Young Australian Indigenous Students' Growing Pattern Generalisations: The Role of Gesture when Generalising

Jodie Miller
Australian Catholic University
<Jodie.Miller@acu.edu.au>

This paper explores how young Indigenous students' (Year 2 and 3) generalise growing patterns. Piagetian clinical interviews were conducted to determine how students articulated growing pattern generalisations. Two case studies are presented displaying how students used gesture to support and articulate their generalisations of growing patterns. This paper presents a hypothesised cultural learning semiotic model that was a result of the interactions that occurred between the non-Indigenous researcher, the Indigenous students and the Indigenous Education Officers.

Much research pertaining to young Australian Indigenous students has focused on pedagogical practices that support students' learning (Harris, 1984; Hurst & Sparrow, 2010) and studies concerning the language of instruction (Meaney, Trinick, & Fairhall, 2012). The minimal studies that have been conducted focusing on a specific mathematical concept are predominately in the area of number (e.g., Butterworth & Reeves, 2008), with few conducted specifically on algebra or algebraic thinking (Matthews, Cooper, & Baturo, 2007). Fundamental to the development of algebraic thinking is the ability to recognise patterns (Mulligan & Mitchelmore, 2009). Visual growing patterns are commonly the construct students' encounter when introduced to formal algebra (Warren & Cooper, 2008). To date no study has been conducted within an Australian context that considers how young Australian Indigenous students engage in mathematical generalisation of growing patterns. To this end, the focus of this paper is an exploration in how young Australian Indigenous students engage with growing pattern generalisation.

Literature

The ability to generalise mathematical structures beyond the initial learning experience has been highlighted as one of the important components of mathematics (Warren & Cooper, 2008). Recently, there has been a growing body of literature exploring generalisation with younger students. Results of this research have shown that young students are capable of generalising mathematical structure across a range of contexts (Carraher, Schliemann, Brizuela, & Earnest, 2006; Radford, 2010). Much of this research has considered the types of structures (Mulligan & Mitchelmore, 2009; Rivera & Becker, 2011), how students attend to these structures (Mason, Stephens, & Watson, 2009), and the types of representations (Cooper & Warren, 2011) that assist students to generalise. Furthermore, the studies that have been conducted are primarily in non-Indigenous settings.

While there is agreement in the scholarly community that the ability to generalise is important, how one generalises remains unclear. Generalising mathematical concepts must go beyond just the act of noticing (Radford, 2006). Students must also develop the capacity to address and express concepts algebraically for all elements of the sequence (Radford, 2006). This sequence draws on semiotic emergences, which are processes of 'sign use' between both teachers and students, as students develop generalisations (Radford, 2006).

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice *(Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* pp. 461–468. Sydney: MERGA.

Radford (2010) presents three types of generalisation expressions that he terms as 'layers of generality' namely: factual, contextual and symbolic generalisations. Factual generality is an elementary level of generalisation where students engage heavily in gestures, words and perceptual activities. Within this level, students attend to only the pattern presented and express the generalisation in terms of numerical operation (e.g., 2+3, 3+3). Contextual generalisation has a reduction of semiotic resources (e.g., gestures, rhythm, hands-on materials) as students generalise beyond the presented pattern (e.g., you double the number of the figure and add one). Finally, the symbolic level requires a further semiotic contraction where students replace words with alphanumeric symbols to express the general rule (e.g., $2n+1$). Within each of these levels signs play an important role in students' understanding and communication.

Semiotic signs assist students in developing mathematical understanding. At times, this mathematics may have remained unseen until the use of semiotic signs (Radford, Bardini, & Sabena, 2007). Radford suggests that signs (such as bodily movement, oral language, concrete objects) play the role of making the mathematics apparent, a semiotic means of objectification (Radford, 2003). This semiotic means of objectification can bring about knowledge formation with the use of particular mathematics activities led by semiotic systems, often referred to as semiotic nodes. In relation to mathematical generalisation, gesture has played a crucial role in assisting older students focus on particular structural aspects of the pattern that in turn assists in the expression of generalities (Radford, Bardini, & Sabena, 2007). When considering algebraic language, research has highlighted that it is more than the use of alphanumeric symbolism. Algebraic language is a combination of semiotic nodes/instruments (language, gesture, written) (Radford, Bardini, & Sabena, 2007). Studies by Radford (2006) have found that these nodes become more refined as students move through the learning experiences. It is also essential to consider all semiotic systems as students generalise, as mathematical thinking will not be captured from a written formula (Radford, Bardini, & Sabena, 2007).

