A Simple Derivation of Supersymmetric Extremal Black-Hole Attractors

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Abstract

We present a simple and yet rigorous derivation of the flow equations for the supersymmetric black-hole solutions of all 4-dimensional supergravities based on the recently found general form of all those solutions.

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Introduction

The discovery of the attractor mechanism that drives the scalar fields of supersymmetric extremal black holes to take values that only depend on the electric and magnetic charges on the event horizon [1, 2, 3, 4, 5, 6] has undoubtedly been one of the main breakthroughs of black-hole physics in the recent years. Long after the discovery of the existence of extremal but non-supersymmetric black-hole solutions [7, 8, 9] it was realized that there is an attractor mechanism at work in those black holes as well. [10, 11, 12, 13]. Since the existence of the attractor mechanism in supersymmetric black holes is related to the existence of flow (first-order) equations for the metric function and scalar fields that follow from the Killing spinor equations[2], it was natural to search for flow equations driving the metric function and scalar fields of extremal non-BPS black holes to their attractor values. Those equations, which depend on a "superpotential" function which coincides with the central charge in the BPS case were found in Ref. [15] for $N = 2, d = 4$ supergravity and in Ref. [16] for $N > 2, d = 4$ theories. These developments were based in the approach pioneered in Ref. [5]. Further extensions of these results to non-supersymmetric cases and singular black-hole-type solutions ("small back holes") can be found in Refs. [17, 18, 19, 20, 21, 22].

In this paper we are going to present a simple derivation of those flow equations in all 4-dimensional, ungauged, supergravities which does not make explicit use of supersymmetry and may be valid for extremal non-supersymmetric black holes and other solutions of those theories.

We start by deriving in Section 1 the black-hole flow equations for $N = 2, d = 5$ supergravity coupled to vector supermultiplets as a particularly simple example. In Section 2 we work out the well-known case of $N = 2, d = 4$ supergravity coupled to vector supermultiplets and in Section 3 we generalize our results to general $N, d = 4$ supergravity.

1 $N = 2, d = 5$

sec-d5

The $N = 2, d = 5$ vector supermultiplets contain one 1-form $A^x{}_\mu$ and one real scalar ϕ^x ($x, y, z = 1, \cdots, n$ where n is the number of vector multiplets). The n matter 1-forms are combined with the graviphoton $A^0{}_\mu$ into $A^I{}_\mu$ and the n scalars are described by $\bar{n} = n + 1$ real functions $h_I(\phi)$, $(I, J, K = 0, 1, \cdots n)$. These are subject to the constraints

$$h^I h_I = 1, \qquad h^I dh_I = h_I dh^I = 0. \tag{1.1}$$

eq:const

The metric is the scalar manifold g_{xy} is given by

$$h^I{}_x h_{Iy} = g_{xy}, \qquad h_{Iy} \equiv -\sqrt{3}\partial_y h_I, \qquad h^I{}_x = \sqrt{3}\partial_x h_I. \tag{1.2}$$

eq:defgx

It is well known that the static, spherically-symmetric supersymmetric solutions of these theories are such that the quotients h_I/f, where f is related to the spacetime metric by $f^2 = g_{tt}$, are harmonic functions in Euclidean $\mathbb{R}^4$ [23, 24]. We can write these functions as linear functions of an appropriate coordinate τ

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[2]See, e.g. Ref. [14] for a derivation of the flow equations in $N = 2, d = 4$ supergravity along these lines.

$$h_I/f \equiv l_I - q_I\tau\,, \tag{1.3}$$ `eq:hIf`

where the constants q_I are the electric charges. We are going to assume that we have a field configuration of the above form for some coordinate τ, not necessarily supersymmetric, not necessarily satisfying the equations of motion and not necessarily being a black hole, although supersymmetric black holes would be the prime example to which one can apply the following results.

