coordinates adapted to the isometry generated by k^μ, the coordinate z and the corresponding field $Z(\xi)$ associated to this isometry simply do not occur in the action.

Sometimes, we will consider in our discussion only the purely gravitational part of the KK11 action, i.e. we will set the worldvolume vector field strength $\mathcal{F}_{ij}$ equal to zero and we will ignore the WZ term:

$$S_{\mathrm{KK11}}^{\mathrm{grav.}} = -T_{\mathrm{KK11}} \int d^7\xi \; k^2 \sqrt{|\det\left(\partial_i X^\mu \partial_j X^\nu \Pi_{\mu\nu}\right)|}\,. \qquad (1.8)$$

This part of the action can be written using an auxiliary worldvolume metric γ_{ij} in the Howe-Tucker form

$$S_{\mathrm{KK11}}^{\mathrm{grav.}} = -\frac{T_{\mathrm{KK11}}}{2} \int d^7\xi \sqrt{|\gamma|} \left[k^{4/7}\gamma^{ij}\partial_i X^\mu \partial_j X^\nu \Pi_{\mu\nu} - 5\right]\,. \qquad (1.9)$$

Eliminating γ_{ij} from (1.9) leads to the expression given in (1.8). An alternative form of (1.9), which makes the relation with a gauged sigma-model clear, is obtained using an auxiliary worldvolume vector field C_i:

$$S_{\mathrm{KK11}}^{\mathrm{grav.}} = -\frac{T_{\mathrm{KK11}}}{2} \int d^7\xi \sqrt{|\gamma|} \left[k^{4/7}\gamma^{ij}D_i X^\mu D_j X^\nu g_{\mu\nu} - 5\right]\,, \qquad (1.10)$$

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with the covariant derivative defined as

$$D_i X^\mu = \partial_i X^\mu + C_i k^\mu \,. \tag{1.11}$$

In the remaining part of this letter we will collect evidence in favour of our proposal. We first show that the KK11 action is the source for the KK11 solution. Next, we compare the KK11 action with the kinetic terms of other known actions via dimensional reduction and T-duality. The relations between the KK11 action of M-theory and the different actions of the IIA/IIB theories are given in Figure 1. The worldvolume fields of the corresponding actions are given in Table 1.

Figure 1: Relation between solutions and effective actions of KK monopoles and other extended objects in M theory and type II string theories. Vertical arrows indicate direct dimensional reduction. Vertical dashed lines indicate direct dimensional reduction in the special direction z. Oblique arrows indicate double dimensional reduction. Double arrows indicate T duality relations and wiggly lines indicate that the supergravity solutions are identical, although the effective actions are not.

Object	Worldvolume	Fields	# of d.o.f.	Total
KK11	6+1	X^μ V_i	$11 - 7$ "-1"$= 3$ $7 - 2 = 5$	8
D6	6+1	X^μ V_i	$10 - 7 = 3$ $7 - 2 = 5$	8
KK10A	5+1	X^μ V_i S	$10 - 6$ "-1"$= 3$ $6 - 2 = 4$ 1	8
P5B	5+1	X^μ W_{ijk}	$10 - 6 = 4$ 4	8
M5	5+1	X^μ V_{ij}^+	$11 - 6 = 5$ 3	8
P5A	5+1	X^μ V_{ij}^+ S	$10 - 6 = 4$ 3 1	8
KK10B	5+1	X^μ V_{ij}^+ S T	$10 - 6$ "-1"$= 3$ 3 1 1	8

Table 1: The table gives the worldvolume fields and number of degrees of freedom of the different objects that appear in Figure 1. The "-1" in the fourth column indicates that a scalar degree of freedom is eliminated by gauging an Abelian isometry.

2 The KK11 Action as Source Term

We first demonstrate that the KK11 action is the source of the 11-dimensional KK monopole[7]. For simplicity, we only consider the purely gravitational part. This leads to the following bulk plus source term:

$$
\begin{aligned}
S \;=\;& \tfrac{1}{\kappa} \int d^{11}x \sqrt{|g|}[R] \\
& - \tfrac{T_{KK11}}{2} \int d^7\xi \sqrt{|\gamma|} \left[k^{4/7}\gamma^{ij}\partial_i X^\mu \partial_j X^\nu \Pi_{\mu\nu} - 5 \right] ,
\end{aligned}
\tag{2.1}
$$

where $\kappa = 16\pi G_N^{(11)}$.