As the teaching of mathematics draws on a variety of representations and resources to assist students to engage with mathematical processes, semiotics provides the tools to understand these processes of thought, symbolisation, and communication. Mathematics is described as an intrinsic symbolic activity, the outward manifestations of the processes are communicated using oral, bodily, written and other signs (Radford, 2006). Peirce (1958) defines a sign as anything that is so determined by an object that brings meaning to the interpreter who is making sense of the sign and objects relationship. In the discipline of mathematics is it essential that signs can be both static and dynamic (Saenz-Ludlow, 2007; Radford, 2006). An example of a dynamic sign is gesture or kinaesthetic movement. Within the context of functions, the object can be considered as the relationship that exists between the two variables, and the variables themselves are the signs that give the function meaning. Gestures, forms of dynamic signs, are defined as all of those movements [hands, arms, eyes] that subjects perform during their mathematical activities (McNeill, 1992). From this perspective, our cognitive relation to reality is mediated by signs, which can be objectified. The relationship between language and gesture has been described as 'unsplittable' (McNeill, 1992). When language is not apparent or is mismatched to home language, students will use gesture to assist conversation (Goldin-Meadow, 2002). Additionally, gesture may be the first instance that students display a new thought (Goldin-Meadow, 2002).

Theoretical Framework

This study adopts the theoretical perspectives of (a) semiotics and (b) Indigenous research perspectives. Both theoretical perspectives adopted for this study complement each other. One is about creating dialogue and the other is about interpreting the communication. That is, interpreting the language, gestures and signs that Indigenous students bring to the learning experiences, which is an essential aspect of respecting Indigenous culture and acknowledging the unique contributions it makes to learning.

Semiotics

As individuals build knowledge through language, symbolism, culture and social encounters the theory of semiotics provides a lens to interpret these interactions. Semiotics is the study of cultural sign processes, analogy, communication, and symbols (Peirce, 1958). Furthermore, mathematics as a discipline is considered to be abstract and heavily based on perceivable signs. Mathematics has been described as an intrinsic symbolic activity that is accomplished through communicating orally, bodily, written or utilising other signs (Radford, 2006). Semiotics had a two-fold role in the study, first it informed the exploration of the teaching and learning activities in the learning experiences, and second, it provided the lens to interpret the signs within and between all social interactions in the learning experiences. In researching these interactions with young Indigenous students, it was important to acknowledge the potential for unique cultural variations with regard to how the outward displays of thought processes may be expressed. To appropriately account for these cultural sensitivities this research acknowledges Indigenous research perspectives as a second theoretical perspective.

Indigenous Research Perspectives

An Indigenous research paradigm is a means of creating dialogue, rather than simply closed observation (Denzin & Lincoln, 2008). This is not to say that through observation information cannot be learnt. More so, when observing an Indigenous culture, there are practices that may not be overtly apparent to the researcher; hence, the importance of including an open dialogue with students. For this particular study, the relationship also needs to be cultivated with Indigenous Education Officers (IEOs) to assist with knowledge that may not be explicitly recognisable to the researcher. It is thus imperative to create space for critical collaborative dialogue within the study; hence, the choice of Piagetian clinical interviews was also used to collect data. The implication of this decolonised approach dictates that the study must be viewed within the bounds of the individual community in which the research takes place and not generalised to the broader Indigenous population.

Method

The research methodology for this study was drawn from Piagetian clinical interviews (Opper, 1977). These interviews were at the conclusion of a teaching experiment that had been conducted with the class. The interviews provided opportunities to trial new ideas and to further discuss with students their mathematical thinking in terms of the activities being presented to them. Interviews were approximately 20 minutes in length. The interviews were video recorded where both students' gesture and the researchers' gestures were captured. All questions were posed to students in a flexible manner. The questions posed and subsequent actions were contingent on the responses given by the student. The interviews mirrored the dimensions associated with Piagetian Clinical interviews, namely, endeavouring to avoid leading the student in a particular direction, but at the same time making the most of the opportunities to formulate and test hypothesis about students' understanding.

