Generating Ideas for Numeracy Tasks across the Curriculum

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The purpose of this article is to provide insight into how teachers identify initial ideas for the design of numeracy tasks. A design-based research approach was employed utilising classroom observations, video-stimulated recall techniques and semi-structured teacher interviews. Data collection and analysis were informed by a rich model of numeracy as well as generic principals of task design synthesised from relevant literature. Data analysis indicated that there were at least two approaches to generating ideas for numeracy tasks both of which were compatible with the principles of task design employed in this study.

The term numeracy is one of a number of terms used internationally (e.g., Australia, Canada, New Zealand, South Africa and the UK) for the capability to make effective and critical use of mathematics in personal, academic, workplace, and civic life (Geiger, Forgasz, & Goos, 2015). Although numeracy is often associated with the mastery of basic arithmetic skills, it is now understood that being numerate, in an increasingly globalised world characterised by rapid technological and economic change, must involve the capability to use mathematics to exercise critical judgement and to explore and bring to resolution real world problems (Steen, 2001).

All Australian students are expected to build the capability to apply mathematics to solve real world problems through engagement with Australian Curriculum (ACARA, 2017) that specifies numeracy as one of the general capabilities to be developed in all subjects, not just in mathematics. However, the lack of advice to teachers on how to design effective numeracy tasks, from within the Australian Curriculum, educational jurisdictions or other sources, threatens the successful implementation of this goal.

Burkhart and Swan (2013) argue for the importance of task design in improving the teaching of mathematics leading to enhanced student learning outcomes. While acknowledging the role quality tasks to effective teaching practice, Schoenfeld (2009) argues that task design principles are rarely made explicit and so it is difficult to for others, including teachers, to adopt effective approaches to task creation and adaptation. This situation is confounded by a lack of insight, due to limited research, into how teachers generate ideas that can serve as the basis for the design of mathematical activities, especially those that promote students' numeracy capability.

The purpose of this paper is to report on an aspect of a three-year longitudinal study that aimed to generate new understandings about how teachers design and implement effective numeracy tasks. The aspect attended to here is the processes teachers employed to generate ideas for numeracy tasks. In doing so the following research question will be addressed:

What processes do teachers utilise when generating ideas in initial stages of designing numeracy tasks?

In responding to this question, the following will be described and discussed: underpinnings of task design; methodological approach; two illustrative classroom vignettes; and findings and opportunities for further research.

2018. In Hunter, J., Perger, P., & Darragh, L. (Eds.). Making waves, opening spaces (*Proceedings of the 41st annual conference of the Mathematics Education Research Group of Australasia*) pp. 314-321. Auckland: MERGA.

Principles of Task Design as Lenses for Introspection

Two complementary theoretical perspectives were used to guide the initial stages of the larger study from which data for this paper is drawn: (1) the 21_{st} Century Model of Numeracy (Goos, Geiger, & Dole, 2014); and (2) generic principles of task design. These perspectives were employed as guidelines for teachers' attempts to design numeracy tasks and as a means of structuring teachers' introspection on the processes they employed to think of initial ideas for tasks. These perspectives are outlined below.

The 21_{st} Century Numeracy Model (Goos, Geiger, & Dole, 2014) has been validated via a series of research projects as a basis for: auditing curriculum documents for numeracy opportunities; structuring teachers' design of numeracy tasks; planning for the implementation of numeracy tasks; and examining teachers' learning trajectories in relation to effective numeracy practice (e.g., Geiger, Forgasz, & Goos, 2015; Goos, Dole, & Geiger, 2011). There are four dimensions central to the model – contexts, mathematical knowledge, tools, and dispositions that are embedded in a critical orientation to the use of mathematics. A critical orientation has been established as the dimension central to sustaining students' interest in and persistence with an activity; providing the reason for activation of the other model dimensions and requiring the use of inquiry approaches when seeking solutions (Geiger, Forgasz, & Goos, 2015). These dimensions are summarised in Table 1.

Table 1
Dimensions of 21st Century Numeracy Model

Contexts	The use of mathematics to act in and on the world, thus in a range of real world situations both within schools and beyond school settings.
Mathematical Knowledge	Concepts and skills; problem solving strategies; estimation capacities.
Dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge.
Tools	Use of material (e.g., models, measuring instruments), representational (e.g., symbol systems, graphs, maps, tables) and digital (e.g., computers, applications, internet) tools to mediate and shape thinking.
Critical Orientation	Use of mathematical information and activity to: make decisions and judgements; form opinions; add support to arguments; challenge an argument or position.

