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Exact SU(2) × U(1) stringy black holes

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Extreme magnetic dilaton black holes are promoted to exact solutions of heterotic string theory with unbroken supersymmetry. With account taken of α' corrections this is accomplished by supplementing the known solutions with SU(2) Yang-Mills vectors and scalars in addition to the already existing Abelian U(1) vector field. The solution has a simple analytic form and includes multiple black holes. The issue of exactness of other black-hole-type solutions, including extreme dilaton electrically charged black holes and Taub–Newman-Unti-Tamburino solutions, is discussed.

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Charged four-dimensional extreme dilaton black holes [1] with a stringy dilaton coupling $a=1$ have attracted a lot of attention in recent years. In particular it was established that they have half of supersymmetries unbroken when embedded into $d=4$, $N=4$ pure supergravity without matter [2]. As solutions of quantum four-dimensional supergravity they are subject to some supersymmetric nonrenormalization theorem. In particular, it has been argued that the entropy of extreme black holes does not get quantum corrections [2,3].

In addition to considering these solutions in the framework of four-dimensional quantum supergravity one may address the issue of stringy α' corrections for these black holes. The stringy α' corrections related to the Lorentz anomaly in the general context of unbroken supersymmetry of the heterotic string have been analyzed in [4]. For supersymmetric string solitons and multimonopole solutions α' corrections were studied in [5–7]. These solutions with SU(2) Yang-Mills fields are considered to be exact solutions of the heterotic string theory. The α' corrections of generalized supersymmetric gravitational wave solutions as well as dual wave solutions have been studied in [8,9]. These solutions require SO(8) Yang-Mills fields for unbroken supersymmetry and exactness in the heterotic string theory. The extreme black holes with $a=\sqrt{3}$ without Yang-Mills fields were found to be exact solutions of the type II superstring theory in [7].

The purpose of this Rapid Communication is to investigate whether $a=1$ charged dilaton black holes, which solve the leading-order equations of motion of the string effective action, can be promoted to exact solutions of heterotic string theory. We will try to supplement them with a non-Abelian Yang-Mills field that cancels the gravitational part of the α' corrections. This will be an efficient use of the Green-Schwarz mechanism of the cancellation of the Lorentz and Yang-Mills anomalies, as well as supersymmetry anomaly [10,4] for a given configuration. For this purpose it is useful to work directly in the critical dimension of the heterotic string: $d=10$. The analysis of unbroken supersymmetry as well as the issue of α' corrections is much simpler in $d=10$. Therefore we will first uplift the four-dimensional dilaton black holes to $d=10$ and analyze the α' corrections there. In the case of the magnetic extreme dilaton black hole it will be possible to find the appropriate ten-dimensional Yang-Mills configuration to cancel the gravitational part of the α' corrections. The non-Abelian field is completely determined by the gravitational part of the uplifted black hole. By adding the Yang-Mills field to the solution we will also achieve the restoration of unbroken supersymmetry which becomes anomalous [10,4] if only gravitational α' corrections of the heterotic string are taken into account. Under plausible assumptions, the solution obtained in this way is free of α' corrections.

By the end of this analysis, we will dimensionally reduce our exact solution to $d=4$. There we will get an additional SU(2) scalar and vector as part of the non-Abelian magnetic black hole.

The issue of α' corrections for extreme magnetic dilaton black holes in four-dimensional geometry has been studied in [11], where it was shown that if the gravitational α' corrections are taken into account, the solution gets corrections. We will see later that it is the same balance between the gravitational α' corrections and Yang-Mills contribution responsible for the cancellation of anomalies which has allowed us to preserve the gravitational part of the black hole intact.

For extreme electric black holes this procedure of promoting a lowest order solution of a heterotic string theory to an exact one fails. Our procedure involves the embedding of the spin connection into a gauge group. The corresponding gauge group for the electric dilaton black hole turns out to be noncompact. The dilatonic Israel-Wilson-Perjés (IWP) solutions [12] could be made exact if certain relations between the Newman-Unti-Tamburino (NUT) charge and the mass hold.

Let us start with the extreme magnetic dilaton black hole [1] in four dimensions. In this paper we will work in stringy frame[1]:

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[1]Our conventions are essentially those of Ref. [9], but we reserve the indices i,j,k,l,m for the three-dimensional space and Yang-Mills indices. Yang-Mills indices can be identified for being always the last ones in upper position. Also we underline curved indices and $\epsilon_{123}=\epsilon^{123}=+1$.