The central charge in these theories is given by

$$\mathcal{Z}[\phi, q] \equiv h^I(\phi)q_I\,. \tag{1.4}$$

We are going to find the flow equations obeyed by $f(\tau)$ and the scalar fields $\phi^x(\tau)$ using just basic relations of real special geometry. First, using Eqs. (1.1) we write the differential of f^{-1} as

$$df^{-1} = d(h^I h_I/f) = h^I d(h_I/f)\,, \tag{1.5}$$

from which we get the first component of the flow equations

$$\frac{df^{-1}}{d\tau} = \mathcal{Z}[\phi(\tau), q]\,. \tag{1.6}$$ `eq:d5flo`

Using now the same constraints plus the definition Eq. (1.2) we can write for the differential of ϕ^x

$$d\phi^x = h^{Ix}h_{Iy}d\phi^y = -\sqrt{3}h^{Ix}dh_I = -\sqrt{3}h^{Ix}d(fh_I/f) = -\sqrt{3}fh^{Ix}d(h_I/f)\,, \tag{1.7}$$

from which we get the remaining n components of the flow equations:

$$\frac{d\phi^x}{d\tau} = fg^{xy}\partial_y\mathcal{Z}[\phi(\tau), q]\,. \tag{1.8}$$ `eq:d5flo`

The variables $\phi^x(\tau)$ and the solutions will be attracted to the fixed points ϕ^x_{fixed} at which the r.h.s. vanishes, i.e. where the attractor equations

$$\partial_y\mathcal{Z}[\phi^x_{\text{fixed}}, q] = 0\,, \tag{1.9}$$

are satisfied. The solutions of these equations give ϕ^x_{fixed} as functions of the electric charges q_I and at the point $\tau = \tau_{\text{fixed}}$, ϕ^x takes the value $\phi^x_{\text{fixed}}(q)$, independently of the constants l_I. Furthermore, at the attractor point

$$\left.\frac{df^{-1}}{d\tau}\right|_{\tau=\tau_{\text{fixed}}} = \mathcal{Z}[\phi_{\text{fixed}}(q), q] \equiv \mathcal{Z}_{\text{fixed}}(q)\,. \tag{1.10}$$

The derivation of the flow equations (1.6),(1.8) that we have presented and the properties that follow (the attractor mechanism) holds for any field configuration of the form Eq. (1.3), irrespectively of the meaning of the function f or the coordinate τ and of the physical properties of the

configuration. If the field configuration describes a 5-dimensional supersymmetric black-hole solution, then one can show that there is an event horizon at τ_{fixed} and the attractor mechanism relates the central charge to the black-hole entropy [4]. The general 5-dimensional flow equations for $N = 2$ theories have been derived in Ref. [25], in Ref. [26] using timelike dimensional reduction techniques and in Ref. [27].

2 $N = 2, d = 4$

Let us now consider $N = 2, d = 4$ supergravity coupled to n vector supermultiplets. Each of them contains a 1-form $A^i{}_\mu$ and one complex scalar Z^i $i = 1, \cdots, n$. The Z^i parametrize a special Kähler manifold. The n matter 1-forms are combined with the graviphoton into $A^\Lambda{}_\mu$ $(\Lambda, \Sigma = 0, 1, \cdots, n)$ while the n complex scalars are combined into the $2\bar{n} = 2(n + 1)$ components of the symplectic section $\mathcal{V} \equiv (\mathcal{L}^\Lambda, \mathcal{M}_\Lambda)$. These are subject to the constraints

$$i\langle \mathcal{V} \mid \mathcal{V}^* \rangle \equiv i(\mathcal{L}^{*\Lambda}\mathcal{M}_\Lambda - \mathcal{M}_\Lambda^* \mathcal{L}^\Lambda) = 1, \qquad \langle \mathcal{D}_i \mathcal{V} \mid \mathcal{V}^* \rangle = 0, \qquad (2.1)$$

where $\mathcal{D}_i$ is the Kähler-covariant derivative and $\mathcal{V}$ has Kähler weight 1.

The supersymmetric black-hole solutions of these theories [1, 2, 28, 29, 30, 31, 32] are such that the components of the real symplectic vector $\mathcal{I} \equiv \Im\mathrm{m}(\mathcal{V}/X)$, where X is a Kähler-weight[3] 1 complex function related to the spacetime metric by $|X|^{-2} = 2g_{tt}$, are given again by linear functions of some coordinate τ

$$\mathcal{I} \equiv \mathcal{I}_0 - \tfrac{1}{\sqrt{2}}\mathcal{Q}\tau, \qquad (2.2)$$

where $\mathcal{I}_0$ and $\mathcal{Q}$ are constant symplectic vectors[4]. The components of $\mathcal{Q}$, (p^Λ, q_Λ), are the magnetic and electric charges of the solution.

