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RESEARCH REPORTS (P-Y)

CONCEPTION TO CONCEPT OR CONCEPT TO CONCEPTION? FROM BEING TO BECOMING

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Previous approaches to mathematics knowing and learning have attempted to account for the complexity of students' individual conceptions of a mathematical concept. Those approaches primarily focused on students' conceptual development when a mathematical concept comes into being. Recent research insights indicate that some students give meaning not only to states/objects that have a being but also to states/objects that are yet to become. In those cases, conceptual development is not meant to reflect an actual concept (conception-to-concept fit), but rather to create a concept (concept-to-conception fit). It is argued that the process of generating a concept-to-conception fit, in which ideas that express a yet to be realized state of the concept are created, might be better referred to meaning-making than sense-making.

INTRODUCTION

Consideration of mathematical concept formation has a long history in, and is certainly an important branch of, cognitive psychology in mathematics education (see Skemp, 1986). Previous research has focused on the complexity of students' conceptions and their conceptual development when a mathematical concept comes into being. Students have been regarded as active sense-makers in mathematical concept formation (von Glasersfeld, 1995), that is, students actively seek comprehensibility of a mathematical concept. Students might, in this process, develop conceptions (from Latin *concipere*, 'to conceive') of a mathematical concept that are construed by a researcher (or educator) as a way a mathematical concept is perceived (or regarded) as it seems to be (for a discussion on conception and concept, see Simon, 2017). Recent research, however, suggests that students not only activate conceptions to make sense of how they perceive (or regard) a mathematical concept that comes into being in a certain context but also to imagine (or envision) a mathematical concept that is yet to become. In those cases, conceptual development is not meant to reflect an actual concept, but rather to create a concept.

The purpose of this paper is to clarify in which respects this act of creation differs from sense-making construed as an act of comprehension. In doing so, a theoretical background is briefly outlined that orients the general discussion of concept formation and sense-making. Afterward, key insights from recent research are summarized that foreground the act of creation in concept formation. Then, critical differences between two different states that a mathematical concept can have ('making it being' and

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'making it becoming') are discussed which allow to conclude that the act of creation might be better understood as meaning-making than sense-making.

THEORETICAL BACKGROUND: ON CONCEPT AND CONCEPTION

The work presented here is framed in theoretical assertions made by Scheiner (2016) with regard to mathematical concept construction. In Scheiner's (2016) view, the meaning of a mathematical concept comes into being in the ways that an individual interacts with the concept; or more precisely, in the ways that an individual interacts with objects that in a Fregean (1892a) sense fall under a concept. (A mathematical concept might be best described as an organic, multidimensional, structured gestalt, whose dimensions emerge from an individual's interactions with it.) As such, a concept does not have a fixed meaning. Rather, the meaning of a concept is relative (a) to the senses$_F$ that are expressed by representations that refer to objects coming under a concept and (b) to an individual's system of ideas$_F$ (the subscript F indicates that these terms refer to Frege, 1892b). Frege (1892b) revealed the fundamental distinction between reference and sense$_F$ as two semantic functions of a representation (an image, sign, or description): a reference of a representation is the object to which a representation refers, whereas a sense$_F$ of a representation describes a certain state of affairs in the world, namely, the way that some object is presented. Thus, it seems to follow that we may understand Frege's notion of an idea$_F$ in the manner in which we make sense of the world. Ideas$_F$ can interact with each other and form more compressed knowledge structures, called conceptions. A general outline of this view is provided in Fig. 1.