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$$ds^2=dt^2-e^{4\phi}d\mathbf{x}^2\,,$$

$$F_{\underline{ik}}=2\partial_{[\underline{i}}V_{\underline{k}]}=\pm\epsilon_{ikl}\ \partial_{\underline{l}}e^{2\phi}\,,$$

$$e^{2\phi}=1+\sum_s\frac{2M_s}{|\mathbf{x}-\mathbf{x}_s|}\,. \tag{1}$$

The Abelian vector field $V=V_{\underline{i}}dx^{\underline{i}}$ is defined up to a gauge transformation. For example, for a single black hole we can take

$$\mathbf{V}=\pm\frac{2M(0,0,z)\wedge(x,y,z)}{\rho(\rho^2-z^2)}\,,\quad F_{\underline{ik}}=\mp 2M\epsilon_{ikl}\frac{x^{\underline{l}}}{\rho^3}\,,$$

$$e^{2\phi}=1+\frac{2M}{\rho}\,,\quad \rho^2=\mathbf{x}^2=x^2+y^2+z^2\,. \tag{2}$$

The ten-dimensional uplifted version of this geometry[2] can be easily obtained by using supersymmetric dimensional reduction [14,15]. In the ten-dimensional configuration there are no vector fields, but there are nondiagonal components of the metric as well as a two-form field:

$$ds^2_{(10)}=dt^2-e^{4\phi}d\mathbf{x}^2-(dx^{\underline{4}}+V_{\underline{i}}dx^{\underline{i}})^2-dx^{\underline{I}}dx^{\underline{I}}.$$

$$B_{(10)}=\tfrac{1}{2}B_{\mu\nu}\ dx^\mu\wedge dx^\nu=-V_{\underline{i}}dx^{\underline{i}}\wedge dx^{\underline{4}},$$

$$\partial_{\underline{i}}\partial_{\underline{i}}e^{2\phi}=0\ ,e^{2\phi}=1+\sum_s\frac{2M_s}{|\mathbf{x}-\mathbf{x}_s|}\,,$$

$$2\partial_{[\underline{i}}V_{\underline{k}]}=\pm\epsilon_{ikl}\ \partial_{\underline{l}}e^{2\phi}. \tag{3}$$

This uplifted extreme magnetic black hole only solves the zero-slope limit of the equations of motion of the heterotic string effective action. Fortunately, as we will show later, it is possible to promote this zero slope limit solution to one exact at all orders in α'. For this purpose the solution has to be supplemented by a ten-dimensional SU(2) Yang-Mills field, which in this case is

$$A_{\underline{k}}{}^{\underline{i}}=-\epsilon_{ikl}\partial_{\underline{l}}e^{-2\phi},\quad A_{\underline{4}}{}^{\underline{i}}=\pm 2\partial_{\underline{i}}e^{-2\phi}\,. \tag{4}$$

In what follows we are going to prove that *the ten-dimensional configuration (3), (4) is an exact solution of the heterotic string theory effective action with unbroken supersymmetry.*

The idea of identifying the gravitational spin connection with the Yang-Mills field was suggested a long time ago [16]. It turns out to be extremely effective in our case. In the context of heterotic string theory the idea of the embedding of the spin connection into the gauge group was developed in [17] and applied in [5,6]. We would like to stress however the important fact discovered by Bergshoeff and de Roo [4] that there are two spin connections with torsion, $\Omega_\pm=\omega(e)\mp\frac{3}{2}H$ that play different roles: Ω_+ controls the classical part of supersymmetry transformation rule of gravitino, whereas Ω_- is responsible for α' corrections to the action and to the supersymmetry rules, related to Lorentz and supersymmetry anomalies. Because of that, the proper embedding of the spin connection into the gauge group is

$$\Omega_{\mu-}{}^{ab}=A_\mu{}^{ab}\ . \tag{5}$$

This means that it is $\Omega_{\mu-}{}^{ab}$ which has to be identified with the gauge field to cancel anomalies as well as corrections to the supersymmetry rules and to solutions. Heterotic string α' corrections to supersymmetry transformation laws as well as to equations of motion enter via T tensors and Chern-Simons terms, defined in [4] and described in detail in [8,9]. If the corresponding T tensors and Chern-Simons terms defined in Eqs. (49)–(52) of [8] do not vanish for a given configuration, the supersymmetry of a given configuration which is unbroken in the lowest order is destroyed unless the torsionful spin connection $\Omega_{\mu-}{}^{ab}$ is embedded into the gauge group. In our previous work [8] we have shown that *there are two necessary conditions for having unbroken supersymmetry and the absence of α' corrections to a given configuration.*

(i) The configuration has to solve the lowest-order equation of motion. The corresponding Yang-Mills equations (8) will be defined below.