We are going to assume that we have a field configuration of the above form for some coordinate τ, not necessarily supersymmetric, not necessarily satisfying the equations of motion and not necessarily being a black hole, and we are going to find flow equations for $X(\tau)$ and the complex scalar fields $Z^i(\tau)$ using basic relations of special geometry.

Let us define the central charge

$$\mathcal{Z}[Z(\tau), \mathcal{Q}] \equiv \langle \mathcal{V} \mid \mathcal{Q} \rangle = p^\Lambda \mathcal{M}_\Lambda - q_\Lambda \mathcal{L}^\Lambda. \qquad (2.3)$$

Since $\mathcal{V}/X$ has zero Kähler weight, using Eqs. (2.1)

$$\mathcal{D}X^{-1} = i\langle \mathcal{V} \mid \mathcal{V}^* \rangle \mathcal{D}X^{-1} = \langle \mathcal{D}(\mathcal{V}/X) \mid \mathcal{V}^* \rangle = \langle d(\mathcal{V}/X) \mid \mathcal{V}^* \rangle. \qquad (2.4)$$

We now need to use a less trivial property, proved in an appendix of Ref. [14] using the homogeneity of the prepotential

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[3]The function X appears naturally in the spinor-bilinear method [32] and plays a role analogous to that of the function f in the $N = 2, d = 5$ case. Furthermore, since X has the same Kähler weight as $\mathcal{V}$, the quotient is Kähler gauge-independent. This independence is necessary for the prescription to construct the most general black-hole solutions to be consistent.

[4]The factor $1/\sqrt{2}$, necessary for a correct normalization for the charges, was omitted in Ref. [14].

$$\langle\, d(\mathcal{V}/X) \mid \mathcal{V}^*/X^* \,\rangle = 2i\langle\, d\mathcal{I} \mid \mathcal{V}^*/X^* \,\rangle\,, \qquad (2.5)$$

which leads us to

$$\mathcal{D}X^{-1} = 2\langle\, \mathcal{V}^* \mid d\mathcal{I} \,\rangle\,, \qquad (2.6)$$

from which we get the first component of the flow equations

$$\mathcal{D}_\tau X^{-1} = -\sqrt{2}\mathcal{Z}^*[Z(\tau), \mathcal{Q}]\,. \qquad (2.7)$$

Using the property $-i\langle\, \mathcal{D}_i\mathcal{V} \mid \mathcal{D}_{i^*}\mathcal{V}^* \,\rangle = \mathcal{G}_{ii^*}$ and the previous ones

$$
\begin{aligned}
dZ^i &= i\mathcal{G}^{ij^*}\langle\, \mathcal{D}_{j^*}\mathcal{V}^* \mid \mathcal{D}_j\mathcal{V} \,\rangle dZ^j = iX\mathcal{G}^{ij^*}\langle\, \mathcal{D}_{j^*}\mathcal{V}^* \mid \mathcal{D}_j(\mathcal{V}/X) \,\rangle dZ^j \\[2mm]
&= iX\mathcal{G}^{ij^*}\langle\, \mathcal{D}_{j^*}\mathcal{V}^* \mid \partial_j(\mathcal{V}/X) \,\rangle dZ^j = iX\mathcal{G}^{ij^*}\langle\, \mathcal{D}_{j^*}\mathcal{V}^* \mid d(\mathcal{V}/X) \,\rangle \qquad (2.8) \\[2mm]
&= iX\mathcal{G}^{ij^*}\mathcal{D}_{j^*}\langle\, \mathcal{V}^* \mid d(\mathcal{V}/X) \,\rangle = -2X\mathcal{G}^{ij^*}\mathcal{D}_{j^*}\langle\, \mathcal{V}^* \mid d\mathcal{I} \,\rangle\,,
\end{aligned}
$$

from which we get the remaining components of the flow equations[5]

$$\frac{dZ^i}{d\tau} = \sqrt{2}X\mathcal{G}^{ij^*}\mathcal{D}_{j^*}\mathcal{Z}[Z(\tau), \mathcal{Q}]\,. \qquad (2.9)$$

The first component of the flow equations, for the black-hole case, is customarily written in terms of the component g_{tt} of the metric (or the function $U = \frac{1}{2}\log g_{tt}$) [1, 2, 3, 5]. Those expressions can be obtained from Eq. (2.6), which is more general.