Generic principles for effective tasks design were synthesised from relevant research literature. A brief outline of the literature underpinning these principles is presented below and summarised in Table 2. More detailed descriptions of the generation of these principles have been reported elsewhere (e.g., Geiger, 2016).

The fit to circumstance of tasks with local conditions and constraints is important for effective implementation as most tasks are developed for specific curriculum and school contexts (Kieran, Doorman, & Ohtani, 2013). Such circumstances also include

considerations such as the pedagogies adopted to implement tasks and teaching resources available within a school.

Challenge is important for students if real learning is to take place. Most guidelines for improving learning outcomes stress the need for teachers to extend students' thinking by posing extended, realistic, and open-ended problems (e.g., City, Elmore, Fiarman, & Teitel, 2009). By posing such challenging tasks, teachers provide opportunity for students to take risks, to justify their thinking and to work with other students (Sullivan, 2011). Challenge also includes opportunities for students to make decisions and judgments and so exercise and develop their capacities to use mathematics critically (Goos, Geiger, & Dole, 2014).

While it is important for students to engage with learning experiences that offer challenge, tasks must also appear to be achievable, that is, challenging yet accessible (e.g., Sullivan, Clarke, & Clarke, 2013). Further, for tasks to be accessible they must be transparent; that is, it is clear what students are expected to do, and there must be points of entry where every student can begin an activity (Burkhart & Swan, 2013).

Table 2
Generic Principles for Effective Task Design

Fit to circumstance	Accommodating curriculum requirements and other affordances or constraints within a school setting, for example, teaching materials available within a particular school.
Challenge	Extending students' thinking by including elements of challenge in tasks provides opportunity for reasoning, risk taking, and the justification decisions.
Challenging yet accessible	Tasks must feel achievable to all students regardless of their prior history of achievement.
Complementary pedagogies	The pedagogical approach must match the demands and instructional intention of the task.
Transparency	In order for students to engage fully with tasks, activities must not only be accessible but also transparent in relation to expected outcomes – there is clarity around what is required of students to achieve success.
Opportunity to make decisions and judgements	The opportunity to make decisions and judgements introduces a critical demand into a task and provides purpose for students to engage with an activity.

Research Design

Methodological Approach

A design-based research approach was employed with the teacher professional learning component based on a framework devised by Loucks-Horsley, Love, Stiles, Mundry and Hewson (2003) that situates effective professional learning within teachers' own school-based contexts. The selection of this approach was appropriate as the study involved iterative cycles of intervention and improvement utilised to enhance teaching practice

while working in school classrooms – known to be complex and contextually rich (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003).

Research was carried out in three iterative phases; each consisting of three interventions in the form of teacher/research workshops between which researchers conducted school visits where teacher designed numeracy tasks were implemented. Teacher/researcher workshops were initially based on input from the researcher about the nature of numeracy, the generic principles of task design, and emersion activities in the form of exemplar numeracy task. Over time, these workshops evolved to opportunities for the cooperative development (researcher/teachers) of new principles of design and implementation. During researcher visits, numeracy tasks designed by teachers were trialled with lessons video-recorded. Video-stimulated recall interviews with teachers were conducted as soon as convenient after an observation in order to discuss critical events or phases during a lesson. Additionally, pre- and post semi-structured interviews were carried out in association with each classroom observation. Interview questions was structured around the elements of the 21st Century Model of Numeracy and the generic principles of task design. A particular focus of these interviews was on how teachers had generated the ideas for the tasks they implemented. Data was synthesised into case studies of teachers and students that was used to identify changes over time. This paper is based on data drawn from classroom observations and teacher interviews.

Participants

Five teachers from both Queensland and Victorian schools were recruited for Phase 1 of the project with an additional five teachers, one from each of the Phase 1 schools, agreeing to participate in Phases 2 and 3. Phase 1 teachers were purposively selected (Burns, 2000), firstly for their capability to design rich numeracy learning tasks, established through previous collaboration in numeracy-based researchers projects, and secondly for representation across learning areas. The quality of tasks developed through previous collaborations is evidenced by publication in peer reviewed research articles (e.g., Geiger, Goos, & Dole, 2013). Phase 2 teachers were recruited by Phase 1 teachers from their own schools, providing mutual support for each other's contributions as the project progressed. Phase 1 and 2 teachers represented a range of learning areas and sectors of schooling – Secondary English (1), History (1) Mathematics (1), Music (1), Science (1) Technology and Design (1), and Early Childhood/ Primary teaching (4). Because of space limitations, this paper reports on the accounts of two teachers; the first an early childhood teacher and the second a secondary teacher of English. These teachers were chosen because they reported different approaches to generating ideas for numeracy tasks.