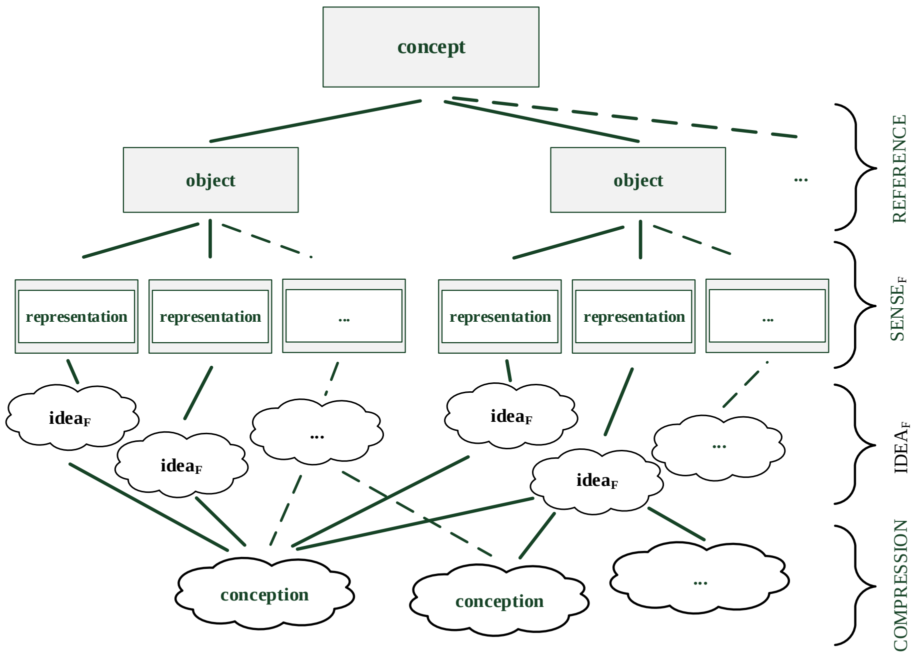

Fig. 1: On reference, sense$_F$, idea$_F$, and compression

(reproduced from Scheiner, 2016, p. 179)

There are several ways that individuals can make sense of a mathematical concept; the focus here is on extracting meaning and giving meaning (Pinto, 1998). Pinto and Tall (1999) stated with respect to sense-making of a formal concept definition,

> "Giving meaning involves using various personal clues to enrich the definition with examples often using visual images. Extracting meaning involves routinizing the definition, perhaps by repetition, before using it as a basis for formal deduction." (p. 67)

Tall (2013) explicated that these two approaches are related to a 'natural approach' that builds on the concept image and a 'formal approach' that builds formal theorems based on the formal definition. Scheiner (2016) broadened the original conceptualization provided by Pinto (1998), emphasizing that individuals can extract meaning from objects and give meaning to objects; or more precisely, extract meaning from their interactions with objects and give meaning to their interactions with objects. Further, extracting meaning was linked to the manipulation of objects and reflections of instances that appear in senses$_F$ when objects are manipulated – a phenomenon often discussed in terms of reflective abstraction, that is, abstraction of actions on mental objects (see e.g., Dubinsky, 1991). Giving meaning was related by Scheiner (2016) to attaching meaning to instances of objects that appear in senses$_F$ – a phenomenon that has been considered in terms of structural abstraction, that is, abstraction of "the richness of the particular [that] is embodied not in the concept as such but rather in the objects that falling under the concept […]. This view gives primacy to meaningful, richly contextualized forms of (mathematical) structure over formal (mathematical) structures" (Scheiner, 2016, p. 175). Scheiner (2016) offered a theoretical grounding for coordinating extracting meaning and giving meaning by putting in dialogue reflective abstraction and structural abstraction. Earlier, Tall (2013) discussed the relations of structural and operational abstraction and the natural and formal approach that evolve into a wider framework of the long-term development in mathematical thinking. (Structural abstraction focuses on the structure of objects, and operational abstractions on actions that become operations that are symbolized as mental objects (Tall, 2003).) The research presented in this paper has built on these theoretical interpretations of extracting meaning and giving meaning, and the assumed relationship between them.

RESEARCH BACKGROUND: GIVING MEANING REVISED

Recently, Scheiner and Pinto (2017a, 2017b) reanalyzed students' reasoning and sense-making of the limit concept of a sequence using theoretical innovation that involved contextuality, complementarity, and complexity of knowledge, plus knowledge development, and knowledge usage when giving meaning.

In their case study, Scheiner and Pinto (2017a) discussed giving meaning as a sense-making strategy in which ideas$_F$ are activated to give meaning to instances of an object that are actualized in certain, or even new, contexts. They described that the context in which an object is actualized might trigger the activation of ideas$_F$; however, it seems that it is not the context but the knowledge system that determines what is activated. (This does not mean that a knowledge system determines the meaning of a mathematical concept nor the form of interaction with objects that fall under a concept.) This is to say, it is not the context that determines the interpretation or meaning of an object, but the ideas$_F$ that are attached to instances of an object that orient an individual in giving meaning when making sense of certain contexts. As such, individuals do not construct a mental image of an 'external reality' that appears in the senses$_F$, but rather they give meaning to a sense$_F$ of an instance by attaching an idea$_F$ to it. Scheiner and Pinto's (2017a) analysis also suggests that this attachment is highly context dependent, that is to say, individuals might attach different ideas$_F$ to the same object that is actualized in different contexts.

In a cross-case analysis, Scheiner and Pinto (2017b) foregrounded that the attachment, however, seems to take place in such a way as to create and maintain coherence in a student's reasoning. However, the authors did not interpret coherence within the meaning of an established body of knowledge, but rather in the meaning of a student's usage. As such, coherence is not so much an attribution of the interconnectedness of the pieces of a created knowledge system, but of activity: students, who give meaning, activate ideas$_F$ that are coherent with their reasoning. This suggests that what seems to matter are coherence in reasoning and functionality of an individual's knowledge system, rather than any sort of correctness that mirrors a pre-specified 'reality' of the mathematical concept. This leads one to suppose that students are not concerned with creating a knowledge system that best reflects a given reality, but they are concerned with creating a reality that best fits with their knowledge system.