3   Arbitrary $N \geq 2, d = 4$

In Ref. [33] a formulation of all $N \geq 2, d = 4$ supergravities coupled to vector supermultiplets was given that allows to treat simultaneously all of them[6]. This formulation was recently used in Ref. [34] to determine the form of all the timelike supersymmetric solutions (including black holes) of these theories in a unified way. We are going to use this formulation in order to derive flow equations for the metric function and the scalars of all these theories.

The scalars of these theories are described by two sets of symplectic vectors: $\mathcal{V}_{IJ} = \mathcal{V}_{[IJ]}, \mathcal{V}_i$, where $I, J, K = 1, \cdots, N$ and $i = 1, \cdots, n$,

$$\mathcal{V}_{IJ} = \begin{pmatrix} f^\Lambda{}_{IJ} \\ h_{\Lambda IJ} \end{pmatrix}\,, \qquad \mathcal{V}_i = \begin{pmatrix} f^\Lambda{}_i \\ h_{\Lambda i} \end{pmatrix}\,, \qquad (3.1)$$

n being the number of vector supermultiplets (none for $N > 4$). The theory contains $N(N - 1)/2 + n$ 1-forms $A^\Lambda{}_\mu$ $\Lambda = 1, \cdots, N(N - 1)/2 + n$ the first $N(N - 1)/2$ of which are the

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[5]For a derivation of these equations for black holes, from the Killing spinor equations, see Ref. [14].

[6]Some details are N-dependent, but their treatment or use can be postponed until the end of the analysis.

graviphotons that we could have labeled by $A^{IJ}{}_\mu = -A^{JI}{}_\mu$ and the rest of which are the matter 1-forms. In all cases, the symplectic vectors satisfy the constraints

$$
\begin{aligned}
\langle \mathcal{V}_{IJ} \mid \mathcal{V}^{*\,KL} \rangle &= -2i\delta^{KL}{}_{IJ}, \\[2mm]
\langle \mathcal{V}_i \mid \mathcal{V}^{*\,j} \rangle &= -i\delta_i{}^j,
\end{aligned}
\tag{3.2}
$$

with the rest of the symplectic products vanishing. In the $N = 2$ case, these vectors are related to the objects used in the previous section by

$$
\mathcal{V}_{IJ} = \mathcal{V}\varepsilon_{IJ}, \qquad \mathcal{V}_i = \mathcal{D}_i\mathcal{V}. \tag{3.3}
$$

Using them one can construct the scalar Vielbeine

$$
P_{IJKL} = P_{[IJKL]} \equiv -i\langle d\mathcal{V}_{IJ} \mid \mathcal{V}_{KL} \rangle, \qquad P_{iIJ} = P_{i[IJ]} \equiv -i\langle d\mathcal{V}_{IJ} \mid \mathcal{V}_i \rangle. \tag{3.4}
$$

To construct supersymmetric black-hole solutions one must choose first a time-independent, rank-2 complex antisymmetric matrix M_{IJ} satisfying $M_{[IJ}M_{K]L} = 0$ and

$$
M^{[IJ}\mathfrak{D}M^{KL]} = 0, \tag{3.5}
$$

where $\mathfrak{D}$ s the $U(N)$-covariant derivative. in the $N = 2$ case $M_{IJ} = X\varepsilon_{IJ}$ where X is the complex Kähler weight 1 function we introduced in the previous section. M_{IJ} is related to the spacetime metric by $g_{tt} = |M|^{-2}$ where $|M|^2 \equiv M^{PQ}M_{PQ}$. The supersymmetric black-hole solutions are such that the components of the real symplectic vector

$$
\mathcal{I} \equiv \Im\mathrm{m}\mathcal{V} \equiv \Im\mathrm{m}\left(\mathcal{V}_{IJ}M^{IJ}/|M|^2\right), \tag{3.6}
$$

are harmonic functions in Euclidean $\mathbb{R}^3$, so they can be written as linear functions of some coordinate τ

$$
\mathcal{I} \equiv \mathcal{I}_0 - \tfrac{1}{\sqrt{2}}\mathcal{Q}\tau. \tag{3.7}
$$

where, again $\mathcal{Q}$ is the symplectic vector of all magnetic and electric charges of the theory.