Participants

The research was conducted in one Year 2/3 classroom of an urban Indigenous school in North Queensland. All students in this study identified themselves as Aboriginal or Torres Strait Islander people and they spoke a mixture of Aboriginal English and Standard Australian English. For the intention of this study three students have been considered for deeper analysis and have formed three separate case studies. These three students were selected in consultation with the teacher and Indigenous education officers. Students attended school regularly, were good communicators, and identified as Aboriginal (S1 female, S6 male) and Torres Strait Island (S2 male). The students were selected to provide a range of mathematical achievements as identified by the classroom teacher (S1 high achiever, S2 average achiever, S3 low achiever). Two Indigenous Education Officers provided guidance during all phases of the research. Within this study the researcher worked in collaboration with the Indigenous Education Officers to:

- identify differing cultural interpretations of gesture and actions within the class and interview interactions;
- ensue a mathematical context relevant to students was used in the teaching episodes;
- assist students with communicating their ideas, particularly with regard to language;
- provided on-going analysis for each of the individual video recordings of the teaching experiments and for students who participated in the one on one interviews.

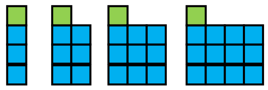
Data Analysis

In this study, data analysis was contemporaneous and formative during data collection. It occurred after interviews, and this analysis informed the next stage in the data collection process and assisted in refining conjectures (Confrey & Lachance, 2000). At the conclusion of the study the initial video-footage was transcribed to capture students' verbal responses. These transcriptions were then analysed to consider emerging key mathematical themes from the lessons and interviews. Then, semiotics was utilised as a lens to reanalyse the data. The evolving data were reanalysed focusing on semiotic bundles (signs, gestures, language) of both the student and researcher in the lessons and interviews. This analysis provided an interpretation of the learning interactions between the researcher and students. Of particular importance were the students' physical gestures including the manipulation of hands-on materials. Finally, data were reanalysed in line with the cultural perspectives provided by the Indigenous Education Officers. This process was repeated for all lessons and student interview data.

Findings and Discussion

Examples of Gesture to Articulate Pattern Generalisations

The following is taken from a lesson where 'Sally' is discussing the structure of a growing pattern for a near generalisation. That is a generalisation for small pattern terms (e.g., 12th pattern term). Figure 1 displays the pattern Sally was discussing.



1 2 3 4

Figure 1. Growing pattern presented in Lesson 3 of teaching experiment.

While describing the growing pattern for the 7th term, she stated:

> I would have seven of [gesture drawing an imaginary loop around row of three using her finger] … not sure [gesture looking away, head down] … of the word [gesture drawing an imaginary loop around row of three using her finger] … of three and a teacher [gesture indicating to the green tile].

It is at this point that Sally demonstrated a sense of how the growing pattern was structured; however, she had difficulty accessing the mathematical language of 'row' or 'groups' to describe the structure. The Indigenous Education Officers supported this, as they both confirmed that Sally was having difficulty with the language. Both the non-Indigenous researcher and the Indigenous Education Officers observed Sally replacing the mathematical language with gesture.

Further evidence of this was observed during a clinical interview with 'Jaydin'. The researcher constructed the first three terms of the growing pattern (see Figure 2). Jaydin was able to identify that the three green tiles remained the same in each pattern. He also noticed that each pattern was constructed of groups of five, "They are all in a line of five." The excerpt below demonstrates how Jaydin predicted terms beyond the pattern presented.

131 J	What do you think the fourth one will look like?
132 R1	There are four blue tiles, five of them [*J gestures down the lines of five*] and three green ones [*J point to the desk where the three yellow tiles would be placed on the pattern*].
133 J	What about number ten what would it be?
134 R1	Ten blue tiles [*J gestures down the lines of five*] and 3 around it.
135 J	What about 100?
136 R1	100 blue tiles [*J gestures down the lines of five*] and 3 around it.

Jaydin relied on gesture to support his explanation. He had difficulty with the mathematical language and therefore used gesture to communicate his understanding of the structure. Jaydin was also able to identify the pattern term if given the structure of the pattern, for example, "If there were 25 rows of 5 blue tiles and 3 around it, what position would it be in the pattern?" Jaydin was able to identify that it was position 25. He also did this for position 80.

Additionally, Jaydin used gesture while generalising the rule for the growing pattern. When asked to provide an explanation for how to construct the pattern for any number, Jaydin stated:

> Any number [J gesture – using his five fingers to make imaginary lines of tiles] in the blue tiles and three around it.

In this explanation Jaydin is missing the mathematical language of 'groups of five', 'rows of five', or 'times five". The video data presented Jaydin gesturing beside a pattern on the desk to show the line of five. Figure 2 illustrates the gesture Jaydin used in his generalisation.