Two Approaches to Generating Ideas for Numeracy Tasks

In this section, two vignettes are presented by way of illustration of how two teachers adopt different approaches to generating initial ideas as the basis for numeracy tasks embedded in learning areas other than mathematics.

Vignette 1 – Looking Through the curriculum

Olive is an early childhood teacher working in a school within the Catholic education sector. The school was located in a satellite city 45 km from a state capital. The observation on which this vignette is based took place during Phase 1 (Year 1) of the project. Olive's school had recently acquired an adjacent block of land on which they were

planning to build additional classrooms as part of a school expansion. While plans for new buildings were underway, Olive had obtained permission to involve her students in creating a *prayer garden* in the backyard of this property. As part of this initiative, she had developed a series of tasks for her group of preparatory (prep) students (typically 5-6 years of age) as part of an activity rotation that integrated geography, religious studies and mathematics.

In one of these tasks, students were asked to determine if a long rectangular bench-seat could be moved to a different position within the garden without moving the seat itself as this was too heavy for young students. No formal measuring tools (e.g., tape measures, rulers) could be utilised as the students had not yet learned about formal units of measure. Instead, 30 cm by 30 cm square tiles were provided as measuring tools for informal units.

After the teacher had explained the task, students discussed among themselves how they could go about the activity. After this discussion, students used the tiles to determine a measure of the bench seat length by placing the tiles end-to-end across the top of the seat. Once the length of the bench seat had been covered, the tiles were gathered up and moved by the students to the proposed new site for the seat. Once in position at one end of the designated space, students laid out the tiles end-to-end on the ground until they reached the other end of the area in question. Left over tiles were then piled up at the end of the space. When asked, students concluded that there was not enough space to move the bench seat to the proposed space and that another place would need to be found.

After the lesson, Olive was asked how she had thought of the task. She replied by saying her starting point was reflecting on the objectives of relevant curriculum documents she needed to address at that time of the year (O – Olive; I – Investigator).

- O: Yeah, I guess I always like to look at my curriculum; I know that we have to meet the needs, obviously, of the curriculum and make sure, yeah, I was looking, yeah, through and I saw that this idea of spaces and how we can change a space to meet a new purpose.
- I: So, you had the idea of a prayer garden, but then you were also looking through your geography thinking well I'm going to do that next.
- O: Yes, as well, yeah.
- I: And you put the two together.
- O: It's a hands-on approach and the kids can visually see it as well, and they can be a part of it. I think that's more meaningful to them as well. It's very easy to talk about it and show photos, but unless they're actually experiencing it, I think, yeah.
- I: How did you bring the maths into it then? Was that a forced thing or did that just come to you?
- O: No, I think it did just kind of came. I think it's knowing your curriculum and knowing what needs you need to meet. I think that's the starting point.
- I: And knowing lots of bits of curriculum at the same time.
- O: Yes, absolutely, yeah.

At the same time, she also considered what resources or aspects of the environment could be utilised and eventually brought curriculum and the resources offered by the built environment together.

- O: I just think it's probably more engaging for students if they can be a part of that environment and get their hands dirty and now they're talking about ripping down fences and taking out poles and weeding.
- I: Are you conscious of just seeing things like that, and just thinking what to do with it?
- O: Yeah, it's an opportunity. I think you've got to look at things and think it's an opportunity.

Olive believed it was her familiarity with curriculum documents that allowed her to pick out relevant strands from both geography, religious education and mathematics and bring these together through the opportunity made available by the purchase of the new property. When asked if this was typical of the way she developed numeracy tasks, she replied that she had worked hard to be thoroughly familiar with the curriculum requirements for any year level she was teaching and that she used this as a lens when looking to create new activities. While maintaining a focus on the curriculum, Olive also took advantage of a local resource in the form of a new aspect of the built environment.

Vignette 2 – Archiving Ideas

Richard is an English teacher in a government secondary school situated in a regional centre. He was observed during Phase 3 (Year 3) of the project conducting a Year 9 lesson in which students were required to write a letter to a new pen pal who lived on Horn Island – located off the northern Australian coastline in the Torres Strait. As preparation for this assignment, Richard had asked his students to research a number of aspects related to the island including:

- population
- land area
- number of schools
- distance from their home to Horn Island
- means of transport and travelling time from their home to Horn Island
- frequency of transport to and from the island

Richard had asked students to gather this information in order to help them understand the life circumstances of their new pen pals and so provide them with starting points for their first letter. While gather this information, students were asked to identify the advantages and disadvantages of living on Horn Island. Students worked enthusiastically on this task through the lesson, regularly expressing surprise at what they found, for example, the population of Horn Island is only 539 people, a small fraction of the population of the country town in which students lived. The lesson concluded before the task was completed with Richard telling students it would be continued the next day.