To determine the Einstein equations we have to take into account the metric factors hidden in factors like k^2. The rule is that k^μ is independent of the metric. The metrics that have to be varied in k^2 and Π are shown explicitly below:

$$
\begin{cases}
k^2 \;=\; -k^\mu k^\nu g_{\mu\nu} , \\[2mm]
\Pi_{\mu\nu} \;=\; g_{\mu\nu} + \left(k^\alpha k^\beta g_{\alpha\beta} \right)^{-1} k^\rho g_{\rho\mu} k^\sigma g_{\sigma\beta} .
\end{cases}
\tag{2.2}
$$

The equations of motion for $g_{\mu\nu}$ and X^ν are

$$
\begin{aligned}
G^{\alpha\beta} + \tfrac{T_{KK11}\kappa}{2\sqrt{|g|}} \int d^7\xi \sqrt{|\gamma|} k^{4/7}\gamma^{ij} \Big\{ -\tfrac{2}{3}k^{-2}k^\alpha k^\beta \Pi_{ij} + \partial_i X^\alpha \partial_j X^\beta \\
-2k^{-3}k^\alpha k^\beta k_i k_j - 2k^{-2}k^{(\alpha}\partial_i X^{\beta)}k_j \Big\} \delta^{(10)}(x-X) \;=\; 0 , \\
\tilde{\nabla}^2 X^\rho + \tilde{\Gamma}_{\mu\nu}{}^\rho \partial_k X^\mu \partial^k X^\nu \;=\; 0 ,
\end{aligned}
\tag{2.3}
$$

where $\tilde{\Gamma}$ are the Christoffel symbols of the "metric"

$$
\tilde{g}_{\mu\nu} = k^2 \Pi_{\mu\nu} ,
\tag{2.4}
$$

and $\tilde{\nabla}^2$ is the Laplacian with respect to the worldvolume metric

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[7]Results concerning singular sources in General Relativity are known to be highly coordinate-dependent. In particular, in isotropic coordinates the sources of extremal black holes and branes seem to be placed at the horizon, which is non-singular but looks like a point in this coordinate system. The reason for this is that the coordinates chosen do not cover the region where the physical singularity is and the flux lines of the different fields seem to come out of the point that represents the horizon. Our results have to be understood in the same sense. We thank G.W. Gibbons and P.K. Townsend for discussions on this point.

$$\tilde{\gamma}_{ij} = k^{-4/7}\gamma_{ij}\,, \tag{2.5}$$

while the equation of motion for γ_{ij} simply implies

$$\gamma_{ij} = k^{4/7}\Pi_{ij}\,, \qquad \tilde{\gamma}_{ij} = \Pi_{ij}\,. \tag{2.6}$$

Observe that in the source term for the Einstein equation, instead of the usual 11-dimensional Dirac delta function $\delta^{(11)}$ we have written a 10-dimensional Dirac delta function $\delta^{(10)}$. The reason is that the KK monopole effective action does not depend on all the coordinates, as we have seen. In adapted coordinates, it will not depend on Z, and, therefore, a factor of $\delta(z - Z)$ is absent.

We find that the only non-vanishing components of the Einstein tensor with upper indices for the KK monopole metric (1.1) are

$$\begin{cases} G^{zz} & = & -H^{-1}\partial^2 H\,, \\ \\ G^{ij} & = & \frac{1}{2}\eta^{ij}H^{-2}\partial^2 H\,. \end{cases} \tag{2.7}$$

For the embedding coordinates we make the following ansatz:

$$X^i = \xi^i\,, \qquad Z = X^m = 0\,, \tag{2.8}$$

which justifies that the indices i, j can be used both as worldvolume indices as well as the first 7 target-space indices. The above ansatz tells us that the extended object worldvolume lies in the position $x^m = z = 0$. With this ansatz $\tilde{\gamma}_{ij} = \eta_{ij}$ and the equation of motion for the scalars X^μ is satisfied.