"*Lines of 5*" [Gesture line up table] … "*and three all around it*"

Figure 2. The gesture Jaydin used while generalising the pattern structure.

Conclusions and Implications

This study demonstrates that young Indigenous students are able to generalise geometric growing patterns without engaging in alphanumeric notation. Evidently, to hold this view on algebraic thinking one must adopt the notion that the importance lies in how the general is considered, rather than algebraic thinking being merely the use of notations (Radford, 2006). Considering Radford's (2010) levels of generalisation, these students are demonstrating contextual and arguably symbolic generalisation. These young Indigenous students used gesture to supplement the language used in communicating their ideas about growing patterns. This resonates with past research, which suggests gestures are utilised by students to communicate their understandings prior to acquiring specific content terminology (e.g., mathematical terms) (Kendon, 1997; Roth, 2001). The use of gestures appeared more pronounced for these young Indigenous students. It was observed that they used gesture as an adjunct to language for communicating ideas about growing patterns and generalising the growing patterns. It is paramount for teachers to understand student gesture in Indigenous contexts and provide opportunities for students to gesture as they engage in and explain their own mathematical knowledge. At times, there is a mismatch between young Indigenous students' home language and Western mathematical language used in class. If there is an over emphasis on requiring young Indigenous students to verbalise their mathematics using Australian Standard English then we may in fact miss what these students actually know and understand. Rather, the communication of mathematics should be seen as an embodied process drawing from gestural cues, manipulation of hands-on materials and language combine. Students demonstrated that they had the requisite knowledge to generalise growing patterns however, their form of communication drew on more facets than communicating in Standard Australian English. This platform served to assist students to develop an understanding of the language of mathematics.

In exploring the construction of shared knowledge, culture played a pivotal role in the learning interactions and the decoding and encoding of signs between the teacher, student, and Indigenous Education Officer. Each person within the class brings their own cultural perspectives. However, by teachers having an awareness of students' culture enables them to better interpret the learning and the semiotic interactions. Essentially, teachers need to consider the role of culture when interpreting the semiotic signs and how students perceive their own (teacher) semiotic signs. This present study provided first-hand experiences of bringing a non-Indigenous teacher (the researcher) into an Indigenous classroom context to explore elements of Western mathematics. Throughout this process a shift in understanding of what was happening occurred. This shift involved moving from conveying content to transacting knowledge through shared dialogue. When working at the interface between divergent cultures, this model poses significant challenges. Figure 3 is a depiction of the knowledge interactions that where experienced and observed in the present research.

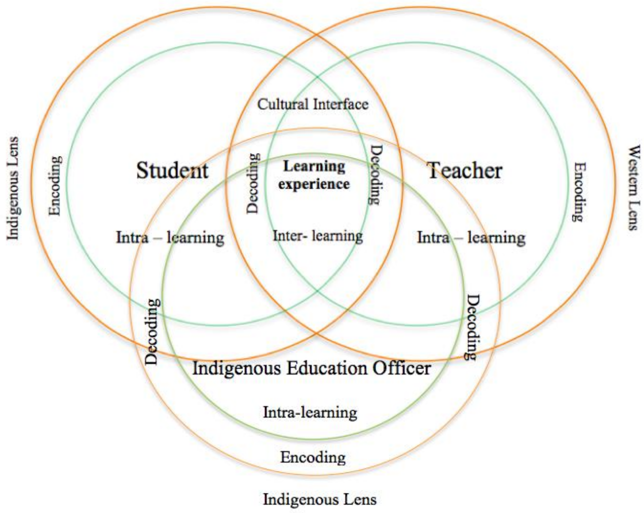


Figure 3. Hypothesised Cultural Semiotic Learning Model: Knowledge interactions when engaging in learning experiences for Non-Indigenous teachers, Indigenous education officers and Indigenous students.

In considering learning as a shared dialogue and experience, the figure illustrates the teacher arriving at the interaction with knowledge of mathematics largely from a Western perspective. At the same time, students bring knowledge from their own life and experiences, not heavily dominated by Western mathematical language. The Indigenous Education Officers complete the triumvirate, not by acting as interpreters; but rather as a facilitators for encoding and decoding knowledge from the Western and Indigenous domains to create a new and shared knowledge. This created shared space is where empowerment occurs (Denzin & Lincoln, 2008), an essential facet to Indigenous students' learning. This model has implications for both teaching and research. First, it highlights the important role of the Indigenous Education Officer in assisting the teacher to be aware of cultural signs in the teaching and learning process. If these cultural signs are missed or misinterpreted the true understanding of students' knowledge will potentially be unseen. Second, from a research perspective, this model emphasises how data analysis needs to be in conjunction with Indigenous Education Officers to have a deeper understanding of students' interactions.