(ii) All T tensors and Chern-Simons terms have to vanish for the configuration.

For the special case of gravitational waves, supplemented by a two-form and non-Abelian field, which we called supersymmetric string waves [8], we have shown that both above conditions are met. We have also found that for the most general of those solutions the Yang-Mills group is SO(8).

Here we start with the configuration without Yang-Mills field which solves the lowest-order equations of motion for the metric, dilaton, and the two-form field. The lowest-order equation of motion for the Yang-Mills field follows from the effective action of the heterotic string which includes first-order α' corrections:

$$S^{(1)}=\tfrac{1}{2}\int d^{10}xe^{-2\phi}\sqrt{-g}\,[-R+4(\partial\phi)^2-\tfrac{3}{4}H^2+\tfrac{1}{2}T]\,, \tag{6}$$

where

$$T=2\alpha'\,[R_{\mu\lambda}{}^{ab}(\Omega_-)R^{\mu\lambda}{}_{ba}(\Omega_-)-\operatorname{tr}F_{\mu\lambda}F^{\mu\lambda}]\ . \tag{7}$$

The trace is in the vector representation of the SO(32) group and the H^2 term does contain the first-order correction term in α' given by the combination of Yang-Mills and Lorentz Chern-Simons terms. The detailed form of the Yang-Mills equation is given in Eq. (62) of [8], where also other details of the procedure, described here shortly, can be found. The lowest-order Yang-Mills equation can be presented in a very compact form

$$\alpha'\ D_a{}^+(e^{-2\phi}F^{ab})=0\,, \tag{8}$$

where $D_a{}^+$ is the Yang-Mills and general covariant derivative associated with the connections A_μ and $\Omega_{\mu+}{}^{ab}$.

In what follows, we are going to exhibit the corresponding spin connections for the uplifted extreme magnetic dilaton black hole and use them to rescue the unbroken super-

[2]The uplifting of the single magnetic dilaton black hole in spherically symmetric coordinate system was performed in [13].

symmetry, which is damaged by the gravitational part of α' corrections not balanced by the Yang-Mills field contribution.

Our configuration corresponds to a decomposition of the manifold $M^{1,9}\to M^{1,5}\times M^4$ and the tangent space SO (1,9)→SO(1,5)×SO(4). The curved manifold is only M^4 since $M^{1,5}$ is flat. The zehnbein basis is given by the sechsbein part $e_{\underline{I}}{}^{I}=e_I{}^{I}=1$ and the vierbein part $e_{\underline{i}}{}^{j}=e^{2\phi}\delta_{\underline{i}}{}^{j}$, $e_{\underline{i}}{}^{4}=V_{\underline{i}}$, $e_{\underline{4}}{}^{i}=0$, $e_{\underline{4}}{}^{4}=1$. The nonvanishing components of $\Omega_{c\pm}{}^{ab}$ in the above basis are

$$\Omega_{4-}{}^{ij}=\Omega_{i+}{}^{4j}=\pm\epsilon_{ijk}\partial_{\underline{k}}e^{-2\phi},$$

$$\Omega_{k-}{}^{im}=\Omega_{k+}{}^{im}=2\delta_{k[i}\partial_{\underline{m}]}e^{-2\phi}. \tag{9}$$

Having found the torsionful spin connection we may perform the embedding of the spin connection into the gauge group. We will relate the spin connection to a gauge field A^k_μ as

$$\Omega_{\mu-}{}^{ij}=\epsilon^{ijk}A^k_\mu\ . \tag{10}$$

The choice of the gauge group is SU(2), according to Eqs. (9). This gauge group is a subgroup of the gauge group of the heterotic string SO(32) (or $E_8\times E_8$).