Taking into account the form of the metric given in Eq. (1.1) we find

$$k^\mu = \delta^{\mu z}\,, \quad k_i = 0\,, \quad \Pi_{ij} = g_{ij} = \eta_{ij}\,. \tag{2.9}$$

At this stage we find that most of the components of the Einstein equation are automatically satisfied. The only non-trivial ones are the $zz-$ and $ij-$components. Using Eqs. (2.7) we find that both lead to the same equation:

$$\partial^2 H = -T_{KK11}\,\kappa\delta^{(3)}(\vec{x})\,, \tag{2.10}$$

which is solved by

$$H = 1 + \frac{T_{KK11}\kappa}{4\pi}\frac{1}{|\vec{x}|}\,. \tag{2.11}$$

We thus conclude that the KK11 action is indeed the source of the 11-dimensional Kaluza-Klein monopole.

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3 Dimensional Reduction

In this section we will first show that the direct dimensional reduction of the KK11 action leads to the D-6-brane action and next that the double dimensional reduction of the (purely gravitational part of the) same action leads to the KK10A effective action. In this section we will indicate 11-dimensional (10-dimensional) fields by double (single) hats.

3.1 Direct Dimensional Reduction

To perform the direct dimensional reduction it is convenient to use coordinates adapted to the gauged isometry. In such a coordinate system we have

$$\hat{\hat{k}}^{\hat{\hat{\mu}}} = \delta^{\hat{\hat{\mu}}z} \,, \tag{3.1}$$

and the only non-vanishing components of $\hat{\hat{\Pi}}_{\hat{\mu}\hat{\nu}}$ are

$$\hat{\hat{\Pi}}_{\hat{\mu}\hat{\nu}} = \hat{\hat{k}}^{-1} \hat{g}_{\hat{\mu}\hat{\nu}} \,. \tag{3.2}$$

Furthermore

$$\hat{\hat{g}}_{zz} = -\hat{\hat{k}}^2 = -e^{\frac{4}{3}\hat{\phi}} \,. \tag{3.3}$$

Substituting the latter two equations into the KK11 action gives

$$S_{\mathrm{D6}} = -T_{\mathrm{KK11}} \int d^7\hat{\xi} e^{-\hat{\phi}} \sqrt{|\det\,(\hat{g}_{\hat{i}\hat{j}} + \hat{\mathcal{F}}_{\hat{i}\hat{j}})|} + \mathrm{WZ} \,, \tag{3.4}$$

which is precisely the action for the D-6-brane. Our results suggest the identification

$$T_{\mathrm{KK11}} = T_{\mathrm{D6}} \,. \tag{3.5}$$

3.2 Double Dimensional Reduction

We next perform a double dimensional reduction of the KK11 action. We only give the reduction for the purely gravitational part. Thus, our starting point is

$$S^{\mathrm{grav.}} = -\frac{T_{KK11}}{2} \int d^7\hat{\xi} \sqrt{|\hat{\gamma}|} \left[\hat{\hat{k}}^{4/7} \hat{\gamma}^{\hat{i}\hat{j}} \hat{D}_{\hat{i}} \hat{\hat{X}}^{\hat{\hat{\mu}}} \hat{D}_{\hat{j}} \hat{\hat{X}}^{\hat{\hat{\nu}}} \hat{\hat{g}}_{\hat{\mu}\hat{\nu}} - 5 \right] \,, \tag{3.6}$$

where the covariant derivative is defined as

$$\hat{D}_i \hat{X}^{\hat{\mu}} = \hat{\partial}_i \hat{X}^{\hat{\mu}} + \hat{C}_i \hat{\hat{k}}^{\hat{\mu}} \,. \tag{3.7}$$