We are going to show that, for any field configuration of the above form there are flow equations for the metric function and the scalar Vielbeine.

We define the central charges

$$
\mathcal{Z}_{IJ}[\phi(\tau), \mathcal{Q}] \equiv \langle \mathcal{V}_{IJ} \mid \mathcal{Q} \rangle = p^\Lambda h_{\Lambda\,IJ} - q_\Lambda f^\Lambda{}_{IJ}, \tag{3.8}
$$

$$
\mathcal{Z}_i[\phi(\tau), \mathcal{Q}] \equiv \langle \mathcal{V}_i \mid \mathcal{Q} \rangle = p^\Lambda h_{\Lambda\,i} - q_\Lambda f^\Lambda{}_i. \tag{3.9}
$$

Then, using the above constraints and the definitions of the Vielbeine

$$
\begin{aligned}
\mathfrak{D}\frac{M^{IJ}}{|M|^2} &= \tfrac{i}{2}\mathfrak{D}\left(\frac{M^{KL}}{|M|^2}\langle\,\mathcal{V}_{KL}\mid\mathcal{V}^{*\,IJ}\,\rangle\right) = \tfrac{i}{2}\mathfrak{D}\langle\,\mathcal{V}\mid\mathcal{V}^{*\,IJ}\,\rangle = \tfrac{i}{2}\langle\,d\mathcal{V}\mid\mathcal{V}^{*\,IJ}\,\rangle \\[2mm]
&= \tfrac{i}{2}\langle\,d\mathcal{V}^*\mid\mathcal{V}^{*\,IJ}\,\rangle - \langle\,d\mathcal{I}\mid\mathcal{V}^{*\,IJ}\,\rangle = \tfrac{i}{2}\frac{M_{KL}}{|M|^2}\langle\,d\mathcal{V}^{*\,KL}\mid\mathcal{V}^{*\,IJ}\,\rangle + \tfrac{1}{\sqrt{2}}\langle\,\mathcal{Q}\mid\mathcal{V}^{*\,IJ}\,\rangle d\tau \\[2mm]
&= -\tfrac{1}{\sqrt{2}}\mathcal{Z}^{*\,IJ}[\phi(\tau),\mathcal{Q}]d\tau + \tfrac{1}{2}P^{*\,KLIJ}\frac{M_{KL}}{|M|^2}\,.
\end{aligned}
\tag{3.10}
$$

Using this identity we can compute

$$
M^{[IJ}\mathfrak{D}\frac{M^{KL]}}{|M|^2} = -\tfrac{1}{\sqrt{2}}M^{[IJ}\mathcal{Z}^{*\,KL]}[\phi(\tau),\mathcal{Q}]d\tau + \tfrac{1}{2}P^{*\,MN[IJ}\mathcal{J}^K{}_M\mathcal{J}^{L]}{}_N\,,
\tag{3.11}
$$ eq:compu

where

$$
\mathcal{J}^I{}_J \equiv 2|M|^{-2}M^{IK}M_{JK}\,,
\tag{3.12}
$$

is a rank-2 projector $\mathcal{J}^2 = \mathcal{J}$, $\mathcal{J}^I{}_I = 2$. M_{IJ} and $\mathcal{J}$ project over and $N = 2$ subspace of the theory and induces a decomposition of all objects into $N = 2$ representations. In particular, the projected Vielbeine $P^{*\,MN[IJ}\mathcal{J}^K{}_M\mathcal{J}^{L]}{}_N$ and $P_{i\,KL}\mathcal{J}^K{}_I\mathcal{J}^L{}_J$ would correspond to scalars in $N = 2$ vector supermultiplets. The remaining components of the scalar Vielbein would correspond to $N = 2$ hyperscalar which do not have any attractor behavior and do not allow for regular black hole solutions when they are excited [35] and therefore will not be considered any further[7].