Using the spin connections defined above we have confirmed the unbroken supersymmetry in the zero slope limit for the uplifted black holes directly in terms of the ten-dimensional gravitino and dilatino supersymmetry rules. The Killing spinor is a constant spinor, satisfying one of the constraints ($\gamma_5\equiv\gamma_1\gamma_2\gamma_3\gamma_4$)

$$(1\pm\gamma_5)\epsilon_\pm=0\,. \tag{11}$$

The integrability condition for the unbroken supersymmetry is expressed in the simplest way as the (anti-)self-duality condition for the torsionful curvatures in M^4:

$$R_{\mu\nu}{}^{ij}(\Omega_+)=\mp\epsilon_{ijm}R_{\mu\nu}{}^{m4}(\Omega_+)\,. \tag{12}$$

By performing the embedding we complied with the condition (ii). However, how do we know that the spin connection of the uplifted magnetic black hole does solve the Yang-Mills equation of motion? In view of the fact that non-Abelian black hole solutions have not yet been constructed in an analytic form, we have to study this problem very carefully. Callan, Harvey, and Strominger [5] have made a crucial observation about the embedding spin connection into the gauge group. Because of the exchange identity for the torsionful curvatures, the embedding automatically guarantees that the unbroken supersymmetry of the solution remains unbroken after the addition of the Yang-Mills field. Indeed by identifying $\Omega_{c-}{}^{ab}$ with the Yang-Mills field we checked directly that the gaugino supersymmetry rule also has an unbroken supersymmetry with the Killing Spinor satisfying constraint (11). The (anti-)self-duality of the non-Abelian field strength

$$F_{ik}{}^{j}=\mp\epsilon_{ikm}F_{m4}{}^{j} \tag{13}$$

is also valid [it follows from $R_{ij}{}^{\mu\nu}(\Omega_-)=\mp\epsilon_{ijm}R_{m4}{}^{\mu\nu}(\Omega_-)$] and provides the integrability condition for the unbroken supersymmetry of gaugino. However, supersymmetry in general may be not sufficient to show that the configuration above solves the Yang-Mills equation of motion. We have checked that for our configuration unbroken supersymmetry does mean that the Yang-Mills equation of motion is satisfied. We have also checked directly that our solution (3), (4) solves[3] Eq. (8). The lowest-order equations of motion for the metric, dilaton, and two-form field are satisfied, since we have performed a supersymmetric uplifting of the magnetic black hole. Therefore the condition (i) of exactness is met for our solution.

Thus we conclude that under the assumptions specified in [8,9] our ten-dimensional configuration (3), (4), in addition to being a solution of the lowest-order equations of motion, is an exact solution with unbroken supersymmetry of the effective equations of motion of the heterotic string to all orders in α'. The main assumption about the exactness of a solution is the following. A solution which has unbroken supersymmetry is subject to a supersymmetric nonrenormalization theorem in absence of supersymmetry anomalies. The known supersymmetry anomaly [10], related to Lorentz anomaly, is taken care of by embedding the spin connection in the gauge group, which results in the necessary presence of a specific non-Abelian field in the solution.

To support our space-time arguments for the exactness of this solution we have analyzed the appropriate superconformal world-sheet σ model following the analysis performed in [5]. By using the work of Howe and Papadopoulos [18] we have found that the corresponding σ model for our special background has (4,1) world-sheet supersymmetry. The model can be presented in terms of (1,1) superfields [18]. For this model the additional 3 supersymmetries exist when the background satisfies some conditions including the existence of 3 complex structures. The complex structures are constructed from the chiral Killing spinors, responsible for the unbroken space-time supersymmetry of our background. The bilinear combinations of these spinors are covariantly constant with respect to the proper spin connection. The self-duality of the gauge field strength, which is also a condition for additional world-sheet supersymmetry of our class of backgrounds, is also met. Thus the σ model discussed above for our special background has (4,1) world-sheet supersymmetry. In the heterotic string theory, this property of the background is believed to provide the exactness property of the background [18,5].

The analogous procedure can be performed for other black-hole-type solutions, known to be supersymmetric in $d=4$. The most surprising result of this analysis concerns the extreme electric black hole. The extreme electric dilaton black hole was uplifted to $d=10$ in [15]. Meanwhile, Horowitz and Tseytlin have proved that the extreme electric dilaton black holes as well as their generalizations [12] are exact solutions of bosonic string theory [19]. In heterotic string theory we know that after we get the torsionful spin connection of the uplifted configuration we have to identify the corresponding non-Abelian field to nullify the above-mentioned T tensors and the combination of Lorentz and Yang-Mills Chern-Simons terms to restore the unbroken su-

[3]The correct normalization for the SU(2) Lie algebra with our conventions is $[T_i,T_j]=-\frac{1}{2}\epsilon_{ijk}T_k$.