The most remarkable issue, however, is that Scheiner and Pinto's (2017a, 2017b) analyses point to the idea that students might even give meaning to states that are yet to become. This means though an object does not appear in a sense$_F$, an individual might create an idea$_F$ of a potential instance of that object. That is, students might give meaning beyond what is apparent. It is proposed that the creation of such ideas$_F$ is of the nature of what Koestler (1964) described as 'bisociation', and Fauconnier and Turner (2002) elaborated as 'conceptual blending'.

Koestler's (1964) central idea is that any creative act is a *bisociation* of two (or more) unrelated (and seemingly incompatible) frames of thought (called matrices) into a new matrix of meaning by way of a process involving abstraction, analogies, categorization, comparison, and metaphors. More recently, Fauconnier and Turner (2002) elaborated and formalized Koestler's idea of bisociation into what they called *conceptual blending*. The essence of conceptual blending is to construct a partial match, called a cross-space mapping, between frames from established domains (known as inputs), in order to project selectively from those inputs into a novel hybrid frame (a blend), comprised of a structure from each of its inputs, as well as a unique structure of its own (emergent structure).

The point to be made here is that unrelated ideas$_F$ can be transformed into new ideas$_F$ that allow 'setting the mind' (see Dörfler, 2002) not only to actual instances but also to potential instances that might become 'reality' in the future. In those cases, conceptual development is not merely meant to reflect an actual concept, but rather to create a concept (see Lakoff and Jonson (1980) on the power of (new) metaphors to create a (new) reality rather than simply to give a way of conceptualizing a preexisting reality:"changes in our conceptual system do change what is real for us and affect how we perceive the world and act upon those perceptions" (pp. 145-146.)). It is reasonable to assume that students transform ideas$_F$ to express a yet to be realized state of a concept.

DISCUSSION: ON 'MAKING IT BEING' AND 'MAKING IT BECOMING'

The research insights outlined in the previous section assert construing two different states that a mathematical concept can have: (1) a mathematical concept is given and comes into being in the dialogue of extracting meaning and giving meaning (in short, *making it being*) and (2) a mathematical concept is created and comes into becoming in the process of transforming ideas$_F$ (in short, *making it becoming*).

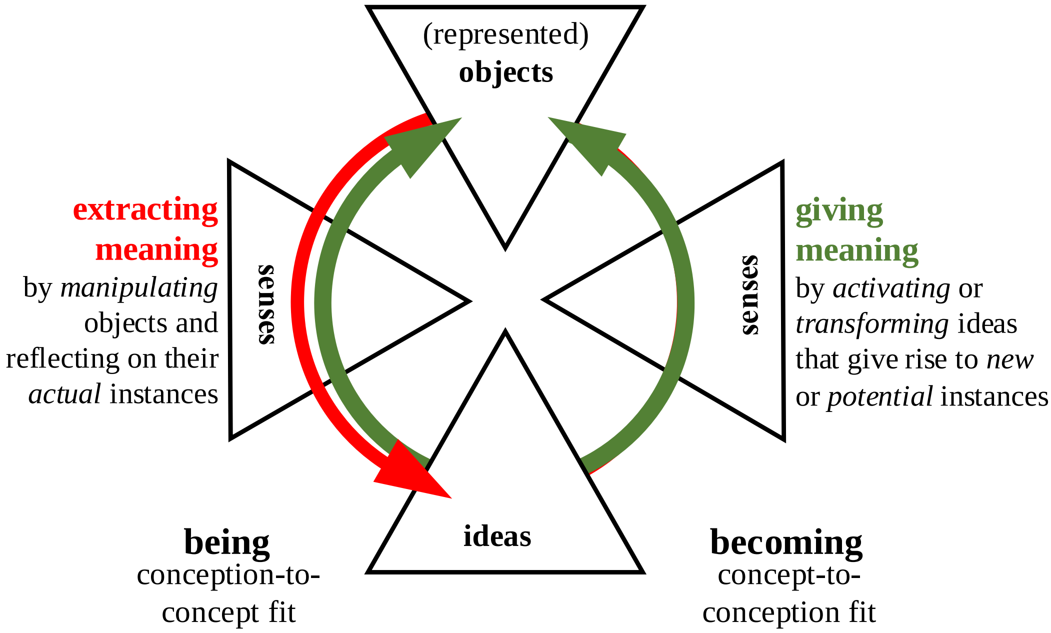

Fig. 2: From being to becoming

In making it being, extracting meaning and giving meaning can occur simultaneously: an individual might extract meaning by manipulating objects and reflecting on the actual instances of such objects, while at the same time an individual gives meaning to the instances that appear in the senses$_F$ by activating and attaching ideas$_F$ (see Fig. 2). With respect to giving meaning, an individual might either activate already available ideas$_F$ to attach meaning to instances or an individual might create new ideas$_F$ in the moment by transforming ideas$_F$ to gain new insight that allows attaching new meaning to an object of consideration.