Usually, in a double dimensional reduction a worldvolume coordinate σ and a target-space coordinate y associated to an isometry are simultaneously eliminated using an ansatz of the form

$$Y(\hat{x}^{\hat{i}}) = \sigma \,, \tag{3.8}$$

while the other coordinates $\hat{X}^{\hat{\mu}}$ are independent of σ so

$$\partial_\sigma \hat{X}^{\hat{\mu}} = \delta^{\hat{\mu}y} \,. \tag{3.9}$$

In the present case it is natural to make an ansatz that respects the gauge symmetry of the gauged sigma-model. We therefore take

$$\hat{D}_\sigma \hat{X}^{\hat{\mu}} = \delta^{\hat{\mu}y} \,. \tag{3.10}$$

However, this would eliminate the component $\hat{C}_\sigma$ and for consistency we have to make sure that this is consistent with its algebraic equation of motion

$$\hat{\hat{k}}_{\hat{\mu}} \hat{D}_\sigma \hat{X}^{\hat{\mu}} = 0 \,. \tag{3.11}$$

This implies that we must take

$$\hat{\hat{k}}_y = 0 \,. \tag{3.12}$$

For simplicity, we furthermore take

$$\hat{C}_\sigma = 0 \,. \tag{3.13}$$

We now split the $(6+1)$-dimensional worldvolume metric as follows[8]:

$$\begin{cases} \hat{\gamma}_{ij} &= \ell^{-1/2}\gamma_{ij} - \ell^2 a_i a_j \,, \\[2mm] \hat{\gamma}_{i\sigma} &= -\ell^2 a_i \,, \\[2mm] \hat{\gamma}_{\sigma\sigma} &= -\ell^2 \,, \end{cases} \qquad \begin{cases} \hat{\gamma}^{ij} &= \ell^{1/2}\gamma^{ij} \,, \\[2mm] \hat{\gamma}^{i\sigma} &= -\ell^{1/2} a^i \,, \\[2mm] \hat{\gamma}^{\sigma\sigma} &= -\ell^{-2} + l^{1/2} a^2 \,. \end{cases} \tag{3.14}$$

Substituting our ansatz for the coordinate Y and for the worldvolume metric into the action we get

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[8]The worldvolume indices are $\hat{i} = (i, \sigma)$ so $\hat{\xi}^6 = \sigma$

$$S^{\text{grav.}} = -\frac{T_{KK11}}{2} \int d^6\xi d\sigma \sqrt{|\gamma|} \left\{ \hat{\hat{k}}^{4/7} \left[\gamma^{ij} D_i \hat{X}^{\hat\mu} D_j \hat{X}^{\hat\nu} \hat{\hat{g}}_{\hat\mu\hat\nu} \right. \right.$$

$$\left. \left. -2a^i D_i \hat{X}^{\hat\mu} \hat{\hat{g}}_{\hat\mu y} - (\ell^{-2} - \ell^{1/2} a^2) \hat{\hat{g}}_{yy} \right] - 5 \right\} , \quad (3.15)$$

where we have used

$$\hat{\hat{k}}^{\hat\mu} = \hat{k}^{\hat\mu} , \quad \hat{C}_i = C_i , \Rightarrow \hat{D}_i \hat{X}^{\hat\mu} = D_i \hat{X}^{\hat\mu} . \quad (3.16)$$

We next eliminate the σ-components of the worldvolume metric (ℓ, a_i) by using their algebraic equations of motion and obtain

$$S^{\text{grav.}} = -\frac{T_{KK11}}{2} l_{11} \int d^6\xi \sqrt{|\gamma|} \left[\hat{k}^{2/3} e^{-\frac{2}{3}\hat\phi} \gamma^{ij} D_i \hat{X}^{\hat\mu} D_j \hat{X}^{\hat\nu} \hat{g}_{\hat\mu\hat\nu} - 4 \right] , \quad (3.17)$$

where we have used the relation between the 11-dimensional metric and the 10-dimensional metric and dilaton

$$e^{\frac{4}{3}\hat\phi} = -\hat{\hat{g}}_{yy} ,$$

$$\hat{g}_{\hat\mu\hat\nu} = e^{\frac{2}{3}\hat\phi} \left[\hat{\hat{g}}_{\hat\mu\hat\nu} - \hat{\hat{g}}_{\hat\mu y} \hat{\hat{g}}_{\hat\nu y} / \hat{\hat{g}}_{yy} \right] , \quad (3.18)$$