When asked how he had come up with the idea for the lesson, Richard replied that he was always on the lookout for opportunities to promote students' understanding of the challenges faced by others in the world (R - Richard).

R: So, the background to the lesson was that for sometime I've wanted to have my students communicate with people from indigenous backgrounds. So that's just a general goal I had. And by pure fluke my son just happened to get a job on Horn Island flying teachers around. So, I thought that opportunity was staring me in the face to ask my son if he would mention to a teacher the idea of the students at Mt Erin writing to their students. And he did that and so I decided this would be a terrific project for the kids to be involved in where they would explore the differences lifestyles between Torres Straight Islanders and people from around this area.

In the above except, Richard also indicates that he was searching for ideas that could promote broader educational goals than those specific to curriculum documents. In this case it was his son's engagement with the people of Horn Island that served as the basis for an idea which he 'parked' until he could use it in his classroom at some future date. He made the connection with this archived idea and the English unit on writing to a pen pal when looking forward through his teaching program for the semester (R – Richard; I - Investigator).

I: How did you think of the idea?

- R By firstly starting with the decision to look for a way of connecting students with other students with an indigenous background. So, I started with... that was my goal.
- I: Okay so was that independent of curriculum?
- R: Independent of curriculum.
- I: So, you thought sometime during the year you want to do this?
- R Yeah as soon as possible... so I ended up thinking "I'm going to have this as a goal" so from that goal I was going to look for a way of addressing it
- I Okay and then, so you're working the way you often have that you get an idea and you park it until you see some part of the curriculum coming up where it will fit. Have I got that right?
- R Yes

The task Richard developed from the idea allowed him to bring into the classroom an aspect of teaching he valued – an understanding of others through awareness of their life circumstances. At the same time, while this was the driver for identifying an initial idea for a task, Richard shaped the resulting activity to satisfy specific curriculum requirements as well as the global aims of the subject he was teaching – letter writing and the promotion of empathy for others.

Discussion and Conclusion

In identifying ideas to be used as the basis for numeracy activities embedded in learning areas other than mathematics, Olive and Richard used two quite different approaches.

Olive relied on a deep knowledge of curriculum across learning areas to generate ideas. In this approach she looked to bring forth ideas by making connections between the specific curriculum objectives of different learning areas. In the vignette reported here, she found a connection between geography, religious education and mathematics. These aspects of curriculum were brought together in fitting to circumstance her teaching with an opportunity provided by the built environment. The problem she posed for students, provided developmentally sensitive challenge yet was accessible through the tools she provided which allowed students to bring the critical question embedded in the task to resolution. She worked with students as a whole group and one-on-one to ensure the demands of the task were transparent while employing an investigative pedagogy consistent with the problems solving focus of the activity. Students were provided the opportunity to make a decision about whether the bench-seat could be moved to another place in the garden.

By contrast, Richard did not use the curriculum as a lens through which to look out into the world for a teaching idea but rather as a framework or overarching plan to which he could attach teaching ideas he had already identified as having potential at some future time. In his approach curriculum requirements were viewed as a way of facilitating a broader educational purpose rather than providing an initial direction for task development. This meant that he fit to circumstance the idea he had previously identified to the requirements of curriculum. He challenged students to make use of mathematics-based evidence to argue for the advantages and disadvantages of living on Horn Island. In developing such arguments, students were required to make judgements and to form opinions. Because the task required students to connect the life circumstances of a pen pal on Horn Island to that of their own community and the demands of the task related to mathematical knowledge were limited, they found the task both assessable and transparent.

The above analysis indicates that while Olive and Richard took different approaches to identifying initial ideas for development into tasks, the activities they developed were consistent with the generic principles of task design they had been encouraged to use when designing tasks. While this paper reports on only two cases, they provide tentative evidence that initial ideas for numeracy tasks across the curriculum can be generated in different ways yet can still be shaped to fit the broad specifications of the principles of generic task design. In the cases reported here, task design was initiated in two different ways – drawing intensively on curriculum objectives or upon broader values and beliefs.

These findings indicate that further research is required to establish what other approaches can be utilised by teachers to generate numeracy tasks for implementation across the curriculum. This research could also seek to establish if teachers' approaches generating ideas are invariant or adaptable in relation to the circumstances of their practice.

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