persymmetry. The corresponding nonvanishing components of the spin connection turn out to be

$$\Omega_{4-}{}^{0i} = \pm e^{4\phi}\partial_i e^{-2\phi}. \tag{14}$$

The Yang-Mills field which will serve to restore the unbroken supersymmetry and fix $T_{44}=T=0$ has to belong to a boosts part of the noncompact group SO(1,3). This is not part of SO(32) or $E_8\times E_8$ gauge system of the heterotic string in critical dimension. Alternatively, the non-Abelian gauge field may belong to SU(2) group, but the field has to be imaginary. Both solutions do not seem to be acceptable. The black-hole–wave duality analysis performed in [15] had the strange feature that the electric black hole was dual to a wave with an imaginary component in the metric. This mysterious property has found a complete explanation now. We have found the system of coordinates where the corresponding wave solution, promoted to the exact one in the presence of the gauge fields, had the metric and the two-form field real, but the gauge field is still imaginary. This we have found now from the direct analysis without using black-hole–wave duality. In conclusion, the uplifted electrically charged black hole cannot be promoted to an exact solution of the heterotic string theory via spin embedding, does not have an unbroken supersymmetry and does not solve the field equations with account taken of the α' corrections in the frame for which the the space-time supersymmetric embedding is known [4]. The situation seems to be better for the uplifted IWP axion-dilaton geometries [12], at least for NUT charges which are not smaller than the mass of the solution. Our conclusion about electric black holes is not in disagreement with the analysis of Horowitz and Tseytlin [19]. Indeed, we were looking for exact solutions of heterotic string theory with unbroken supersymmetry which are supposed to be stable. They have found that by changing the renormalization scheme for the β functions one can find a possibility without a gauge field to keep the zero slope limit of these solutions exact due to the null properties of one of the spin connections. The issue of α' corrections to unbroken supersymmetry of the solutions without a gauge field remains unclear, since it is not known if the effective action which was used in [19] has a supersymmetric embedding. We are trying to understand the relation between those two effective actions.

As we have seen, in ten dimensions the simple addition of an SU(2) Yang-Mills field is enough to turn the uplifted extreme magnetic dilaton black hole into an exact solution. The resulting configuration will also be exact upon reduction to four-dimensions. The four dimensional metric, axion, dilaton, and U(1) vector field are unchanged. They are still those of the leading-order solution. But, in addition to them, the four dimensional exact solution has an SU(2) scalar Φ^i whose origin is the fourth component of the ten-dimensional gauge field $A_4{}^i$, and an SU(2) vector field that we call W to distinguish it from the ten-dimensional A whose 1,2,3 components are identical in flat indices but not in curved. This is due to the presence of nondiagonal elements V_i in the ten-dimensional metric (3).

The four-dimensional exact SU(2)×U(1) dilaton (multi)-black hole is, therefore,

$$\begin{aligned} ds^2 &= dt^2 - e^{4\phi}d\mathbf{x}^2, \\ e^{2\phi} &= 1+\sum_s \frac{2M_s}{|\mathbf{x}-\mathbf{x}_s|}, \\ F_{ik}(V) &= \pm \epsilon_{ikl}\partial_l e^{2\phi}, \\ \Phi^i &= \pm 2\partial_i e^{-2\phi}, \\ W_j{}^i &= -2\epsilon_{ijk}\partial_k e^{-2\phi}. \end{aligned} \tag{15}$$

Since the dimensional reduction has been done in a way compatible with supersymmetry, the four-dimensional exact SU(2)×U(1) black hole will have half of unbroken supersymmetries of $N=4$ supergravity interacting with $N=4$ Yang-Mills theory with account taken of stringy α' corrections.

As different from the pure SU(2) solutions presented in Ref. [6], the self-duality of the SU(2) gauge field strength in the Euclidean four-dimensional space $x^1,\ldots,x^4$ does not translate into the conventional form of the Bogomolnyi bound but into an analogous balance of forces expression that involves the U(1) field strength F:

$$G_{jk}(W) + \Phi F_{jk}(V) = \pm \epsilon_{jkl}D_l\Phi. \tag{16}$$

Again this is due to the nondiagonal elements of the ten-dimensional metric [the U(1) vector field V] which do not occur in the *Ansätzes* used in Ref. [6].