References

Butterworth, B., & Reeve, R. (2008). Verbal counting and spatial strategies in numerical tasks: Evidence from Indigenous Australia. *Philosophical Psychology*, *21*, 443–457.

Carraher, D., Schliemann, A., Brizuela, B., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics Education*, *37*(2), 87–115.

Confrey, J., & Lachance, A. (2000). Transformative teaching experiments through conjecture-driven research design. In A. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 231–265). Mahwah, NJ: Lawrence Erlbaum Associates.

Cooper, T. J., & Warren, E. (2011). Years 2 to 6 students' ability to generalise: Models, representations and theory for teaching and learning. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives. Advances in Mathematics Education* (pp. 187–214). Berlin Heidelberg, Germany: Springer-Verlag. doi:10.1007/978-3-642-17735-4_12

Denzin, N. K., & Lincoln, Y. S. (2008). Critical methodologies and Indigenous inquiry. In N. K. Denzin, Y. S. Lincoln, & L. T. Smith (Eds.), *Handbook of critical and Indigenous methodologies* (pp. 1–30). London, UK: SAGE Publications.

Goldin-Meadow, S. (2002). Constructing communication by hand. *Cognitive Development*, *17*, 1385–1405. doi:10.1016/S0885-2014(02)00122-3

Harris, S. (1984). *Culture and learning: Tradition and education in north-east Arnhem Land.* Canberra, ACT: Australian Institute of Australian Studies.

Hurst, C., & Sparrow, L. (2010). The mathematical needs of urban Indigenous primary children: A Western Australian snapshot. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 272–279). Fremantle, WA: MERGA.

Kendon, A. (1997). Gesture. *Annual Review of Anthropology*, *26,* 109–128.

Mason, J., Stephens, M., & Watson, A. (2009). Appreciating mathematical structure for all. *Mathematics Education Research Journal*, *21*(2), 10–32.

Matthews, C., Cooper, T., & Baturo, A. (2007). Creating your own symbols: Beginning algebraic thinking with Indigenous students. In J. Woo, H. Lew, K. Park, & D. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 249–256). Seoul, South Korea: PME.

McNeill, D. (1992). Gesture and language dialectic. *Acta Linguistica Hafniensia, 34*(1), 7-37.

Meaney, T., Trinick, T., & Fairhall, U. (2012). *Collaborating to meet language challenges in Indigenous mathematics classrooms*. New York, NY: Springer.

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal*, *21*(2), 33–49.

Opper, S. (1977). Piaget's clinical method. *Journal of Children's Mathematical Behavior*, *1*(4), 90–107.

Peirce, C. S. (1958). *Collected papers, Vol. I–VIII.* Cambridge, MA: Harvard University Press.

Radford, L. (2003). Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. *Mathematical Thinking and Learning, 5*(1), 37–70.

Radford, L. (2006). Algebraic thinking and the generalization of patterns: A semiotic perspective. In S. Alatorre, J. Cortina, M. Saiz, & A. Mendez (Eds.), *Proceedings of the 28th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 2–21). Mexico: PME.

Radford, L. (2010). Layers of generality and types of generalization in pattern activities. *PNA*, *4*(2), 37–62.

Radford, L., Bardini, C., & Sabena, C. (2007). Perceiving the general: The multisemiotic dimension of students' algebraic activity. *Journal for Research in Mathematics Education*, *38*(5), 507–530.

Rivera, F. D., & Becker, J. R. (2011). Formation of pattern generalization involving linear figural patterns among middle school students: Results of a three-year study. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives. Advances in mathematics education* (pp. 323–366). Berlin Heidelberg, Germany: Springer-Verlag.

Roth, M. W. (2001). Gestures: Their role in teaching and learning. *Review of Educational Research*, *71*(3), 365–392.

Saenz-Ludlow, A. (2007). Signs and the process of interpretation: Sign as an object and as a process. *Studies in Philosophy and Education, 26*, 205–223.

Warren, E., & Cooper T. J. (2008). Generalising the pattern rule for visual growth patterns: Actions that support 8 year olds' thinking. *Educational Studies in Mathematics, 67*, 171–185.