In making it becoming, giving meaning means not only attaching ideas$_F$ to actual instances of an object but also creating new ideas$_F$ for potential instances. As such, ideas$_F$ can also be transformed in order to give meaning to instances that are yet to become (see Fig. 2). This means an individual might set her or his mind to future possibilities in which the object might be realized. In such cases, the mind would shape the future in a way that individuals might work to move the present to an intended future. That is, rather than creating conceptions that reflect a seemingly given concept, individuals might create a meaning of a concept that best reflects their conceptions of the concept. That is, individuals might create new forms of meaning, suggesting that the meaning of a mathematical concept varies on its actual use and intentions, rather than having an inherent meaning.

The differences between "making it being" and "making it becoming" can be discussed around at least three related issues:

(1) *Different states of the meaning of a mathematical concept*

In making it being, students treat objects that fall under a concept as states that have a being. Here students seem to understand the meaning of a mathematical concept as given. As such, an individual might extract meaning from manipulating objects and give meaning to actual instances of such objects. The meaning of a concept, then, emerges (from Latin *emergere*, 'to become visible') in the dialogue of extracting meaning and giving meaning.

In making it becoming, students create new ideas$_F$ by transforming previously created ideas$_F$ that are directed to objects that are yet to become. They transform ideas$_F$ to create future possibilities. Here the meaning of a mathematical concept is created that is to say, the meaning evolves (from Latin *evolvere*, 'to make more complex') in transforming various ideas$_F$.

(2) *Different functions of senses$_F$*

In making it being, senses$_F$ are construed as bearers of actual instances of an object that seems to have a being prior to students' attempts to know it. That is, the seeming 'objectivity' of an object appears in such senses$_F$.

In making it becoming, objects are not seen as preceding students' attempts to know them. Senses$_F$ are not construed as bearers of instances of an object but rather as triggers to transform ideas$_F$ to create new, potential instances of an object.

(3) *Different directions of fit*

Making it being is meant to reflect the concept as it is actualized, suggesting a conception-to-concept direction of fit: students extract meaning that reflects the concept and give meaning that fits the concept as it is assumed to be.

Making it becoming is meant to create the concept, suggesting a concept-to-conception direction of fit: students express a yet to be realized state of the concept, that is, they express a way that the concept can, or should, be. Students create the meaning of a concept that fits their conceptions.

CONCLUSION: ON SENSE-MAKING AND MEANING-MAKING

Sense-making was discussed in this paper in terms of extracting meaning and giving meaning. Extracting meaning and giving meaning were construed as interactions with objects to seek comprehensibility of a mathematical concept when it is actualized. Individuals can make sense if their conceptions fit the concept as it is assumed, or pre-specified, to be. As such, sense-making is an act of comprehension that consists of creating conceptions that best reflect a given concept.

Recent research, however, prompts one to rethink how students give meaning in the immediate context. In addition to attaching activated ideas$_F$ (already existing in the knowledge system) to actual instances of a mathematical concept, ideas$_F$ can also be transformed to attach new meaning to potential instances of a mathematical concept that, in this process, comes into becoming.

While with respect to the former it is assumed that students might make sense of the objects that fall under a particular concept primarily within their existing knowledge system, the latter allows an individual to journey toward a new meaning of a concept. It is asserted that this might be better referred to as *meaning-making*.

In consequence, sense-making is here understood as an act of comprehension, while meaning-making is construed as an act of creation. In a nutshell:

(1) A student might intend to *comprehend* a meaning of a mathematical concept in a way that best *reflects* the concept as it is. The meaning of a concept *emerges* (comes into *being*) by a continuous dialogue of the *sense-making* of extracting meaning and giving meaning.

(2) A student might intend to *create* a meaning of a mathematical concept that best *fits* student's conceptions. The meaning of a concept *evolves* (comes into *becoming*) by *meaning-making* via transforming ideas$_F$.

It is hoped that this distinction better brings to light critical issues and underlying cognitive processes in students' sense-making and meaning-making. The research insights outlined above and the theorizing provided here allow one to sharpen the distinction between making sense when the meaning of a mathematical concept comes into being and making meaning when the meaning of a mathematical concept comes into becoming. This nuance of sense-making and meaning-making might better highlight the critical differences of 'making it being' and 'making it becoming' with respect to the different states of the meaning of a mathematical concept, the different functions of senses$_F$, and the different directions of fit.

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