The fields of a single SU(2)×U(1) black hole are

$$\begin{aligned} ds^2 &= dt^2 - e^{4\phi}d\mathbf{x}^2, \\ F_{\underline{ik}} &= P\epsilon_{ikl}\ x^{\underline{l}}/\rho^3, \quad P=\mp 2M, \\ e^{2\phi} &= 1+2M/\rho, \\ \Phi^i &= -2P\frac{x^{\underline{i}}}{\rho(\rho+2M)^2}, \\ W_j{}^i &= \mp 2P\epsilon_{ijk}\frac{x^k}{\rho(\rho+2M)^2}. \end{aligned} \tag{17}$$

The asymptotic behavior of our non-Abelian fields is the same as in stringy monopole solution [6], they fall off faster than the fields of 't Hooft–Polyakov monopole. By performing a gauge transformation of the type $g\partial_4 g^{-1}$ one may also change the asymptotic value of the scalar filed so that it does not vanish at $\rho\to\infty$.

The conclusion is that by addition of an appropriate Yang-Mills field in some cases it is possible to get solutions of the heterotic string effective action with unbroken supersymmetry which are exact at all orders in α'. This we have done for the extreme magnetic dilaton black holes and we have found that the same procedure does not work for the electric ones and for some of Taub-NUT solutions. The procedure which we have used can be applied to other four-dimensional solutions with unbroken supersymmetry. Our expectation is that all of these solutions which will be found exact can be obtained in a closed analytic form. One may hope that in this way more information will be obtained about axion-dilaton black-hole–type solutions and new restrictions on their parameters may emerge. This may lead to better understanding of the charge quantization of the black holes due to the possibility that the non-Abelian charges will play an important role.

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[1] G. W. Gibbons, Nucl. Phys. **B207**, 337 (1982); G. W. Gibbons and K. Maeda, *ibid.* **B298**, 741 (1988); D. Garfinkle, G. T. Horowitz, and A. Strominger, Phys. Rev. D **43**, 3140 (1991).

[2] R. Kallosh, A. Linde, T. Ortín, A. Peet, and A. Van Proeyen, Phys. Rev. D **46**, 5278 (1992).

[3] C. Hull (in preparation).

[4] E. Bergshoeff and M. de Roo, Nucl. Phys. **B328**, 439 (1989).

[5] C. G. Callan, Jr., J. A. Harvey, and A. Strominger, Nucl. Phys. **B359**, 611 (1991); **B367**, 60 (1991).

[6] R. Khuri, Nucl. Phys. **B387**, 315 (1992); J. Gauntlett, J. Harvey, and J. Liu, *ibid.* **B409**, 363 (1993).

[7] M. Duff, R. Khuri, R. Minasyan, and J. Rahmfeld, Nucl. Phys. **B418**, 195 (1994).

[8] E. Bergshoeff, R. Kallosh, and T. Ortín, Phys. Rev. D **47**, 5444 (1993).

[9] E. Bergshoeff, I. Entrop, and R. Kallosh, Phys. Rev. D **49**, 6663 (1994).

[10] R. E. Kallosh, Phys. Lett. **159B**, 111 (1985).

[11] M. Natsuume, Phys. Rev. D **50**, 3949 (1994).

[12] R. Kallosh, D. Kastor, T. Ortín, and T. Torma, Phys. Rev. D **50**, 6374 (1994); C. Johnson and R. C. Myers, *ibid.* **50**, 6512 (1994); D. Gal'tsov and O. Kechkin, this issue, *ibid.* **50**, 7394 (1994).

[13] W. Nelson, Phys. Rev. D **49**, 5302 (1994).

[14] A. Chamseddine, Nucl. Phys. **B185**, 403 (1981) .

[15] E. Bergshoeff, R. Kallosh, and T. Ortín, Phys. Rev. D **50**, 5188 (1994); E. Bergshoeff, R. Kallosh, and T. Ortín, "Duality Versus Supersymmetry and Compactification," Report No. UG-894, SU-ITP-94-19, QMW-PH-94-13, hepth/9410230, 1994 (unpublished).

[16] F. Wilczek, in *Quark Confinement and Field Theory*, edited by D. R. Stump and D. H. Weingarten (Wiley-Interscience, New York, 1977); J. Charap and M. Duff, Phys. Lett. **69B**, 445 (1977).

[17] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985).

[18] P. Howe and G. Papadopoulos, Nucl. Phys. **B289**, 264 (1987); **B381**, 360 (1992).

[19] G. T. Horowitz and A. A. Tseytlin, Phys. Rev. D **50**, 5204 (1994); "A new class of exact solutions in string theory," Report No. Imperial /TP/93-94/54, UCBTH-94-31, hep-th/9409021, 1994 (unpublished).