and the following relation between the 11- and 10-dimensional Killing vectors[9]

$$\hat{\hat{k}}^2 = e^{-\frac{2}{3}\hat\phi} \hat{k}^2 . \quad (3.19)$$

Furthermore, we define

$$l_{11} = \int d\sigma . \quad (3.20)$$

The action (3.17) is the purely gravitational part of the KK10A monopole action and, thus, we find

$$T_{KK11} l_{11} = T_{KK10A} . \quad (3.21)$$

Eliminating from the action the worldvolume metric γ_{ij} by using its equation of motion, we get

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[9]It is in this identity (and only in this identity) where the condition $\hat{\hat{k}}_y = 0$ plays a role.

$$S = -T_{KK10A} \int d^6\xi e^{-2\hat\phi}\hat{k}^2\sqrt{\det |D_i\hat{X}^{\hat\mu}D_j\hat{X}^{\hat\nu}\hat{g}_{\hat\mu\hat\nu}|}\,, \qquad (3.22)$$

which shows that both the factor $e^{-2\hat\phi}$ characteristic of a solitonic object and the factor $\hat{k}^2$ characteristic of a KK monopole are present in front of the kinetic term.

4 T-duality

As a final piece of evidence in favour of the KK11 action we consider T-duality. More explicitly, we will show that the heterotic Kaluza-Klein monopole (KKh) is T-dual to the solitonic five-brane (P5h)[10]. Before doing this we first wish to comment on the Buscher's T-duality rules [11]. The standard derivation of the Buscher's rules goes via a worldsheet duality transformation on the isometry scalar in the string effective action. A disadvantage of this derivation that it only applies to strings but not to five-branes since the dual of a scalar is a scalar only in two dimensions. However, it turns out that there is an alternative way of deriving the Buscher's rules which is more suitable for our purposes. Combining the fact that a wave is T-dual to a string and that the corresponding source terms are given by a massless particle and a string, respectively, one can show that the massless particle is T-dual to the string via reduction to d=9 dimensions [6]. This way of formulating Buscher's T-duality is identical to the way the type II T duality between the $R \otimes R$ fields is treated [12][11]. It is in this sense that we show below that the KKh and P5h actions are T-dual to each other.

Our starting point is the 10-dimensional heterotic five-brane (P5h) action given by

$$S_{\text{P5h}} = -T_{\text{P5h}} \int d^6\xi e^{-2\hat\phi}\sqrt{|\det(\partial_i\hat{X}^{\hat\mu}\partial_j\hat{X}^{\hat\nu}\hat{g}_{\hat\mu\hat\nu})|} + \text{WZ}\,, \qquad (4.1)$$

A direct dimensional reduction of the P5h action leads to an action involving an extra worldsheet scalar S:

$$S_{\text{P5h}} = -T_{\text{P5h}} \int d^6\xi\ e^{-2\phi}k^{-1}\sqrt{|\det(\partial_i X^\mu\partial_j X^\nu g_{\mu\nu} - k^2 F_i F_j)|} + \text{WZ}, \quad (4.2)$$

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[10]The T-duality between KK monopole and five–brane solutions corresponding to the effective action has been considered in the context of the magnetic ciral null model [9] and p-brane bound states [10].

[11]An interesting feature of this alternative derivation of Buscher's rules is that the duality rule of the dilaton is needed already at the classical level.

with

$$F_i = \partial_i S - A_i \,. \tag{4.3}$$

On the other hand, the heterotic KK monopole action KKh is given by

$$S_{\text{KKh}} = -T_{\text{KKh}} \int d^6\xi \, e^{-2\hat{\phi}} \hat{k}^2 \sqrt{|\det(\, \partial_i \hat{X}^{\hat{\mu}} \partial_j \hat{X}^{\hat{\nu}} \hat{\Pi}_{\hat{\mu}\hat{\nu}} - \hat{k}^{-2} \hat{F}_i \hat{F}_j)|} + \text{WZ} \,, \tag{4.4}$$

with

$$\hat{F}_i = \partial_i \hat{S} - \partial_i \hat{X}^{\hat{\mu}} \hat{k}^{\hat{\nu}} \hat{B}_{\hat{\mu}\hat{\nu}} \,. \tag{4.5}$$