Since the l.h.s. of Eq. (3.11) vanishes, we get the flow equation for scalars ($N = 4, 6, 8$)

$$
P^{*\,MN[IJ}\mathcal{J}^K{}_M\mathcal{J}^{L]}{}_N = \sqrt{2}M^{[IJ}\mathcal{Z}^{*\,KL]}[\phi(\tau),\mathcal{Q}]d\tau\,.
\tag{3.13}
$$

We can also compute from Eq. (3.10)

$$
d|M|^{-2} = \frac{M_{IJ}}{|M|^2}\mathfrak{D}\frac{M^{IJ}}{|M|^2} + \frac{M^{IJ}}{|M|^2}\mathfrak{D}\frac{M_{IJ}}{|M|^2} = -\tfrac{1}{\sqrt{2}}\left[\frac{M_{IJ}}{|M|^2}\mathcal{Z}^{*\,IJ} + \frac{M^{IJ}}{|M|^2}\mathcal{Z}_{IJ}\right][\phi(\tau),\mathcal{Q}]d\tau\,,
\tag{3.14}
$$

which leads to the component flow equation for the metric function

$$
\frac{d}{d\tau}|M|^{-1} = -\tfrac{1}{\sqrt{2}}\mathfrak{Re}\left(\frac{M^{IJ}\mathcal{Z}_{IJ}}{|M|}\right)\,.
\tag{3.15}
$$

The final set of components of the flow equation ($N = 2, 3, 4$) follows from

$$
\begin{aligned}
\tfrac{1}{2}\frac{M^{IJ}}{|M|^2}P_{iIJ} &= -\tfrac{i}{2}\frac{M^{IJ}}{|M|^2}\langle\,d\mathcal{V}_{IJ}\mid\mathcal{V}_i\,\rangle = -\tfrac{i}{2}\langle\,d\mathcal{V}\mid\mathcal{V}_i\,\rangle = \langle\,d\mathcal{I}\mid\mathcal{V}_i\,\rangle - \tfrac{i}{2}\langle\,d\mathcal{V}^*\mid\mathcal{V}_i\,\rangle \\[2mm]
&= \tfrac{1}{\sqrt{2}}\mathcal{Z}_i[\phi(\tau),\mathcal{Q}]d\tau\,.
\end{aligned}
\tag{3.16}
$$

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[7]See also [40, 41, 42].

and takes the final form

$$P_{i\,KL}\mathcal{J}^{K}{}_{I}\mathcal{J}^{L}{}_{J} = \sqrt{2}M_{IJ}\mathcal{Z}_{i}[\phi(\tau),\mathcal{Q}]d\tau\,.$$

The equations for the critical (attractor) points of $N = 8, d = 4$ supergravity, both supersymmetric and non-supersymmetric where given in Ref. [6] and some flow equations for the $N = 8$ theory based on different Ansatze were given in Refs. [36, 37, 38, 39]. It would be interesting to compare them with those that are determined from the above flow equations, although more information about the matrix of functions M_{IJ} is necessary.

4   Conclusions

After considering two examples ($N = 2, d = 4, 5$ supergravity) we have derived the general flow equations for supersymmetric black holes in all $N \geq 2, d = 4$ supergravities using a procedure that only uses basic properties of the scalar manifolds of those theories and an Ansatz for certain combinations of the scalar functions and some auxiliary functions (M_{IJ}) inspired in the general form of the supersymmetric black-hole solutions. we have derived the flow equations for $N \geq 2$ in a form which is manifestly duality-covariant.

The procedure used here may apply to many more solutions (supersymmetric of not) which share the form of the Ansatz. For instance, it should apply to non-supersymmetric extremal black holes and may also apply, for instance, to cosmological solutions in which the coordinate τ is timelike. This derivation, which depends on so few assumptions, may shed new light on the reasons underlying the attractor mechanism in black holes and other supergravity solutions.

The generalization of these derivations to higher dimensions should be straightforward, using the formulation of Ref. [33].

Acknowledgments

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