A reduction of the KKh action over the z-direction gives

$$S_{\text{KKh}} = -T_{\text{KKh}} \int d^6\xi \, e^{-2\phi} k \sqrt{|\det(\, \partial_i X^\mu \partial_j X^\nu g_{\mu\nu} - k^{-2} F_i' F_j')|} + \text{WZ} \,. \tag{4.6}$$

Furthermore, we have

$$F_i' = \partial_i S - B_i \,. \tag{4.7}$$

Combining the above reductions we see that the P5h and KKh actions reduce to two actions in nine dimensions that differ by the following interchanges:

$$k \leftrightarrow k^{-1} \,, \qquad A \leftrightarrow B \,, \tag{4.8}$$

which are exactly Buscher's rules in nine-dimensional language [13]. This proves the T-duality between the P5h and KKh actions.

5 Discussion

We have proposed an effective action for the 11-dimensional Kaluza-Klein monopole that is given by a gauged sigma-model. We collected the following pieces of evidence in favour of this proposal:

1. The KK11 action is the source of the KK11 solution.

2. The action reduces to the D-6-brane action via direct dimensional reduction and gives the KK10A action via double dimensional reduction.

3. The truncated 10-dimensional KKh action is T-dual to the P5h action.

It should be not too difficult to derive the Wess-Zumino term and to include it in the calculations performed in this letter [6].

It is natural to consider the kappa-symmetric extension of the KK11 action. In fact a natural ansatz for the relevant projection operator is (omitting the double hats)

$$\Gamma = \frac{1}{7!\sqrt{|\gamma|}}\epsilon^{i_1\cdots i_7} D_{i_1} X^{\mu_1} \cdots D_{i_7} X^{\mu_7} \Gamma_{\mu_1\cdots\mu_7}\,. \tag{5.1}$$

This form of the projection operator leads to the following Killing spinor condition for unbroken supersymmetry [14]:

$$(1 - \Gamma_{01\dots6})\epsilon = 0\,. \tag{5.2}$$

This result for the Killing spinor can be used as another argument for a 6-brane interpretation of the 11-dimensional Kaluza-Klein monopole [15].

One might also wonder whether our results shed new light on the evasive 11-dimensional 9-brane (see also [7]). A standard argument against the 9-brane is that the corresponding 10-dimensional worldvolume field theory does not allow multiplets containing a single scalar to indicate the position of the 9-brane. A way out of this is to assume that the 9-brane is really a 8-brane with an extra isometry in one of the 2 transverse directions, leading to a gauged sigma-model. Now, we are dealing with a nine-dimensional field theory which naturally contains a vector multiplet with a single scalar. In this context we note that the 11-dimensional origin of the D-2-brane requires a worldvolume duality transformation of the Born-Infeld (BI) vector into a scalar [16]. This only works for massless backgrounds, i.e. $m = 0$, due to the presence of a topological mass term for $m \neq 0$. However, for $m \neq 0$ it is still possible to dualize *on-shell* leading to the following line element for the eleventh scalar:

$$\partial_i X^{11} - m V_i\,. \tag{5.3}$$

In other words, the general line element is given by

$$\partial_i X^\mu - V_i k^\mu\,, \tag{5.4}$$

with $k^\mu = m\delta^{\mu 11}$. We thus end up with something which is similar to the line element of the gauged sigma-model we considered in this letter:

$$\partial_i X^\mu - C_i k^\mu\,. \tag{5.5}$$

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It would be interesting to pursue this line of thought further and see whether it leads to a proper formulation of the long sought for 11-dimensional 9-brane.

Finally, we believe that gauged sigma-models will play a relevant role as source terms for many purely gravitational solutions of 11-dimensional supergravity. The gauging procedure generically seems to allow for new potentially supersymmetric effective actions without the need for higher dimensions.

Acknowledgments

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